Matlab program for the

Analysis of radioisotope

Decay

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Mar. 20, 2015

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Abstract

MAD is a project focused on constructing a mathematical model for the radioactive isotopes dating of alpha decay and beta decay of an element, using Matlab ecosystem.

Unstable atoms which has more or less neutrons than its protons are called isotope. Given time, these atoms emit ionizing radiation and sending energy into the air, a process called radioactive decay. Through this process, the atom altered its own structure, forming a different kind of atom.

Since radioactive decay is a long-term process, and often one that can yield dangerous result, being able to model the process can be useful when looking for a certain amount of the atom, or when preparing for the worst, i.e how long should a nuclear laboratory be isolated after a nuclear leak.

Keywords: radioisotope dating, radioactive decay, mad.

Introduction

There are three types of radioactive decay: alpha, beta, and gamma. Since gamma decay only redistributes local electrons, it does not change the identity of an atom. Any element that decays via alpha or beta decay, and whose final product contains less than four different elements could be modeled using the result of this project.

Studying the decay of an element allow precise prediction of its age, in turn making the task of forecasting how long such process occurs available.

Test results were derived from Bodner's study of Potassium-Argon dating.

Theory

An atom transforms into different atoms via radioactive decay at a certain rates. Each transformation in turn adhere to a certain linear constant. Thus for a three-atoms-chain model, a certain element, K, decays into two different element, P_1, P_2, would each depends on a certain constant, λ_1 and λ_2 . (These constants could be found in Michael's paper and the University of Toronto's Physic department pages). If the rate of change (mol/billion year) of each elements P_1, P_2 is denoted as $\frac{dP1}{dt}$ and $\frac{dP2}{dt}$, and the amount of element K is denoted as K(t), the following system is derived:

$$\frac{dP1}{dt} = \lambda_1 * K(t)$$

$$\frac{dP1}{dt} = \lambda_2 * K(t)$$

According to the Conservation of Mass principal, the amount of P_1 and P_2 generated should be the same as the amount of K decreased. Thus the change of K in mol per billion year can be modeled using the function:

$$\frac{dK}{dt} = -(\lambda_1 + \lambda_2) * K(t)$$

Solving this system of three equation using Matlab requires the use of making a matrix.

An anonymous function was derived to model the system of equation:

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y = @(t,K)[  -(l2+l1).*K(1);

             l1.*K(1);

             l2.*K(1);];
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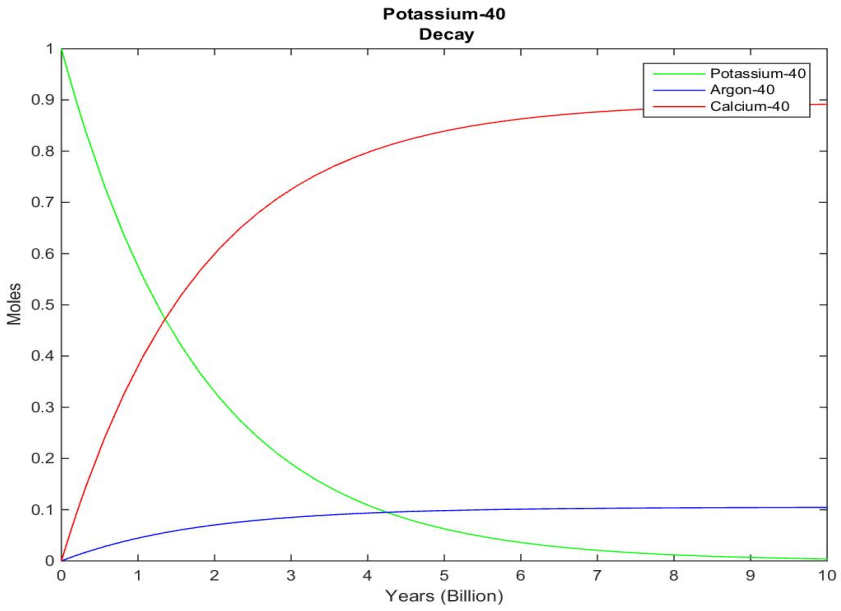
With l stands for lambda (λ), using ode45, with time interval from 0 to 10, and an initial vector of [1 0 0] denoting that there is 1 mol of element K, and none of P_1 and P_2 from the beginning, the model returned the result discussed below. The test data models the decay process of Potassium-40 to Calcium-40 and Argon-40.

Conclusion



Data from Berkley Labs shows that the process of decay through beta emission results in the bulk of the resulting product; Calcium-40. The accepted measurements produce the data through which Calcium-40 yields 89.28% of the final product where Argon-40 forms the final 10.72%. This is the situation for a “perfect” decay. This is slightly problematic due to the nature

of the system of differential equations used and the real world and the limit by which the Potassium-40 can decay along with a rare case beyond the scope of the problem. The first example of the systems of equations results in logarithmic functions and by definition this will result in the amount of Potassium-40 never actually reaching zero since the limit as time goes to infinity only approaches zero. This is also evident in nature where 100% yield of any reaction is rarely achieved. These pieces of information should be kept in mind and are present in the results of this program. Potassium-40, after 10 billion years, decays (through the Matlab program) to Calcium-40 with a total yield of 89.17% of the product where Argon-40 is present in quantities of 10.44% the total yield. This results in quantities of Potassium-40 still present by .39% of the final products.

Within the confines of the procedure and techniques used within the course certain variables were excluded, such as the expulsion of a gamma ray, neutrino and the beta particle capture which would occur in the third creation of Argon-40 in quantities of .001%. Despite the realistic nature of the results due to the experimentally gained data of the constants used (done by professionals and not by our team) and the exclusion of the rare case, the results of the program are very accurate and represent the nature of Radioisotope decay well. Since the amount of initial Potassium-40 can be changed in the program we can assume the presence of 1 mole of the initial isotope. This one mole yields .8917 moles of Calcium-40 and .1044 moles of Argon-40. The actual half-life of Potassium-40 is given as 1.251x10^9 years and should represent the time when potassium-40 reaches the .5 mark on the y-axis (Amount of moles). The error given by the program for the half life can be attributed to the method used to define a continuous function off of the interpolated values for the decay of Potassium-40. The interp1 command was used with

the ‘spline’ method and this results in the most accurate (still, with very slight error) function to depict the decay. The error within accepted values gives the results:

Calcium-40: .12% error

Argon-40: 2.6% error

Half-life: .17% error

The results are well within the usual error accepted error of science classes here at Green River College, where any value of error under 5% is looked highly upon. Overall the results match very well with the “actual” expected results.

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