ENGR 250 Numerical Methods in Matlab
Winter 2015

Exam 1 Practice Problems

Written component of Exam 1 will take place on Friday, February 6th. It will be comprised of free response and computational questions. The following problems are designed to give you an idea of what you might see.

1. Root Solving Methods

a) Free Response

1) Describe the difference between open and bracketed root solving methods
2) Why is the bisection method called the bisection method?
3) What are some differences between the secant and modified secant methods?
4) What is the primary difference between the Newton-Raphson method and either secant method.
5) What is the primary difference between the bisection method and the false position method?
6) What requirements exist for selecting initial upper and lower bounds for a bracketing method?
7) Describe (insert root solving method here) in words.

b) Computation

1) Use the following root finding methods to find the root of $f(x) = x^2 - 2$:
    a. Bisection method, start with initial brackets of [0,2], and compute three iterations.
    b. False position method, same brackets, 3 iterations
    c. Newton-Raphson, initial guess of $x = 2$, 3 iterations
    d. Secant method, initial guesses of $x = 3$ and $x = 2$, 2 iterations
    e. Modified Secant method, initial guess of $x = 2$, $\delta = 0.05$, 2 iterations

Note: I wouldn't make you do all of that on one test, but you should know how to do all of that for a test.

2. Solving $Ax = b$

a) Free response (sort of)

1) Given the following system of equations:

$$\begin{cases} 3x + 4y + 6x = 4 \\ -2x - 5y + 2x = 5 \\ x + y + z = 1 \end{cases}$$

Rewrite the equation in the form $Ax = b$. Or, just tell me what A, x, and b are in this case.

b) Computation

1) Solve the equation $LUx = b$, with

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, U = \begin{bmatrix} 1/2 & -2 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} \; b = \begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix}$$

Remember, to solve this system, first solve $Ly = b$, then solve $Ux = y$. (I would certainly give you this reminder on an exam.)

The above problem requires you to solve 2 systems. I could also just ask you to solve the system

$$Lx = b,$$

which would be easier.

3. Eigenvalue problems

a) Free response

1) An eigenvalue λ of a vector A satisfies what matrix equation?

b) Computation

2) Find the eigenvalues of the following matrices

$$A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 \\ 4 & -6 \end{bmatrix}$$