ENGR 250 Numerical Methods using MATLAB
2015 Winter Quarter

Homework Assignment #6

Due : March 11(Wednesday)

You should submit your M-file, named **HW6_*yourEmailAccount***, by email "pandrist@greenriver.edu"!

1. ODE IVP

Solve the following differential equation from $t = 0$ to 3:

$$\frac{dy}{dt} = -10.8y$$

with the initial condition of $y(0) = 1$.

- save $y(t), 0 \leq t \leq 5$ on ***HW6_1.dat*** file

2. Pendulum ODE

Consider a pendulum of length l moving in the $x-y$ plane. The pendulum is fixed with a pin on one end and a mass at the other. Note that $g = 9.81m/s^2$ and $l = 5.5m$. The differential equation governing its motion is given by

$$\ddot{\theta} + \frac{g}{l} sin(\theta) = 0$$

In a class in ODEs, this equation is typically linearized to the following result so that an analytical solution can be obtained

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

But this is a class in numerical methods! We are not hindered by such petty obstacles. Solve the system using both differential equations and compare the results. Let $\theta(0) = \pi/2$ and $\dot{\theta}(0) = 0$ in both cases.

- Save $\theta_{true}(t)$, the solution for the true DiffEQ $0 \leq t \leq 10$ on **HW6_2.dat**
- Save $\theta_{linear}(t)$, the solution for the linear DiffEQ $0 \leq t \leq 10$ on **HW6_3.dat**
- Calculate the difference, $\theta_{true}(t) - \theta_{linear}(t)$, and save the vector on **HW6_4.dat**.

Use linear interpolation of $\theta_{linear}(t)$ if necessary.
Also, while it won't be graded, you should graph both to see how different the linear approximation is from the true solution.

3. Stiff ODE

The following second-order ODE is considered to be stiff.

$$\frac{d^2y}{dx^2} = -2001\frac{dy}{dx} - 2000.5y$$

With 'ode45' command and the initial condition $y(0) = 1$ and $y'(0) = 0$,

- Save the $y(t)$ $(0 \leqq t \leqq 10)$ on **HW6_5.dat** file
- Save the $\dot{y}(t)$ $(0 \leqq t \leqq 10)$ on **HW6_6.dat** file

With 'ode23s' command and the initial condition $y(0) = 1$ and $y'(0) = 0$,

- Save the $y(t)$ $(0 \leq t \leq 10)$ on **HW6_7.dat** file
- Save the $\dot{y}(t)$ $(0 \leq t \leq 10)$ on **HW6_8.dat** file

4. Concentration ODE

The following equations defines the concentrations of three reactions:

$$\begin{aligned} \frac{dc_a}{dt} &= -c_b - c_c \\ \frac{dc_b}{dt} &= c_a + 0.2c_b \\ \frac{dc_c}{dt} &= 0.2 + c_c(c_a - 5.4) \end{aligned}$$

If the initial conditions are $c_a = 1$, $c_b = 1$, and $c_c = 1$, find the concentrations from 0 to 5 seconds.

- Save the $c_a(t)$ on **HW6_9.dat** file
- Save the $c_b(t)$ on **HW6_10.dat** file
- Save the $c_c(t)$ on **HW6_11.dat** file