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# ODEs MATH325

Course Outline:

*Based on Lectures from Winter, 2024 by Prof. Antony Humphries.*

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# 1 Introduction

## 1.1 Definitions

↪ **Definition 1.1: Diffferential equation**

A *diffferential equation* (DE) is an equation with derivatives. *Ordinary* DE's (ODE) will be covered in this course; other types (PDE's, SDE's, DDE's, FDE's, etc.) exist as well but won't be discussed. ODE's only have one independent variable (typically, $y = f(x)$ or $y = f(t)$).

⊛ **Example 1.1: A Trivial Example**

$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x$. Integrating both sides:

$$\int \frac{\mathrm{d}y}{\mathrm{d}x}\,\mathrm{d}x = \int 6x\,\mathrm{d}x \implies y(x) = 3x^2 + C.$$

⊛ **Example 1.2: Another One**

$$\frac{\mathrm{d}^2u}{\mathrm{d}t^2} = 0 \implies y = at + b.$$

↪ **Definition 1.2: Order**

The order of a differential equation is defined as the order of the highest derivative in the equation.

## 1.2 Initival Values

**Remark 1.1.** *Note the existence of arbitrary constants in the previous examples, indicating infinite solutions. We often desire unique solutions by fixing these coefficients. For first order ODEs, we simply specify a single initial condition (say, some $y(x_0) = \alpha_0$). For higher order ODEs of degree $n$, we can either specify $n - 1$ initial conditions for $n - 1$ derivatives (say, $y(x_0) = \alpha_0$, $y'(x_0) = \beta_0$), or boundary conditions (say, $y(x_0) = \alpha_0, y(x_1) = \alpha_1$) where values for the solution itself are specified.*

⊛ **Example 1.3: A Less Trivial Example**

$\frac{\mathrm{d}y}{\mathrm{d}x} = y$. We cannot simply integrate both sides as before, as we have no way to know what $\int y\,\mathrm{d}x$ (the RHS) is equal to. We can fairly easily guess that $y = e^x$ is a solution; its derivative is equal to itself, hence it does indeed solve the equation. This is not the only solution; indeed, given $y = ce^x$, we have

$$\frac{\mathrm{d}y}{\mathrm{d}x} = ce^x = y = ce^x.$$

Luckily, we were rather limited in how many places constants could appear; this doesn't always hold.

## 1.3 Physical Applications

⊛ **Example 1.4: Simple Pendulum**

Let $\theta$ be the angle of a pendulum of mass $m$ from vertical and length $l$. Then, we have the equation of motion

$$ml\ddot{\theta} = -mg\sin\theta \implies \ddot{\theta} + \frac{g}{l}\sin\theta = 0 \implies \ddot{\theta} + \omega^2\sin\theta = 0.$$

Take $\theta$ small, then, $\sin\theta \approx \theta$. Then, $\ddot{\theta} + \omega^2\theta = 0$. This is linear simple harmonic motion, and has periodic solutions; how do we know this is a valid solution to the non-linear model?

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