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Algebra I, II

MATH235

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1 Fundamentals

1.1 Sets

1.1.1 Definition

A **set** can be considered as a collection of elements; more intuitively, you can consider something a set if you can determine wheteher a given object belongs to it. Typically sets are defined as $A = \{1, 2, \ldots\}$, by a property $A = \{x \mid x\%2 = 0\}$, or with an appropriate verbal description.

1.1.2 Set Operations

There are a number of ways to "combine" sets:

- **Union**: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- **Intersection**: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- **Difference**: $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$

Lemma 1.1

$A = (A \setminus B) \cup (A \cap B)$

Proof: To prove set equivalencies, we must prove that both RHS $\subseteq$ LHS and LHS $\subseteq$ RHS; meaning, the LHS and RHS are subsets of each other, and are thus equal.

First, to prove LHS $\subseteq$ RHS, let $a \in A$. If $a \notin B$, then $a \in A \setminus B$, and $a \in$ RHS. Else, if $a \in B$, then $a \in A \cap B$ and $a \in$ RHS. Thus, LHS $\subseteq$ RHS.

Next, to prove RHS $\subseteq$ LHS, let $a \in$ RHS. If $a \in A \setminus B$, then $a \in A =$ LHS. Else, $a \in A \cap B$, and thus $a \in A =$ LHS. Thus, RHS $\subseteq$ LHS. Since LHS $\subseteq$ RHS and RHS $\subseteq$ LHS, LHS = RHS. ■

1.1.3 Indexed Sets

Let I be a set. If for every $i \in I$, we have a set B_i, we say that we have a *collection* of sets B_i indexed by I. We write $\{B_i : i \in I\}$.

Example 1.1. *Let $I = \{1, 2, 3\}$, and $B_i = \{1, 2, 3, 4\} \setminus \{i\}$ (B_i is the set of all numbers from 1 to 4, excluding i), for $i \in I$. We thus have $B_1 = \{2, 3, 4\}$ (etc.).*

This concept of indexing allows us to introduce repeated unions/intersections. For instance, we can write

$$\bigcup_{i \in I} B_i = B_1 \cup B_2 \cup B_3 = \{1, 2, 3, 4\}.$$

Similarly,

$$\bigcap_{i \in I} B_i = \{4\}.$$[1]

Example 1.2. *Let* $I = \mathbb{R}$*, and* $B_i = [i, \infty] = \{r \in \mathbb{R} : r \geq i\}$*. Then,* $\bigcup_{i \in \mathbb{R}} B_i = \mathbb{R}$ *and* $\bigcap_{i \in \mathbb{R}} B_i = \emptyset$*.*

[1] You can somewhat consider these "large" unions/intersections as analogous to summations Σ and products Π.

2 Appendix