State of Rhode Island June 23, 2018

Asymptomatic Covid Rate Ceiling

Author Louis Millette

1 Abstract

I am going to prove that the probability of having covid at any point given you have no covid symptoms is equal to: the probability that you have Covid19 symptoms times the probability you have Covid19 given you have symptoms, divided by (1 over the probability of having no symptoms given you have Covid19) times the probability you have no covid symptoms.
Using the definitions below:

$$C|B = \frac{A * (C|A)}{(\frac{1}{B|C}) * B}$$

This is important because it allows us to find the probability of an asymptomatic member of society having covid, without taking a sample of that population.

Definitions

- let A be the probability that a random member of the population has Covid19 symptoms.
- let B be the probability that a random member of the population does not have Covid19 symptoms.
- let C be the probability that a random member of the population tests positive for Covid19 for the first time.

Assumption

For the purposes of this proof, let "a random member of the population" exclude people who have an active Covid19 case and know it. People who know they have contagious Covid19 are (hopefully) not going into public places until they've been cleared. For the purpose of this proof, lets assume A is know. This assumes most people who come down with symptoms indicative of covid get a covid test, in the state of Rhode Island.

$B = 1 - A$ Follows as these numbers are compliments.

If most people who get tested are symptomatic, we can assume $C|A$ is known also.

What if most people who get tested are not symptomatic? We can still use a ceiling estimate of $C|A$ given the proportion of the people who are tested that we think are symptomatic, as follows:

- Lets say we think only 50% of the people the state gives daily covid19 tests to are symptomatic.

- We know how many positive cases there are out of the tested population, but not how many of the positive cases were from people who had covid symptoms.
- Lets assume ALL of the people who tested positive were symptomatic, giving an absolute ceiling for the value of $C|A$
- because the value of $C|B$ is strictly non-decreasing when $C|A$ is positive and non-decreasing, our ceiling estimate of $C|A$ also gives a ceiling estimate of $C|B$.

This ceiling estimate of $C|A$ works until there are more positive cases then assumed symptomatic tests. Fortunately, another ceiling exists for our estimate of $C|B$:

- Lets say we think only 50% of the people the state gives daily covid19 tests to are symptomatic.
- We know how many positive cases there are out of the tested population, but not how many of the positive cases were from people who had covid symptoms.
- Lets assume ALL of the people who tested positive were Asymptomatic, giving an absolute ceiling for the value of $C|B$: $\frac{(NumberOfPositiveCovidTests)}{(PercentOfAsymptomaticTests)X(NumberOfPeopleTested)}$

Using the lower of the 2 ceilings, we may proceed to find a useful and novel ceiling for C—B.

$B|C = 0.78$. This is based on a study conducted in england, where and entire town received covid tests. It found that 78% of the people who had covid, didn't have any symptoms. This provides a good estimate for the constant probabilty that someone doesn't habe covid symptoms, given they test positive for covid.

Proof

What we know, from the assumptions:
1. A
2. B
3. $C|A$
4. $B|C = 0.78$

why $B \cap C = \frac{C \cap A}{\frac{1}{B|C}}$:

5. $B|C = \frac{B \cap C}{C} = 0.78$:Definition of conditional probability line 4
6. $C = (C \cap A) + (C \cap B)$:Law of total probability
7. $\frac{B \cap C}{B|C} = C$:Line 5 + division
8. $\frac{B \cap C}{B|C} = (C \cap A) + (C \cap B)$:Substitution of line 7 with def. of C in line 6
9. $\frac{B \cap C}{B|C} - (C \cap B) = C \cap A$:Substitution
10. $B \cap C = \frac{C \cap A}{\frac{1}{B|C}}$:Division

11. $C|A = \frac{C \cap A}{A}$:The definition of conditional probability
12. $C \cap A = A * C|A$:Multiplication
13. $C|B = \frac{C \cap B}{B}$:Definition of conditional probabilty
14. $C|B = \frac{\frac{C \cap A}{\frac{1}{B|C}}}{B}$:Substitution of line 13 and line 10
15. $C|B = \frac{\frac{A*(C|A)}{\frac{1}{B|C}}}{B}$:Substitution of line 14 and line 12
16. $C|B = \frac{A*(C|A)}{(\frac{1}{B|C})*B}$:Reorganization of line 15 for clarity