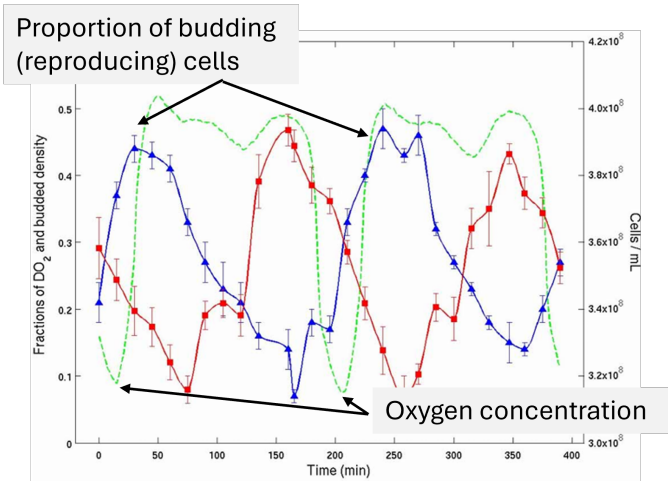
Pattern Formation in Cell Metabolism

Motivation

- Oscillations in yeast metabolism
- Oscillations in cell division
- Hypothesis: pattern formation through temporal clustering [1]



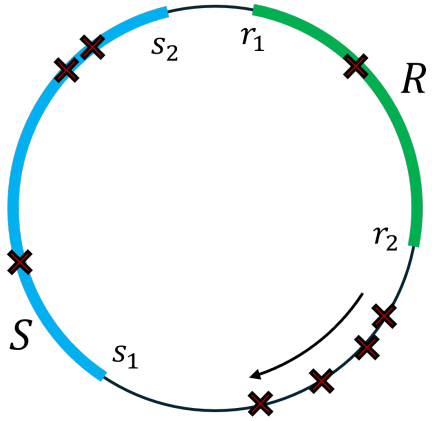
Modeling

- Particles in signaling region (S) influence particles in responsive region (R) [2]
- N-particle system of ordinary differential equations (NODE) for some feedback function $f : \mathbb{R} \to \mathbb{R}$ (ex. $f : x \mapsto -\alpha x$, $\alpha < 1$)

$$\left\{ \dot{z}_i = 1 + \chi_R(z_i) f\left(\frac{1}{N}\sum_{j=1}^{N} \chi_S(z_j)\right) + \sqrt{2\epsilon^2} dW_i \right.$$

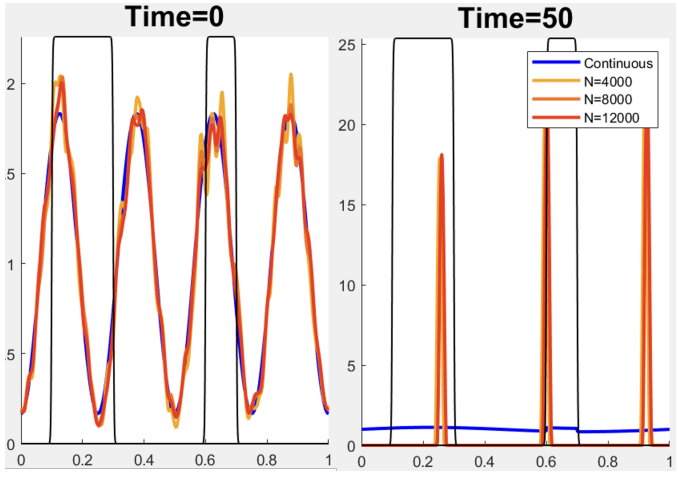
- Mean-field partial differential equation (MFPDE)

$$\partial_t u + \partial_x \left((1 + f(\chi_R \otimes \chi_S u))\, u \right) + \epsilon \Delta u = 0$$



Simulation

- Expectation: NODE $\to$ MFPDE as $N \to \infty$
- Simulation: systems exhibit qualitatively distinct behavior



Direction

- Derive mean-field limit with linear feedback [3]
- Compare time-periodic solutions of both particle and continuum models
- Learn MFPDE dynamics from NODE data with WSINDy [4]

[1] Eric Boczko, Tomas Gedeon, Christ Stowers, and Todd Young. "ODE, RDE and SDE models of cell cycle dynamics and clustering in yeast" *J. Bio. Dyn.*, **2010**, 4(4):328-345

[2] Todd Young, Bastien Fernandez, Richard Buckalew, Gregory Moses, and Erik Boczko. "Clustering in cell cycle dynamics with general response/signaling feedback" *J. Theor. Biol.*, **2012**, 292:103-115

[3] Vinh Nguyen and Roman Shvydkoy. "Continuous model of opinion dynamics with convictions" *Discrete Cont. Dyn. Syst*, **2023**, 43(11):410-4026

[4] Daniel Messenger and David Borts."Weak SINDy for partial differential equations" *Comput. Phys*, **2021**, 443:110525