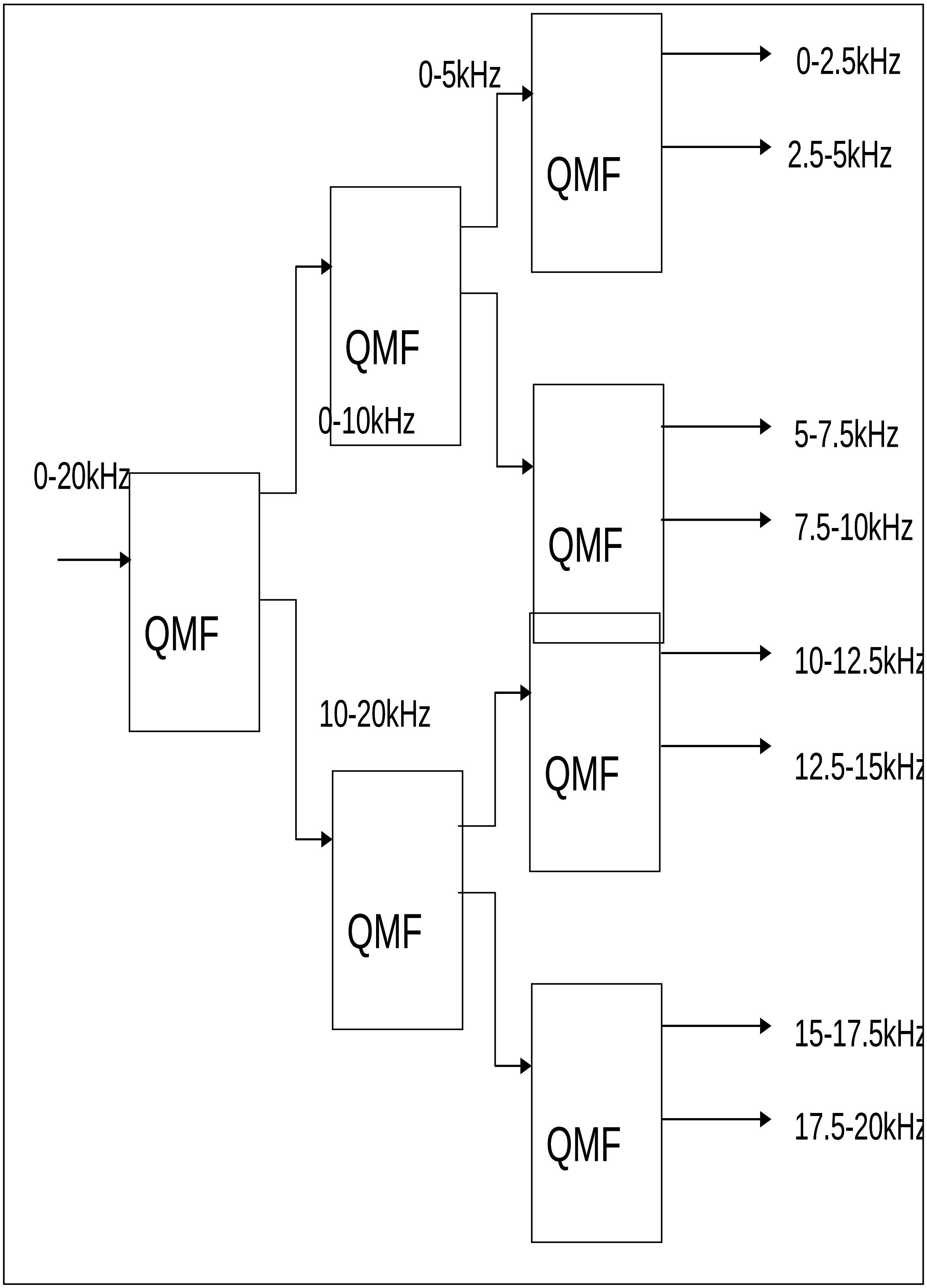
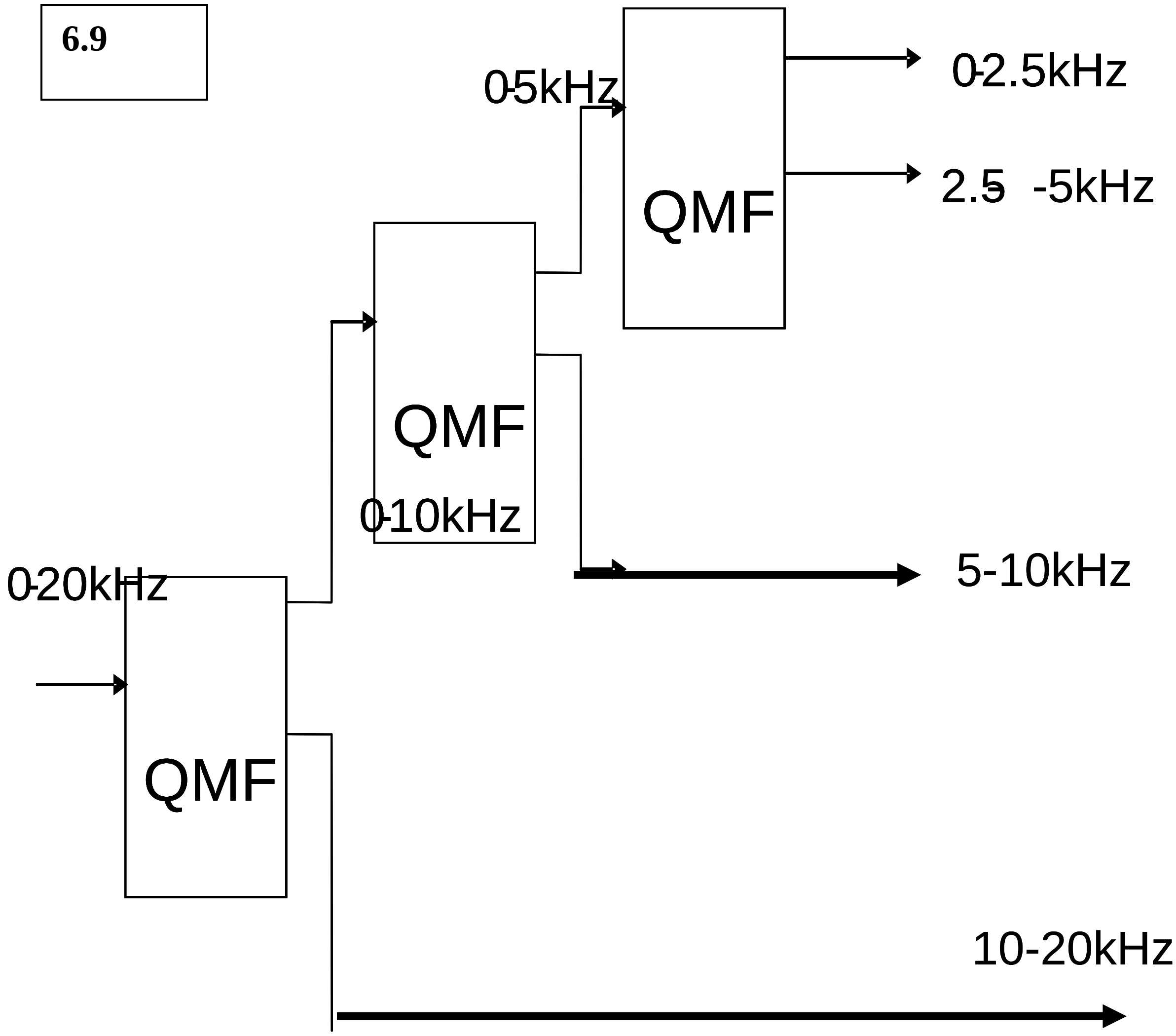
EEE509 – DSP Algorithms & Software – Hw5 Solutions

6.8



6.9


0-20kHz
QMF
0-10kHz
QMF
0-5kHz
QMF
0-2.5kHz
2.5 -5kHz
5-10kHz
10-20kHz

PROBLEM 7.2:

The inverse DFT of *Y(k)* can be obtained as the circular convolution of $x(n)$ and $h(n)$, i.e.,

$$\begin{aligned} IDFT(Y(k)) = y(n) &= x(n) \otimes h(n) \\ &= 1\times 1 + 2\times 1 + 3\times 1 + 4\times 1 \\ &= 10, \quad \forall n \end{aligned}$$

The linear convolution of $x(n)$ and $h(n)$ is given by

$$\begin{aligned} &y_L(n) = x(n) * h(n) = \sum_{l=0}^{3} x(l)h(n-l) \\ &\therefore y_L(0) = 1\times 1,\ y_L(1) = 1\times 1 + 1\times 2 = 3, \\ &\ \ y_L(2) = 1\times 1 + 1\times 2 + 1\times 3 = 6,\ y_L(3) = 1\times 1 + 1\times 2 + 1\times 3 + 1\times 4 = 10. \end{aligned}$$

Note that $y(3)= y_L(3)=10$. In general, in N point circular convolution only the last output matches with the output of the linear convolution.

PROBLEM 8.6:

The impulse response and transfer function of the filter is

$$h(n) = 0.7^n u(n) \qquad\qquad H(z) = \frac{1}{1-0.7z^{-1}}.$$

The mean of the output at steady state is

$$\begin{aligned} \mu_y &= \sum_{k=0}^{\infty} h(k)\mu_x = H(e^{j\Omega})\big|_{\Omega=0}\, \mu_x = \frac{1}{1-0.7}\times 0 \\ &= 0 \end{aligned}.$$

The autocorrelation of the output is given by

$$\begin{aligned} r_{yy}(m) &= \sum_{k=0}^{\infty}\sum_{i=0}^{\infty} 0.7^{k+i}\delta_{xx}(m-k+i) \\ &= \sum_{i=0}^{\infty} 0.7^{m+2i}; \quad m \geq 0 \end{aligned}.$$

Using autocorrelation symmetry, we can write

$$r_{yy}(m) = r_{yy}(-m) = 1.96\times 0.7^{|m|};\ \forall \mathrm{m}.$$

Therefore, variance can be calculated as

$$\sigma_y^2 = r_{yy}(0) = 1.96.$$

PROBLEM 8.7:

We proceed with $\sigma_y^2 = E\left[(y-\mu_y)^2\right] = E[y^2] - \mu_y^2$, where we plug in $y(n) = \sum_{k=0}^{L-1} x(n-k)/L$ and obtain

$$\sigma_y^2 = \frac{1}{L^2}\sum_{k=0}^{L-1}\sum_{i=0}^{L-1} r_{xx}(i-k) - S^2 \ . \qquad \textbf{(Error! No text of specified style in document.}.1)$$

Since $x = S + w$ and $r_{xx}(m) = E\left[x(n+m)x(n)\right] = S^2 + \delta(m)$, (**Error! No text of specified style in document.**.1) can written as

$$\begin{aligned}\sigma_y^2 &= \frac{1}{L^2}\left\{\sum_{k=0}^{L-1}\sum_{i=0}^{L-1} S^2 + \sigma_w^2\delta(i-k)\right\} - S^2 \\ &= \frac{\sigma_w^2}{L}\end{aligned} . \qquad \textbf{(Error! No text of specified style in document.}.2)$$