

STOCHASTIC OPTIMIZATION FOR MULTISPECIES FISHERIES IN THE BARENTS SEA

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ABSTRACT. We present a multispecies stochastic model that suggests optimal fishing policy for two species in a three-species predator–prey ecosystem in the Barents Sea. We employ stochastic dynamic programming to solve a three-dimensional model, in which the catch is optimized by using a multispecies feedback strategy. Applying the model to the cod, capelin, and herring ecosystem in the Barents Sea shows that the optimal catch for the stochastic interaction model is more conservative than that implied by the deterministic model. We also find that stochasticity has a stronger effect on the optimal exploitation policy for prey (capelin) than for predator (cod).

KEY WORDS: Barents Sea, ecosystem-based management, multispecies, stochastic optimization.

1. Introduction. The Barents Sea is one of the most productive ocean areas in the world (O’Brien et al. [2004], Aanestad et al. [2009]) and represents a highly diverse Arctic ecosystem (Larsen et al. [2001]). The major commercial fish species in the Barents Sea include capelin (*Mallotus villosus*), herring (*Clupea harengus*), and their main predator, Northeast Arctic cod (*Gadus morhua*). Of the three species, only capelin and cod are harvested in the Barents Sea. However, the inflow and outflow of herring affect the growth of these main commercial species through their predator–prey relationships. The relationships between these species are highly dynamic (Bogstad et al. [1997]). There are biological, physical, and economic interactions among these species in the ecosystem. The optimal catch of these two commercial species in the Barents Sea is affected by the interactions among the species. Management problem exists for the species that interact among themselves and at different trophic levels (May et al. [1979]). The most common approach—single-species management in multispecies fisheries—ignores not only the ecological relationships among species but also the technological and economic relationships (Kasperski [2010]). This may generate misleading results and policy errors that cause stocks to be over- or underexploited (Fleming and Alexander [2003], Hoff et al. [2010]). Therefore, an ecosystem-based fisheries management (EBFM) approach is required. This approach has gained importance in recent years (Scandol et al. [2005], Zhou et al. [2010]). EBFM is a holistic approach to maintaining ecosystem quality and sustaining the associated benefits (Brodziak and Link [2002],

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Zhou et al. [2010]). Fisheries may affect the marine ecosystem through their effect on, for example, fish populations and their growth habits, interactions among fish species, fish migration, food preferences, and more importantly management. Among many aspects of fisheries management, fish harvesting is a key determinant. An ecosystem-based management strategy for marine fisheries would combine sustainable exploitation of fish resources with minimal impact on the ecosystem (Witherell et al. [2000]). The EBFM strategy is based on conservative and precautionary harvest limits, comprehensive monitoring and enforcement, by-catch controls, gear restrictions, temporal and spatial distribution of fisheries, and marine protected areas (Witherell et al. [2000]). EBFM requires a multispecies ecosystem model for sustainable harvesting of fisheries that looks beyond the sustainable yield for individual species. Multispecies management involves examining the big picture to address the long-term consequences of current decisions. Therefore, it constitutes an important approach to sustainable management (Hollowed et al. [2000]). Interest in multispecies interaction in the fisheries model has been increasing in recent years (Agnarsson et al. [2008]). However, most researchers have focused on predator–prey interactions between two species. The development of three-species interaction models is in its infancy, but has attracted recent interest (for example, Samanta et al. [2003], Aanestad et al. [2009], Das et al. [2012], Nieminen et al. [2012], Sharma and Samanta [2013]).

In addition to being affected by interactions among species, the growth of individual species is also affected by physical uncertainties and random shocks (Sarkar [2009]). Random shocks, such as environmental disturbances and variability, zooplankton abundance, and predator distribution and migration, affect the recruitment and growth of fish stocks (Misund et al. [1998]). The migration of herring stock in the Barents Sea is highly variable (Lindstrøm et al. [2002]), which creates uncertainty about the growth of the stock, its predators, and its prey in the ecosystem. There is a lack of three-species interaction models with stochastic stock dynamics because multispecies stochastic bioeconomic models are difficult to solve analytically (Kar and Chaudhuri [2004], Posch and Trimborn [2010]) and computationally, particularly in the case of nonlinear dynamic models in higher dimensions (Munos and Zidani [2005], Singh et al. [2006]). In this study, we develop a multispecies model that not only considers interactions among species but also takes into account physical and biological uncertainties by incorporating stochasticity into the stock-growth dynamics. Incorporating stochasticity into the model makes it more realistic. This is because the Barents Sea ecosystem is highly volatile, partly because of variability in the abundance of immature herring year classes (Lindstrøm et al. [2002]). Bogstad et al. [1997] found that herring is most likely to be affected by changes in the abundance of its top predators, such as minke whales, which further influence the growth of the targeted species. Capelin and herring also compete for food during times of food scarcity in the Barents Sea (Huse and Toresen [1996]). In addition to cod and herring predation, capelin are also predated in large quantities

by whales, seals, and other fish, such as haddock (Ushakov and Prozorkevich [2002]). Similarly, cod recruitment and survivability is directly affected by the climate and food availability. A higher temperature at spawning time and a high capelin biomass have a positive effect on cod recruitment. However, a temperature increase lowers the capelin biomass by reducing herring recruitment, which affects the stock of cod recruitment negatively through cannibalism (Hjermann et al. [2007]). Thus, fluctuations in the distribution and abundance of herring have an impact on the growth of its prey and predators, which are the commercially exploitable species.

We apply stochastic dynamic programming (SDP) to derive optimal feedback control rules, which is a central tool for tackling stochastic optimization problems (Sydsæter et al. [2005]). Dynamic programming is a mathematical technique for solving certain types of sequential decision problems in which a sequence of decisions must be made, with each decision affecting future decisions (Howard [1966]). Although the dynamic programming technique has tremendous methodological appeal in solving intertemporal economic problems, it has not been as widely used as anticipated for empirical analysis of natural resource problems (Howitt et al. [2002]). This may be because of the associated computational difficulties. To solve our stochastic dynamic problem, we use a numerical approximation scheme, “the probabilistic approach,” which yields a finite difference approximation of the associated Hamilton–Jacobi–Bellman (HJB) equation.

Our main contribution is to develop a multispecies bioeconomic model incorporating stochastic dynamics, which is essential for EBFM. The contribution of the study is twofold; first we show that the dynamic programming approach can be used for empirical models in higher dimensions, and that solving an SDP problem in such a higher dimension can be useful for bioeconomic modeling; and second, we obtain empirical results on optimal management in a complex biological ecosystem that harbors three species.

2. The bioeconomic model. Our model is a surplus production model (Schaefer [1957]) in continuous time. The general biological interdependent deterministic growth functions for three interacting species are specified by adding one species to the model of Sandal and Steinshamn [2010]. Let x , y , and z denote capelin, cod and herring respectively, where capelin is preyed upon by both cod and herring. Cod preys upon both capelin and herring. Herring preys upon capelin but is preyed upon by cod. The deterministic growth increments of these species are:

$$(1) \quad \begin{aligned} \frac{dx}{dt} &= [f(x, y, z) - u_x], \\ \frac{dy}{dt} &= [g(x, y, z) - u_y], \\ \frac{dz}{dt} &= [j(x, y, z)]. \end{aligned}$$

The functions $f(x, y, z)$, $g(x, y, z)$, and $j(x, y, z)$ are the biological growth functions of the three fish species, capelin, cod, and herring, respectively, u_x and u_y represent the harvest rates of species x and y ; there is no herring harvest in the Barents Sea.

A three-species interaction model incorporating SDP can be formulated by adding stochastic terms to equation (1) as follows:

$$(2) \quad \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} f(x, y, z) - u_x \\ g(x, y, z) - u_y \\ j(x, y, z) \end{pmatrix} dt + \begin{pmatrix} \sigma_{11}(x, y, z) & \sigma_{12}(x, y, z) & \sigma_{13}(x, y, z) \\ \sigma_{21}(x, y, z) & \sigma_{22}(x, y, z) & \sigma_{23}(x, y, z) \\ \sigma_{31}(x, y, z) & \sigma_{32}(x, y, z) & \sigma_{33}(x, y, z) \end{pmatrix} \begin{pmatrix} dB_1 \\ dB_2 \\ dB_3 \end{pmatrix}.$$

In equation (2), the term $\sigma_{ij}(x, y, z)$ is the diffusion term and represents volatility in the growth models. The term dt is the time increment and the term $dB_{(\cdot)}$ is Brownian motion, which is identically and independently distributed with mean zero and variance dt . Although it is likely that the stochastic events are correlated among species, we assumed it to be small enough to be neglected; that is, $\sigma_{ij}(x, y, z) = 0$ for all $i \neq j$. This simplifying assumption does not hinder our ability to observe the effects of stochasticity (Stefansson [2001]).

We further simplify the model by letting volatility be a linear function of its own stock level: $\sigma_{11}(x, y, z) = \sigma_1 x$; $\sigma_{22}(x, y, z) = \sigma_2 y$; and $\sigma_{33}(x, y, z) = \sigma_3 z$. This assumption is made to simplify the numerical approximation and because complicated functional volatility dependence is not widely known. Because the species are prey and predator, the stochastic processes that directly affect prey may not directly affect predators and vice versa. The stochastic process for one species indirectly affects the other species through interaction in the drift (deterministic) part. Epidemics are a typical example. Equation (2) can now be written as:

$$(3) \quad \begin{aligned} dx &= [f(x, y, z) - u_x] dt + \sigma_1 x dB_1, \\ dy &= [g(x, y, z) - u_y] dt + \sigma_2 y dB_2, \\ dz &= [j(x, y, z)] dt + \sigma_3 z dB_3. \end{aligned}$$

In equation (3), the term $\sigma_{(\cdot)}(\cdot)dB_{(\cdot)}$ represents the stochastic part of the stock growth relationship. The biological growth functions and their coefficients, obtained from Aanestad et al. [2009], are as follows:

$$(4) \quad \begin{aligned} f(x, y, z) &= r_1 x \left(1 - \frac{x}{k_1} - b_1(y + z) \right), \\ g(x, y, z) &= r_2 y \left(1 - \frac{y}{k_2} + b_2(1 + x)\sqrt{z} \right), \\ j(x, y, z) &= \tilde{r}_3(x, y) z \left(1 - \frac{z}{\tilde{k}_3(x, y)} + \frac{b_3 x(x + y)}{z} \right) + C, \end{aligned}$$

where, $\tilde{r}_3(x, y) = \frac{r_3}{x+y}$, $\tilde{k}_3(x, y) = k_3 \tilde{r}_3(x, y)$, and $r_i, k_i, b_i, C > 0$ are parameters.

In equation (4), the growth function for each species is concave with respect to its own species. The coefficients r_i and k_i represent the intrinsic growth rates and carrying capacity in the logistic functions. The additional coefficient $b_i > 0$ is the interaction coefficient, which adjusts the biomass in the multispecies model. The capelin biomass decreases because of predation by cod and herring, whereas that of cod increases because of predation on capelin and herring. The term $(1+x)\sqrt{z}$ implies that the biomass of capelin is more important than the biomass of herring. The growth function for herring differs from the standard logistic growth function. The terms $\tilde{r}_3(\cdot)$ and $\tilde{k}_3(\cdot)$ depend on the biomass of cod and capelin. The coefficient $C > 0$ implies that herring cannot become extinct because of the nature of its outflows and inflows in the Barents Sea. We assume that stock and harvest rates are nonnegative.

Substituting the growth functions from equation (4) into equation (3) yields our basic biological model:

$$\begin{aligned}
 (5) \quad dx &= \left[r_1 x \left(1 - \frac{x}{k_1} - b_1 (y + z) \right) - u_x \right] dt + \sigma_1 x dB_1, \\
 dy &= \left[r_2 y \left(1 - \frac{y}{k_2} + b_2 (1 + x) \sqrt{z} \right) - u_y \right] dt + \sigma_2 y dB_2, \\
 dz &= \left[\frac{r_3}{x+y} z \left(1 - \frac{z}{k_3 \frac{x+y}{x+y}} + \frac{b_3 x(x+y)}{1+z} \right) + C \right] dt + \sigma_3 z dB_3.
 \end{aligned}$$

Equation (5) implies that all species grow stochastically and interact with each other. However, what if only one species has stochastic growth? To understand the effect of stochastic growth of one species on optimal exploitation in a multispecies ecosystem, one can simplify equation (5) by setting the other two stochastic parameters to zero. For example, setting $\sigma_2 = \sigma_3 = 0$ in equation (5) enables the stochasticity in capelin growth to be analyzed. Similarly, setting $\sigma_1 = \sigma_2 = 0$ allows the stochastic effect of herring in a multispecies ecosystem to be modeled. However, it is worth noting that assuming stochasticity in any one species makes all other species stochastic because of the predator–prey interaction terms (xy, yz, xz) in the growth functions.

The second part of the bioeconomic model comprises economic component, which represents the net revenue from the harvest of capelin and cod:

$$(6) \quad \pi(x, y, u_x, u_y) = \pi_x(x, u_x) + \pi_y(y, u_y).$$

The functions $\pi_x(x, u_x)$ and $\pi_y(y, u_y)$ represent net revenues from capelin (x) and cod (y), respectively. The following revenue functions are from Sandal and Steinshamn [2010]:

$$\begin{aligned}
 (7) \quad \pi_x(x, u_x) &= p_x(u_x) u_x - c_x(x, u_x), \\
 \pi_y(x, u_y) &= p_y(u_y) u_y - c_y(y, u_y),
 \end{aligned}$$

where $p(\cdot)$ and $c(\cdot)$ are the inverse demand function and the cost function, respectively, which are specified as follows: Substituting (7) and (8) into equation (6) yields the following profit function:

$$(8) \quad \begin{aligned} p_x(u_x) &= p_1, \\ c_x(x, u_x) &= q_1 u_x^{\alpha_1}, \\ p_y(u_y) &= p_2 - p_3 u_y, \\ c_y(y, u_y) &= \frac{q_2 u_y^{\alpha_2}}{y}, \end{aligned}$$

$$(9) \quad \pi(x, y, u_x, u_y) = p_1 u_x - q_1 u_x^{\alpha_1} + p_2 u_y - p_3 u_y^2 - \frac{q_2 u_y^{\alpha_2}}{y},$$

Here $p_1, p_2, q_1, q_2, \alpha_1, \alpha_2$ are parameters for capelin and cod prices and costs.

Because the prey is a schooling species, we assume that the unit cost of the harvest is independent of the stock size. Hence, our revenue function includes only the predator stock and the harvest levels of prey and predators. The revenue function given by equation (9) can simply be written as a function of three arguments $\pi(y, u_x, u_y)$.

Given the growth and profit functions, management's objective is to maximize the expected net present value of the return from the harvest schedule over an infinite time horizon. This can be achieved by maximizing the following function:

$$(10) \quad J(u_x, u_y) = E \left[\int_0^\infty e^{-\delta t} \pi(y, u_x, u_y) dt \right].$$

The nonnegative parameter δ is the discount rate and E is the expectations operator. Along with the dynamic constraints and appropriate boundary conditions, the dynamic optimization problem can be written as

$$(11) \quad \begin{aligned} V(x_0, y_0, z_0) &= \max_{h_x, h_y \geq 0} J(y, u_x, u_y), \\ x(t=0) &= x_0, \\ y(t=0) &= y_0, \\ z(t=0) &= z_0. \end{aligned}$$

The optimal solution for the predator-prey model can be obtained by solving the HJB equation below (Kushner and Dupuis [2001]), given appropriate boundary conditions. Because the equations below are valid for any feasible initial conditions, we replace (x_0, y_0, z_0) with (x, y, z) as follows:

$$\begin{aligned} \delta V(x, y, z) = & \max_{u_x, u_y \geq 0} \{ \pi(y, u_x, u_y) + V_a^T(x, y, z) F(x, y, z, u_x, u_y) \\ & + \frac{1}{2} \text{tr}[\sigma(x, y, z) \sigma^T(x, y, z) V_{aa}(x, y, z)] \}, \end{aligned} \quad (12)$$

$$\text{where } V_a(x, y, z) = \begin{pmatrix} V_x(x, y, z) \\ V_y(x, y, z) \\ V_z(x, y, z) \end{pmatrix}, \quad F(x, y, z, u_x, u_y) = \begin{pmatrix} f(x, y, z) - u_x \\ g(x, y, z) - u_y \\ j(x, y, z) \end{pmatrix},$$

$$\begin{aligned} \sigma(x, y, z) &= \begin{pmatrix} \sigma_1 x & 0 & 0 \\ 0 & \sigma_2 y & 0 \\ 0 & 0 & \sigma_3 z \end{pmatrix}, \text{ and } V_{aa}(x, y, z) \\ &= \begin{pmatrix} V_{xx}(x, y, z) & V_{xy}(x, y, z) & V_{xz}(x, y, z) \\ V_{yx}(x, y, z) & V_{yy}(x, y, z) & V_{yz}(x, y, z) \\ V_{zx}(x, y, z) & V_{zy}(x, y, z) & V_{zz}(x, y, z) \end{pmatrix} \text{ are matrices.} \end{aligned}$$

The HJB equation given by (12) can be rearranged and simplified as follows:

$$\begin{aligned} \delta V = & \max_{u_x, u_y \geq 0} \left\{ \pi(y, u_x, u_y) + (f(x, y, z) - u_x) V_x + (g(x, y, z) - u_y) \right. \\ & \left. V_y + j(x, y, z) V_z + \frac{1}{2} \sigma_1^2 x^2 V_{xx} + \frac{1}{2} \sigma_2^2 y^2 V_{yy} + \frac{1}{2} \sigma_3^2 z^2 V_{zz} \right\}. \end{aligned} \quad (13)$$

The subscripts on V denote partial derivatives with respect to the index ($i = x, y, z$). The optimal solution can be derived by solving the HJB equation (13). Because it is difficult, even impossible, to solve the HJB equation with its boundary conditions analytically, we apply SDP to solve it based on numerical approximation methods. Although a number of techniques are available, the Markov chain approximation approach, which is based on probability theory, is one of the most effective methods (Song [2008]).

3. Numerical approximation. We employ the Markov chain approximation approach of Kushner and Dupuis [2001]. The basic idea of the Markov chain approximation approach is to discretize the HJB control problem. Let the HJB control problem (13) for a multidimensional space be rewritten as:

$$\delta V(x) = \max_{u \geq 0} \left\{ \pi(x, u) + V_x(x)^T F(x, u) + \frac{1}{2} \text{tr}[a(x) V_{xx}(x)] \right\}, \quad (14)$$

where $V_x(x)$ and $F(x, u)$ are matrices of dimension $1 \times n$, $a(x) = \sigma(x)\sigma(x)^T$ is the covariance matrix, with $\sigma(x)$, and $V_{xx}(x)$ being matrices of dimension $n \times n$.

The numerical method is based on a direct discretization of the HJB equation (14). Let e_i be the unit basis of R^n for $i = 1, \dots, n$ and let $h > 0$ be a uniform step size, or the approximation parameter. Let $V(x)$ be the numerical solution of the value function. Then, by applying standard finite difference approximations (Kushner and Dupuis [2001]), the first and second derivatives of $V(x)$ are given by equations (15) and (16), respectively:

$$(15) \quad V_{x_i}^{\pm}(x) = \frac{V(x \pm e_i h) - V(x)}{h},$$

$$(16) \quad V_{x_i x_j}(x) = -\frac{[2V(x) + V(x + e_i h - e_j h) + V(x - e_i h + e_j h)]}{2h^2} + \frac{[V(x + e_i h) + V(x - e_i h) + V(x + e_j h) + V(x - e_j h)]}{2h^2}.$$

The next step is to define the positive and negative parts of the drift terms as follows:

$$(17) \quad F_i^+(x, u) = \max(0, F_i(x, u)),$$

$$F_i^-(x, u) = \max(0, -F_i(x, u)),$$

where $F_i^+(x, u)$ is the positive term and $F_i^-(x, u)$ is the flow in a positive or negative direction.

Next, we define the transition probabilities and the interpolation interval, given by (18) and (19), respectively, as follows:

$$(18) \quad P(x, x \pm e_i h | u) = \left[\frac{1}{2} a_{ii}(x) - \frac{\sum_{j:j \neq i} |a_{ij}(x)|}{2} + F_i^+(x, u) h \right] / Q^h(x, u),$$

$$P(x, x + e_i h + e_j h | u) = P(x, x - e_i h - e_j h | u) = [a_{ij}^+(x)] / 2Q^h(x, u),$$

$$P(x, x - e_i h + e_j h | u) = P(x, x + e_i h - e_j h | u) = [a_{ij}^-(x)] / 2Q^h(x, u),$$

$$P(x, x | u) = 1 - \sum_{y \neq x} P(x, y | u),$$

where $P(x, x | u)$ denotes the probability of remaining at the same point, and $\sum_{y \neq x} P(x, y | u)$ is the sum of the values in (18). The implied stochastic interpolation time is

$$(19) \quad \Delta t = \frac{\left[\frac{h^2}{1 + \delta h^2} \right]}{Q^h(x, u)},$$

where $Q^h(x, u) = \sum_i a_{ii}(x) - \frac{\sum_{i,j:j \neq i} |a_{ij}(x)|}{2} + h \sum_i F_i(x, u)$ is the normalization coefficient. Defining $\beta(x, u) = 1 - \delta \Delta t$, with δ being the discount rate, implies that the proper discrete form of (14) (the Markov chain) is given by:

$$(20) \quad V(x) = \max_u \left\{ \pi(x, u) \Delta t(x, u) + \beta(x, u) \sum_y P(x, y|u) V(y) \right\}.$$

Given the transition probabilities and interpolation intervals, the optimal value function in the policy space is obtained and updated by using value iteration and policy iteration of the HJB equation. We iterate the value function by starting with an initial guess, $V_0(x)$, for a given policy, such as $u = u_0$, using value and policy iterations of $V_{n+1}^k(x) = L(x, u_k)[V_n^k(x)]$ and $u_{k+1,n} = \operatorname{argmax}\{L(u)[V_n^k(x)]\}$, respectively. In this context, $L(x, u)$ represents the Markov chain operator. The policy iterations (given $u_k(x)$) converge to the true value $\tilde{V}^k(x)$ for the given policy, and the sequence $\{\tilde{V}^k(x)\}$ converges to our problem's value function $V(x) = \tilde{V}(x)$.

Iteration of the value function is performed with an initial guess, $V_0(x) = 0$, for a given policy $u = u_k$ using the following value iteration:

$$(21) \quad V_{n+1}^k(x) = L(x, u_k) V_n^k(x),$$

where the Markov chain operator L is defined by $L(x, u)V(x) = [\beta(x, u)(\sum_i P(x, x \pm e_i h|u) \cdot V(x, x \pm e_i h|u)) + \pi(x, u) \cdot \Delta t] \cdot V^*$ is obtained when n and k are sufficiently large; specifically, $V_n^k \rightarrow [n \rightarrow \infty](V^k)^* \rightarrow [k \rightarrow \infty]V$. Similarly, the policy iteration is

$$(22) \quad u_{k+1,n} = \operatorname{argmax}_u [L(x, u), V_n^k(x)].$$

The new policy $u_{k+1,n}$ is then employed in the value function iteration.

A mixture of value and policy iterations are carried out until the value function converges to the optimal value function. The function to which this procedure converges is the unique value function for the infinite horizon problem, and the policy associated with the optimal value function is the optimal policy. Uniqueness follows from the contraction operator nature of our approach (Kushner and Dupuis [2001]). Iterations are stopped by using the following stopping criterion (22) for the policy and value functions:

$$(23) \quad \begin{aligned} & \|u_{k+1,n+1}(x) - u_{k,n+1}(x)\| < \varepsilon_1, \\ & \|V_{n+1}^{k+1}(x) - V_n^{k+1}(x)\| < \varepsilon_2, \end{aligned}$$

where ε_i are predefined threshold values. To ensure convergence of the value function, we perform policy and value iterations until the largest absolute difference in the value function is no more than $\varepsilon_1 = \varepsilon_2 = 10^{-6}$ from one iteration to the next.

4. Specification of the model parameters. The biological and economic parameters were obtained from Aanestad et al. [2009] and Sandal and Steinshamn [2010]. These parameters were estimated from Barents Sea stock and harvest data

TABLE 1. Biological parameter value.

| Notation | Specification | Value |
|----------|-------------------------------|----------------|
| r_1 | Growth parameter for capelin | 1.8515 |
| r_2 | Growth parameter for cod | 0.5490 |
| r_3 | Growth parameter for herring | 1380.7 |
| k_1 | Carrying capacity for capelin | 7890.1 |
| k_2 | Carrying capacity for cod | 3191.3 |
| k_3 | Carrying capacity for herring | $1.0572e^7$ |
| b_1 | Interaction coefficient | $1.1142e^{-4}$ |
| b_2 | Interaction coefficient | $2.3e^{-6}$ |
| b_3 | Interaction coefficient | $1.87e^{-6}$ |

Source: Aanestad et al. [2009].

TABLE 2. Economic parameter values.

| Notation | Specification | Value |
|------------|---------------------------------|---------|
| p_1 | Price of capelin stock | 1.0 |
| q_1 | Cost of capelin exploitation | 0.07 |
| α_1 | Harvest cost parameter | 1.4 |
| p_2 | Price of cod stock | 12.65 |
| p_3 | Strength of demand in cod stock | 0.00839 |
| q_2 | Cost of capelin exploitation | 5848.1 |
| α_2 | Harvest cost parameter | 1.0 |
| δ | Discount rate | 0.05 |

Sources: Aanestad et al. [2009] and Sandal and Steinshamn [2010].

for 1973–2005, obtained from the International Council for the Exploration of the Seas (ICES). The values of the biological and economic parameters are given in Tables 1 and 2, respectively.

5. Results and discussion. In this section, the feedback exploitation policy implied by our numerical results is described with the aid of multidimensional graphical surface plots. The exploitation policies in the deterministic and stochastic models are discussed and compared. Because the numerical output has four dimensions, which is impossible to illustrate graphically, we fix one of the species at a certain stock level to construct our three-dimensional surface plots.

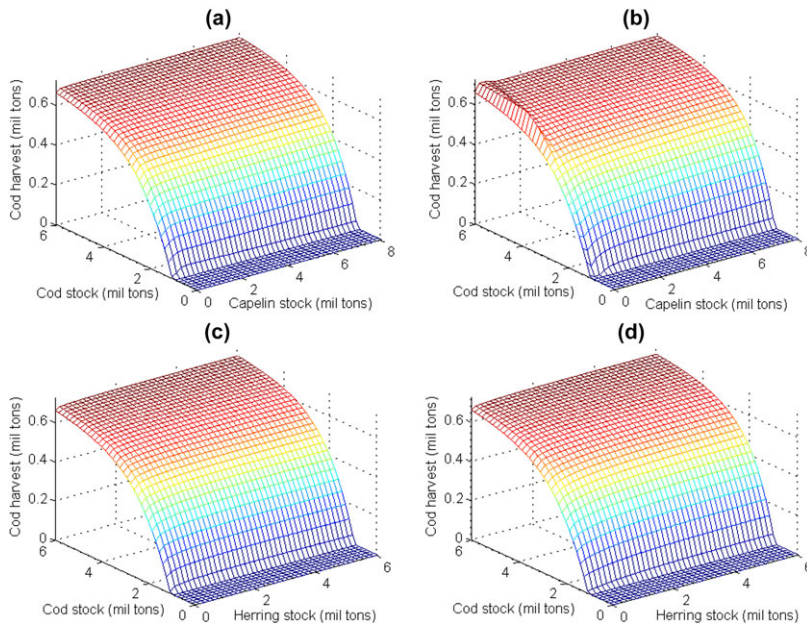


FIGURE 1. Optimal exploitation policy for cod in a three-species interaction ecosystem: (a) 1.5 million tons of herring; (b) 6 million tons of herring; (c) 2 million tons of capelin; and (d) 6 million tons of capelin.

5.1. The optimal harvest.

5.1.1. Deterministic model. The deterministic optimal exploitation policy for cod and capelin is obtained by employing dynamic equation (1) and is presented in Figures 1 and 2, respectively.

The fishing moratorium on cod in the deterministic growth model is found to be nearly 1 million tons when there are large capelin and herring stocks in the ecosystem. When the cod stock biomass exceeds 1 million tons, its exploitation should be increased. For example, with an increase in the cod biomass from 2 million tons to 6 million tons, its optimal exploitation can be increased from 0.36 million tons to 0.62 million tons. We also observe that the abundance of biomass of other stocks, such as capelin and herring, in the ecosystem has no strong influence on the cod exploitation policy (Figures 1a–d). This may be because there are two prey stocks for cod and both stocks serve as food substitute for its growth.

However, the optimal exploitation policy for capelin is greatly influenced by the presence of its predators in the ecosystem (Figure 2). The greater the influx of juvenile herring, the more capelin is affected. The exploitation of capelin should be

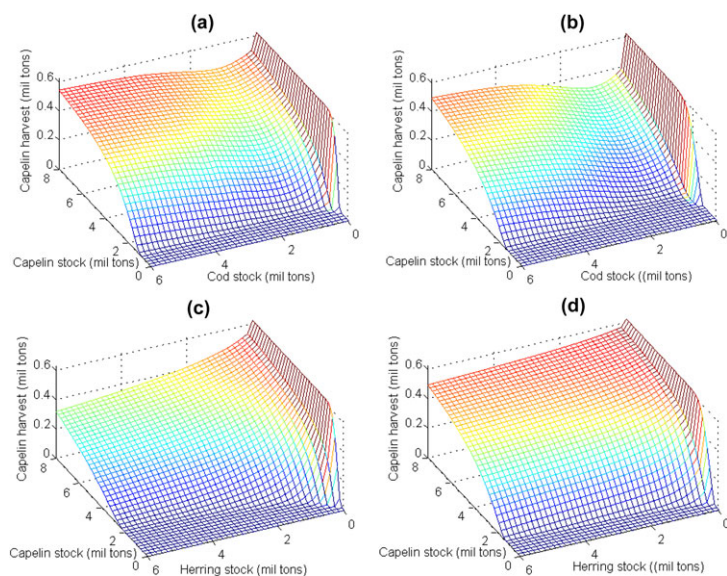


FIGURE 2. Optimal exploitation policy for capelin in a three-species interaction ecosystem: (a) low herring; (b) high herring; (c) low cod; and (d) high cod. Note: x -axis reversed for clarity.

more conservative with a large herring biomass than with a small biomass because a large herring biomass could consume more capelin.

Capelin harvest should also be less conservative when there is low herring and large cod biomass relative to low cod biomass because a large cod preys on herring and thus reducing the impact on capelin. However, when cod stocks are low, capelin should be exploited conservatively. This is because immature cod need capelin for their growth (Hamre [2003]), and excessive harvesting could make capelin collapse because of cod and herring predation.

When there is a no herring and a high cod biomass in the ecosystem, one should exploit the capelin as in a single-species model (Figure 2d). This is because there is no gain from the capelin conservation when the cod stock is high.

When cod levels are low and herring biomass is increasing, one should be conservative to avoid the risk of capelin collapse because large inflows of herring may wipe out capelin fry (Mehl [1991], Hamre [2003]) if capelin is harvested without considering multispecies interactions in the ecosystem. This may be because (i) a low level of cod leaves herring unaffected and the large herring predares capelin rigorously, (ii) but also due to food competition in the ecosystem (Kar and Chaudhuri [2004]).

5.1.2. Stochastic model. In this section, we discuss the results from dynamic equation (5), in which we choose different levels of stochasticity to compare the results with deterministic model. Because we have no knowledge about the stochastic parameters in stock growth in the Barents Sea, we used various levels of stochasticity. Furthermore, we assume uncorrelated stochasticity because the stochastic processes that affect prey directly may not directly affect predators, and vice versa. The effect of stochasticity on the optimal exploitation policies for cod and capelin is described in the next subsection.

5.1.2.1. The effect of stochasticity on cod. To observe the effect of stochasticity on cod exploitation policy, we compare different cases. We find that cod harvest policies are similar whether only one or all species are stochastic. This may be because if one species is stochastic, all species are affected due to the interaction terms. Given the similarity in the results, we only describe results based on the stochasticity of all species.

When the level of stochasticity is low ($\sigma_1 = \sigma_2 = \sigma_3 = 0.1$), there is no pronounced effect on the optimal policy (results not shown). With increasing stochasticity in all species, the exploitation level should be decreased. For example, when the level of stochasticity increases to 0.5 ($\sigma_1 = \sigma_2 = \sigma_3 = 0.5$), with zero capelin and a high level of herring stock in the ecosystem, it is optimal to delay the exploitation until the stock level increases. This is because the lack of capelin could slow the growth of cod (perhaps young cod prefer capelin to herring). At an increased level of stochasticity ($\sigma_1, \sigma_2, \sigma_3 > 0.9$), the moratorium moves towards a smaller stock level. This suggests that when stochasticity is high, myopic exploitation is optimal.¹ As in the deterministic model, the levels of other stocks (prey) have less influence on the optimal cod exploitation policy (Figure 3).

5.1.2.2. The effect of stochasticity on capelin. To analyze the effect of stochasticity on the capelin exploitation policy, we examine different cases. First, we consider stochasticity in all species, then we investigate the case in which only capelin is stochastic.² We find that capelin is more sensitive to stochasticity at different biomass levels, and also at different levels of stochasticity than at stochasticity at different species.

Case I: All Species are Stochastic. Stochasticity has a large effect on the exploitation policy for capelin compared with that for cod. If all three species are stochastic, the exploitation policy should be conservative with increased stochasticity ($\sigma_1, \sigma_2, \sigma_3 = 0.9$). The reason is that when these species are stochastic, there is a high risk of stock collapse (Poudel et al. [2012]), which dictates conservative exploitation to avoid the collapse of the stock. When herring stock is large, there is a higher risk of capelin collapse because of herring predation coupled with stochasticity. Therefore, one should be more conservative (Figures 4b and d) than with low herring (Figures 4a and c).

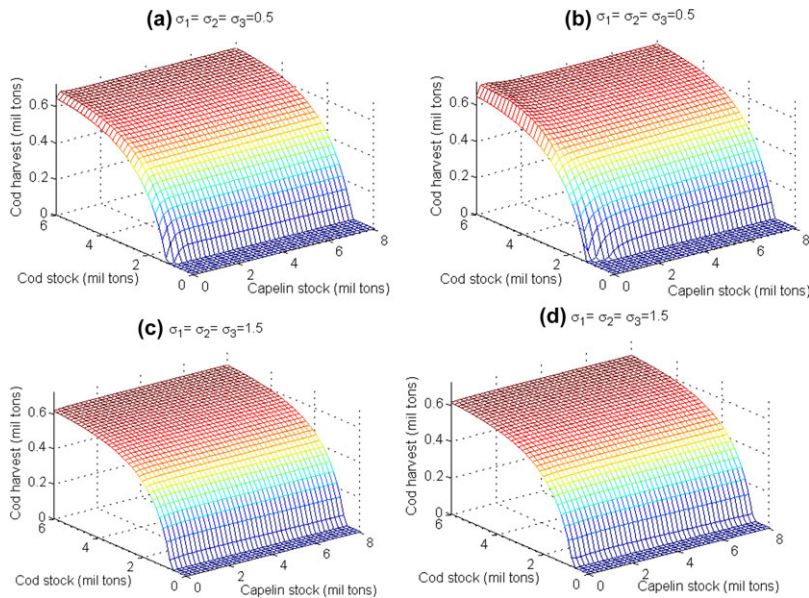


FIGURE 3. The effect of stochasticity on the optimal cod exploitation policy in the three-species interaction model: (a) low herring, low stochasticity; (b) high herring, low stochasticity; (c) low herring, high stochasticity; and (d) high herring, high stochasticity.

For a substantially high level of stochasticity ($\sigma_1, \sigma_2, \sigma_3 > 0.9$), it is optimal to exploit capelin myopically when the stock is small but to be conservative when the stock is high. At low stock, because it is virtually impossible to avoid stock collapse through predation and stochasticity, it is profitable to exploit as much as possible. However, at a high stock level, conservative exploitation can avoid stock collapse. In the case of high stochasticity, it is optimal to be more conservative when the herring stock in the ecosystem is high than when it is low (Figures 4e and f).

Case II: Only Capelin is Stochastic. In this section, we determine the optimal capelin policy when capelin stock is stochastic and the other stocks are deterministic. The numerical results are obtained by setting the stochastic parameters for herring and cod to zero and choosing different levels of stochasticity for the capelin stock (Figure 5).

We find that the capelin exploitation should be conservative with increasing stochasticity ($\sigma_1 = 0.9$). There are at least two reasons; first, because stochastic stock has a high risk of collapse; and second, by saving the stochastic capelin for cod food, one can increase revenue from the cod harvest, particularly when the cod stock is low and the herring stock is high.

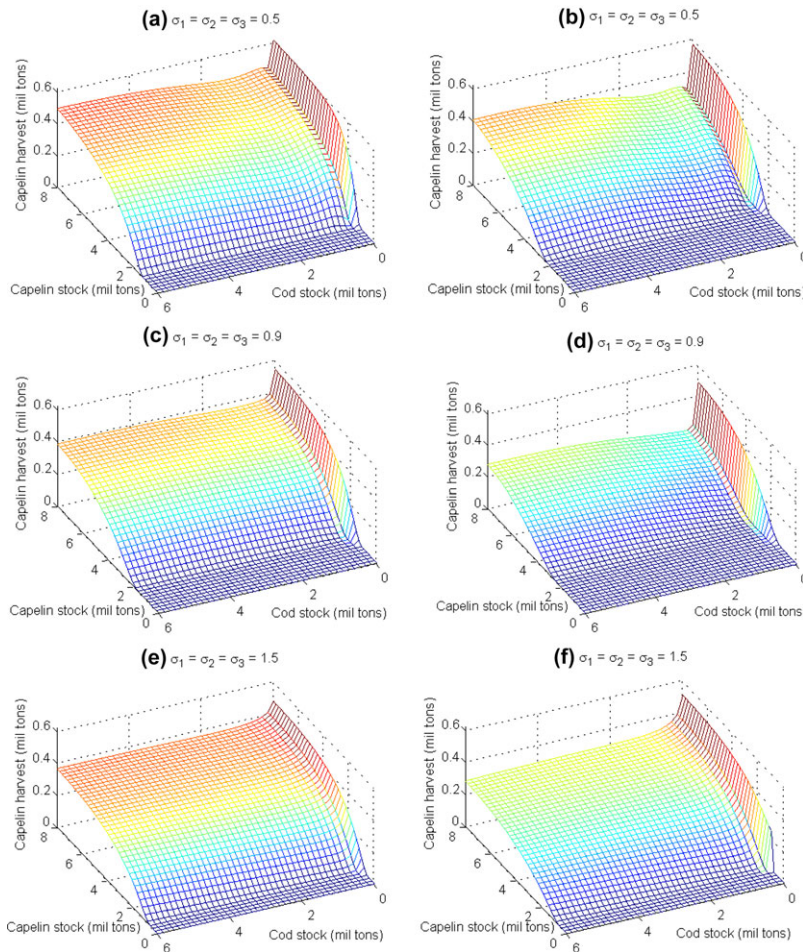


FIGURE 4. The feedback optimal exploitation policy for capelin in the three-species interaction model at different herring stock levels under stochasticity: (a) low herring (1.5 million tons); (b) high herring (6 million tons); (c) low herring; (d) high herring; (e) low herring; and (f) high herring.

Note: x -axis is reversed for clarity.

When stochasticity is substantially high ($\sigma_1 = 1.5$), one should be conservative except if capelin stocks are low. However, at this stochasticity level, exploitation should be myopic when there is no cod. This is because, in the absence of cod, the combined effects of herring predation and stochasticity stock could drive the stock to extinction.

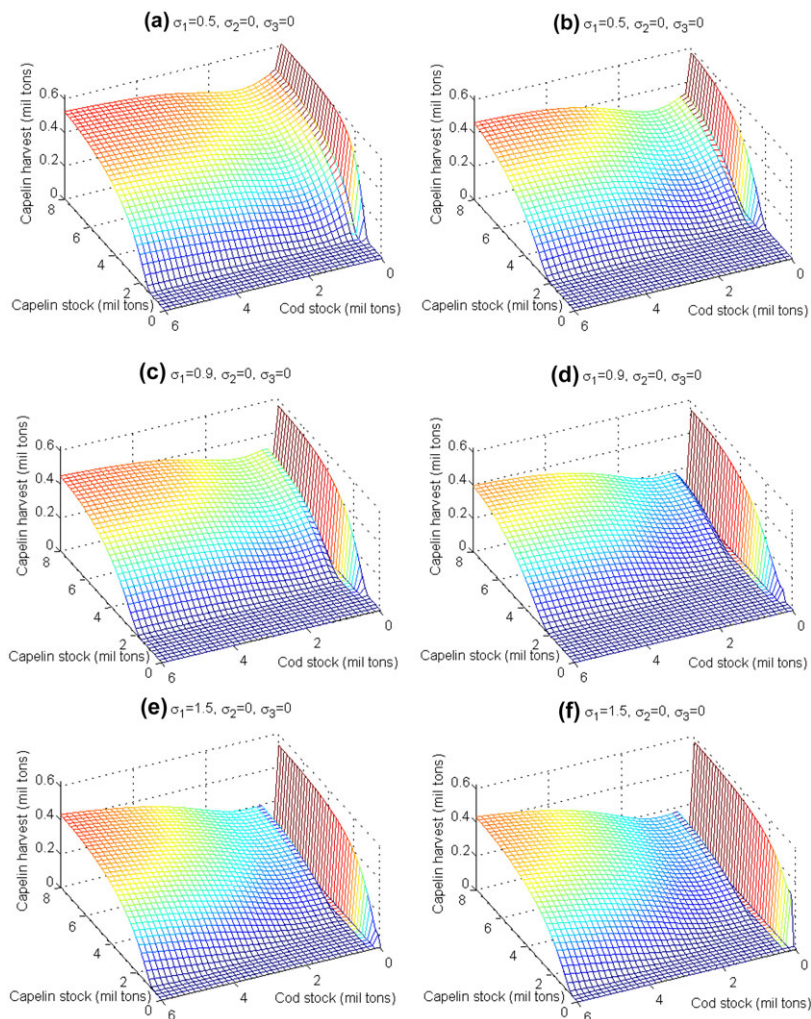


FIGURE 5. Effect of capelin stochasticity on capelin exploitation policy: (a) low herring with $\sigma_1 = 0.5$; (b) high herring with $\sigma_1 = 0.5$; (c) low herring with $\sigma_1 = 0.9$; (d) high herring with $\sigma_1 = 0.9$; (e) low herring with $\sigma_1 = 1.5$; and (f) high herring with $\sigma_1 = 1.5$. Note: x -axis reversed for clarity.

The main result of the analysis is that at low level of stochasticity there is little or no effect on the exploitation policy in the multispecies ecosystem. Although an assumption of very high stochasticity is only important for precautionary principle, an assumption of a moderate level of stochasticity is highly relevant for fishery management. We find that at moderate level of stochasticity, both aggressive and

TABLE 3. The biological equilibrium (BE) and long-term sustainable optimal state (LSOS) in the deterministic model.

| Species and harvest (million tons) | Type of equilibrium | |
|---------------------------------------|---------------------|-------|
| | BE | LSOS |
| Capelin stock | 2.711 | 3.025 |
| Capelin harvest | – | 0.228 |
| Cod stock | 4.046 | 3.156 |
| Cod harvest | – | 0.560 |
| Herring stock | 1.844 | 2.012 |

conservative policy becomes optimal and depends on the different interacting stock levels in the ecosystem.

5.2. The long-term sustainable optimal state (LSOS). In the previous section (Section 5.1), we derived optimal feedback solution in a three-species ecosystem. Given that optimal solution, we estimate long-term stock (harvest) levels, which we call LSOS. To derive the LSOS, we conduct Monte Carlo simulations for given optimal solutions. We simulate the system forward in time for a range of initial stock levels with all species present in the system. (For example, the LSOS for capelin is computed based on the optimal feedback solution for capelin, cod and herring.) For the deterministic and stochastic models, the LSOS is defined below. (The LSOS in single- and two-species contexts has been discussed by Poudel et al. [2013] and Poudel et al. [2014].)

Deterministic LSOS: In the deterministic setting, the simulated paths approach a certain level over time, which is defined as the LSOS, or steady state, or the equilibrium level that can be achieved after a certain period of time if the stocks are managed optimally.

Stochastic LSOS: In the stochastic setting, there is no equilibrium but most paths converge to the same level after a certain period.³ This is a region around a zero-drift level, which is defined as the stochastic LSOS (Poudel et al. [2013]). In other words, the LSOS can be defined as the optimal stochastic stationary state (Smith [1986]).

The LSOS is also compared with the biological equilibrium (BE). The latter is an equilibrium that can be achieved without taking into consideration harvest (that is, it is a natural equilibrium level). The LSOS is reported in Tables 3–5, both for the deterministic and stochastic models. Table 3 shows the BE and LSOS results for the stocks and their harvesting in the deterministic model.

TABLE 4. Biological equilibrium (BE) levels under different degrees of stochasticity.

| Stock (million tons) | Degree of stochasticity | | | | |
|----------------------|-------------------------|----------------|----------------|----------------|----------------|
| | $\sigma = 0.1$ | $\sigma = 0.3$ | $\sigma = 0.5$ | $\sigma = 0.7$ | $\sigma = 0.9$ |
| Capelin | 2.744 (0.35) | 3.059 (1.06) | 3.485 (0.17) | 2.824 (0.18) | 1.988 (0.16) |
| Cod | 4.011(0.32) | 3.727 (0.94) | 3.102 (0.14) | 2.131(0.16) | 1.046 (0.13) |
| Herring | 1.812 (0.27) | 1.545 (0.73) | 1.294 (1.06) | 0.827(0.12) | 0.645 (1.19) |

Note: Figures in parentheses are standard deviations.

TABLE 5. The biological equilibrium (BE) and long-term sustainable optimal state (LSOS) under different degree of stochasticity.

| Species and harvest (in million tons) | Degree of stochasticity | | | |
|--|-------------------------|---------------|----------------|----------------|
| | $\sigma = 0.1$ | | $\sigma = 0.3$ | |
| | BE | LSOS | BE | LSOS |
| Capelin stock | 2.744 (0.354) | 3.084 (0.351) | 3.059 (1.061) | 3.357 (1.036) |
| Capelin harvest | – | 0.221 (0.050) | – | 0. 209 (0.138) |
| Cod stock | 4.011 (0.321) | 3.136 (0.292) | 3.727 (0.938) | 3.000 (0.815) |
| Cod harvest | – | 0.547 (0.032) | – | 0.443 (0.130) |
| Herring stock | 1.812 (0.277) | 1.938 (0.290) | 1.545 (0.734) | 1.662 (0.783) |

Note: Figures in parentheses are standard deviations.

The BE is lower than the LSOS in capelin and herring but is higher in cod. This contrast is interesting. The difference arises because, in general, the LSOS is expected to be lower than the BE due to harvest. However, this can be explained as follows. The cod LSOS is smaller than the BE; that is, when we harvest cod, there is a low level of predators for capelin and herring in the ecosystem, which allows them to grow to a higher equilibrium level.

We also derive BE levels for different degrees of stochasticity (Table 4). Interestingly, for increased stochasticity, the BE for capelin increases until a certain level of stochasticity (such as $\sigma = 0.5$) but then decreases when stochasticity is high ($\sigma \geq 0.7$). The main reason is that because of stochastic downward drag, equilibrium levels of both predators (cod and herring) are lower at higher stochasticity. This means that there is a smaller predator stock in the ecosystem to prey upon capelin, which allows capelin stocks to grow to a higher level. This persists

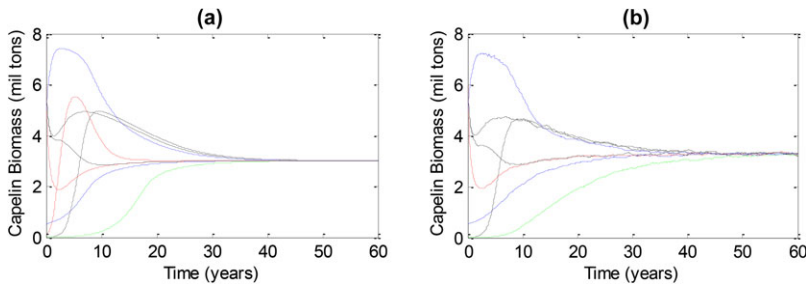


FIGURE 6. Evolution of capelin stock over time toward the LSOS from a combination of several initial conditions for capelin, cod, and herring: (a) deterministic model; and (b) stochastic model. Note: The initial conditions are the combination of different stock levels. In y -axis, the initial conditions for capelin ranges from 0 to 6 million tons but with different combination of initial cod and herring level.

until capelin stock is no longer affected by its own stochastic drag. However, when stochasticity is high, capelin's own stochastic downward drag lowers its BE level.

The stochastic LSOS stock levels and their exploitation and the stochastic BE stock levels in our model are presented and compared in Table 5. It is generally expected that the LSOS is lower than the BE because of the harvest. This can be observed in the case of cod, for which the LSOS is smaller than the BE. By contrast, for capelin, the BE is lower than the LSOS. The interesting point behind this result is that because of the decreased LSOS for cod, there is a low predation, which allows capelin to increase at the LSOS.

5.3. Stock evolution behavior. We have illustrated how different stocks evolve over time as they approach the LSOS in the prey–predator ecosystem. Several different optimal paths for various combinations of initial biomass levels of capelin, cod, and herring are shown in Figures 6 and 7 for deterministic and stochastic stocks over time. The paths in the stochastic setting were obtained by taking an average of a thousand realizations from the same point.

5.3.1. Capelin evolution over time. Capelin evolution to the LSOS is shown in Figure 6. For any combination of initial biomass of capelin, cod, and herring, the stock reaches the LSOS in about 40 years in the deterministic model (Figure 6a) but it takes more than 60 years in the stochastic model. However, unharvested stock approaches the LSOS earlier when predator levels in the ecosystem are low. During the evolution process, there is overshooting and undershooting. Overshooting occurs when there is fast growth of the stock and an absence of harvest or predation; undershooting takes place if a large stock is heavily harvested because such harvesting is less costly.

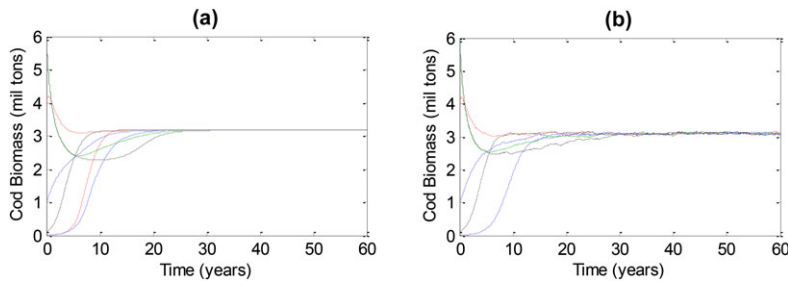


FIGURE 7. Evolution of cod stock over time toward the LSOS from a combination of several initial conditions for capelin, cod, and herring: (a) deterministic model and (b) stochastic model. The initial conditions are the combination of different stock levels. In y -axis, the initial conditions for cod ranges from 0 to 6 million tons but with different combination of initial capelin and herring level.

When the initial capelin stock is high (5 million tons) and stocks of cod and herring in the ecosystem are low, overshooting occurs because of the low level of predation. In this case, it takes longer to reach the LSOS because market conditions mean that it is suboptimal to exploit heavily. However, when the initial capelin stock is high (5 million tons) and there are high levels of cod and herring in the ecosystem, undershooting occurs because of both exploitation and predation. However, if there is only one predator in the ecosystem, overshooting may occur because of low predation. A small capelin stock (0.1 million tons) with high cod and high herring levels leads to slow growth towards the LSOS compared with a medium but unharvested stock.

Stock evolution trends in the stochastic and deterministic models are similar, but the stochastic stock takes longer to reach the LSOS than does the deterministic evolution.

5.3.2. Cod evolution over time. The evolution of cod is shown in Figure 7. A large cod stock reaches the LSOS earlier than a small initial stock. There is no overshooting but undershooting occurs when cod and capelin stocks are high but herring levels in the ecosystem are low. At high stock levels, both cod and capelin can be harvested extensively, but because of the low level of herring in the ecosystem, the lack of food for cod generates competition and cannibalism, which persists until herring and capelin stocks increase in the ecosystem. A low cod stock reaches the LSOS more quickly if a high capelin stock is accompanied by a high herring stock. This is because the harvesting of capelin leaves cod short of food, which retards its growth. However, cod's own initial stock level influences its evolution directly. A low initial stock takes longer to reach LSOS. Under stochasticity, although the LSOS is lower, its characteristics are similar to the LSOS in the deterministic growth model. Given the same initial level, a stochastic stock takes longer to reach

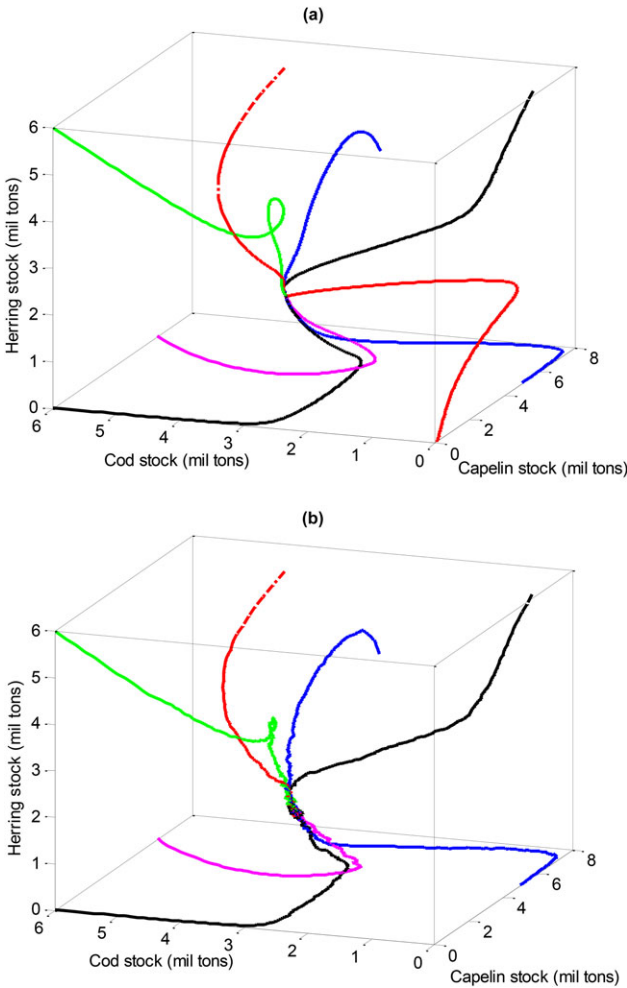


FIGURE 8. Dynamic paths towards the LSOS for capelin, cod, and herring: (a) deterministic paths and (b) stochastic paths (mean of 1000 realizations).

the LSOS than a deterministic one (Figure 7b). However, stochastic cod evolves more quickly than capelin. This may be because the cod stock has a negative effect on the growth of prey as stochastic cod needs more food.

5.3.3. Paths toward the LSOS. The dynamic paths from different initial combinations of cod, capelin, and herring are shown in Figure 8. The combination of initial biomass level determines how the paths approach the LSOS. The figure shows seven different combinations of initial biomass levels and how they reach the

LSOS in the long term. Although it is difficult to interpret the movement in detail because of presentational complexities, general dynamic patterns can be observed. Generally, the tendency toward the LSOS appears similar in both the deterministic and stochastic models except that the LSOS is lower in the stochastic case.

6. Concluding remarks. The contribution of the study is twofold; first we demonstrated that the dynamic programming approach can be used for empirical models in higher dimensions; and second, we obtain empirical results on optimal management in a complex biological ecosystem that harbors three species.

We solved a three-dimensional stochastic model to determine an optimal feedback strategy for Barents Sea species. We used stochastic dynamic programming for our numerical approximations. Stochastic multispecies models, particularly higher dimension, are rare in the economic literature. Hence, our work is a step advance in dealing with complex three-dimensional models, and an important step toward the ultimate goal of devising ecosystem-based fisheries management when there are more than three species and uncertainty. However, in practice dealing with higher dimensions increases complexity with calculation, presentation and interpretation of the results.

An interesting result from the empirical application is that juvenile herring in the Barents Sea must be considered when exploiting capelin; otherwise, a higher herring level may cause capelin stocks to collapse, perhaps to extinction. A conservative exploitation for capelin is optimal in the multispecies ecosystem.

Our findings suggest that a low level of stochasticity has little effect in a multispecies ecosystem. Moreover, stochasticity affects prey more than predators. Higher stochasticity coupled with high level predators has significant impact on the prey stock level. Whether or not our empirical results describe reality accurately, we argue that incorporating, rather than avoiding, uncertainty will boost the chances of successfully achieving conservation and management goals, as suggested by Halpern et al. [2006].

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ENDNOTES

1. Myopic exploitation policy refers to exploitation that ignores the future.
2. We also investigate many different cases such as only cod is stochastic, only herring is stochastic and cases with two species are stochastic. Interestingly, the effect of cod and herring stochasticity has a little influence on the capelin exploitation policy even for a relatively high degree of stochasticity, therefore; we do not discuss the details.
3. In this study, each path is obtained by taking an average of 1000 stochastic realizations.

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