Propositional Inference by Resolution – Blood Types and Tests

Problem Statement

There are four blood types, A, B, AB, and O. Every person has one of those blood types. There are two blood tests – S and T. (Only) types A and AB blood test positive with the S test. (Only) types B and AB test positive with the T test. Show that if a person's blood tests negative both with S and T, they must have type O blood.

Propositions and Their Meanings

- A – the person has type A blood
- B – the person has type B blood
- AB – the person has type AB blood
- O – the person has type O blood
- S – the person's blood tests positive using test S
- T – the person's blood tests positive using test T

Axioms

1. Everybody has one of the four blood types: `A V B V AB V O`
2. (Only) types A and AB blood test positive with the S test: `S <=> (A V AB)`
3. (Only) types B and AB blood test positive with the T test: `T <=> (B V AB)`

Goal Statement

4. If a person tests negative for both S and T, then they have type O blood: `¬S ^ ¬T => O`

Converting to CNF

Identify the clauses that will be used in the resolution proof. The clauses are identified with a label like **C1:** and set in bold face

(Axiom 1) : A V B V AB V O

C1: **`A V B V AB V O`** `-- This formula is already in CNF`

(Axiom 2): S <=> (A V AB)

```
            (S => (A V AB)) ^ ((A V AB) => S)   -- Convert <=> to two =>
```

Note now we have two conjuncts so we can handle them separately

```
            (S => (A V AB))     -- First conjunct
C2:         ¬S V A V AB         -- Definition of =>;  this is a disjunction

            (A V AB) => S       -- Second conjunct
            ¬ (A V AB) V S      -- Definition of =>
            (¬A ^ ¬AB) V S      -- Move negation to propositions
            (¬A V S) ^ (¬AB V S)  -- Distribute V over ^; have two conjuncts
C3:         ¬A V S
C4:         ¬AB V S
```

(Axiom 3): T <=> (B V AB) -- almost identical to (2)

```
            (T => (B V AB)) ^ ((B V AB) => T)   -- Convert <=> to two =>
```

Note now we have two conjuncts so we can handle them separately

```
            (T => (B V AB))     -- First conjunct
C5:         ¬T V B V AB         -- Definition of =>;  this is a disjunction

            (B V AB) => T       -- Second conjunct
            ¬ (B V AB) V T      -- Definition of =>
            (¬B ^ ¬AB) V T      -- Move negation to propositions
            (¬B V T) ^ (¬AB V T)  -- Distribute V over ^; have two conjuncts
C6:         ¬B V T
C7:         ¬AB V T
```

(4 Goal): (¬S ^ ¬T) => O

```
            ¬((¬S ^ ¬T) => O)       -- Negate the goal
            ¬(¬ (¬S ^ ¬T) V O)      -- Definition of =>
            (¬S ^ ¬T) ^ ¬O          -- Move outer negation in;
                                    --   we now have three conjuncts, done
NG1:        ¬S
NG2:        ¬T
NG3:        ¬O
```

Resolution Proof

Clause #	Clause	Derivation
C1	A V B V AB V O	
C2	¬S V A V AB	
C3	¬A V S	
C4	¬AB V S	
C5	¬T V B V AB	
C6	¬B V T	
C7	¬AB V T	
NG1	¬S	
NG2	¬T	
NG3	¬O	
1	A V B V AB	C1 + NG3
2	S V B V AB	1 + C3
3	B V AB	2 + NG1
4	T V AB	3 + C6
5	AB	4 + NG2
6	S	5 + C4
7	∅	6 + NG1

Therefore we can say that the axioms plus the negated goal is unsatisfiable, and therefore the goal formula (¬S ^ ¬T) => O follows from axioms (1) – (3)