Time Series Decomposition

Outline

- Components of a time series
- STL decomposition
- Moving average
- Forecast with decomposition

Time series components

- Trend: long term increase or decrease in the data.
- Seasonal: a seasonal pattern exists when a series is influenced by seasonal factors.
- Cyclic: A cyclic pattern exists when data exhibit rises and falls that are not of fixed period.

Time series components

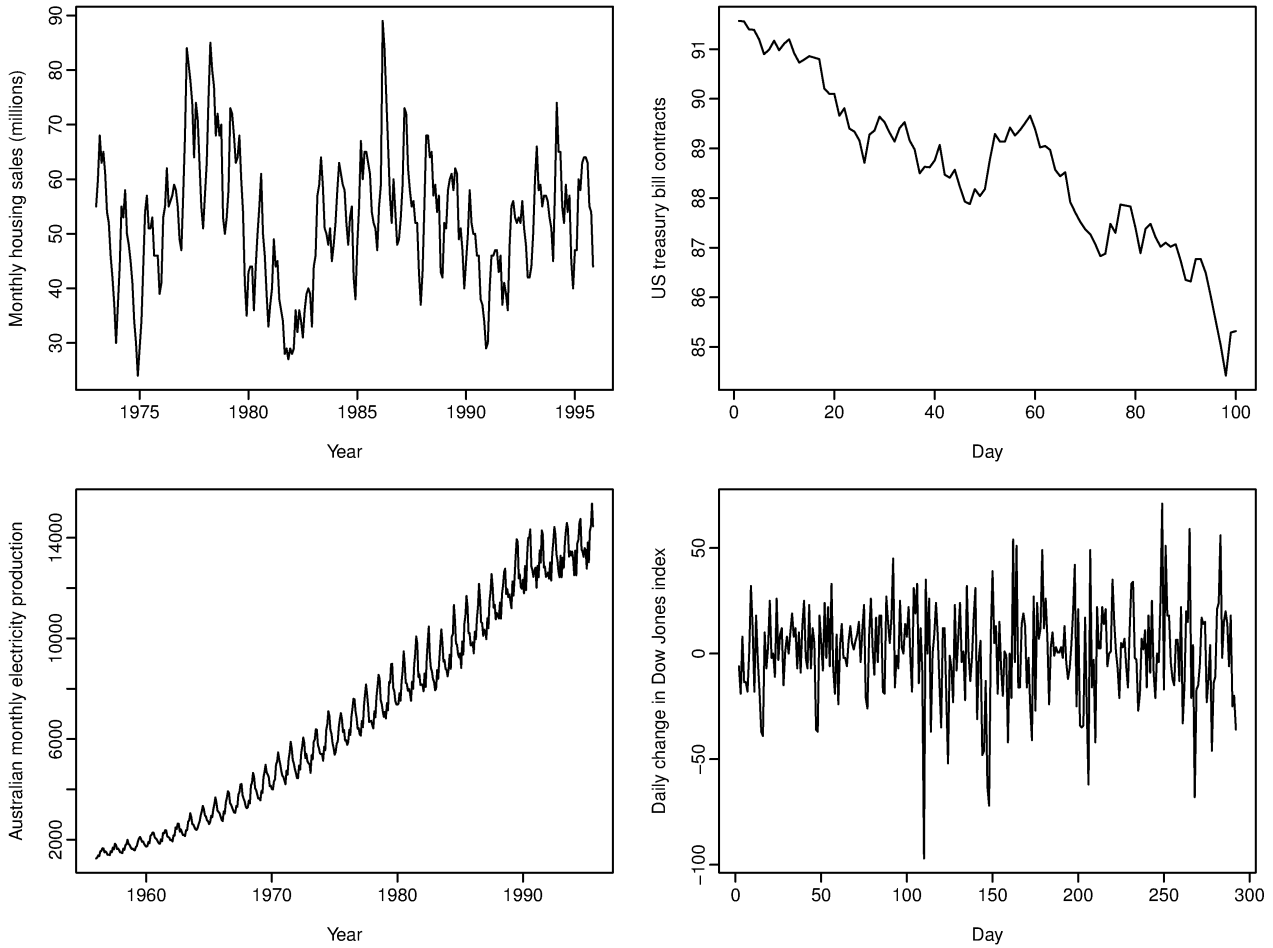


Figure: Examples with different combinations of the components.

TS decomposition

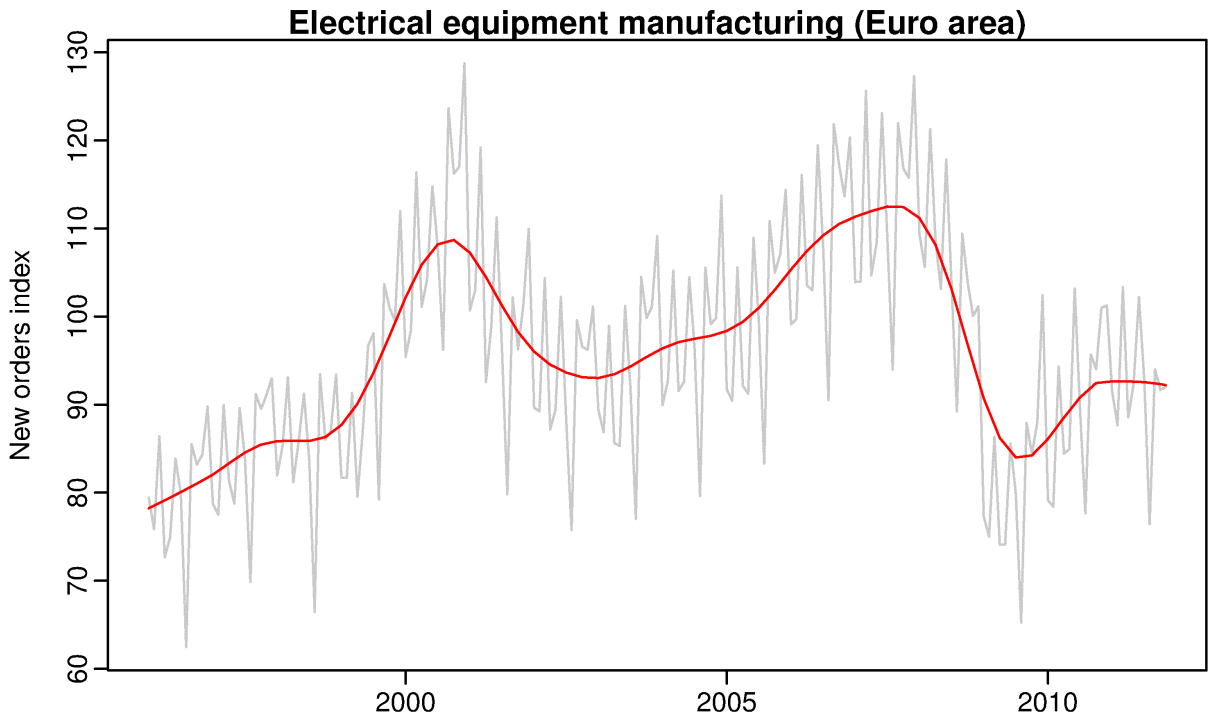
- Classical decomposition

$$X_t = S_t + T_t + E_t$$

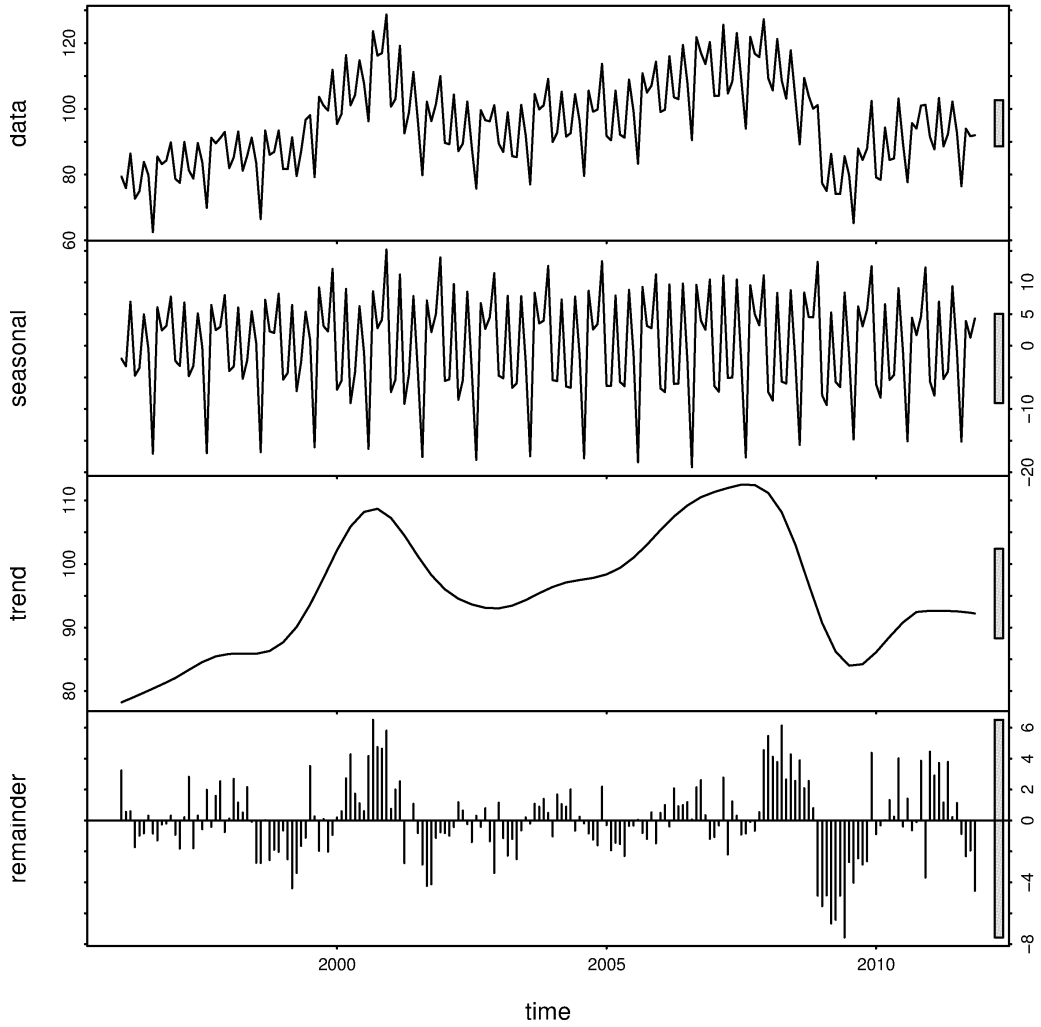
- Trend (T_t) - Long term movement in the mean.
- Seasonal variation (S_t) - Cyclical fluctuations due to calendar.
- Errors (E_t) - random and all other unexplained variations.

Electrical equipment manufacturing example.

```
fit <- stl(elecequip, s.window=5)
plot(elecequip, col="gray",
 main="Electrical equipment manufacturing",
 ylab="New orders index", xlab="")
lines(fit$time.series[,2],col="red",ylab="Trend")
```

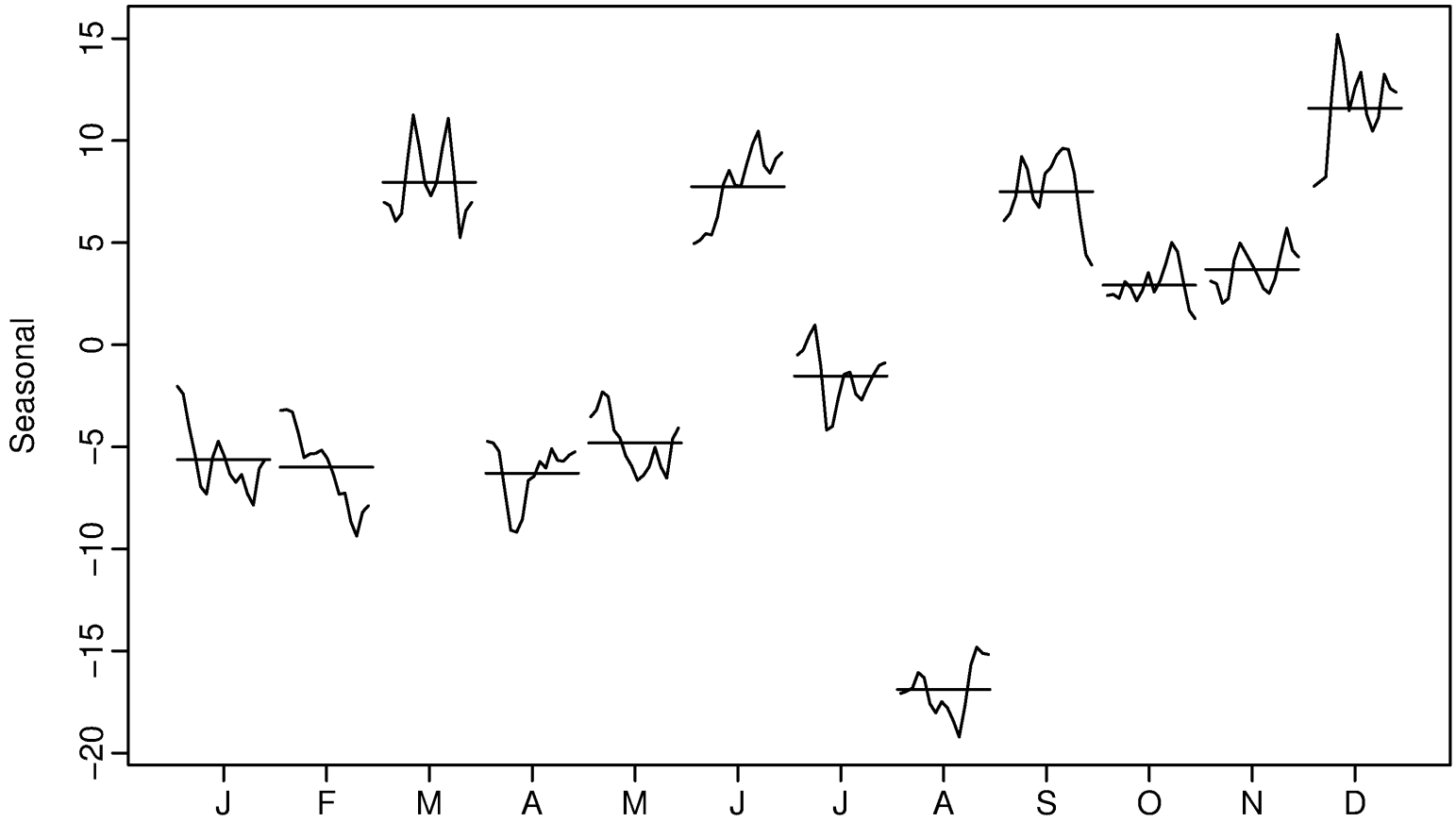


Additive decomposition using STL



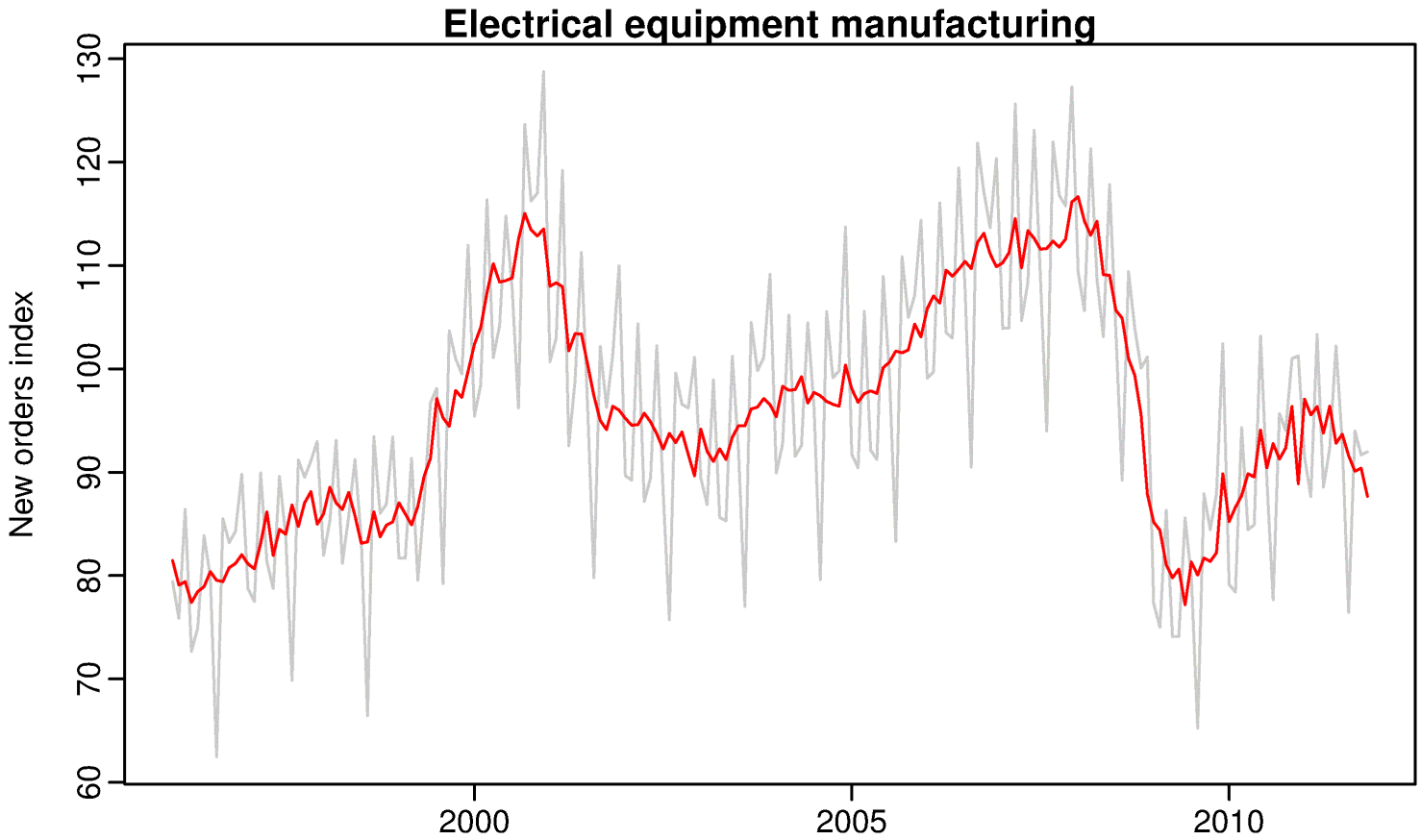
Seasonal subplot

```
monthplot(fit$time.series[,"seasonal"], main="", ylab="Seasonal"
```



Seasonal adjusted data

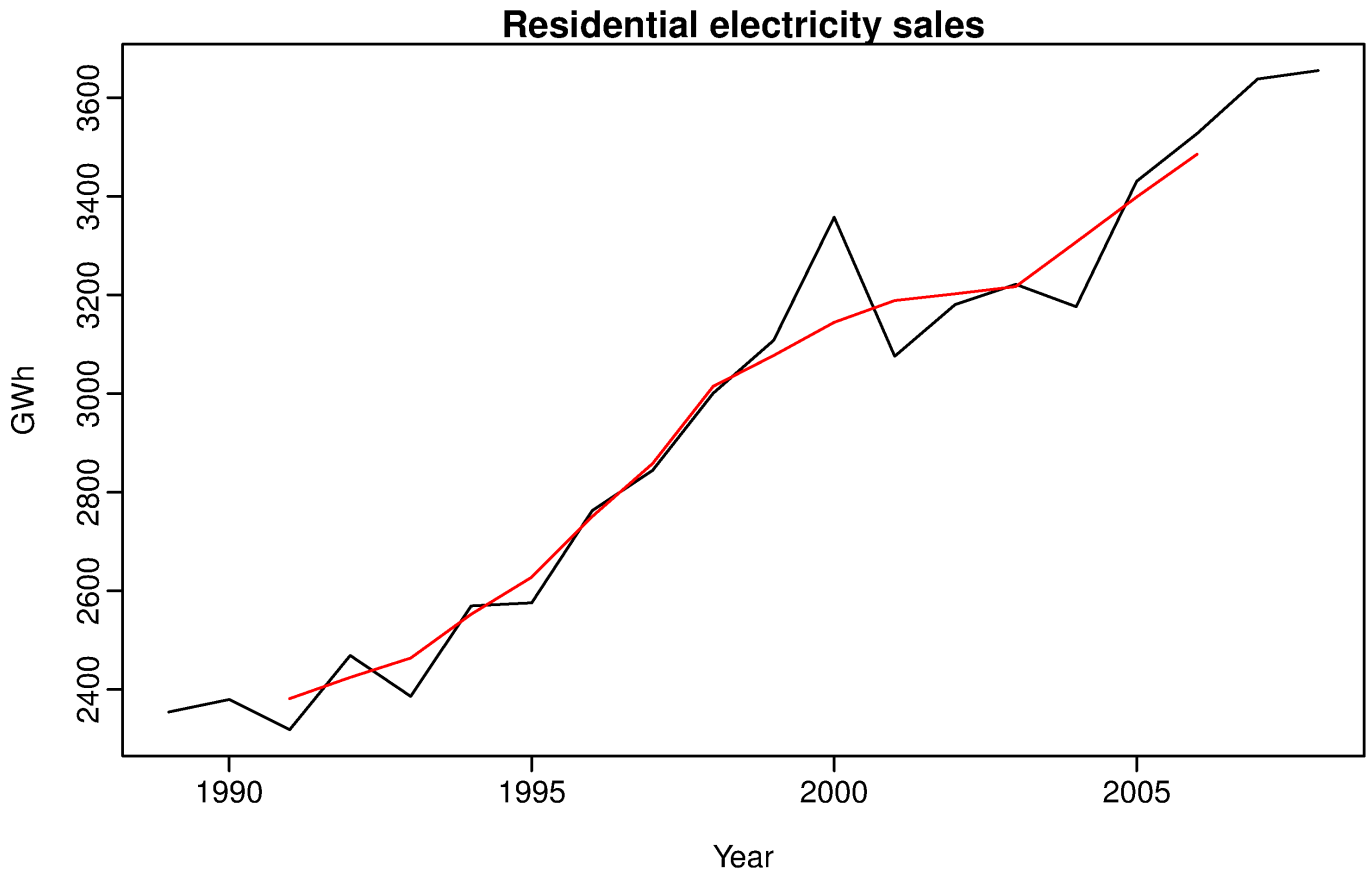
We can obtain seasonal adjusted data by $y_t - S_t$.



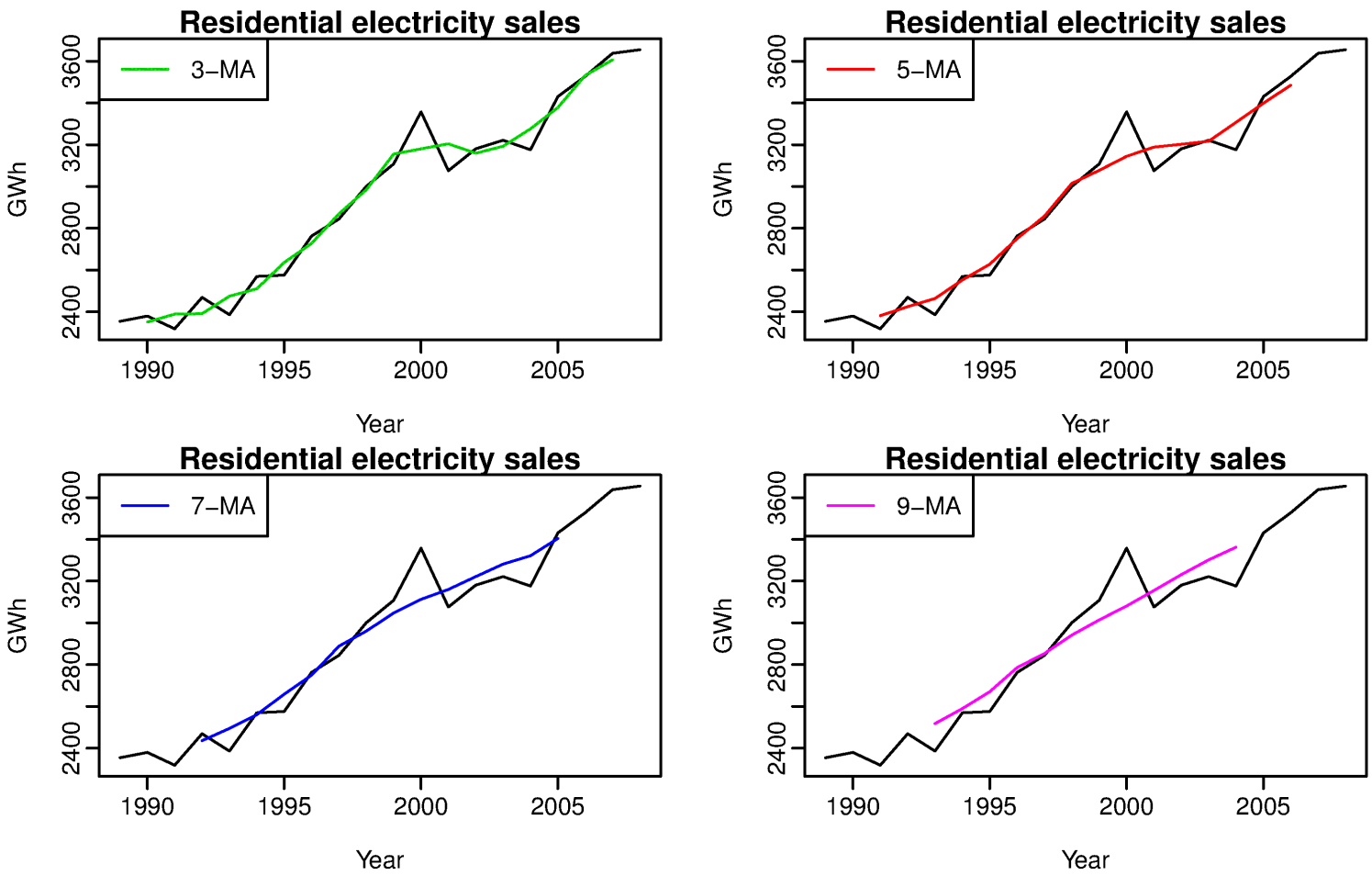
Moving average smoothing

A moving average of order m can be written as

$$\hat{T}_t = \frac{1}{m} \sum_{j=-k}^{k} y_{t+j}, \quad m = 2k + 1.$$



Moving average at different orders



Moving average of moving average

- One reason for MA of MA is to make an ever-order MA symmetric.
- A 2×4 -MA can be written as

$$\frac{1}{2}[\frac{1}{4}(y_{t-2} + y_{t-1} + y_t + y_{t+1}) + \frac{1}{4}(y_{t-1} + y_t + y_{t+1} + y_{t+2})]$$

$$= \frac{1}{8}y_{t-2} + \frac{1}{4}y_{t-1} + \frac{1}{4}y_t + \frac{1}{4}y_{t+1} + \frac{1}{8}y_{t+2}.$$

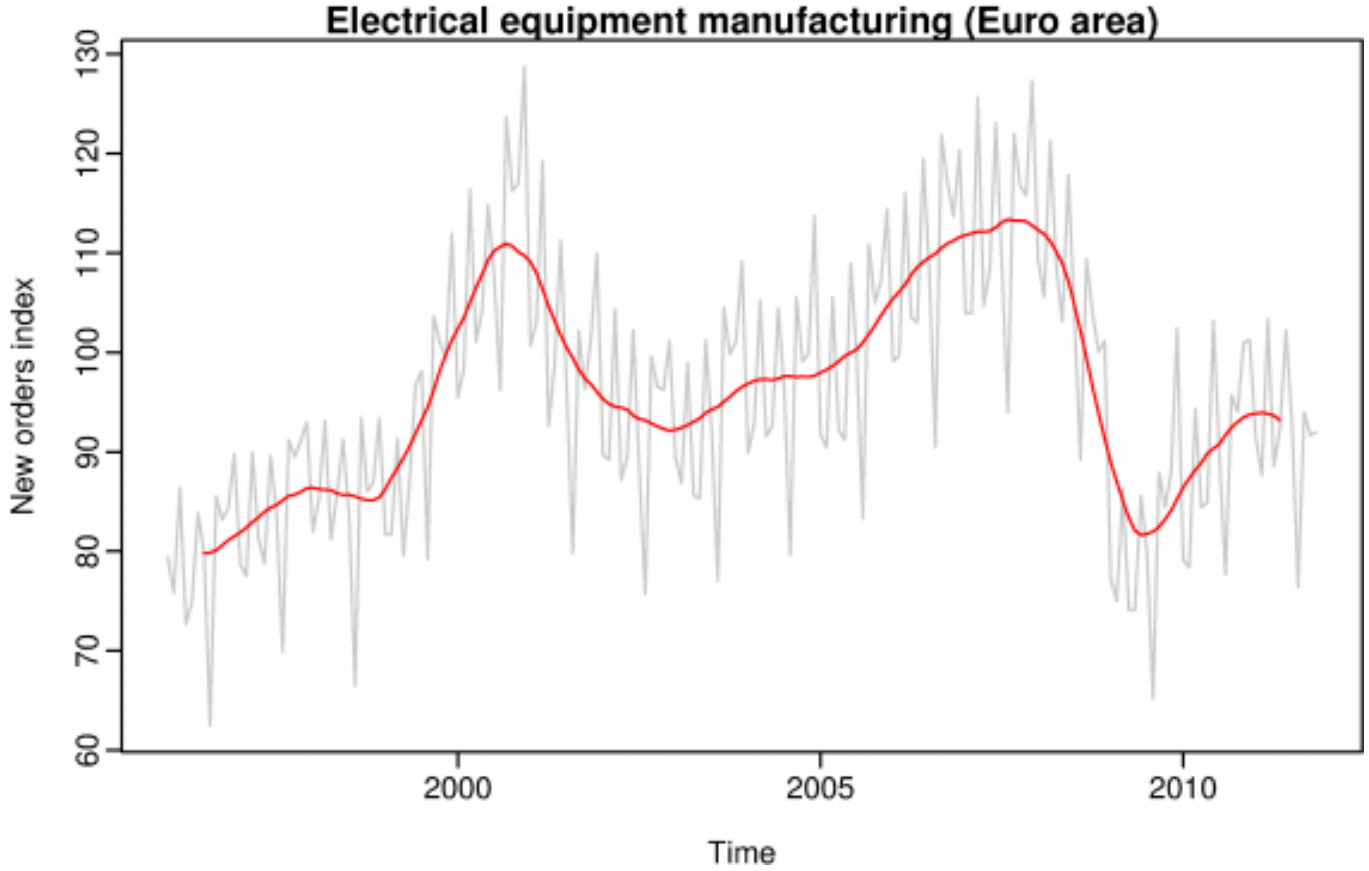
- In general, an ever order MA should be followed by an even one, and vice versa.

Estimating trend cycle with seasonal data

- The 2×4-MA can be used to estimate quarterly data.
- $2 \times m$-MA v.s. $m + 1$MA.
- If the seasonal period is even and of order m, use $2 \times m$ MA to estimate the trend cycle.
- If it's odd , then use m-MA to estimate the trend cycle.
- Thus 2×12-MA for monthly trend, and 7-MA for daily trend.

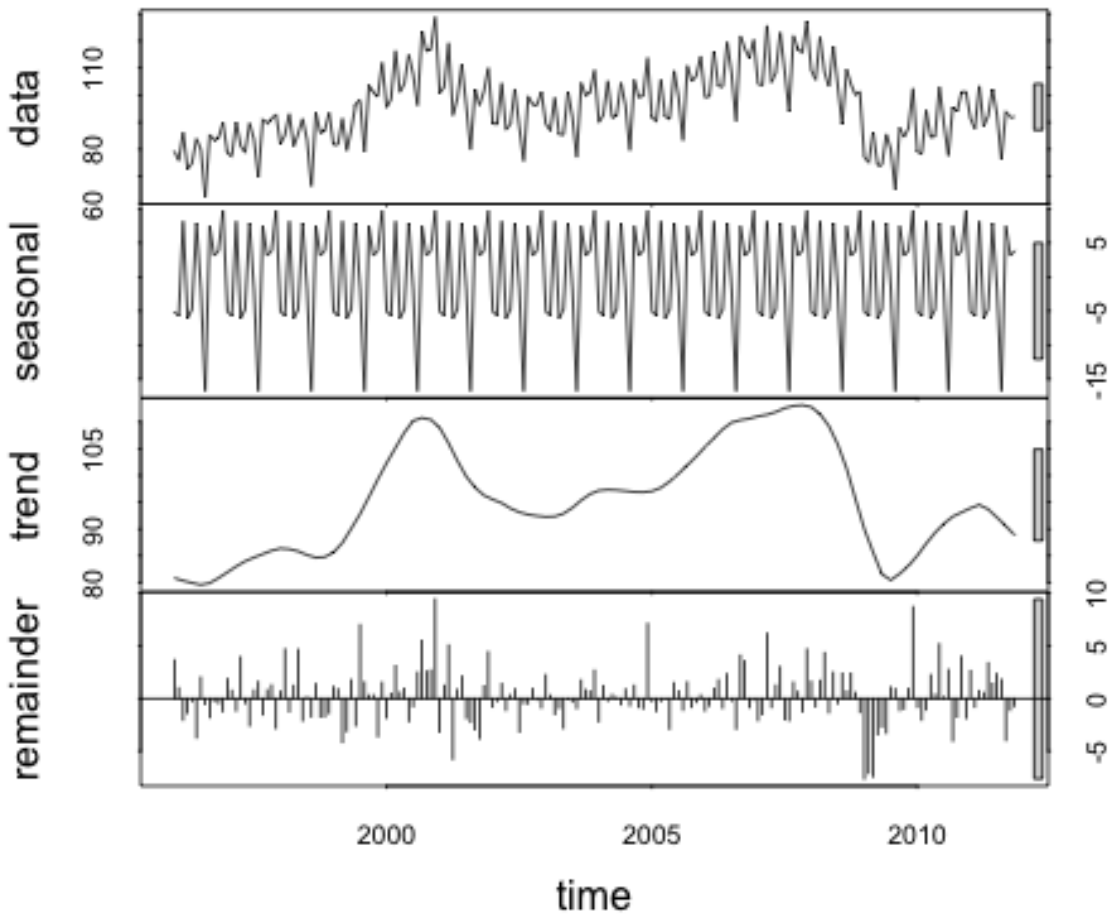
Electrical equipment manufacturing

Application of 2×12-MA.



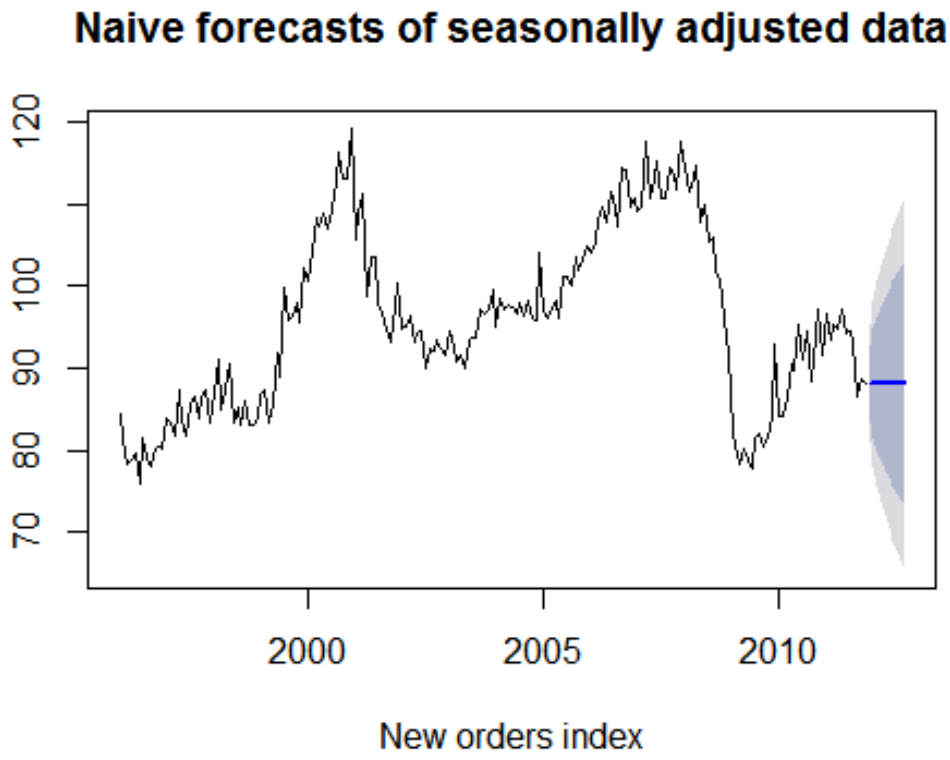
Electrical equipment manufacturing

Application of STL while seasonal component does not change over time. Check STL options.



Forecast with decomposition

- Assuming an additive decomposition, the decomposed time series can be written as $y_t = \hat{S}_t + \hat{A}_t$, where $\hat{A}_t = \hat{T}_t + \hat{E}_t$ is the seasonally adjusted component.
- Forecast the seasonal component and seasonally adjusted component separately.



Forecast with decomposition

Adding back the seasonal component.