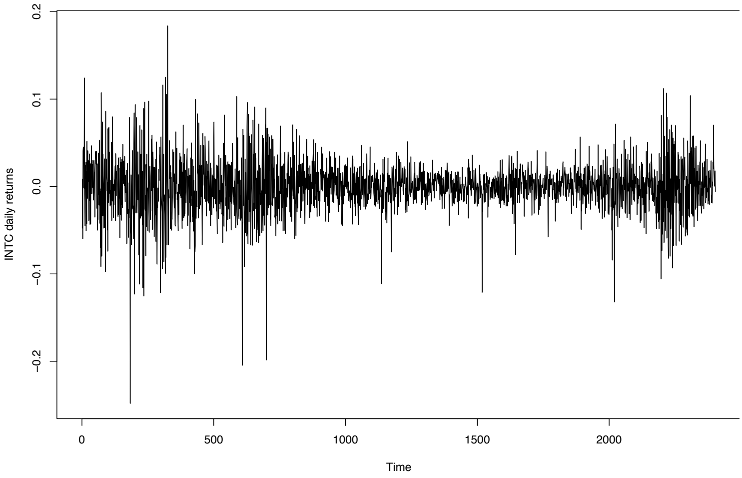
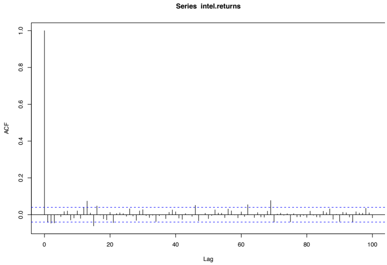
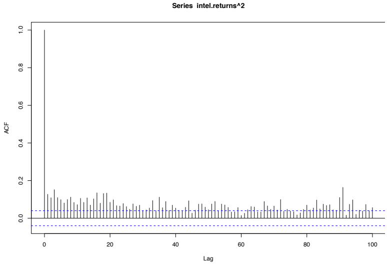
ARCH and GARCH models

Daily log returns of INTC



ACF of the returns and squared returns





Volatility

- The daily log return has very mild autocorrelation, but the squared return has strong autor-correlations.
  Implication: The return may have dynamic and autocorrelated conditional variances.

  Assume that the log return follows certain distribution with mean μ_t and variance σ_t^2. The latter is called stock volatility.

- Volatility is important for Option pricing; Risk management (example on VaR); Interval forecasts ...

- However, volatility is not directly observed.

Volatility

- Formally, volatility is the conditional variance of the return of an asset. Namely

$$\sigma_t^2 = \mathrm{var}(r_t | F_{t-1}),$$

where F_{t-1} is the information available at time $t-1$, including the past returns. The above definition of volatility means the volatility at time t is determined by the information available at time $t-1$.

For stationary series, unconditional variance is a constant, but the conditional variance depends on time.

- Volatility clustering: large price changes happen in clusters.

Heavy tailness

- Heavy tails (heavier than normal, might be close to some t distributions).

```
par(mfrow=c(3,2))
qqnorm(intel.returns)
qqline(intel.returns)
qqplot(rt(2000,6),intel.returns)
qqline(intel.returns)
qqplot(rt(2000,5),intel.returns)
qqline(intel.returns)
qqplot(rt(2000,4),intel.returns)
qqline(intel.returns)
qqplot(rt(2000,3),intel.returns)
qqline(intel.returns)
qqplot(rt(2000,2),intel.returns)
qqline(intel.returns)
```

t with different degrees of freedom

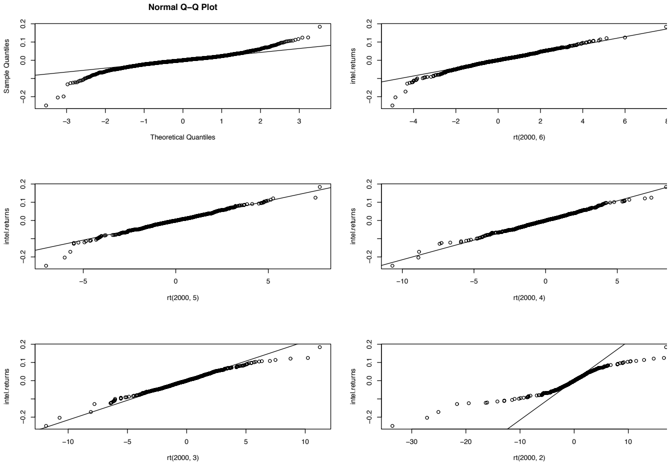


Figure: QQ plots with heavy tail distributions

Additional empirical properties

- Asymmetry: Price of a financial asset responds differently to positive and negative shocks. In fact, the distribution of returns are usually negatively skewed.
- Linear time series model like ARMA(p,q) can not capture these features. For example, ARMA(p,q) can not model the volatility clustering:
- Econometric modeling: ARCH GARCH.

Building a volatility model

- The conditional heteroscedasticity models are concerned with the evolution of the σ_t^2.
- Basic model structure:

$$r_t = \mu_t + a_t, \mu_t = \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{j=1}^{q} \theta_j a_{t-j}.$$

  volatility models are concerned with time-evolution of

$$\sigma_t = \text{var}(r_t | F_{t-1}) = \text{var}(a_t | F_{t-1}).$$

- a_t: the shock or innovation of an asset return at time t. Model for μ_t: mean equation for r_t, for example ARMA(p,q). Model for σ_t^2: volatility model, for example ARCH, GARCH.

Important univariate volatility models

- Autoregressive conditional heteroscedastic (ARCH) model of Engle (1982)
- Generalized ARCH (GARCH) model of Bollerslev (1986)
- IGARCH models
- Exponential GARCH (EGARCH) model of Nelson (1991)
- Threshold GARCH model of Zakoian (1994)
- Conditional heteroscedastic ARMA (CHARMA) model of Tsay (1987)
- Random coefficient autoregressive (RCA) model of Nicholls and Quinn (1982)

Steps for building a volatility model

- If necessary, build a linear model (e.g. ARMA(p,q)) for the return series to remove any linear dependence.
    1. Look at the time series plot of the return precess r_t.
    2. remove the trend and seasonal component
    3. choose orders p and q by ACF, PACF plots and AIC (or AICC, BIC)
    4. Fit an ARMA model to r_t with the orders chosen in the previous step;
- The residual process of the mean equations is a_t. Use the residuals to test for ARCH effect
- Specify a volatility model if ARCH effects are statistically significant and perform a joint estimation of the mean and volatility equations
- Check the fitted model carefully and refine it if necessary

The ARCH model

- An ARCH model of order p is defined as

$$a_t = \sigma_t \epsilon_t, \quad \text{and} \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2,$$

  where $\alpha_0 > 0$ and $\alpha_j \geq 0$, $\{\epsilon_t\} \sim IID(0,1)$ independent of $a_{t-1}, a_{t-2} \ldots$.
  Common distributions of ϵ_t: standard normal, standardized student t, generalized error distribution or skewed student t distribution.
- The conditional variance is weighted average of past squared residuals.

The ARCH model

- ARCH can be expressed as an AR model for a_t^2

$$a_t^2 = \sigma_t^2 \epsilon_t^2 = \sigma_t^2 + \sigma_t^2(\epsilon_t^2 - 1)$$

$$= \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2 + \eta_t,$$

  where η_t are uncorrelated.

- Consider the ARCH(1) case

$$a_t = \sigma_t \epsilon_t, \;\; \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2$$

1. $E(a_t) = 0$.
2. $\mathrm{var}(a_t) = \alpha_0/(1 - \alpha_1)$ if $0 < \alpha_1 < 1$.

The ARCH model

- Advantages of ARCH model
  1. Simple
  2. Generates volatility clustering
  3. Heavy tails
- disadvantages of ARCH model
  1. Can not distinguish positive and negative shocks
  2. Restrictive on parameters. For instance, to ensure finite 4th moment,

$$\sum_{j=1}^{p} \alpha_j < 1/\sqrt{3}.$$

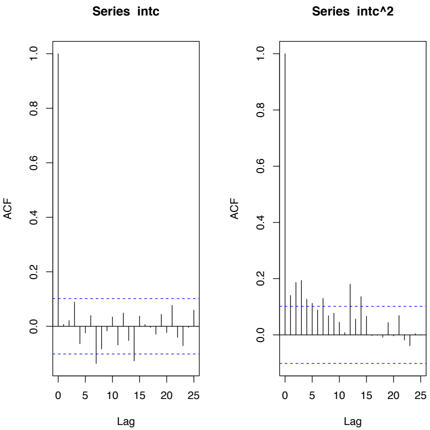


Figure: ACF of INTC monthly return and square returns

Example

- Analysis using the R package fGarch with garchFit command.
- Check the ACF of the monthly return of INTC. Note the autocorrelations on the squared returns.

```
library(fGarch)
da=read.table("m-intc7303.txt",header=T)
intc=log(da[,2]+1)
par(mfcol=c(1,2))
acf(intc)
acf(intc^2)
```

Example

- There seems to be autocorrelation on the squared returns, although not as significant as what we observed for daily data. Peiro A. (2001) showed that the ARCH/GARCH effects are prominent in the daily and weekly data and less for the monthly data.

```
>   Box.test(intc^2,lag=10,type='Ljung')
        Box-Ljung test
data:  intc^2
X-squared = 59.7216, df = 10, p-value = 4.091e-09
>   pacf(intc^2,lag=10) # PACF of the squared return
# to get an idea of the order of ARCH to fit.
```

Example

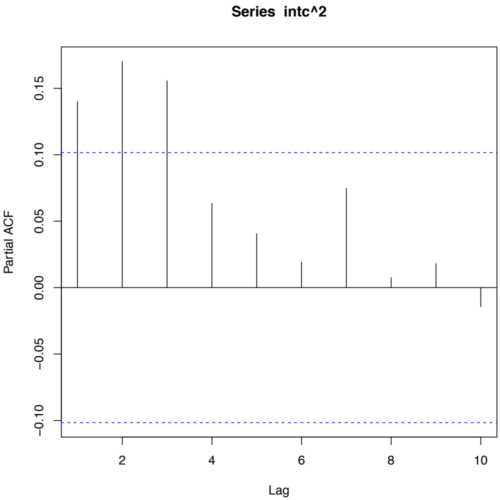


Figure: PACF of INTC monthly square returns

Example

- Now try some ARCH models using fGarch package.

```
>   library(fGarch)
>   m2=garchFit(~garch(3,0),data=intc)  # lots of output
>   m2=garchFit(~garch(3,0),data=intc,trace=F) #no output printed.
>   summary(m2) # Obtain results and model checking statistics
Title:
 GARCH Modelling
Call:
 garchFit(formula = ~garch(3, 0), data = intc, trace = F)
Mean and Variance Equation:
 data ~ garch(3, 0)
Conditional Distribution:
 norm
Coefficient(s):
      mu     omega    alpha1    alpha2    alpha3
0.016572  0.012043  0.208649  0.071837  0.049045
Std. Errors:
 based on Hessian
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.016572    0.006423    2.580  0.00988 **
omega   0.012043    0.001579    7.627  2.4e-14 ***
alpha1  0.208649    0.129177    1.615  0.10626
alpha2  0.071837    0.048551    1.480  0.13897
alpha3  0.049045    0.048847    1.004  0.31536
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

Example

```
Log Likelihood:
 233.4286    normalized:  0.6274962

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  169.7730  0
 Shapiro-Wilk Test  R    W      0.960696  1.970596e-08
 Ljung-Box Test     R    Q(10)  10.97025  0.3598405
 Ljung-Box Test     R    Q(15)  19.59024  0.1882211
 Ljung-Box Test     R    Q(20)  20.82192  0.40768
 Ljung-Box Test     R^2  Q(10)  5.376602  0.864644
 Ljung-Box Test     R^2  Q(15)  22.73460  0.08993975
 Ljung-Box Test     R^2  Q(20)  23.70577  0.255481
 LM Arch Test       R    TR^2   20.48506  0.05844884
Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-1.228111 -1.175437 -1.228466 -1.207193
```

Example

- The three α all seem to be insignificant. Run ARCH(2)

```
>m2=garchFit(~garch(2,0),data=intc)
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.016867    0.006304    2.676  0.00746 **
omega   0.012325    0.001535    8.032 8.88e-16 ***
alpha1  0.255012    0.124200    2.053  0.04005 *
alpha2  0.071680    0.047891    1.497  0.13446
```

- Only α_1 seems significant. Run ARCH(1)

```
>   m2=garchFit(~garch(1,0),data=intc,trace=F)
>   summary(m2)
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.016570    0.006161    2.689  0.00716 **
omega   0.012490    0.001549    8.061 6.66e-16 ***
alpha1  0.363447    0.131598    2.762  0.00575 **
```

- The fitted model is

$$r_t = 0.01657 + a_t, \quad \sigma_t^2 = 0.0125 + 0.363 a_{t-1}^2.$$

Example

- We can change the innovation distribution from normal to t.

```
# Student-t innovations
> m2=garchFit(~garch(1,0),data=intc,trace=F,cond.dist=c("std"))
>  summary(m2)

        Estimate  Std. Error  t value Pr(>|t|)
mu      0.021571    0.006054    3.563 0.000366 ***
omega   0.013424    0.001968    6.820 9.09e-12 ***
alpha1  0.259867    0.119901    2.167 0.030209 *
shape   5.985979    1.660030    3.606 0.000311 ***

# use skewed Student-t innovations
>  m3=garchFit(~garch(1,0),data=intc,trace=F,cond.dist=c("sstd"))
>  summary(m3)

        Estimate  Std. Error  t value Pr(>|t|)
mu      0.017341    0.006294    2.755  0.00587 **
omega   0.013174    0.001820    7.238 4.55e-13 ***
alpha1  0.276969    0.112422    2.464  0.01375 *
skew    0.836063    0.070814   11.806  < 2e-16 ***
shape   6.740548    2.058274    3.275  0.00106 **
```

Example

- Student t with 5.99 degree of freedom

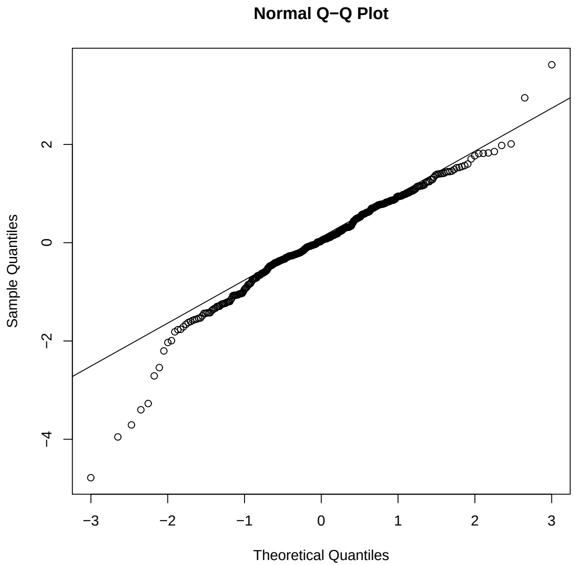
$$r_t = 0.0216 + a_t, \quad \sigma_t^2 = 0.0134 + 0.260 a_{t-1}^2.$$

skewed student t with 6.74 degree of freedom

$$r_t = 0.0173 + a_t, \quad \sigma_t^2 = 0.0132 + 0.277 a_{t-1}^2.$$

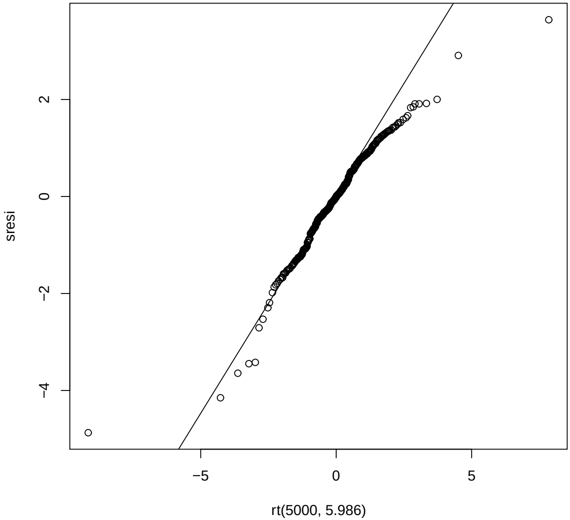
```
m5=garchFit(~garch(1,0),data=intc,trace=F,cond.dist=c("sstd"))
summary(m5)
sresi=residuals(m5,standardize=T)
qqplot(rsstd(5000,0,1,6.74,0.836),sresi)
qqline(sresi)
```

QQ norm plot

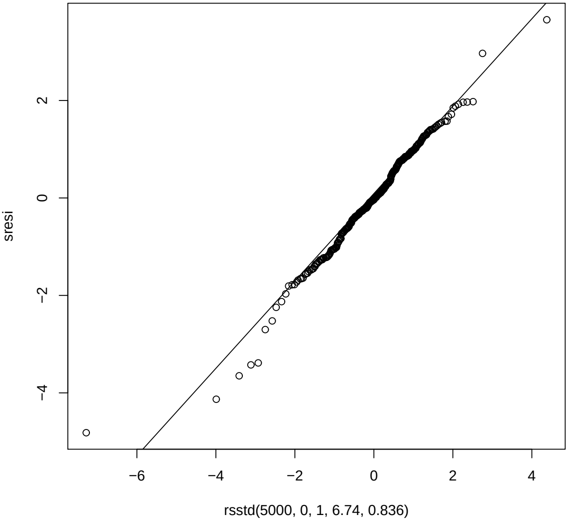


Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
−3
−2
−1
0
1
2
3
−4
−2
0
2

QQ plot



QQ plot



GARCH model

- A natural generalization of ARCH

$$a_t = \sigma_t \epsilon_t, \;\; \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2,$$

  where ϵ_t is i.i.d. with mean 0, standard deviation 1 and independent of a_{t-1}... Also $\alpha > 0$, α_i and $\beta_j \geq 0$, and $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$. Such a model is called GARCH(p,q) model.
- σ_t^2 is weighted average of past volatilities and squared residuals.
- Facilitate the modeling of volatility clustering.
- Captures main features of volatilities by a low order GARCH such as GARCH(1,1).

GARCH model

- Reparametrization of GARCH: Let $\eta_t = a_t^2 - \sigma_t^2 = \sigma_t^2(\epsilon_t^2 - 1)$.
  η_t is an uncorrelated series.
  The GARCH model becomes

$$a_t^2 = \alpha_0 + \sum(\alpha_i + \beta_i)a_{t-i}^2 + \eta_t - \sum_{j=1}^{q} \beta_j \eta_{t-j}.$$

  It is an ARMA model of the series a_t^2.

GARCH(1,1)

- For GARCH(1,1) we have

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2.$$

- Weak stationary condition is $0 \leq \alpha_1, \beta_1 < 1$ and $\alpha_1 + \beta_1 < 1$.
- Volatility clustering.
- Heavy tails: If $1 - 2\alpha_1^2 - (\alpha_1 + \beta_1)^2 > 0$, then $K(a_t) > 3$.
- One-step forecast.

$$\sigma_n^2(1) = \alpha_0 + \alpha_1 a_n^2 + \beta_1 \sigma_n^2$$

GARCH(1,1)

- Two-step forecast

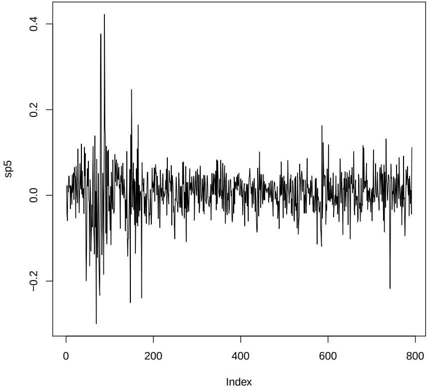
$$\begin{aligned}
\sigma_n^2(2) &= E(\sigma_{n+2}^2 | a_1, \cdots, a_n) \\
&= E(\alpha_0 + \alpha_1 a_{n+1}^2 + \beta_1 \sigma_{n+1}^2 | a_1, \cdots, a_n) \\
&= E(\alpha_0 + \alpha_1 \sigma_{n+1}^2 \epsilon_{n+1}^2 + \beta_1 \sigma_{n+1}^2 | a_1, \cdots, a_n) \\
&= \alpha_0 + \alpha_1 E(\sigma_{n+1}^2 | a_1, \cdots, a_n) E(\epsilon_{n+1}^2 | a_1, \cdots, a_n) \\
&\quad + \beta_1 E(\sigma_{n+1}^2 | a_1, \cdots, a_n) \\
&= \alpha_0 + (\alpha_1 + \beta_1)\sigma_n^2(1).
\end{aligned}$$

- In general, for $h > 1$,

$$\sigma_n^2(h) = \alpha_0 + (\alpha_1 + \beta_1)\sigma_n^2(h-1).$$

GARCH example using fGarch package

```
da=read.table("sp500.txt")   # Load data
 sp5=da[,1]
 plot(sp5,type='l')  # plot the data
```



ACF and PACF for sp500 returns

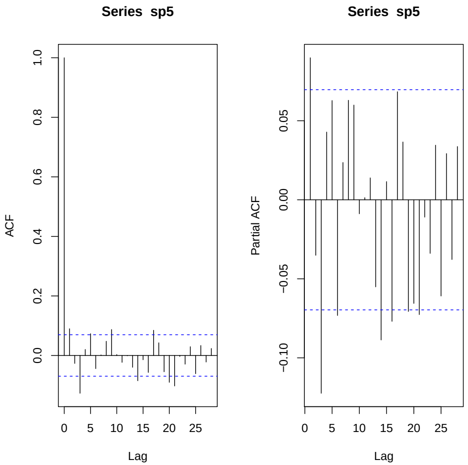


Figure: ACF and PACF of SP500 monthly return

ARMA for sp500 returns

Based on the ACF and PACF plots, we run an AR model to remove serial correlations.

```
acf(sp5)                # compute ACF of the data
pacf(sp5)               # compute PACF of the data
m1=arima(sp5,order=c(3,0,0))  # Fit an AR(3) model to remove
serial correlations.
Box.test(m1$residuals,10,type='Ljung') # check the residuals
     Box-Ljung test
data:  m1$residuals
X-squared = 16.1361, df = 10, p-value = 0.0958
Box.test(m1$residuals^2,10,type='Ljung') # Test for ARCH effect.
        Box-Ljung test
data:  m1$residuals^2
X-squared = 353.7695, df = 10, p-value < 2.2e-16
```

Joint estimations

Fit ARMA+GARCH model to the SP500 monthly returns.

```
 m2=garchFit(~arma(3,0)+garch(1,1),data=sp5,trace=F)
 summary(m2)
garchFit(formula = ~arma(3, 0) + garch(1, 1), data = sp5, trace
         Estimate  Std. Error  t value Pr(>|t|)
mu      7.708e-03   1.607e-03    4.798 1.61e-06 ***
ar1     3.197e-02   3.837e-02    0.833  0.40473
ar2    -3.026e-02   3.841e-02   -0.788  0.43076
ar3    -1.065e-02   3.756e-02   -0.284  0.77677
omega   7.975e-05   2.810e-05    2.838  0.00454 **
alpha1  1.242e-01   2.247e-02    5.529 3.22e-08 ***
beta1   8.530e-01   2.183e-02   39.075  < 2e-16 ***
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Log Likelihood:
 1272.179    normalized:  1.606287
```

Joint estimations

```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  73.04842  1.110223e-16
 Shapiro-Wilk Test  R    W      0.9857969 5.961994e-07
 Ljung-Box Test     R    Q(10)  11.56744  0.315048
 Ljung-Box Test     R    Q(15)  17.78747  0.2740039
 Ljung-Box Test     R    Q(20)  24.11916  0.2372256
 Ljung-Box Test     R^2  Q(10)  10.31614  0.4132089
 Ljung-Box Test     R^2  Q(15)  14.22819  0.5082978
 Ljung-Box Test     R^2  Q(20)  16.79404  0.6663038
 LM Arch Test       R    TR^2   13.34305  0.3446075
Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-3.194897 -3.153581 -3.195051 -3.179018
```

- The estimated model is

$$r_t = 0.032 r_{t-1} - 0.03 r_{t-2} - 0.011 r_{t-3} + 0.0077 + a_t$$

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = 7.98 \times 10^{-5} + 0.853 \sigma_{t-1}^2 + 0.124 a_{t-1}^2.$$

- The unconditional variance of a_t is

$$\frac{7.98 \times 10^{-5}}{1 - 0.853 - 0.124} = 0.00352.$$

- The AR coefficients are all insignificant at 5% level. Thus we run a pure GARCH model

Refined model

```
> m3=garchFit(~garch(1,1),data=sp5,trace=F)
> summary(m3)
 garchFit(formula = ~garch(1, 1), data = sp5, trace = F)
        Estimate  Std. Error  t value Pr(>|t|)
mu     7.450e-03   1.538e-03    4.845 1.27e-06 ***
omega  8.061e-05   2.833e-05    2.845  0.00444 **
alpha1 1.220e-01   2.202e-02    5.540 3.02e-08 ***
beta1  8.544e-01   2.175e-02   39.276  < 2e-16 ***
 Jarque-Bera Test   R    Chi^2  80.32111  0
 Shapiro-Wilk Test  R    W      0.9850517 3.141228e-07
 Ljung-Box Test     R    Q(10)  11.2205   0.340599
 Ljung-Box Test     R    Q(15)  17.99703  0.262822
 Ljung-Box Test     R    Q(20)  24.29896  0.2295768
 Ljung-Box Test     R^2  Q(10)  9.920157  0.4475259
 Ljung-Box Test     R^2  Q(15)  14.21124  0.509572
 Ljung-Box Test     R^2  Q(20)  16.75081  0.6690903
 LM Arch Test       R    TR^2   13.04872  0.3655092
      AIC       BIC       SIC      HQIC
-3.195594 -3.171985 -3.195645 -3.186520
```

Refined model

- The simplified model is

$$r_t = 0.00745 + a_t, \quad \sigma_t^2 = 8.06 \times 10^{-5} + 0.854\sigma_{t-1}^2 + 0.122a_{t-1}^2.$$

- What is the form for 1-step ahead forecast? What is the unconditional variance of a_t?
- In R we can use

```
>   predict(m3,5)  # Perform prediction 1 to 5-step ahead.
  meanForecast  meanError standardDeviation
1  0.007449721 0.05377242        0.05377242
2  0.007449721 0.05388567        0.05388567
3  0.007449721 0.05399601        0.05399601
4  0.007449721 0.05410353        0.05410353
5  0.007449721 0.05420829        0.05420829
```

IGARCH model

- Special case of GARCH model with $\alpha_1 + \beta_1 = 1$.
- IGARCH(1,1)

$$\sigma_t^2 = \alpha_0 + (1 - \beta_1) a_{t-1}^2 + \beta_1 \sigma_{t-1}^2.$$

- Effect of $\sigma_h(1)$ on future volatilities is persistent since

$$\sigma_h^2(l) = \sigma_h^2(1) + (l - 1)\alpha_0.$$

- Key differences with GARCH.
  The unconditional variance of a_t for the usual GARCH model is defined. But for IGARCH model, the unconditional variance of a_t is undefined.

IGARCH model

- IGARCH(1,1) with $\alpha_0 = 0$ using igarch.R.

```
>   Igarch(sp5,volcnt=F)
Maximized log-likehood:  -1258.219

Coefficient(s):
      Estimate  Std. Error  t value Pr(>|t|)
mu    0.0068744   0.0015402  4.46332 8.07e-06 ***
beta 0.9007153   0.0158018 57.00082  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

- The resulting model is

$$r_t = 0.00687 + a_t, a_t = \sigma_t \epsilon_t; \quad \sigma_t^2 = 0.0993 a_{t-1}^2 + 0.9007 \sigma_{t-1}^2.$$

GARCH-M model

- The model is

$$r_t = \mu + c\sigma_t^2 + a_t, \; a_t = \sigma_t \epsilon_t, \;\; \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

  where c is called risk premium. This is expected to be positive.

- Fitting the Garch-M model to the SP500 percentage returns, the estimated model is

$$r_t = 0.743 + 0.048\sigma_t^2 + a_t, \;\; \sigma_t^2 = 0.812 + 0.123 a_{t-1}^2 + 0.854\sigma_{t-1}^2.$$

  The risk premium estimate is not significant at 5% level.

GARCH-M model

```
> garchM(sp5*100)
[1] 2399.822
  0:     2380.0229: 0.422452 0.00561297 0.806149 0.121976 0.8543
  3:     2379.9525: 0.427596 0.00652832 0.806107 0.121466 0.8557
  6:     2378.4838: 0.606807 0.0375055 0.801889 0.126193 0.84683
  9:     2377.9587: 0.673997 0.00166937 0.798191 0.125517 0.8519
 12:     2377.8474: 0.692209 0.0444464 0.802574 0.122278 0.85414
 15:     2377.7922: 0.742796 0.0480959 0.812187 0.122530 0.85350
Maximized log-likehood:  2377.792

Coefficient(s):
       Estimate  Std. Error  t value   Pr(>|t|)
mu    0.7427956   0.1540336  4.82229 1.4192e-06 ***
gamma 0.0480959   0.1408765  0.34140  0.7327991
omega 0.8121873   0.2858128  2.84168  0.0044877 **
alpha 0.1225297   0.0220596  5.55449 2.7843e-08 ***
beta  0.8535072   0.0219079 38.95885 < 2.22e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

Threshold GARCH model

- Another volatility model commonly used to handle the asymmetric distribution is the threshold GARCH model, which is also called GJR model

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i + \gamma_i N_{t-i})a_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2,$$

where

$$N_{t-i} = \begin{cases} 1, & \text{if } a_{t-i} < 0, \\ 0, & \text{if } a_{t-i} \geq 0. \end{cases}$$

APARCH model

- The general APARCH(p,q) model (Ding, Granger and Engle 1993) is

$$
\begin{aligned}
a_t &= \sigma_t \epsilon_t, \\
\epsilon_t &\sim D(0,1); \\
\sigma_t^\delta &= \alpha_0 + \sum_{i=1}^{p} \alpha_i (|a_{t-i}| - \gamma_i a_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta,
\end{aligned}
$$

  where $\delta > 0$ and $-1 < \gamma_i < 1$.
- It adds the flexibility of a varying exponent with asymmetry coefficients.

APARCH model

- The family of APARCH models includes the following special cases:
  - ARCH model $\delta = 2, \gamma_i = 0, \beta_j = 0$.
  - GARCH model $\delta = 2, \gamma_i = 0$.
  - Threshold-GARCH (GJR-GARCH) when $\delta = 2$.
  - T-ARCH when $\delta = 1$.
- In fGarch, we can fix the value of δ using the subcommand such as "include.delta=F, delta = 2". in this case, the APARCH model is similar to the TGARCH model.

Example

Consider the percentage log returns of monthly IBM stock from 1926 to 2009.

```
>   library(fGarch)
>  data=read.table("m-ibm2609.txt",header=T)
>  ibm=log(data$ibm+1)*100 #IBM percentage monthly log return
>  m1=garchFit(~aparch(1,1), data=ibm,trace=F, delta=2,
include.delta=F)
>  summary(m1)
 garchFit(formula = ~aparch(1, 1), data = ibm, delta = 2,
  include.delta = F,trace = F)
Conditional Distribution: norm
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu       1.18659     0.20019    5.927 3.08e-09 ***
omega    4.33663     1.34161    3.232  0.00123 **
alpha1   0.10767     0.02548    4.225 2.39e-05 ***
gamma1   0.22732     0.10018    2.269  0.02326 *
beta1    0.79468     0.04554   17.449  < 2e-16 ***
```

Example

```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  67.07416  2.775558e-15
 Shapiro-Wilk Test  R    W      0.9870144 8.599046e-08
 Ljung-Box Test     R    Q(10)  16.90603  0.07646941
 Ljung-Box Test     R    Q(15)  24.19033  0.06193097
 Ljung-Box Test     R    Q(20)  31.89097  0.04447406
 Ljung-Box Test     R^2  Q(10)  4.591691  0.9167342
 Ljung-Box Test     R^2  Q(15)  11.98464  0.6801912
 Ljung-Box Test     R^2  Q(20)  14.79531  0.7879979
 LM Arch Test       R    TR^2   7.162971  0.8466584

Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
6.615430 6.639814 6.615381 6.624694

>  plot(m1)
```

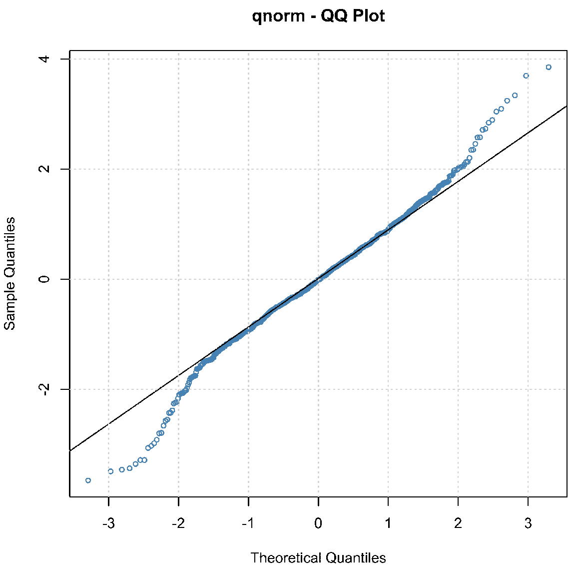


Figure: QQ norm

Example

Judging from the standardized residual QQ plot, the normality assumption is not adequate. We run the APARCH model with t innovations.

```
>m2=garchFit(~aparch(1,1),data=ibm,trace=F,
delta=2,include.delta=F,cond.dist="std")
> summary(m2)
```

```
        Estimate  Std. Error  t value Pr(>|t|)
mu       1.20476     0.18715    6.437 1.22e-10 ***
omega    3.98975     1.45331    2.745 0.006046 **
alpha1   0.10468     0.02793    3.747 0.000179 ***
gamma1   0.22366     0.11595    1.929 0.053738 .
beta1    0.80711     0.04825   16.727  < 2e-16 ***
shape    6.67329     1.32779    5.026 5.01e-07 ***
```

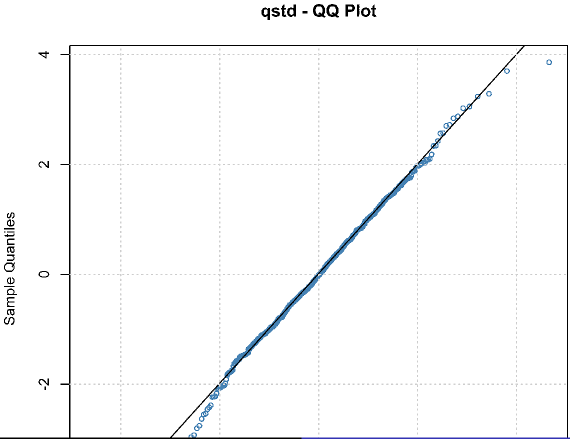
Example

```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  67.82336  1.887379e-15
 Shapiro-Wilk Test  R    W      0.9869699 8.213668e-08
 Ljung-Box Test     R    Q(10)  16.91352  0.07629961
 Ljung-Box Test     R    Q(15)  24.08691  0.06363224
 Ljung-Box Test     R    Q(20)  31.75305  0.04600187
 Ljung-Box Test     R^2  Q(10)  4.553248  0.9189583
 Ljung-Box Test     R^2  Q(15)  11.66891  0.7038973
 Ljung-Box Test     R^2  Q(20)  14.18533  0.8209764
 LM Arch Test       R    TR^2   6.771675  0.872326
Information Criterion Statistics:
     AIC      BIC      SIC     HQIC
6.579782 6.609042 6.579711 6.590898
>  plot(m2)
```

Example

The fitted model on percentage return is

$$
\begin{aligned}
r_t &= 1.2 + a_t, \quad \epsilon_t \sim t_{6.67} \\
\sigma_t^2 &= 3.99 + 0.105(|a_{t-1}| - 0.224 a_{t-1})^2 + 0.807\sigma_{t-1}^2.
\end{aligned}
$$



The fitted models

The fitted model on percentage return are
under normalilty

$$
\begin{aligned}
r_t &= 1.19 + a_t, \\
\sigma_t^2 &= 4.34 + 0.108(|a_{t-1}| - 0.227a_{t-1})^2 + 0.795\sigma_{t-1}^2.
\end{aligned}
$$

under t

$$
\begin{aligned}
r_t &= 1.2 + a_t, \quad \epsilon_t \sim t_{6.67} \\
\sigma_t^2 &= 3.99 + 0.105(|a_{t-1}| - 0.224a_{t-1})^2 + 0.807\sigma_{t-1}^2.
\end{aligned}
$$