# How to define bird flocks

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# What are flocks of birds?

### Definition
Flocks of birds are groups of individuals that share similar characteristics like position, direction and speed.

# How to define Flocks mathematically?

### Problem
Definitions of bird flocks are very vague in literature, because they are a complex concept.

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### Solution
Instead of giving a definition for flocks, we give a definition for the **coherence** of birds and derive a definition for **coherence** of flocks. This **coherence** concept gives us a mathematical parameter of *flockiness* with which it will be easier to define flocks.

# Coherence

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- The **position** of this bird $x_i \in \mathbb{R}^3$ is given in cartesian coordinates.
- Its **velocity** is $v_i \in \mathbb{R}^3$.

# Simple Coherence

### Definition

Let $i, j$ be two birds.

The **simple coherence** between the birds is given by

$$C_{i,j} = \underbrace{\frac{1}{||x_i - x_j||}}_{\text{inverse distance}} \cdot \underbrace{\frac{v_i \cdot v_j}{||v_i|| \cdot ||v_j||}}_{\text{directional correlation}}$$

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### Definition
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### Remark
- As two birds cannot physically be at the same position,
  $x_i \neq x_j \Rightarrow ||x_i - x_j|| \neq 0$
- As birds are not stationary, because if they were, they would
  fall from the sky, $||v_i||, ||v_j|| \neq 0$

# Logarithmic Coherence

### Definition

The **logarithmic coherence** between birds $i, j$ is given by

$$L_{i,j} := \log\left(1 + \frac{1}{||x_i - x_j||}\right) \cdot \frac{v_i \cdot v_j}{||v_i|| \cdot ||v_j||}$$

# Coherence

### Corollary

*Let $i, j$ be birds from a set of birds.*

1. *If the birds are flying in the same direction, coherence is positive.*
2. *If the birds are flying in directions more than 90 degrees offset, the coherence is negative.*
3. *If the birds are close to one another, $|C_{i,j}|$ is bigger.*
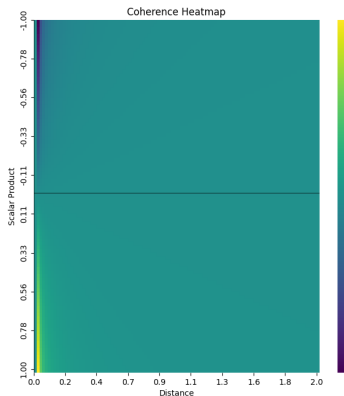4. *If the birds are far from one another, $|C_{i,j}|$ is smaller.*
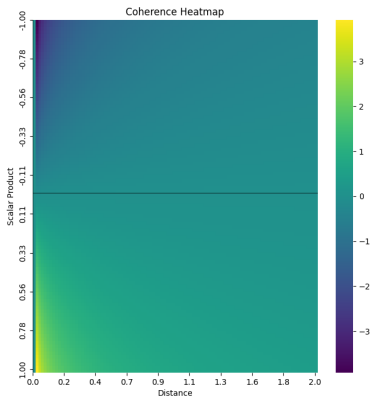
Figure: Coherence Heatmap using **simple coherence**

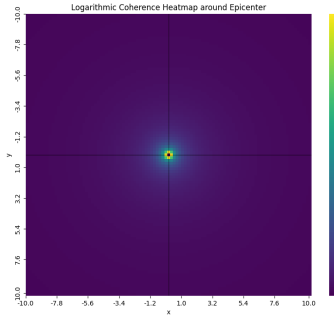Figure: Coherence Heatmap using **logarithmic coherence**

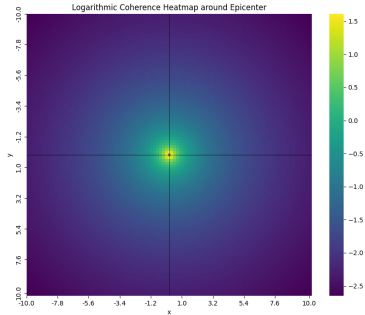Figure: Coherence Heatmap around epicenter using **simple coherence**

Figure: Coherence Heatmap around epicenter using **logarithmic coherence**

# Pros and cons of this method

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1. captures direction and space
2. direction is more important than space

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1. direction and space are not equally important, how to quantify that?
2. other birds behind or in front a bird should be respected more than birds to the side or under
3. Computationally inefficient (n number of birds)
   $\mathcal{O}(n)$ per bird $\rightarrow$ $\mathcal{O}(n^2)$ in total
   with a lot of square roots

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