How to define bird flocks

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What are flocks of birds?

Definition

Flocks of birds are groups of individuals that share similar characteristics like position, direction and speed.

How to define Flocks mathematically?

Problem

Definitions of bird flocks are very vague in literature, because they are a complex concept.

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Solution

Instead of giving a definition for flocks, we give a definition for the **coherence** of birds and derive a definition for **coherence** of flocks. This **coherence** concept gives us a mathematical parameter of *flockiness* with which it will be easier to define flocks.

Coherence

Definition

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- The **position** of this bird $x_i \in \mathbb{R}^3$ is given in cartesian coordinates.
- Its **velocity** is $v_i \in \mathbb{R}^3$.

Simple Coherence

Definition

Let i, j be two birds.

The **simple coherence** between the birds is given by

$$C_{i,j} = \underbrace{\frac{1}{||x_i - x_j||}}_{\textit{inverse distance}} \cdot \underbrace{\frac{v_i \cdot v_j}{||v_i|| \cdot ||v_j||}}_{\textit{directional correlation}}$$

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Remark

- As two birds cannot physically be at the same position, $x_i \neq x_j \Rightarrow ||x_i - x_j|| \neq 0$
- As birds are not stationary, because if they were, they would fall from the sky, $||v_i||, ||v_j|| \neq 0$

Logarithmic Coherence

Definition

The **logarithmic coherence** between birds i, j is given by

$$L_{i,j} := \log(1 + \frac{1}{||x_i - x_j||}) \cdot \frac{v_i \cdot v_j}{||v_i|| \cdot ||v_j||}$$

Coherence

Corollary

Let i, j be birds from a set of birds.

1. *If the birds are flying in the same direction, coherence is positive.*
2. *If the birds are flying in directions more than 90 degrees offset, the coherence is negative.*
3. *If the birds are close to one another, $|C_{i,j}|$ is bigger.*
4. *If the birds are far from one another, $|C_{i,j}|$ is smaller.*

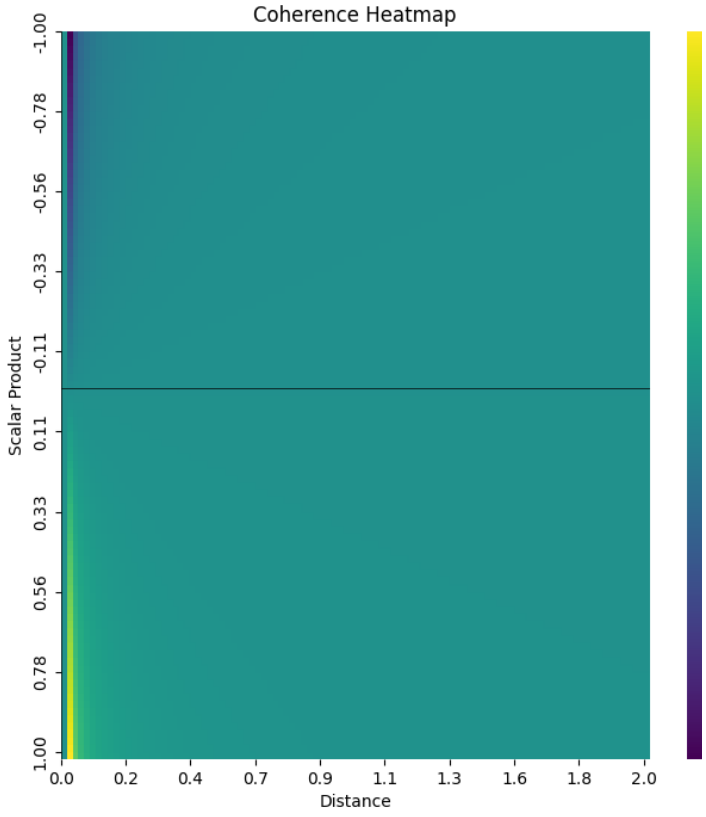


Figure: Coherence Heatmap using **simple coherence**

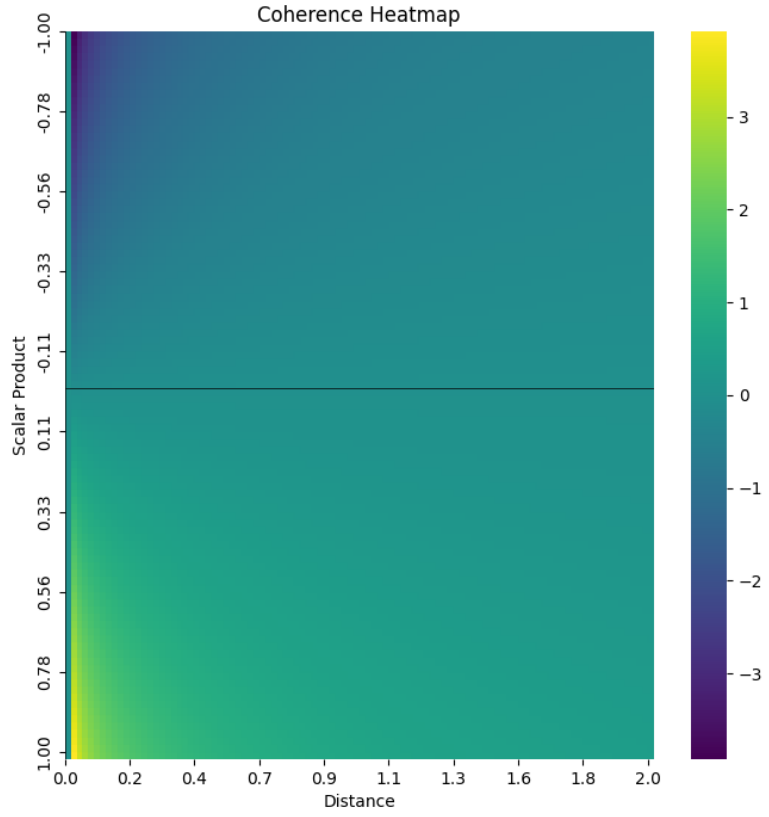
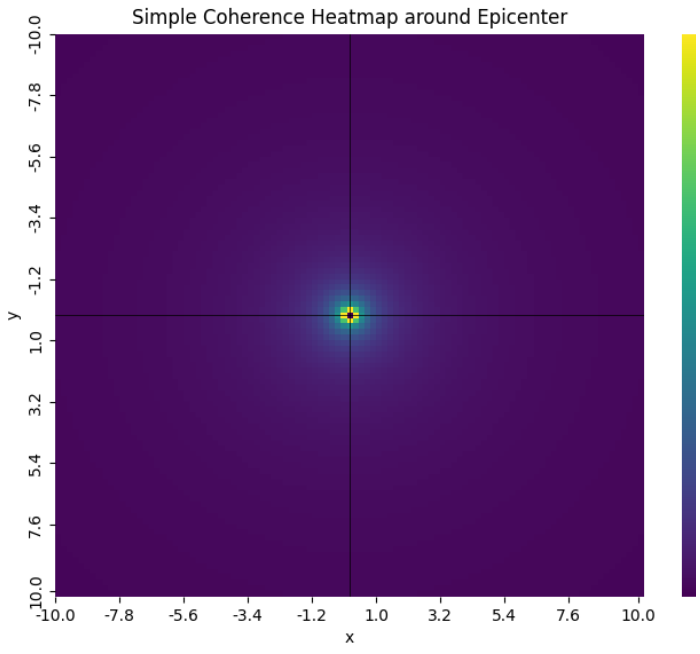


Figure: Coherence Heatmap using **logarithmic coherence**

Heatmaps Coherences around epicenter when flight direction is the same



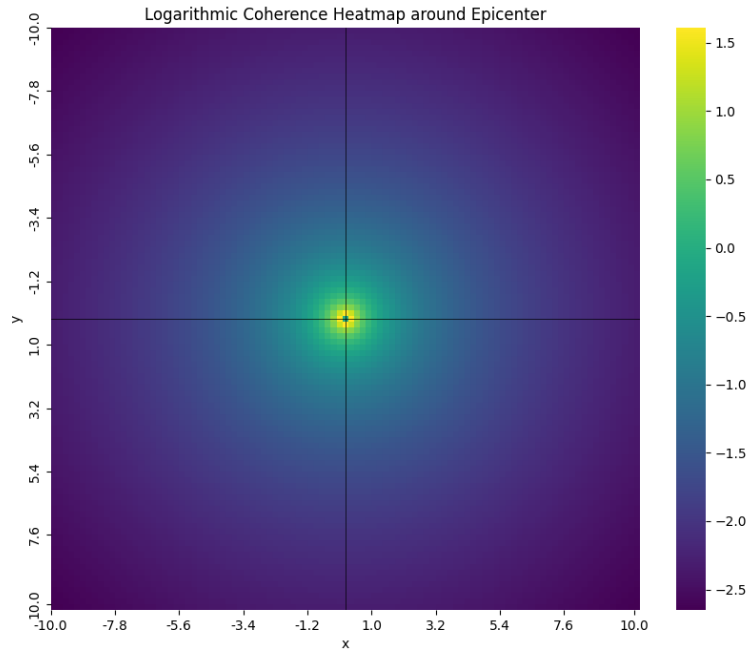


Figure: Coherence Heatmap around epicenter using **simple coherence**

Figure: Coherence Heatmap around epicenter using **logarithmic coherence**

Pros and cons of this method

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1. captures direction and space
2. direction is more important than space

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1. direction and space are not equally important, how to quantify that?
2. other birds behind or in front a bird should be respected more than birds to the side or under
3. Computationally inefficient (n number of birds)
   $\mathcal{O}(n)$ per bird $\rightarrow \mathcal{O}(n^2)$ in total
   with a lot of square roots