

## APPENDIX

## A. Proof of Proposition 1

Combining Eq. (2) and Eq. (5), we have

$$u_m^{se}[\mathbf{p}(r), b, r] = \left[ n_m + \alpha p_m(r) - \beta \sum_{i \in \mathcal{M}/\{m\}} p_i(r) \right] \times [b - \zeta - p_m(r)] + \gamma \left[ \beta \sum_{i \in \mathcal{M}/\{m\}} p_i(r) - n_m - h_m - \alpha p_m(r) \right]^2. \quad (15)$$

According to Eq. (15), Eq. (6) can be transformed into a discrete dynamic system as

$$p_m(r+1) = p_m(r) + \epsilon \frac{\partial u_m^{se}[\mathbf{p}_m(r), b, r]}{\partial p_m(r)}, \forall m \in \mathcal{M}. \quad (16)$$

Let  $p_m(r+1) = p_m(r), \forall m \in \mathcal{M}$ , we can obtain the equilibrium point  $\mathbf{p}^*(p_1^*, \dots, p_M^*)$  that satisfies Eq. (7). Since Eq. (16) is full rank,  $\mathbf{p}^*$  is the only equilibrium point.

## B. Proof of Proposition 2

The Jacobi matrix of the dynamic system  $p_m(r+1) = p_m(r) + \epsilon \frac{\partial u_m^{se}[\mathbf{p}_m(r), b, r]}{\partial p_m(r)}, \forall m \in \mathcal{M}$  is

$$\mathbf{J}_{\alpha, \beta, \gamma, \epsilon} = \begin{bmatrix} 1 - 2\epsilon\alpha(1 + \alpha\gamma) & \cdots & \epsilon\beta(1 + 2\alpha\gamma) \\ \epsilon\beta(1 + 2\alpha\gamma) & \cdots & \epsilon\beta(1 + 2\alpha\gamma) \\ \vdots & \ddots & \vdots \\ \epsilon\beta(1 + 2\alpha\gamma) & \cdots & 1 - 2\epsilon\alpha(1 + \alpha\gamma) \end{bmatrix}. \quad (17)$$

For all  $0 < \alpha < 0.25$  and  $0 < \beta < 1$ , there is

$$\text{tr}(\mathbf{J}_{\alpha, \beta, \gamma, \epsilon}) = M[1 - 2\epsilon\alpha(1 + \alpha\gamma)] > 0. \quad (18)$$

Since  $\text{tr}(\mathbf{J}_{\alpha, \beta, \gamma, \epsilon}) = \sum_{m \in \mathcal{M}} \lambda_{J, m}$ , where  $\lambda_{J, m}$  represents the eigenvalues of matrix  $\mathbf{J}_{\alpha, \beta, \gamma, \epsilon}$ , there exists  $m \in \mathcal{M}$  such that

$$\text{Re}[\lambda_{J, m}] > 0. \quad (19)$$

Therefore, the equilibrium point  $\mathbf{p}^*$  is unstable.

## C. Proof of Proposition 3

Given  $r \in [0, R]$  and  $b \in [0, B]$ , for convenience, we denote  $v[p] = \hat{u}_m^{se}[p, p_m^*, b, r]$  as function of  $p$ . Let

$$p = \alpha^{-1} \beta \sum_{i \in \mathcal{M}/\{m\}} p_i(r) - \frac{1}{\alpha} (h_m + n_m) + \frac{(b - \zeta) + [(b - \zeta)^2 - 4\gamma(b - \zeta)h_m]^{0.5}}{2\alpha\gamma}, \quad (20)$$

we obtain  $v[p] = 0$  and

$$\begin{aligned} \frac{\partial v[p]}{\partial p} &= \alpha(b - \zeta) - 2\alpha\gamma \left[ h_m + n_m - \beta \sum_{i \in \mathcal{M}/\{m\}} p_i^* \right. \\ &\quad \left. + \frac{\beta}{2\gamma} (b - \zeta - 2\gamma h_m + [(b - \zeta)^2 - 4\gamma(b - \zeta)h_m]^{0.5}) \right. \\ &\quad \left. - \beta^2 n_m + \beta^2 \sum_{i \in \mathcal{M}/\{m\}} p_i^* \right] < 0, \end{aligned} \quad (21)$$

which means when  $p_m(r) > p$ ,

$$u_m^{se}[\mathbf{p}(r), b, r] < v(p) = 0. \quad (22)$$

For point  $\mathbf{p}^*$ , there is

$$\begin{aligned} &p_m^* - \delta_m \\ &= \frac{(\alpha\beta + 2\alpha^3\gamma + 2\alpha^2\beta\gamma)(b - \zeta)}{2(\alpha - \beta + \alpha^2\gamma - 2\alpha\beta\gamma)(2\alpha + \beta + 2\alpha^2\gamma + 2\alpha\beta\gamma)}. \end{aligned} \quad (23)$$

We have  $p_m^* > \delta_m$ , because  $p_m^* > 0$ . Then

$$\begin{aligned} b &< \frac{\gamma h_m^2}{n_m + \alpha B} - \zeta + \delta_m \\ &< \frac{\gamma (q_{m,r} + h_m)^2}{n_m + \alpha B} - \zeta + p_m^*. \end{aligned} \quad (24)$$

Therefore,  $(b - \zeta - p_m^*)(n_m + \alpha B) - \gamma(q_{m,r} + h_m)^2 < 0$ . Due to  $\alpha B \geq \alpha p_m(r) - \beta \sum_{i \in \mathcal{M}/\{m\}} p_i(r)$ , we have

$$\begin{aligned} &u_m^{se}[\mathbf{p}^*, b, r] \\ &\leq (n_m + \alpha B)(b - \zeta - p_m^*) - \gamma(q_{m,r} + h_m)^2 < 0. \end{aligned} \quad (25)$$

## D. Proof of Lemma 1

When  $c_m < n_m + \alpha p_m^* - \sum_{i \in \mathcal{M}/\{m\}} c_i$ ,

$$\begin{aligned} \hat{u}_m^{se}[\mathbf{p}^*, b, r] &= c_m(b - \zeta - \hat{p}_m^*) - \gamma[c_m + h_m]^2 \\ &= \hat{q}_{m,r}(b - \zeta - \hat{p}_m^*) - \gamma[\hat{q}_{m,r} + h_m]^2. \end{aligned} \quad (26)$$

When  $c_m \geq n_m + \alpha p_m^* - \sum_{i \in \mathcal{M}/\{m\}} c_i$ , due to capacity constraints of other servers, edge server  $m$  can hire at least  $\hat{q}_{m,r}$  devices without being affected by price competition, where

$$\hat{q}_{m,r} = n_m + \alpha p_m^* - \sum_{i \in \mathcal{M}/\{m\}} c_i. \quad (27)$$

Consider two scenarios in FL system:

(i) if  $q_{m,r} \leq \hat{q}_{m,r}$ , Eq. (27) is the number of mobile device in the system. Hence,

$$u_m^{se}[\mathbf{p}^*, b, r] \geq u_m^{se}[\hat{\mathbf{p}}^*, b, r]. \quad (28)$$

(ii) if  $q_{m,r} > \hat{q}_{m,r}$ , there is

$$\begin{aligned} u_m^{se}[\mathbf{p}^*, b, r] &= \hat{q}_{m,r}[b - \zeta - p_m^*] - \gamma(\hat{q}_{m,r} + h_m)^2 \\ &= \left[ n_m + \alpha p_m^* - \sum_{i \in \mathcal{M}/\{m\}} c_i \right] \times [b - \zeta - p_m^*] \\ &\quad - \gamma \left[ n_m + \alpha p_m^* - \sum_{i \in \mathcal{M}/\{m\}} c_i \right] \\ &\geq \left[ \frac{\sum_{i \in \mathcal{M}/\{m\}} c_i - n_m + \alpha(b - \zeta) - 2\alpha\gamma(n_m + h_m)}{2\gamma\alpha^2 + 2\alpha} \right. \\ &\quad \left. + \frac{2\alpha\gamma \sum_{i \in \mathcal{M}/\{m\}} c_i}{2\gamma\alpha^2 + 2\alpha} - b - \zeta \right] \times \left[ n_m + \beta \sum_{i \in \mathcal{M}/\{m\}} c_i \right. \\ &\quad \left. + \alpha \frac{\sum_{i \in \mathcal{M}/\{m\}} c_i - n_m + \alpha(b - \zeta) - 2\alpha\gamma(n_m + h_m)}{2\gamma\alpha^2 + 2\alpha} \right. \\ &\quad \left. + \frac{2\alpha^2\gamma \sum_{i \in \mathcal{M}/\{m\}} c_i}{2\gamma\alpha^2 + 2\alpha} \right] - \gamma \left[ n_m + h_m - \sum_{i \in \mathcal{M}/\{m\}} c_i \right. \\ &\quad \left. + \alpha \frac{\sum_{i \in \mathcal{M}/\{m\}} c_i - n_m + \alpha(b - \zeta) - 2\alpha\gamma(n_m + h_m)}{2\gamma\alpha^2 + 2\alpha} \right] \\ &\quad \left. + \frac{2\alpha\gamma \sum_{i \in \mathcal{M}/\{m\}} c_i}{2\gamma\alpha^2 + 2\alpha} \right] = u_m^{se}[\hat{\mathbf{p}}^*, b, r]. \end{aligned} \quad (29)$$

Eq. (28) takes the equal sign if and only if  $p_m^* = \hat{p}_m^* = \frac{2\alpha\gamma h_m - \alpha(b-\zeta) + (1-2\alpha\gamma) \sum_{i \in \mathcal{M}/\{m\}} c_i}{2\alpha(\alpha\gamma-1)}$ .

#### E. Proof of Lemma 2.

Denoting  $s_r = p_m(r) - p_m^*$ , there is  $s_{r_1} - s_r = \epsilon \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)}$  and

$$\begin{aligned} \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} &= 2\alpha(1-\alpha\gamma)p_m(r) + 2\alpha\gamma h_m - \alpha b \\ &\quad - \alpha\zeta + (2\alpha\gamma - 1) \left( q_{m,r} - |\mathcal{N}| + \sum_{i \in \mathcal{M}/\{m\}} c_i \right) \\ &= 2\alpha(1-\alpha\gamma) \left[ p_m(r) - \frac{2\alpha\gamma h_m - \alpha p}{2\alpha(1-\alpha\gamma)} \right. \\ &\quad \left. + \frac{(2\alpha\gamma - 1) \left( q_m^r - |\mathcal{N}| + \sum_{i \in \mathcal{M}/\{m\}} c_i \right)}{2\alpha(1-\alpha\gamma)} \right] p_m(r) \\ &= 2\alpha(1-\alpha\gamma) [p_m(r) - p_m^*] = 2\alpha(1-\alpha\gamma) s_r. \end{aligned} \quad (30)$$

Denoting  $l_r = \frac{[p_m(r+1) - p_m^*]^2}{[p_m(r) - p_m^*]^2}$ , based on Eq. (30), we have

$$\begin{aligned} l_r &= \frac{[p_m(r+1) - p_m^*]^2}{[p_m(r) - p_m^*]^2} \\ &= \frac{\left[ s_r + \epsilon \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} \right]^2}{2\alpha(1-\alpha\gamma) s_r^2} \\ &= \frac{[2\alpha(1-\alpha\gamma)]^2 s_r^2 + 2\alpha(1-\alpha\gamma) \epsilon^2 \left[ \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} \right]^2}{2\alpha(1-\alpha\gamma) s_r^2} \\ &\quad - \frac{4\alpha(1-\alpha\gamma) \epsilon s_r \left[ \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} \right]}{2\alpha(1-\alpha\gamma) s_r^2} \\ &= \frac{[2\alpha(1-\alpha\gamma)]^{-1} \left[ \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} \right]^2 - 2\epsilon \left[ \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} \right]^2}{[2\alpha(1-\alpha\gamma)]^{-1} \left[ \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} \right]^2} \\ &\quad + \frac{2\alpha(1-\alpha\gamma) \epsilon^2 \left[ \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} \right]^2}{[2\alpha(1-\alpha\gamma)]^{-1} \left[ \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} \right]^2}. \end{aligned} \quad (31)$$

When  $\epsilon < \frac{1}{\alpha(1-\alpha\gamma)}$ , there is  $2\alpha(1-\alpha\gamma) \epsilon^2 \left[ \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} \right]^2 - 2\epsilon \left[ \frac{\partial u_m^{se}[\mathbf{p}(r), b, r]}{\partial p_m(r)} \right]^2 < 0$ , which means  $l_r < 1$ .

For all  $\epsilon > 0$ , there exists  $\delta = \frac{\epsilon}{M}$  such that when  $\|\mathbf{p}(0) - \mathbf{p}^*\| < \delta$ ,  $\forall r > 0$ ,

$$\begin{aligned} \|p(r)_m - p_m^*\| &\leq \sum_{m \in \mathcal{M}} \|p(r)_m - p_m^*\| \\ &= \sum_{m \in \mathcal{M}} l^r \|p(0)_m - p_m^*\| \\ &\leq \sum_{m \in \mathcal{M}} l^r \|\mathbf{p}(0) - \mathbf{p}^*\| \\ &\leq M l^r \frac{\epsilon}{M} < \epsilon. \end{aligned} \quad (32)$$

Therefore,  $\mathbf{p}^*$  is uniformly stable.

#### F. Derivation of Eq. (13)

According to Eq. (14c), there is  $c_m \geq q_m^r$ . Consequently, the number of disconnected roaming devices in FL system can be expressed as

$$\tilde{D} = \sum_{m \in \mathcal{M}} \sum_{x=c_m-q_{m,r}}^{|\mathcal{N}|-q_{m,r}} \binom{|\mathcal{N}|-q_{m,r}}{x} p^x (1-p)^{|\mathcal{N}|-q_{m,r}-x}. \quad (33)$$

Based on the central limit theorem, we can approximate the binomial distribution as a Gaussian distribution  $N(|\mathcal{N}| - q_m^r, p - p^2)$ . Thus, the expectation of binomial distribution  $\sum_{m \in \mathcal{M}} \sum_{x=c_m-q_{m,r}}^{|\mathcal{N}|-q_{m,r}} \binom{|\mathcal{N}|-q_{m,r}}{x} p^x (1-p)^{|\mathcal{N}|-q_{m,r}-x}$  can be approximated as

$$\begin{aligned} &\int_{c_m-q_m^r}^{|\mathcal{N}|-q_m^r} x \frac{e^{-\frac{x-p(|\mathcal{N}|-q_m^r)}{(p-p^2)(|\mathcal{N}|-q_m^r)}}}{\sqrt{2\pi(p-p^2)(|\mathcal{N}|-q_m^r)}} dx \\ &= \int_{c_m-q_m^r}^{|\mathcal{N}|-q_m^r} \Phi \left( \frac{x-|\mathcal{N}|+q_m^r}{\sqrt{p-p^2}} \right) \frac{x}{\sqrt{p-p^2}} dx \\ &= \int_{\frac{c_m-q_m^r}{\sqrt{p-p^2}}}^{\frac{(|\mathcal{N}|-q_m^r)}{\sqrt{p-p^2}}} x \sqrt{p-p^2} + (|\mathcal{N}| - q_m^r) \\ &\quad \times \Phi(x) \frac{x}{\sqrt{p-p^2}} d \left[ x \sqrt{p-p^2} + (|\mathcal{N}| - q_m^r) \right] \\ &= \int_{\frac{c_m-q_m^r}{\sqrt{p-p^2}}}^{\frac{(|\mathcal{N}|-q_m^r)}{\sqrt{p-p^2}}} \sqrt{p-p^2} x \Phi(x) + (|\mathcal{N}| - q_m^r) \Phi(x) dx \\ &= -\sqrt{p-p^2} \int_{\frac{c_m-q_m^r}{\sqrt{p-p^2}}}^{\frac{(|\mathcal{N}|-q_m^r)}{\sqrt{p-p^2}}} \Phi'(x) dx \\ &\quad + (|\mathcal{N}| - q_m^r) \int_{\frac{c_m-q_m^r}{\sqrt{p-p^2}}}^{\frac{(|\mathcal{N}|-q_m^r)}{\sqrt{p-p^2}}} \Phi(x) dx \\ &= \sqrt{p-p^2} [\Phi(\alpha) - \Phi(\beta)] + (|\mathcal{N}| - q_m^r) [\Psi(\alpha) - \Psi(\beta)]. \end{aligned} \quad (34)$$

#### G. Proof of Theorem 1

It can be immediately inferred from Lemma 1, Lemma 2, and Eq. (14).