Appendix

A. Proof of Proposition 1

Combining Eq. (2) and Eq. (5), we have

$$\begin{aligned} u_m^{se}[\boldsymbol{p}(r), b, r] = & \left[n_m + \alpha p_m(r) - \beta \sum_{i \in \mathcal{M}/\{m\}} p_i(r) \right] \\ & \times [b - \zeta - p_m(r)] + \gamma \Bigg[\beta \sum_{i \in \mathcal{M}/\{m\}} p_i(r) \\ & - n_m - h_m - \alpha p_m(r) \Bigg]^2 . \end{aligned} \tag{15}$$

According to Eq. (15), Eq. (6) can be transformed into a discrete dynamic system as

$$p_m(r+1) = p_m(r) + \epsilon \frac{\partial u_m^{se}[p_m(r), b, r]}{\partial p_m(r)}, \forall m \in \mathcal{M}. \tag{16}$$

Let $p_m(r+1) = p_m(r), \forall m \in \mathcal{M}$, we can obtain the equilibrium point $\boldsymbol{p}^*(p_1^*, \cdots, p_M^*)$ that satisfies Eq. (7). Since Eq. (16) is full rank, $\boldsymbol{p}^*$ is the only equilibrium point.

B. Proof of Proposition 2

The Jacobi matrix of the dynamic system $p_m(r+1) = p_m(r) + \epsilon \frac{\partial u_m^{se}[p_m(r), p, r]}{\partial p_m(r)}, \forall m \in \mathcal{M}$ is

$$\mathbf{J}_{\alpha,\beta,\gamma,\epsilon} = \begin{bmatrix} 1 - 2\epsilon\alpha(1+\alpha\gamma) & \cdots & \epsilon\beta(1+2\alpha\gamma) \\ \epsilon\beta(1+2\alpha\gamma) & \cdots & \epsilon\beta(1+2\alpha\gamma) \\ \vdots & \ddots & \vdots \\ \epsilon\beta(1+2\alpha\gamma) & \cdots & 1 - 2\epsilon\alpha(1+\alpha\gamma) \end{bmatrix}. \tag{17}$$

For all $0 < \alpha < 0.25$ and $0 < \beta < 1$, there is

$$\mathrm{tr}(\mathbf{J}_{\alpha,\beta,\gamma,\epsilon}) = M[1 - 2\epsilon\alpha(1+\alpha\gamma)] > 0. \tag{18}$$

Since $\mathrm{tr}(\mathbf{J}_{\alpha,\beta,\gamma,\epsilon}) = \sum_{m \in \mathcal{M}} \lambda_{\mathbf{J},m}$, where $\lambda_{\mathbf{J},m}$ represents the eigenvalues of matrix $\mathbf{J}_{\alpha,\beta,\gamma,\epsilon}$, there exists $m \in \mathcal{M}$ such that

$$\mathrm{Re}[\lambda_{\mathbf{J},m}] => 0. \tag{19}$$

Therefore, the equilibrium point $\boldsymbol{p}^*$ is unstable.

C. Proof of Proposition 3

Given $r \in [0, R]$ and $b \in [0, B]$, for convenience, we denote $v[p] = \hat{u}_m^{se}[p, p_{-m}^*, b, r]$ as function of p. Let

$$\begin{aligned} p = & \alpha^{-1}\beta \sum_{i \in \mathcal{M}/\{m\}} p_i(r) - \frac{1}{\alpha}(h_m + n_m) \\ & + \frac{(b-\zeta) + [(b-\zeta)^2 - 4\gamma(b-\zeta)h_m]^{0.5}}{2\alpha\gamma}, \end{aligned} \tag{20}$$

we obtain $v[p] = 0$ and

$$\begin{aligned} \frac{\partial v[p]}{\partial p} = & \alpha(b-\zeta) - 2\alpha\gamma \Bigg[h_m + n_m - \beta \sum_{i \in \mathcal{M}/\{m\}} p_i^* \\ & + \frac{\beta}{2\gamma}\left(b - \zeta - 2\gamma h_m + [(b-\zeta)^2 - 4\gamma(b-\zeta)h_m]^{0.5}\right) \\ & - \beta^2 n_m + \beta^2 \sum_{i \in \mathcal{M}/\{m\}} p_i^* \Bigg] < 0, \end{aligned} \tag{21}$$

which means when $p_m(r) > p$,

$$u_m^{se}[\boldsymbol{p}(r), b, r] < v(p) = 0. \tag{22}$$

For point $\boldsymbol{p}^*$, there is

$$\begin{aligned} & p_m^* - \delta_m \\ = & \frac{(\alpha\beta + 2\alpha^3\gamma + 2\alpha^2\beta\gamma)(b-\zeta)}{2(\alpha - \beta + \alpha^2\gamma - 2\alpha\beta\gamma)(2\alpha + \beta + 2\alpha^2\gamma + 2\alpha\beta\gamma)}. \end{aligned} \tag{23}$$

We have $p_m^* > \delta_m$, because $p_m^* > 0$. Then

$$\begin{aligned} b < & \frac{\gamma h_m^2}{n_m + \alpha B} - \zeta + \delta_m \\ < & \frac{\gamma (q_{m,r} + h_m)^2}{n_m + \alpha B} - \zeta + p_m^*. \end{aligned} \tag{24}$$

Therefore, $(b - \zeta - p_m^*)(n_m + \alpha B) - \gamma (q_{m,r} + h_m)^2 < 0$. Due to $\alpha B \geq +\alpha p_m(r) - \beta \sum_{i \in \mathcal{M}/\{m\}} p_i(r)$, we have

$$\begin{aligned} & u_m^{se}[\boldsymbol{p}^*, b, r] \\ \leq & (n_m + \alpha B)(b - \zeta - p_m^*) - \gamma (q_{m,r} + h_m)^2 < 0. \end{aligned} \tag{25}$$

D. Proof of Lemma 1

When $c_m < n_m + \alpha \hat{p}_m^* - \sum_{i \in \mathcal{M}/\{m\}} c_i$,

$$\begin{aligned} \hat{u}_m^{se}[\hat{\boldsymbol{p}}^*, b, r] = & c_m (b - \zeta - \hat{p}_m^*) - \gamma [c_m + h_m]^2 \\ = & \hat{q}_{m,r} (b - \zeta - \hat{p}_m^*) - \gamma [\hat{q}_{m,r} + h_m]^2 . \end{aligned} \tag{26}$$

When $c_m \geq n_m + \alpha \hat{p}_m^* - \sum_{i \in \mathcal{M}/\{m\}} c_i$, due to capacity constraints of other servers, edge server m can hire at least $\hat{q}_{m,r}$ devices without being affected by price competition, where

$$\hat{q}_{m,r} = n_m + \alpha \hat{p}_m^* - \sum_{i \in \mathcal{M}/\{m\}} c_i. \tag{27}$$

Consider two scenarios in FL system:

(i) if $q_{m,r} \leq \hat{q}_{m,r}$, Eq. (27) is the number of mobile device in the system. Hence,

$$u_m^{se}[\boldsymbol{p}^*, b, r] \geq u_m^{se}[\hat{\boldsymbol{p}}^*, b, r]. \tag{28}$$

(ii) if $q_{m,r} > \hat{q}_{m,r}$, there is

$$\begin{aligned} u_m^{se}[\boldsymbol{p}^*, b, r] = & \hat{q}_{m,r} [b - \zeta - p_m^*] - \gamma (\hat{q}_{m,r} + h_m)^2 \\ = & \Big[n_m + \alpha p_m^* - \sum_{i \in \mathcal{M}/\{m\}} c_i \Big] \times [b - \zeta - p_m^*] \\ & - \gamma \Bigg[n_m + \alpha p_m^* - \sum_{i \in \mathcal{M}/\{m\}} c_i \Bigg] \\ \geq & \Bigg[\frac{\sum_{i \in \mathcal{M}/\{m\}} c_i - n_m + \alpha(b-\zeta) - 2\alpha\gamma(n_m + h_m)}{2\gamma\alpha^2 + 2\alpha} \\ & + \frac{2\alpha\gamma \sum_{i \in \mathcal{M}/\{m\}} c_i}{2\gamma\alpha^2 + 2\alpha} - b - \zeta \Bigg] \times \Bigg[n_m + \beta \sum_{i \in \mathcal{M}/\{m\}} c_i \\ & + \alpha \frac{\sum_{i \in \mathcal{M}/\{m\}} c_i - n_m + \alpha(b-\zeta) - 2\alpha\gamma(n_m + h_m)}{2\gamma\alpha^2 + 2\alpha} \\ & + \frac{2\alpha^2\gamma \sum_{i \in \mathcal{M}/\{m\}} c_i}{2\gamma\alpha^2 + 2\alpha} \Bigg] - \gamma \Bigg[n_m + h_m - \sum_{i \in \mathcal{M}/\{m\}} c_i \\ & + \alpha \frac{\sum_{i \in \mathcal{M}/\{m\}} c_i - n_m + \alpha(b-\zeta) - 2\alpha\gamma(n_m + h_m)}{2\gamma\alpha^2 + 2\alpha} \\ & + \frac{2\alpha\gamma \sum_{i \in \mathcal{M}/\{m\}} c_i}{2\gamma\alpha^2 + 2\alpha} \Bigg] = u_m^{se}[\hat{\boldsymbol{p}}^*, b, r]. \end{aligned} \tag{29}$$

Eq. (28) takes the equal sign if and only if $p_m^* = \hat{p}_m^* = \frac{2\alpha\gamma h_m - \alpha(b-\zeta)+(1-2\alpha\gamma)\sum_{i\in\mathcal{M}/\{m\}} c_i}{2\alpha(\alpha\gamma-1)}$.

E. Proof of Lemma 2.

Denoting $s_r = p_m(r) - p_m^*$, there is $s_{r_1} - s_r = \epsilon \frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}$ and

$$\begin{aligned}\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)} &= 2\alpha(1-\alpha\gamma)p_m(r) + 2\alpha\gamma h_m - \alpha b \\ &- \alpha\zeta + (2\alpha\gamma - 1)\left(q_{m,r} - |\mathcal{N}| + \sum_{i\in\mathcal{M}/\{m\}} c_i\right) \\ =& 2\alpha(1-\alpha\gamma)\left[p_m(r) - \frac{2\alpha\gamma h_m - \alpha p}{2\alpha(1-\alpha\gamma)}\right. \\ &\left. + \frac{(2\alpha\gamma-1)\left(q_m^r - |\mathcal{N}| + \sum_{i\in\mathcal{M}/\{m\}} c_i\right)}{2\alpha(1-\alpha\gamma)}\right] p_m(r) \\ =& 2\alpha(1-\alpha\gamma)\left[p_m(r) - p_m^*\right] = 2\alpha(1-\alpha\gamma)s_r.\end{aligned} \tag{30}$$

Denoting $l_r = \frac{[p_m(r+1)-p_m^*]^2}{[p_m(r)-p_m^*]^2}$, based on Eq. (30), we have

$$\begin{aligned} l_r =& \frac{[p_m(r+1) - p_m^*]^2}{[p_m(r) - p_m^*]^2} \\ =& \frac{\left[s_r + \epsilon\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}\right]^2 2\alpha(1-\alpha\gamma)}{2\alpha(1-\alpha\gamma)s_r^2} \\ =& \frac{[2\alpha(1-\alpha\gamma)]^2 s_r^2 + 2\alpha(1-\alpha\gamma)\epsilon^2\left[\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}\right]^2}{2\alpha(1-\alpha\gamma)s_r^2} \\ &- \frac{4\alpha(1-\alpha\gamma)\epsilon s_r\left[\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}\right]}{2\alpha(1-\alpha\gamma)s_r^2} \\ =& \frac{[2\alpha(1-\alpha\gamma)]^{-1}\left[\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}\right]^2 - 2\epsilon\left[\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}\right]^2}{[2\alpha(1-\alpha\gamma)]^{-1}\left[\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}\right]^2} \\ &+ \frac{2\alpha(1-\alpha\gamma)\epsilon^2\left[\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}\right]^2}{[2\alpha(1-\alpha\gamma)]^{-1}\left[\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}\right]^2}.\end{aligned} \tag{31}$$

When $\epsilon < \frac{1}{\alpha(1-\alpha\gamma)}$, there is $2\alpha(1-\alpha\gamma)\epsilon^2\left[\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}\right]^2 - 2\epsilon\left[\frac{\partial u_m^{se}[\boldsymbol{p}(r),b,r]}{\partial p_m(r)}\right]^2 < 0$, which means $l_r < 1$.

For all $\varepsilon > 0$, there exists $\delta = \frac{\varepsilon}{M}$ such that when $\|\boldsymbol{p}(0) - \boldsymbol{p}^*\| < \delta$, $\forall r > 0$,

$$\begin{aligned}\|p(r)_m - p_m^*\| \le& \sum_{m\in\mathcal{M}} \|p(r)_m - p_m^*\| \\ =& \sum_{m\in\mathcal{M}} l^r \|p(0)_m - p_m^*\| \\ \le& \sum_{m\in\mathcal{M}} l^r \|\boldsymbol{p}(0) - \boldsymbol{p}^*\| \\ \le& M l^r \frac{\varepsilon}{M} < \varepsilon.\end{aligned} \tag{32}$$

Therefore, $\boldsymbol{p}^*$ is uniformly stable.

F. Derivation of Eq. (13)

According to Eq. (14c), there is $c_m \ge q_m^r$. Consequently, the number of disconnected roaming devices in FL system can be expressed as

$$\widetilde{D} = \sum_{m\in\mathcal{M}} \sum_{x=c_m-q_{m,r}}^{|\mathcal{N}|-q_{m,r}} \binom{|\mathcal{N}| - q_{m,r}}{x} p^x (1-p)^{|\mathcal{N}|-q_{m,r}-x}. \tag{33}$$

Based on the central limit theorem, we can approximate the binomial distribution as a Gaussian distribution $N(|\mathcal{N}| - q_m^r, p - p^2)$. Thus, the expectation of binomial distribution $\sum_{m\in\mathcal{M}}\sum_{x=c_m-q_{m,r}}^{|\mathcal{N}|-q_{m,r}} \binom{|\mathcal{N}|-q_{m,r}}{x} p^x(1-p)^{|\mathcal{N}|-q_{m,r}-x}$ can be approximated as

$$\begin{aligned}&\int_{c_m-q_m^r}^{|\mathcal{N}|-q_m^r} x\frac{\mathrm{e}^{-\frac{x-p(|\mathcal{N}|-q_m^r)}{(p-p^2)(|\mathcal{N}|-q_m^r)}}}{\sqrt{2\pi(p-p^2)(|\mathcal{N}|-q_m^r)}}dx \\ =& \int_{c_m-q_m^r}^{|\mathcal{N}|-q_m^r} \Phi\left(\frac{x-|\mathcal{N}|-q_m^r}{\sqrt{p-p^2}}\right)\frac{x}{\sqrt{p-p^2}}dx \\ =& \int_{\frac{c_m-q_m^r}{\sqrt{p-p^2}}}^{\frac{(|\mathcal{N}|-q_m^r)}{\sqrt{p-p^2}}} \frac{x\sqrt{p-p^2} + (|\mathcal{N}| - q_m^r)}{\sqrt{p-p^2}} \\ &\times \Phi(x)\frac{x}{\sqrt{p-p^2}} d\left[x\sqrt{p-p^2} + (|\mathcal{N}| - q_m^r)\right] \\ =& \int_{\frac{c_m-q_m^r}{\sqrt{p-p^2}}}^{\frac{(|\mathcal{N}|-q_m^r)}{\sqrt{p-p^2}}} \sqrt{p-p^2}x\Phi(x) + (|\mathcal{N}| - q_m^r)\Phi(x)\,dx \\ =& -\sqrt{p-p^2}\int_{\frac{c_m-q_m^r}{\sqrt{p-p^2}}}^{\frac{(|\mathcal{N}|-q_m^r)}{\sqrt{p-p^2}}} \Phi'(x)\,dx \\ &+ (|\mathcal{N}| - q_m^r)\int_{\frac{c_m-q_m^r}{\sqrt{p-p^2}}}^{\frac{(|\mathcal{N}|-q_m^r)}{\sqrt{p-p^2}}} \Phi(x)\,dx \\ =& \sqrt{p-p^2}\left[\Phi(\alpha) - \Phi(\beta)\right] + (|\mathcal{N}| - q_m^r)\left[\Psi(\alpha) - \Psi(\beta)\right].\end{aligned} \tag{34}$$

G. Proof of Theorem 1

Theorem 1. *The solution $(b, \boldsymbol{c})$ to $\mathbb{P}_2$ is the optimal incentive mechanism that can minimize the loss upper bound, while ensuring non-zero utility and guaranteeing system stability.*

Proof. It can be immediately inferred from Lemma 1, Lemma 2, and Eq. (14). □