CS303A Homework 2

Q1 . (30%) Consider the following sentence:

$$[(Food \Rightarrow Party) \lor (Drinks \Rightarrow Party)] \Rightarrow [(Food \land Drinks) \Rightarrow Party].$$

a. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable. (10 points)

b. Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a). (10 points)

c. Prove your answer to (a) using resolution. (10 points)

Q2. （40%） This exercise uses the function MapColor and predicates In(x, y), Borders(x, y), and Country(x), whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence and tell why.

a. Paris and Marseilles are both in France. (8 points)

(i) In(Paris $\land$ Marseilles, France).

(ii) In(Paris, France) $\land$ In(Marseilles, France).

(iii) In(Paris, France) $\lor$ In(Marseilles, France).

b. There is a country that borders both Iraq and Pakistan. (8 points)

(i) $\exists$ c Country (c) $\land$ Border (c, Iraq) $\land$ Border (c, Pakistan).

(ii) $\exists$ c Country (c) $\Rightarrow$ [Border (c, Iraq) $\land$ Border (c, Pakistan)].

(iii) [$\exists$ c Country (c)] $\Rightarrow$ [Border (c, Iraq) $\land$ Border (c, Pakistan)].

(iv) ∃ c Border (Country (c), Iraq ∧ Pakistan).

c. All countries that border Ecuador are in South America. (8 points)

(i) ∀c Country(c) ∧ Border(c, Ecuador) ⇒ In(c, SouthAmerica).

(ii) ∀c Country(c) ⇒ [Border(c, Ecuador) ⇒ In(c, SouthAmerica)].

(iii) ∀c [Country(c) ⇒ Border(c, Ecuador)] ⇒ In(c, SouthAmerica).

(iv) ∀c Country(c) ∧ Border(c, Ecuador) ∧ In(c, SouthAmerica).

d. No region in South America borders any region in Europe. (8 points)

(i) ¬[∃c, d In(c, SouthAmerica) ∧ In(d, Europe) ∧ Borders(c, d)].

(ii) ∀c,d [In(c, SouthAmerica) ∧ In(d, Europe)] ⇒ ¬Borders(c, d)].

(iii) ¬∀c In(c, SouthAmerica) ⇒ ∃d In(d, Europe)∧¬Borders(c, d).

(iv) ∀c In(c, SouthAmerica) ⇒ ∀d In(d, Europe) ⇒ ¬Borders(c, d).

e. No two adjacent countries have the same map color. (8 points)

(i) ∀x,y ¬Country(x) ∨ ¬Country(y) ∨ ¬Borders(x, y)∨ ¬(MapColor (x) = MapColor (y)).

(ii) ∀x,y (Country(x) ∧ Country(y) ∧ Borders(x, y)∧¬(x=y)) ⇒ ¬(MapColor (x) = MapColor (y)).

(iii) ∀x,y Country(x) ∧ Country(y) ∧ Borders(x, y)∧ ¬(MapColor (x) = MapColor (y)).

(iv) ∀x,y (Country(x) ∧ Country(y) ∧ Borders(x, y)) ⇒ MapColor(x ≠ y).

Q3. (30%)We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1, X_2, and X_3.

a. Draw the Bayesian network corresponding to this setup and define the necessary CPTs.
(15 points)

b. Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once. (15 points)