A PROOF

A.1 Uniqueness of minimal form

Lemma 1. *For any valid materialization plan S it must have one and only one minimal form S'.*

We prove the uniqueness property of minimal forms, as formulated by Lemma 1.

Proof sketch. We proceed by establishing contradictions. Assume that a valid materialization plan S has two distinct minimal forms, denoted as X and Y. To demonstrate the impossibility of such a scenario, we break down the proof by examining various overlapping cases of X and Y.

First, suppose $X \subset Y$, and let $D = Y \setminus X$. Since X is a valid relation set, we can obtain another valid set by removing relations in D from Y, which contradicts to the assumption that Y is in a minimal form. Similarly, the case where $Y \subset X$ also leads to contradiction.

The remaining case is where sets X and Y have at least one element that is unique to each set. Let A be the set of unique elements in X ($X \setminus Y$), and let B be the set of unique elements in Y ($Y \setminus X$), then we have $A \neq \emptyset$ and $B \neq \emptyset$.

Next, we want to show that for each relation a in A, the execution of queries to a depends on at least one relation in B. To see this, consider the fact that S is a valid materialization plan, and that Y is a minimal form of S. Consequently, relational queries to a should be executable given Y. Moreover, if there exists a relation a in A such that a is executable solely using relations in $X \cap Y$, then we can obtain another valid set by removing a from X, which contradicts to the assumption that X is in a minimal form. Therefore, queries to each relation a in A must rely on at least one relation in B.

Similarly, queries to each relation b in B must also rely on at least one relation in A. This yields mutual dependencies between relations in set A and B. This contradicts to the assumption that any DeCon contract has no circular dependencies among relations.

Therefore, we successfully establish the uniqueness of minimal form of a given valid view materialization plan.

A.2 Completeness

Theorem 1. *Completeness. Given a set of relational queries, let B be the base materialization plan, and B' be the minimal form of B (GetMinimal(B)), Algorithm* **??** *outputs all minimal forms of view materialization plans obtainable from Σ:*

$$\bigcup_{i=0}^{K} MinReplace^{i}(B') = GetMinimal(\Sigma_K)$$

To prove Theorem 1, we first establish the following lemma.

Lemma 2. *For all valid materialization plan S, let S' be the minimal form of S:*

$$MinReplace(\{S\}) = MinReplace(\{S'\})$$

Proof sketch. We first prove $MinReplace(\{S'\}) \subseteq MinReplace(\{S\})$. $\forall$ relation $r \in S$, (1) if $r \in S'$, then r can be executed by any replacement choices of S' including the reduced one v; (2) if $r \notin S'$, then r must be calculated using some relations $p \in P$ where $P \subseteq S'$. In this case, $\forall p \in P$, we have $p \in S'$, and therefore p can be executed by any replacement choices of S' including the reduced one v, and thus r can also be executed by v. Therefore, v is also a reduced alternative choice of S.

We next prove $MinReplace(\{S\}) \subseteq MinReplace(\{S'\})$. $\forall$ relation $r \in S'$, since $S' \subseteq S$, we have $r \in S$, and therefore r can be executed by any replacement choices of S including the reduced one u. So u is also a reduced alternative choice of S'.

Following Lemma 2, we further deduce the following lemma:

Lemma 3. *Let Σ be a set of valid materialization plan, We have:*

$$MinReplace(\Sigma) = MinReplace(GetMinimal(\Sigma))$$

Given Lemma 3, we proceed to prove completeness (Theorem 1) of the materialization enumeration algorithm. The proof is conducted via mathematical induction on the number of the application for *MinReplace* method i.

For the base case, where $i = 0$, and $B' = GetMinimal(B)$, which is true following our assumption.

For induction case where $i = n$, the induction hypothesis is as follow:

$$\bigcup_{i=0}^{n} MinReplace^{i}(B') = \{GetMinimal(S) | S \in \Sigma_n\}$$

We then analyze the case where $i = n + 1$, the output of the enumeration algorithm after $n + 1$ applications of the *minReplace* method can be rewritten as follows:

$$\bigcup_{i=0}^{n+1} MinReplace^{i}(B') = \bigcup_{i=0}^{n} MinReplace^{i}(B') \cup minReplace^{n+1}(B')$$

Substituting the right hand side of the above eqaution with the induction hypothesis, we have:

$$\bigcup_{i=0}^{n+1} MinReplace^{i}(B') = \{GetMinimal(S) | S \in \Sigma_n\} \cup minReplace^{n+1}(B') \tag{1}$$

Next, we expand $minReplace^{n+1}(B')$:

$$minReplace^{n+1}(B') = minReplace^{n}(minReplace(B'))$$

By lemma 3, we can further rewrite the right-hand-side:

$$\begin{aligned} minReplace^{n+1}(B') &= minReplace^{n}(minReplace(B)) \\ &= minReplace^{n}(GetMinimal(Replace(B))) \\ &= minReplace^{n}(Replace(B)) \\ &\dots \\ &= minReplace(Replace^{n}(B)) \\ &= GetMinimal(Replace^{n+1}(B)) \\ &= \{GetMinimal(S) \mid S \in Replace^{n+1}(B)\} \end{aligned}$$

Substituting the right-hand-side of the above equation back to Equation 1, we have:

$$\bigcup_{i=0}^{n+1} MinReplace^{i}(B') = \{GetMinimal(S) \mid S \in \Sigma_{n+1}\}$$

which establishes the validity for $i = n + 1$.

Here, we have successfully completed the induction step and prove the completeness property as defined in Theorem 1.