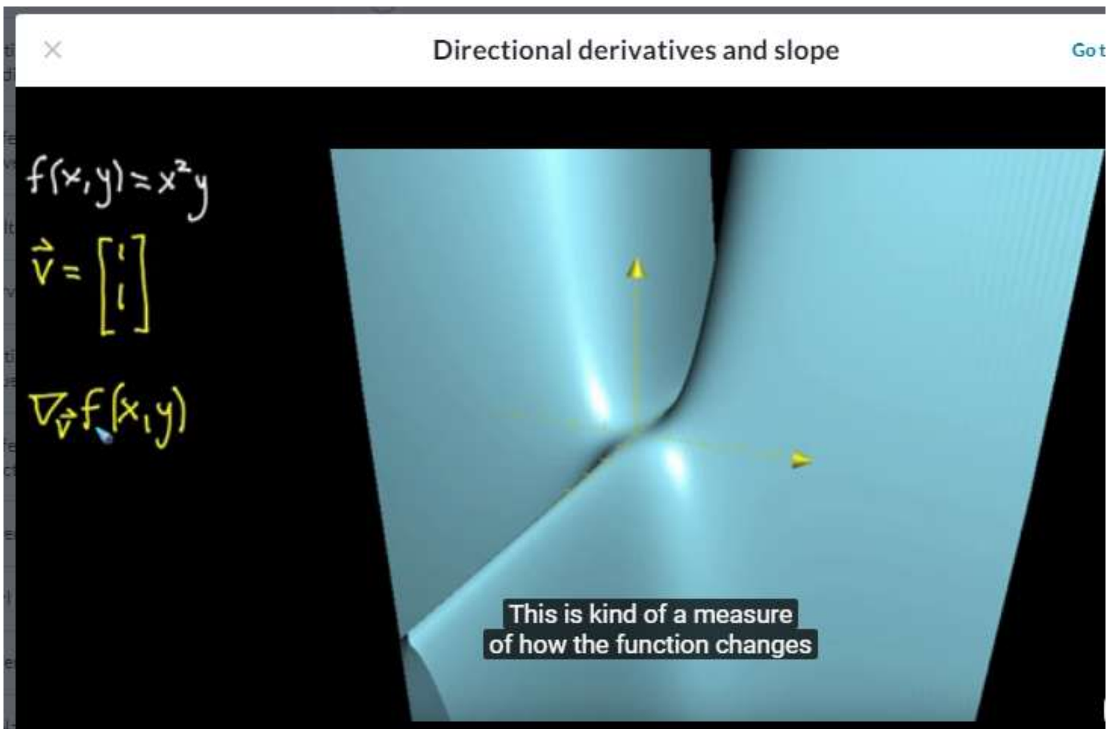
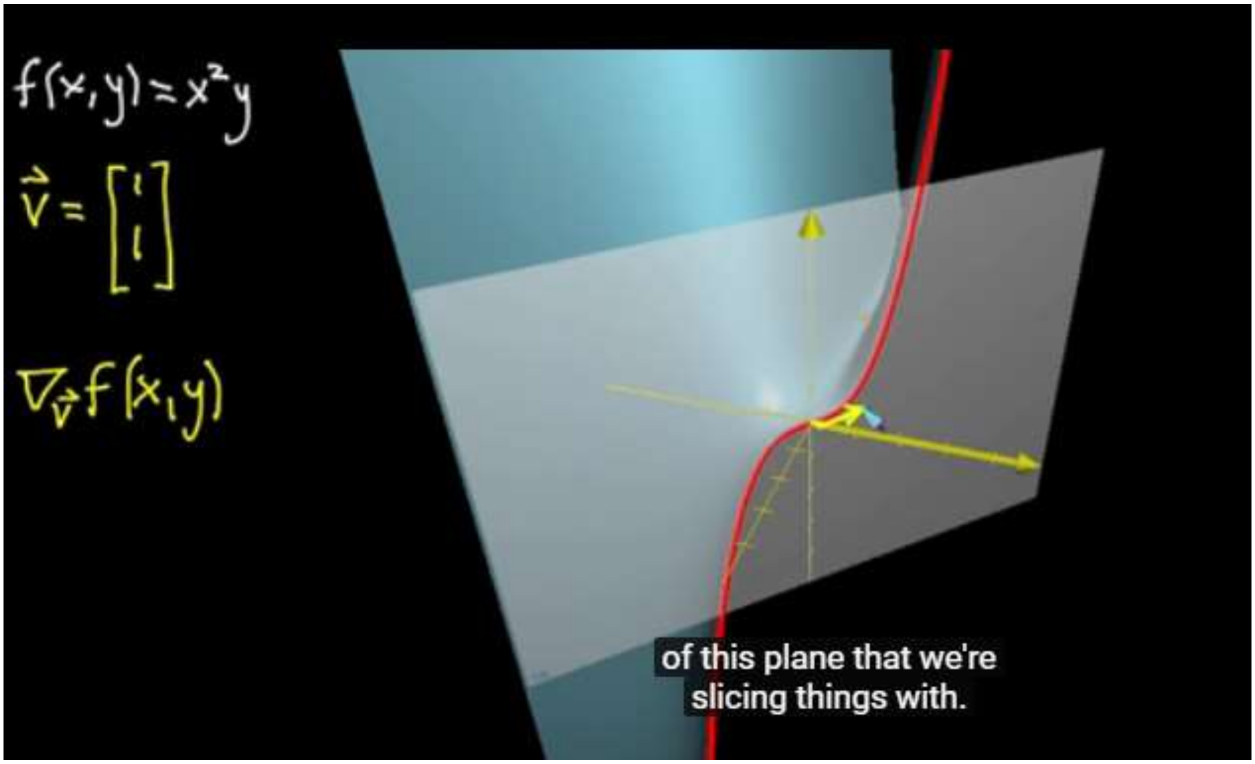
Directional derivatives and slope

- General setup:
  - Vector in input space: vector v
  - Directional derivative which we denote by taking the gradiant and stick the name of the vector as a subscript
    - How the function f(x,y) changes when input moves in the direction of vector v

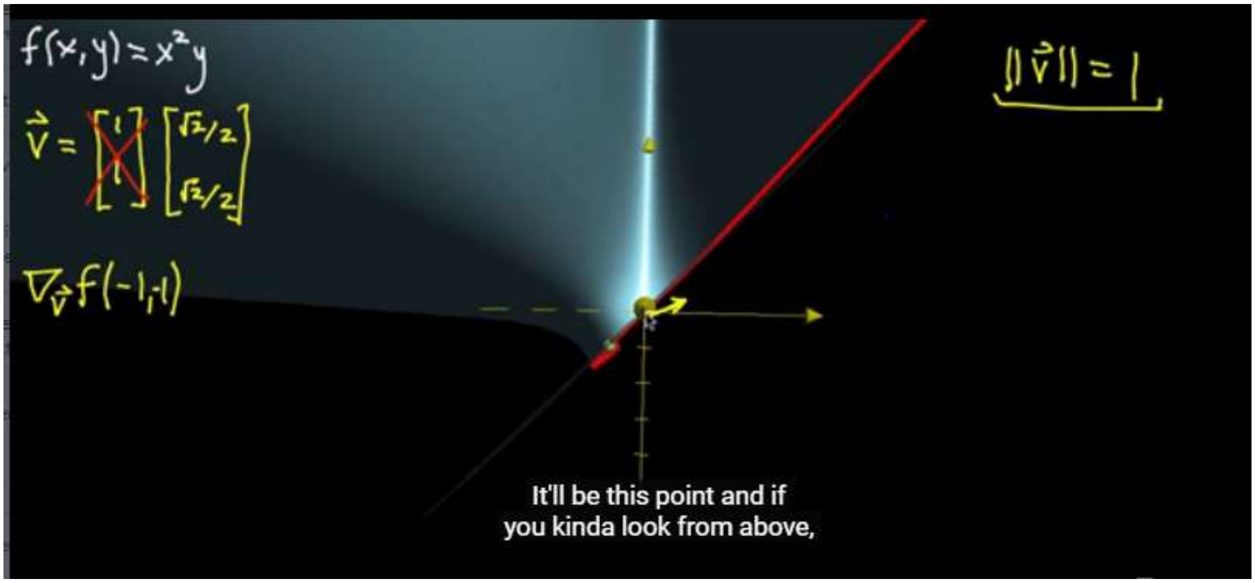


- Showing what is meant by that:
  - Imagine slicing graph by a plane
  - Plane does not have to be parallel to x or y axis (this is what we did for partial derivative, constant x or y value)
    - This plane will tell you what movement in the direction of your vector looks like

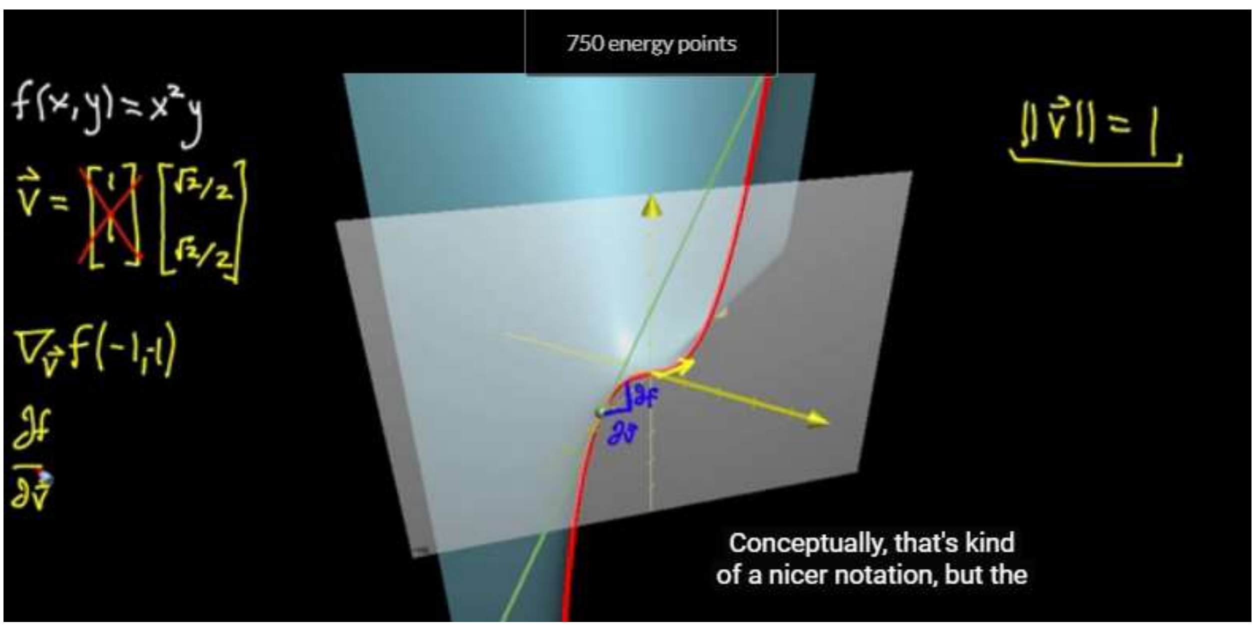


- Red line is where graph intersects that slice
- Vector v lives on x, y plane and determines the direction of plane we're slicing things with

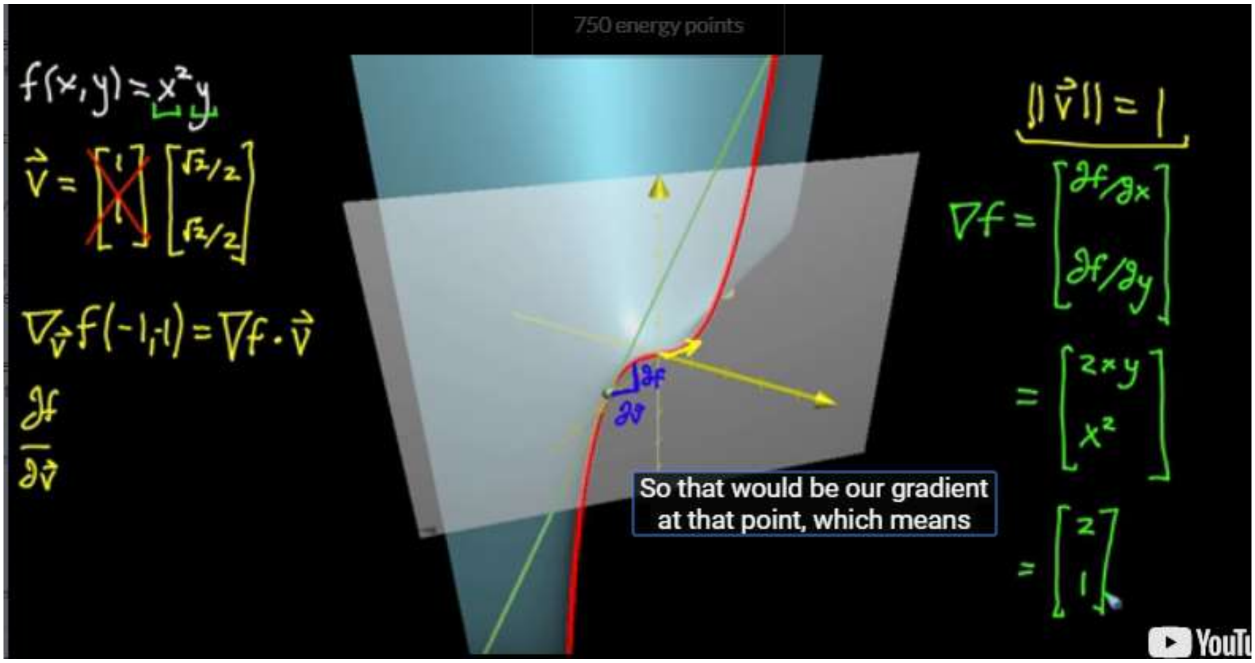
- Interpreting directional derivative at point (-1, -1):
    - It can be interpreted as slope
        - If vector v is a unit vector, it makes the interpretation easier
        - Instead of saying v = [1 1] we say v = [sqrt(2)/2 sqrt(2)/2]
            - Now v is unit vector with magnitude of 1



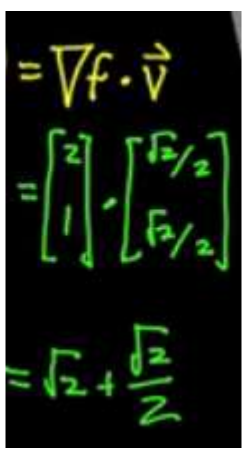
- Can also be written as df/dv at point (-1,-1)
  - Slight nudge in direction of vector v and seeing how the value of function changes
  - Nice notation



- Reason for nabla notation, is because it is indicative of how the directional derivative is actually computed
  - Take the gradient of function f and take the dot product with the vector v
  - Gradient of function f is a vector full of partial derivatives
  - Plug in point (-1,-1) to partial derivative



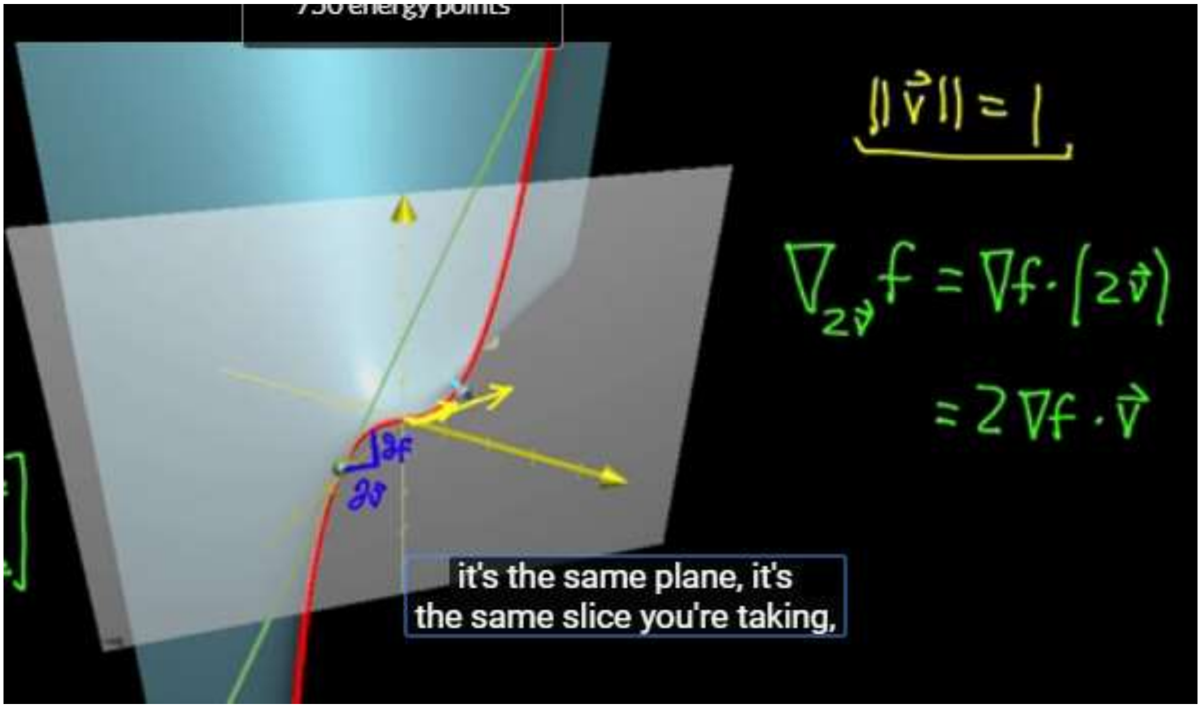
- This would be our gradient at point (-1,-1) which means
  If we want gradient of f times v would be :



- But this only works if v is a unit vector

- What if we scale v by 2? (alluding to formal definition of directional derivative)
  - Directional derivative along 2v of function f
  - You will get twice the value for your derivative
    - This is not what we want
    - The plane we sliced with is not changing direction of plane when we scale
    - It should have the same slope
    - This is an important point to think about when talking about directional derivatives and slopes

Conclusion:



Hence the formula for the slope of a graph in the direction of v is the directional derivative divided by magnitude of v, to make sure you are honing in on the direction of vector v and not the magnitude