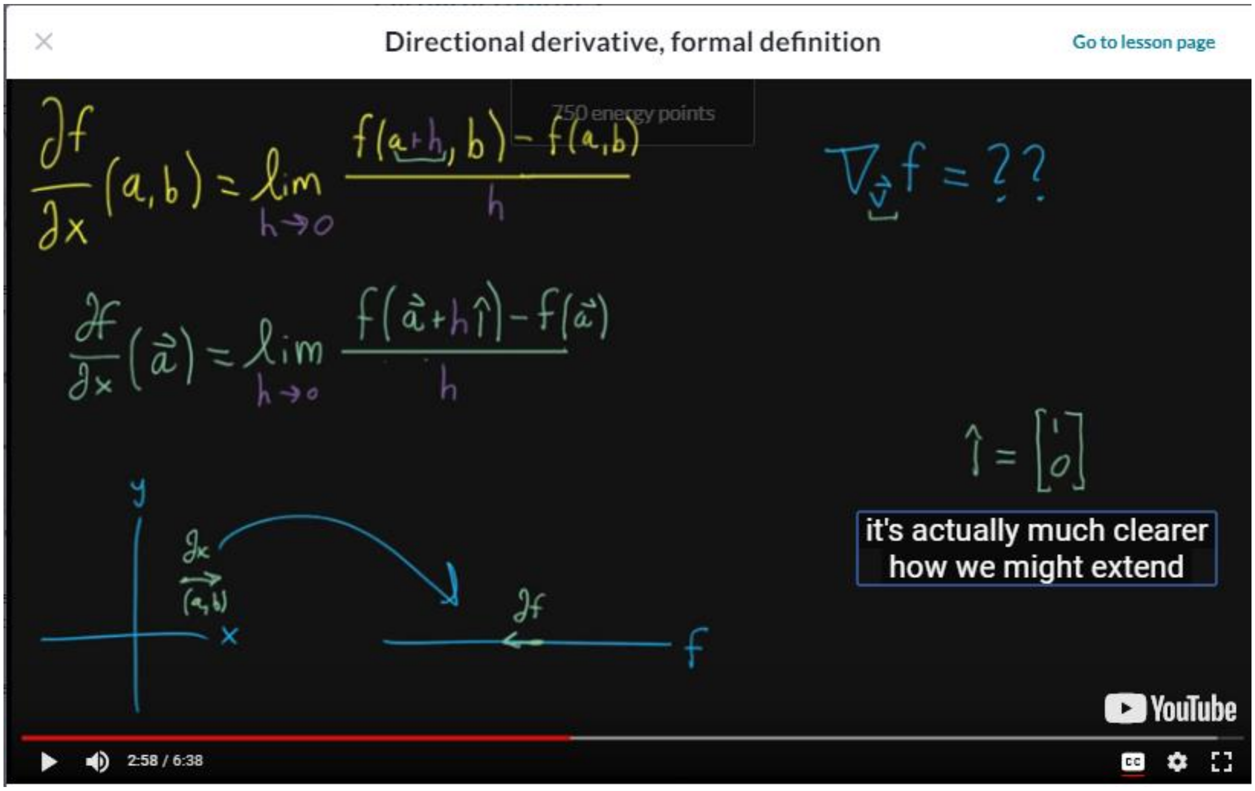
Directional Derivatives of Multivariable Functions



- Let’s start off with the formal definition of the partial derivative
  - We have our input space: X, Y Plane
  - And we think of it somehow mapping over to the real number line which is where your output f lives
  - Take partial derivative at some point (a,b)
    - We nudge it slightly in the X direction
    - See how that nudge influences the function
    - Size of nudge is dx and the impact on function as df
  - Think of h as the change in the input space
    - h is only changing the X component of the function

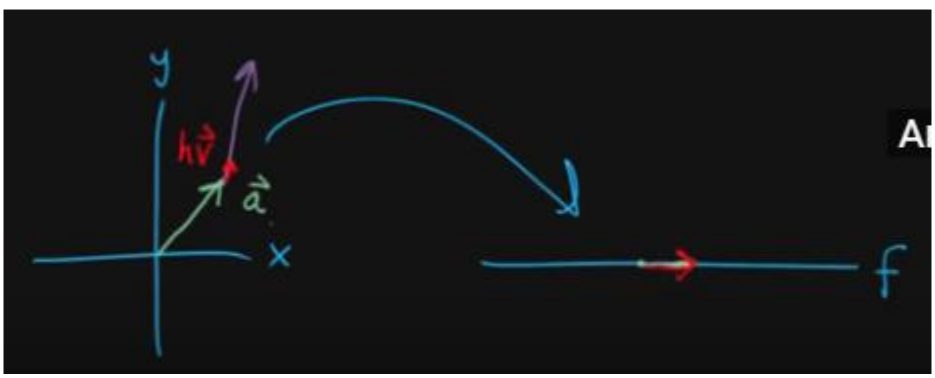
- Partial derivative rewritten (Moving towards directional derivatives, formal definition)
  - Same scenario as first partial derivative example
  - Instead of input of function f being (a,b) we notate the input as a two dimensional vector a
  - But now what do we add or how do we nudge vector a, in the previous example it was clear we could just add it to the first component (X component)
    - We add h times the unit vector in the X direction aka: i hat vector

- Subtract off function with original input, in this case the input is vector a

- Rewritten this way makes it much clearer how we can extend this idea to moving in different directions
- Now all info with respect to what direction you are moving is in the i hat unit vector

$$\nabla_{\vec{v}} f(\vec{a}) = \lim_{h \to 0} \frac{f(\vec{a} + h\vec{v}) - f(\vec{a})}{h}$$

- Formal definition:
  - Function of Original input vector a + h (that little nudge of a value) x vector v (whose direction we care about) – value of function F at that original input vector a



- Vector v is that purple vector that is neither purely in the x or y direction
- h is a scalar that scales down vector v to a tiny little nudge in that direction
  - now what you wonder is what effect this has on your output
  - the ratio between size of resulting nudge to output and the nudge itself is the directional derivative

- *** Notice if you scale value of vector v by 2, you will double the initial nudge and double the resulting nudge or output nudge by 2 even if we scale by same value h