

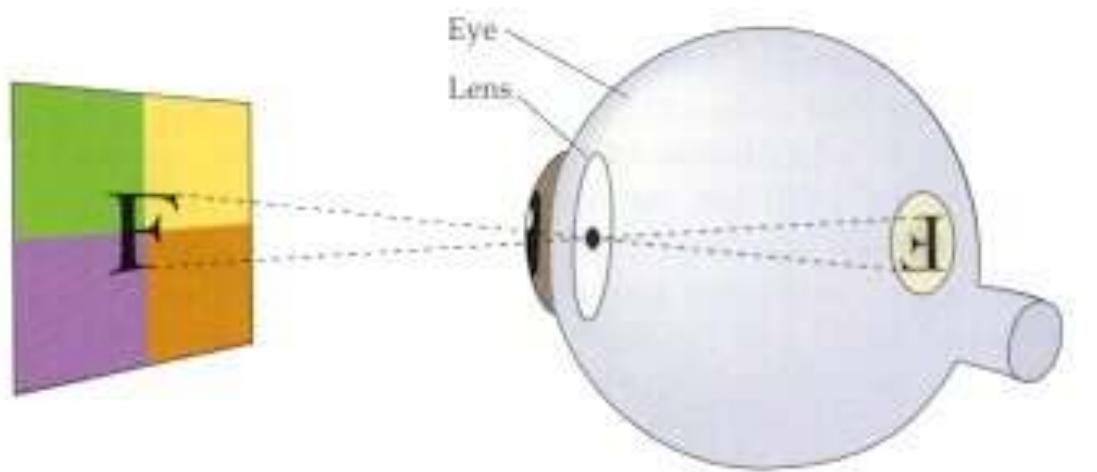
Image Formation (Cont'd)

&

Edge Detection

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(Taught by M. Salzmann)
IC-CVLab
EPFL

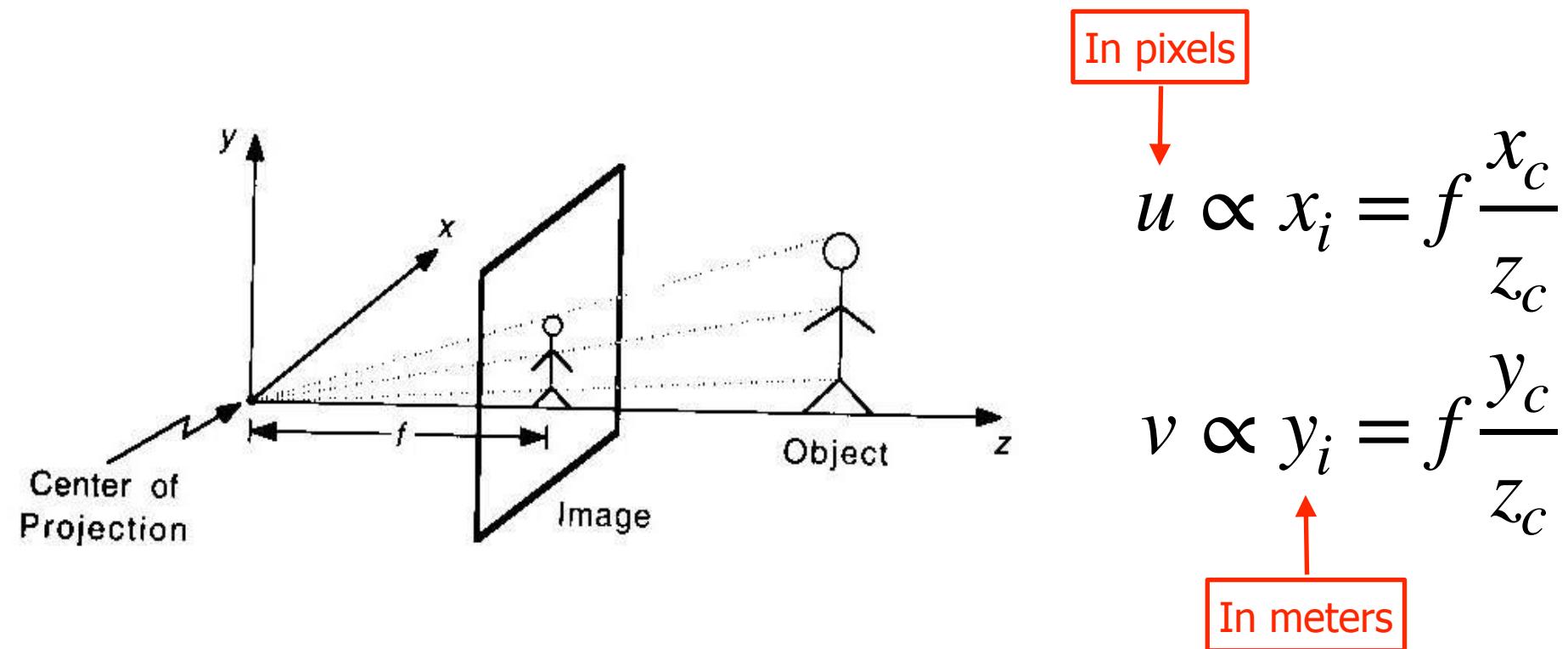
Reminder: Image Formation



Projection from surfaces to 2-D sensor.

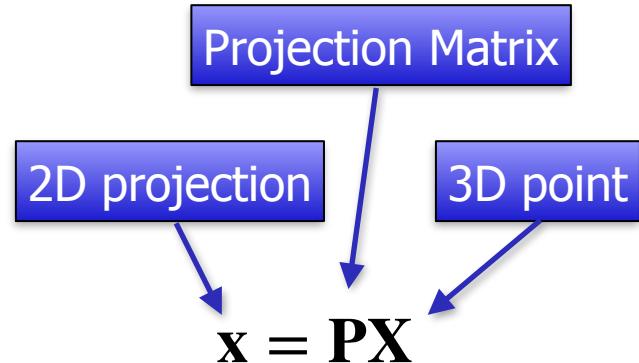
- Where: Geometry
- How bright: Radiometry
- Stored how: Sensing

Reminder: Pinhole Camera Model



→ Reformulate it as a linear operation using homogeneous coordinates.

Reminder: Projection in Homogeneous Coordinates



$$\mathbf{P} = \mathbf{K} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{Rt}$$

$$\text{with } \mathbf{K} = \begin{bmatrix} \alpha_u & s & p_u \\ 0 & \alpha_v & p_v \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{Rt} = \begin{bmatrix} \mathbf{R} & \mathbf{T} \\ \mathbf{0} & 1 \end{bmatrix}, \text{ and } \mathbf{R}^T \mathbf{R} = \mathbf{I}.$$

Intrinsics

Extrinsics

Reminder: Camera Calibration

Internal Parameters:

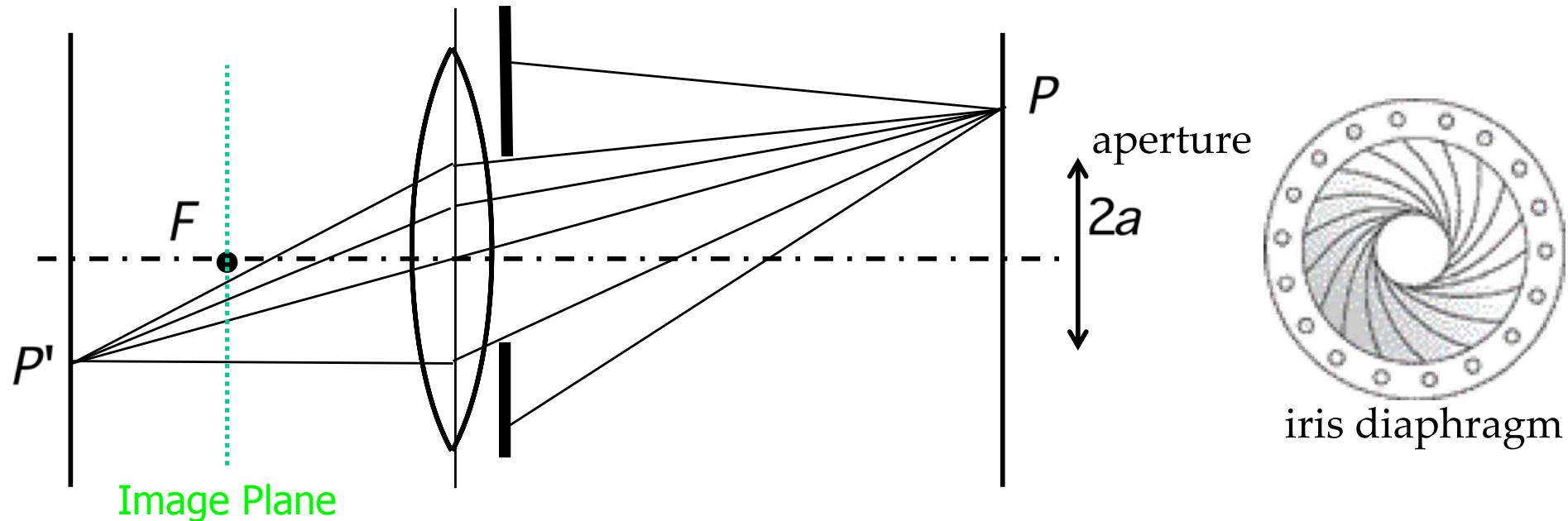
- Horizontal and vertical scaling (2)
- Principal points (2)
- Skew of the axis (1)

External Parameters:

- Rotations (3)
- Translations (3)

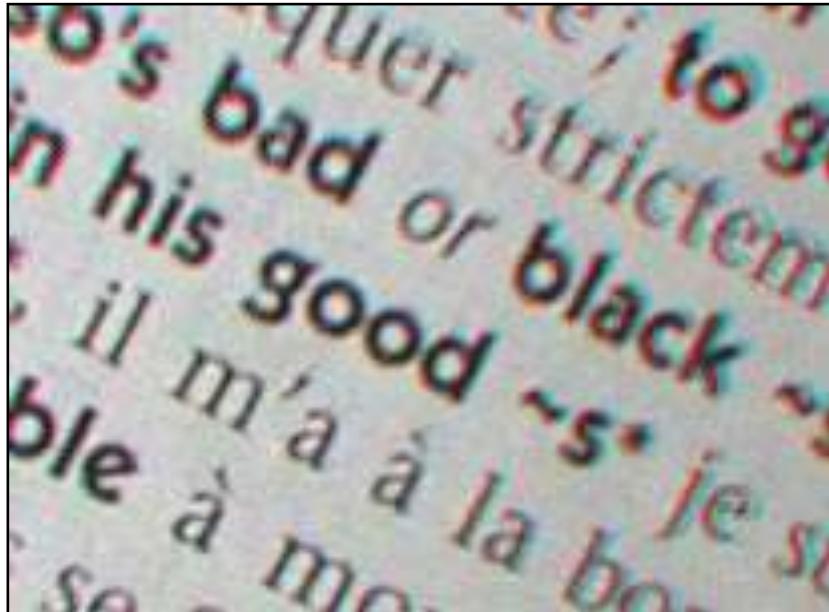
→ There are 11 free parameters to estimate. This is known as **calibrating** the camera.

Reminder: Thin Lens



- Diameter $d=2a$ of the lens that is exposed to light.
- The image plane is not located exactly where the rays meet.
- The greater a , the more blur there will be.

Reminder: Distortions



The lens is not exactly a “thin lens:”

- Different wave lengths are refracted differently,
- Barrel Distortion.

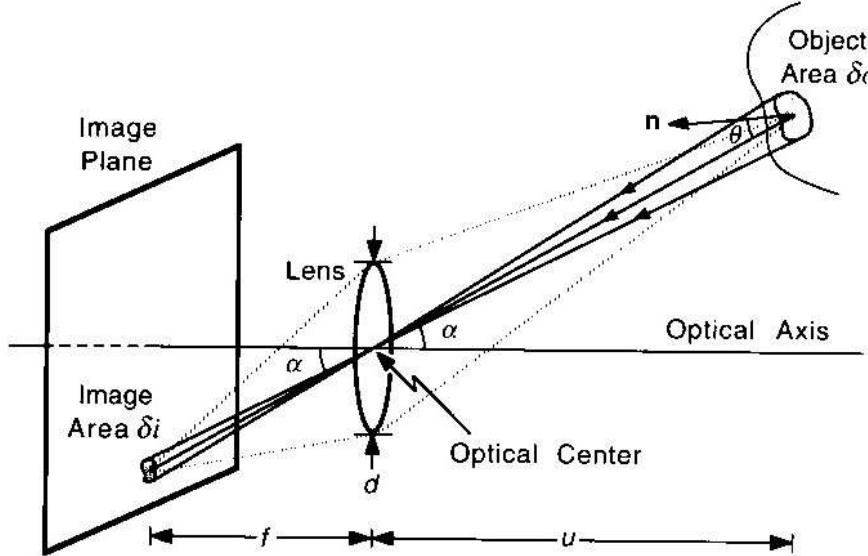
Undistorting



Once the image is undistorted, the camera projection can be formulated as a projective transform.

- The pinhole camera model applies.

Fundamental Radiometric Equation



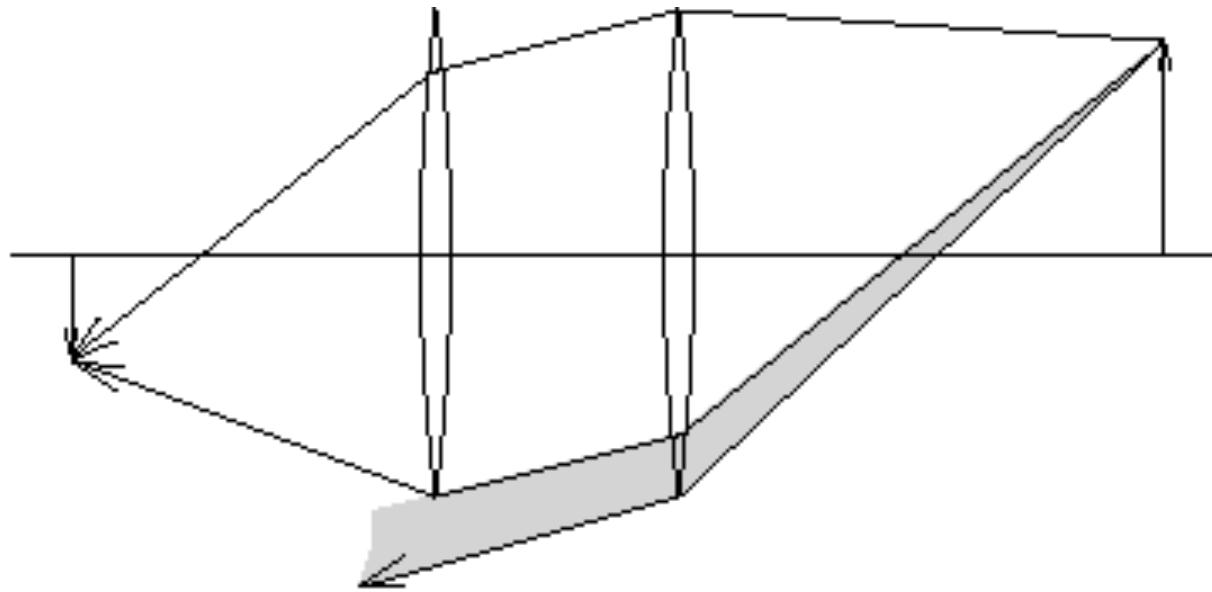
Scene Radiance (Rad) : Amount of light radiation emitted from a surface point (Watt / m² / Steradian).

Image Irradiance (Irr): Amount of light incident at the image of the surface point (Watt / m²).

$$\text{Irr} = \frac{\pi}{4} \left(\frac{d}{f}\right)^2 \cos^4(\alpha) \text{Rad} ,$$

⇒ Irr ∝ Rad for small values of α .

Vignetting



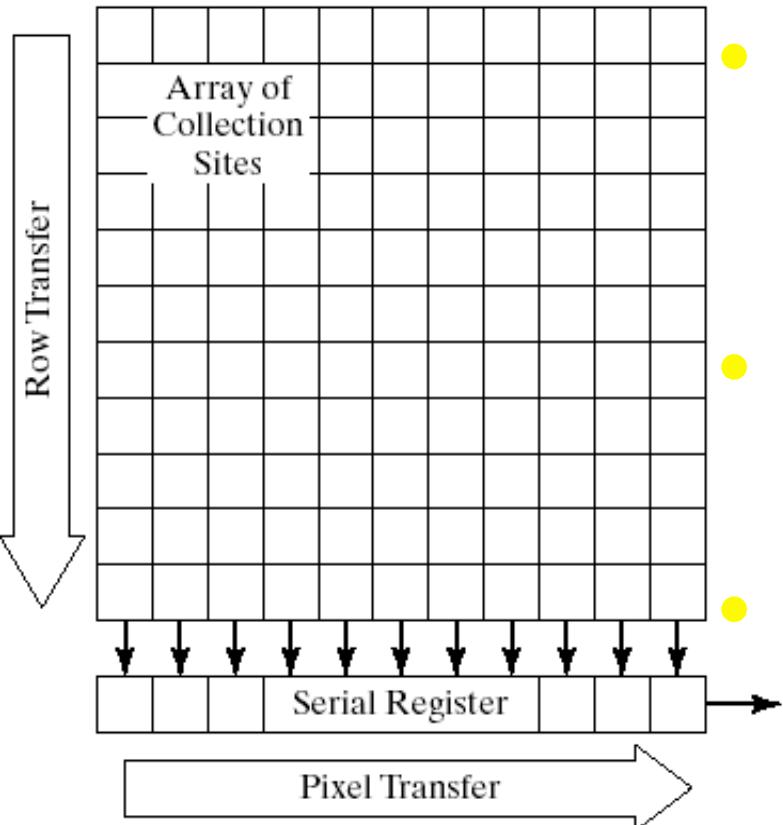
Images can get darker towards their edges because some of the light does not go through all the lenses.

De Vignetting



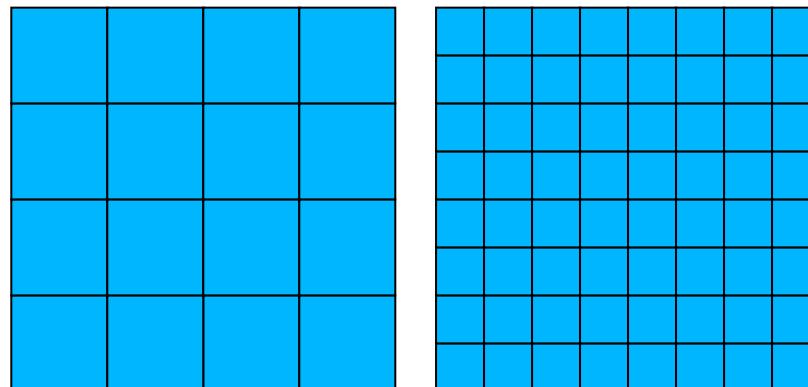
—> As for geometric undistortion, undo vignetting to create an image that an ideal camera would have produced.

Sensor Array



- Photons free up electrons that are then captured by a potential well.
- Charges are transferred row by row wise to a register.
- Pixel values are read from the register.

Sensing



Conversion of the “optical image” into an “electrical image”:

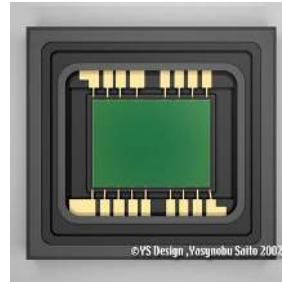
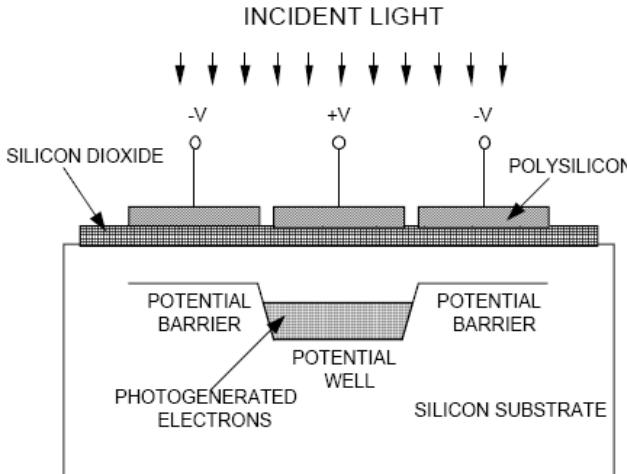
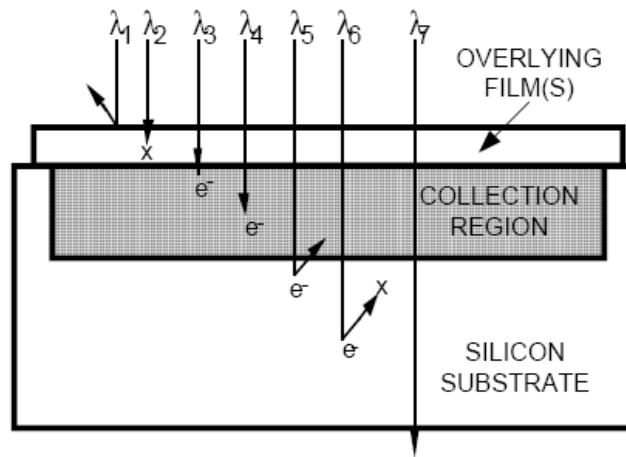
$$E(x, y) = \int_{t_0}^{t_1} \int_0^\Lambda \text{Irr}(x, y, t, \lambda) s(\lambda) dt d\lambda$$

$$I(m, n) = \text{Quantize}\left(\int_{x_0}^{x_1} \int_{y_0}^{y_1} E(x, y) dx dy \right)$$

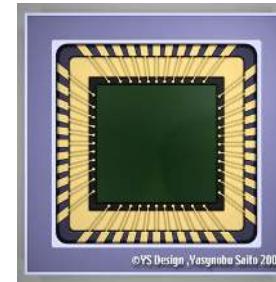
→ Quantization in

- Time
- Space

Sensors



CCD



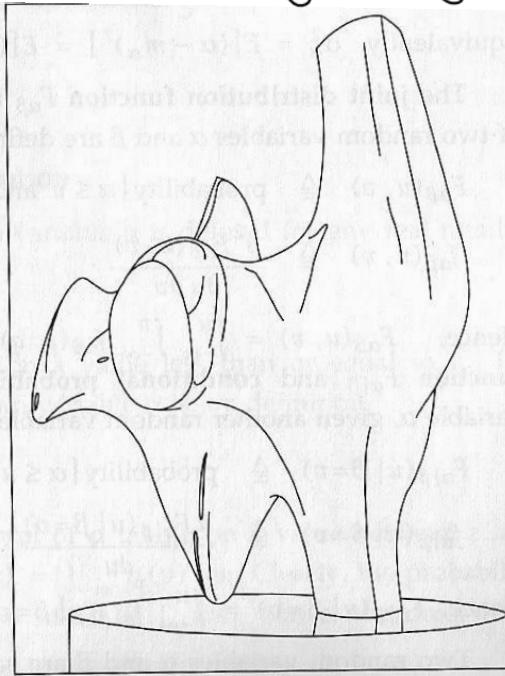
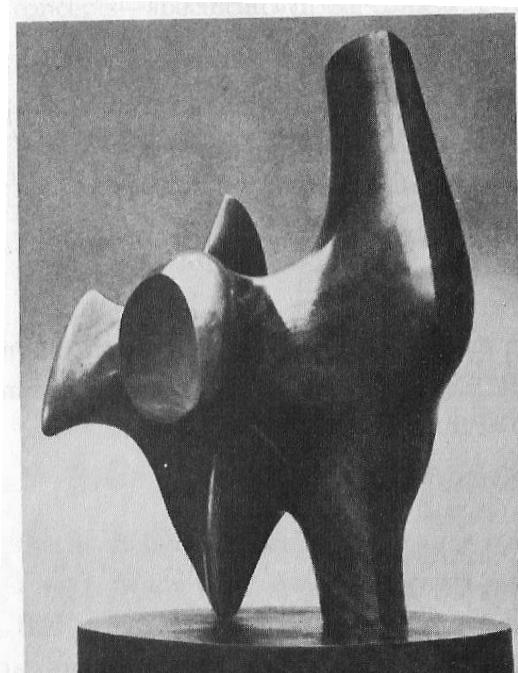
CMOS

- Charged Coupling Devices (CCD): Made through a special manufacturing process that allows the conversion from light to signal to take place in the chip without distortion.
- Complimentary Metal Oxide Semiconductor (CMOS): Easier to produce and similar quality. Now used in most cameras except when quantum efficient pixels are needed, e.g. for astronomy.

In Short

- Camera geometry can be modeled in terms of the pinhole camera model, which is linear in projective space.
- Image radiance is roughly proportional to surface radiance and the two can be used interchangeably for our purposes.

Edge Detection



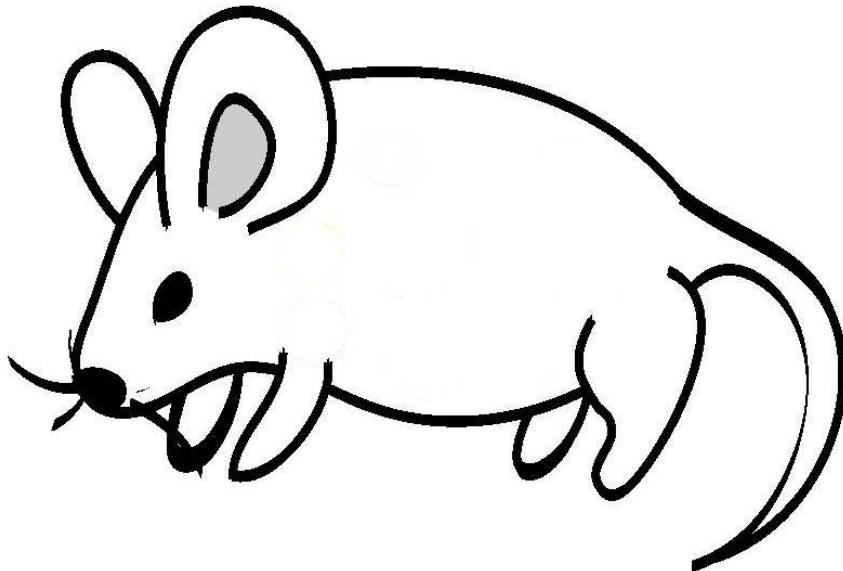
v info

v contours carry a lot of info

Salient contours

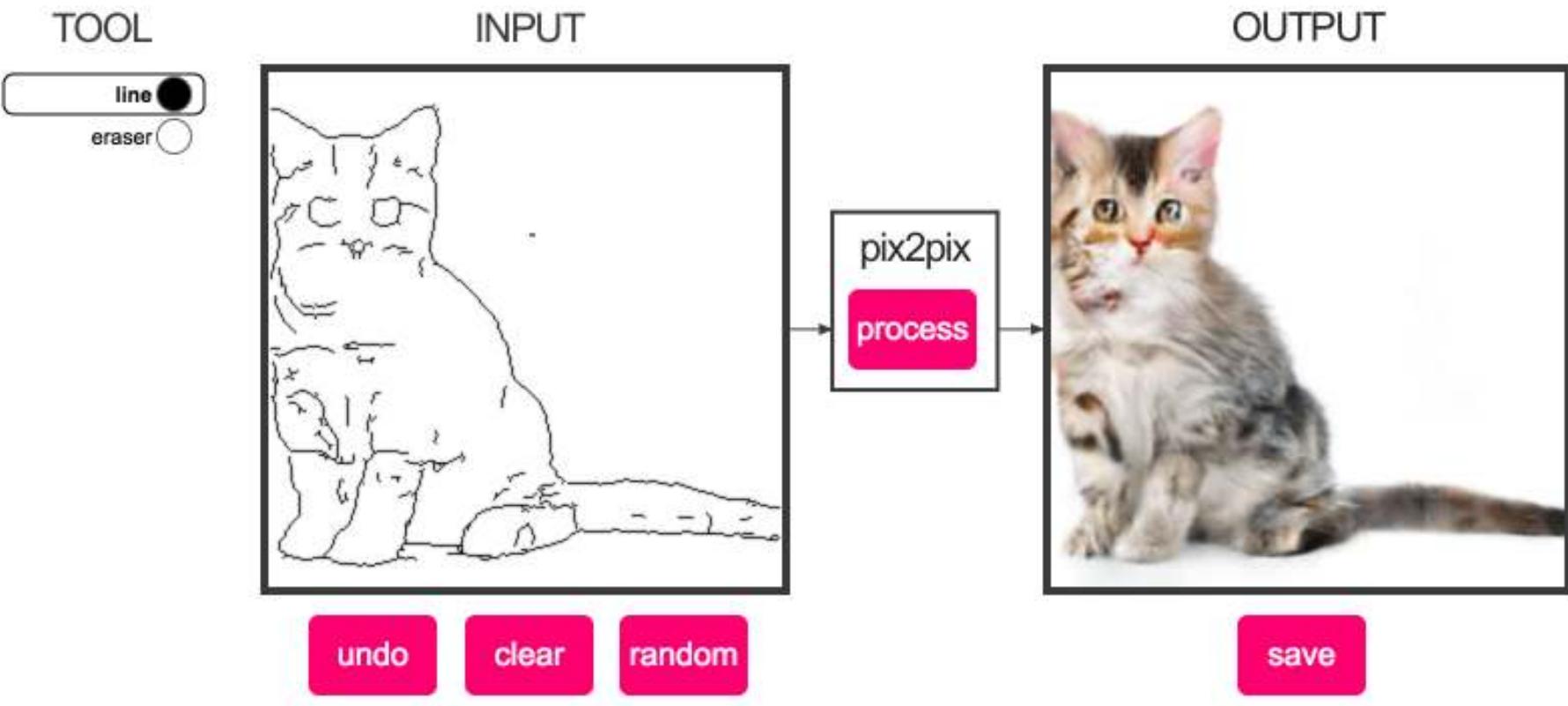
- What's an edge
- Image gradients
- Edge operators

Line Drawings



- Edges seem fundamental to human perception.
- They form a compressed version of the image.

From Edges To Cats



Deep-Learning based generative model.

Maps and Overlays

foods \rightarrow are contaminated



Distance/Rad Between A and B

Corridor

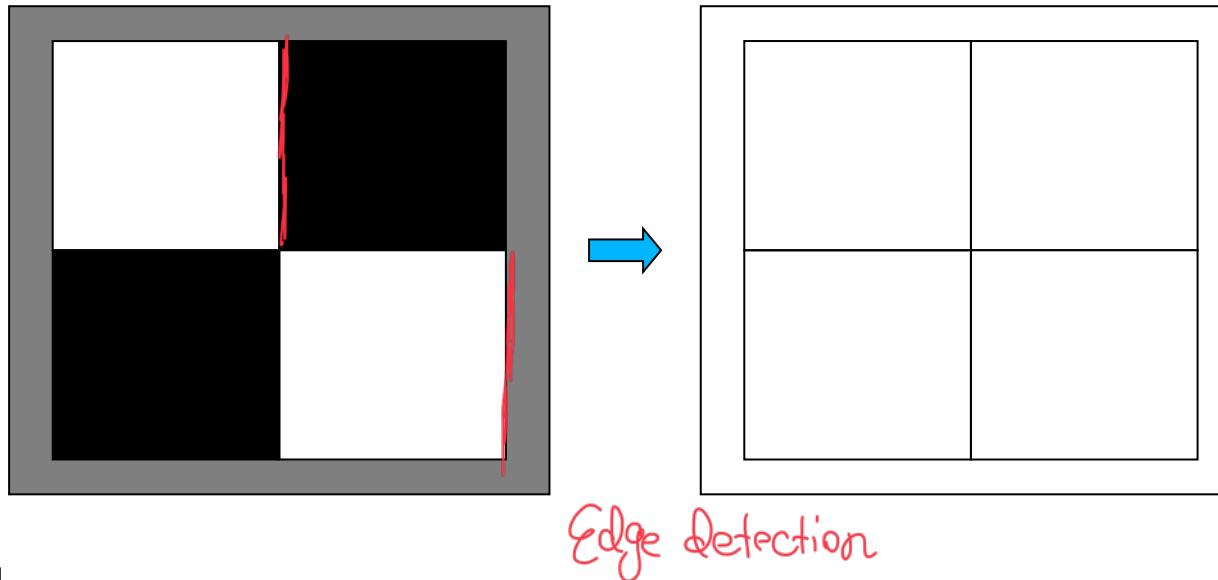
projected in the right place
in Real-time, it calculates projection matrix so that 3D contours get



Corridor



Edges and Regions



Edges:

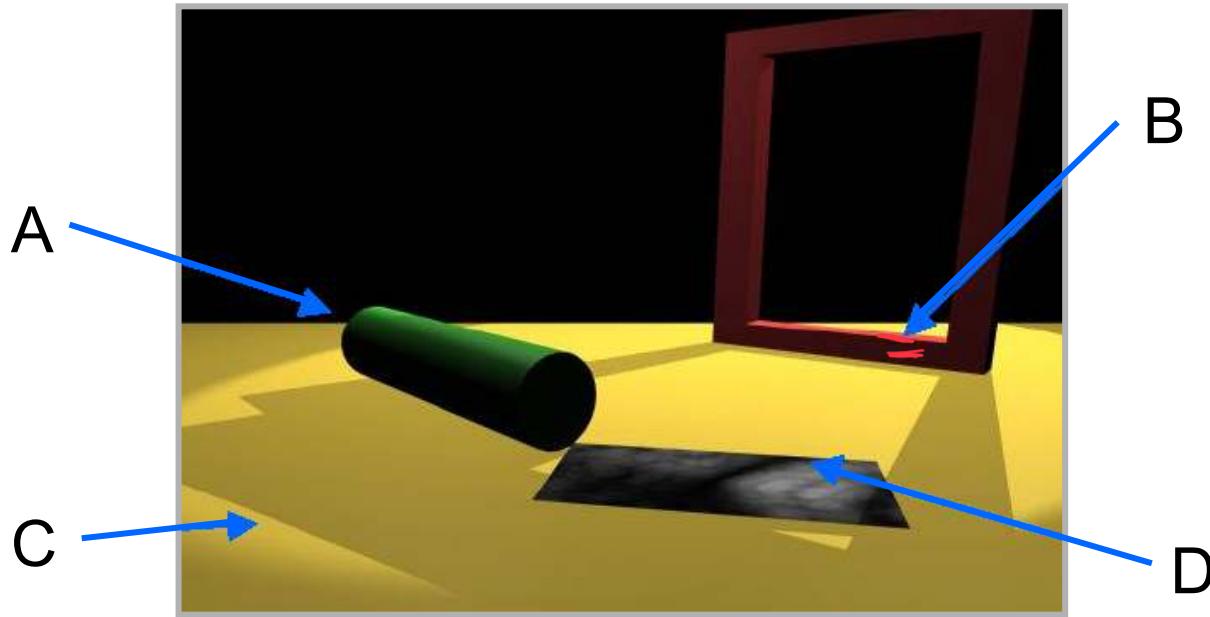
- Boundary between bland image regions.

Regions:

- Homogenous areas between edges.

→ Edge/Region Duality.

Discontinuities



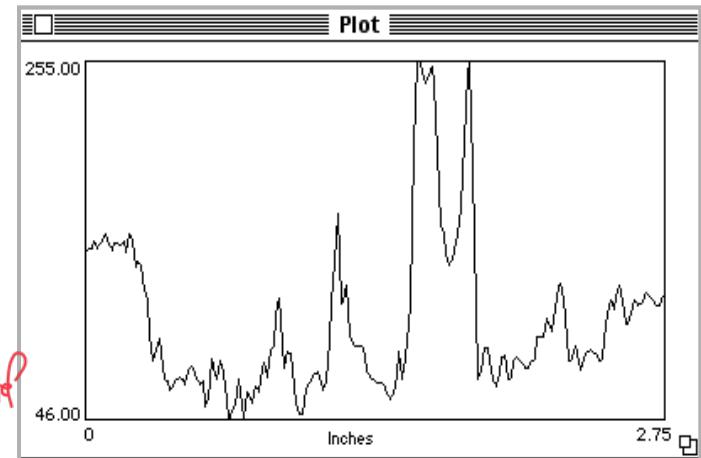
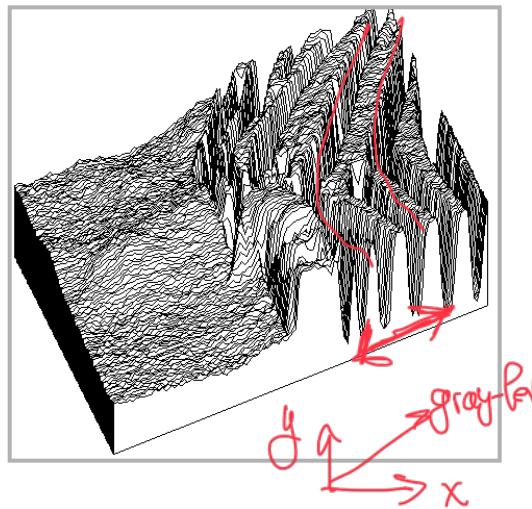
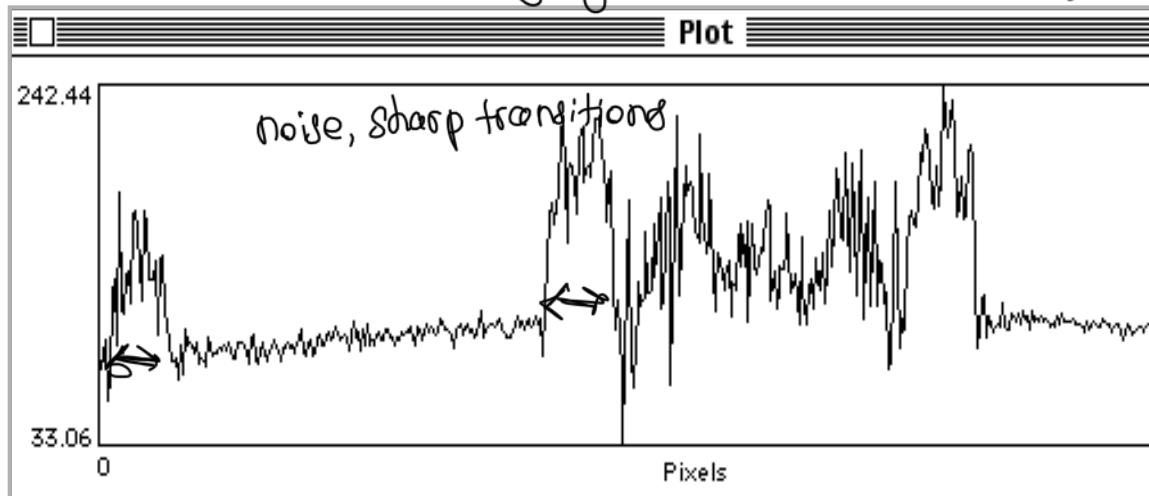
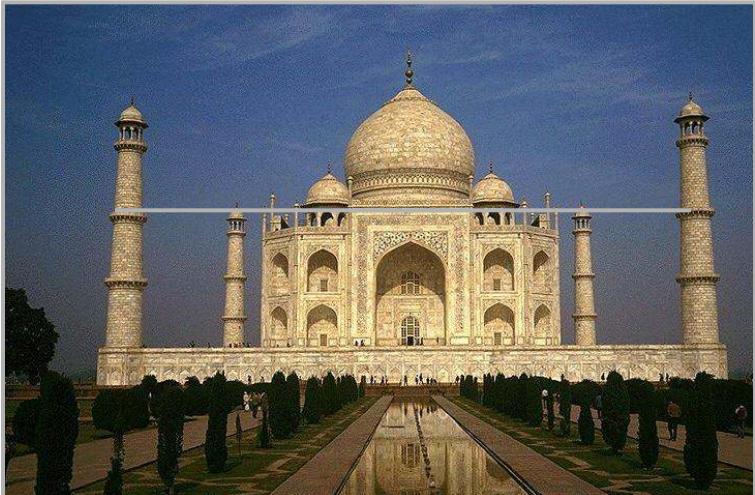
depth changes abruptly in projection

- A. Depth discontinuity: Abrupt depth change in the world
- B. Surface normal discontinuity: Change in surface orientation *change of contrast*
- C. Illumination discontinuity: Shadows, lighting changes
- D. Reflectance discontinuity: Surface properties, markings

→ Sharply different Gray levels on both sides

REALITY

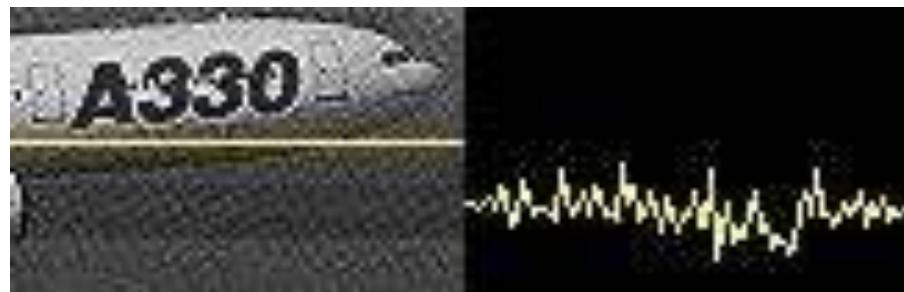
grey Levels (average of R, G, B)



More Reality



*physical finding meaningful
discontinuities in
noisy signals → hard*



Very noisy signals
→ Prior knowledge is required!!

Optional: Illusory Contours



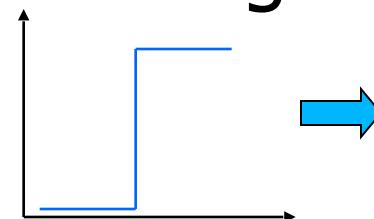
- No closed contour, but we still perceived an edge.
- This will not be further discussed in this class.

Ideal Step Edge

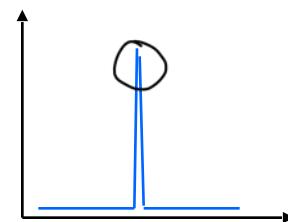
Rapid change in image => High local gradient

$f(x)$ = step edge

Abrupt jump

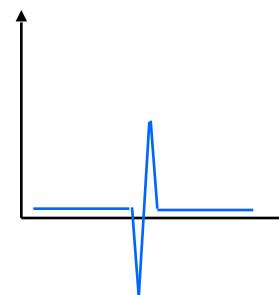


1st Derivative $f'(x)$

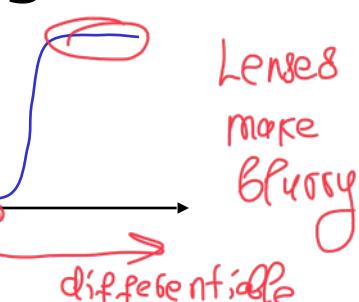


maximum
of 1st derivative

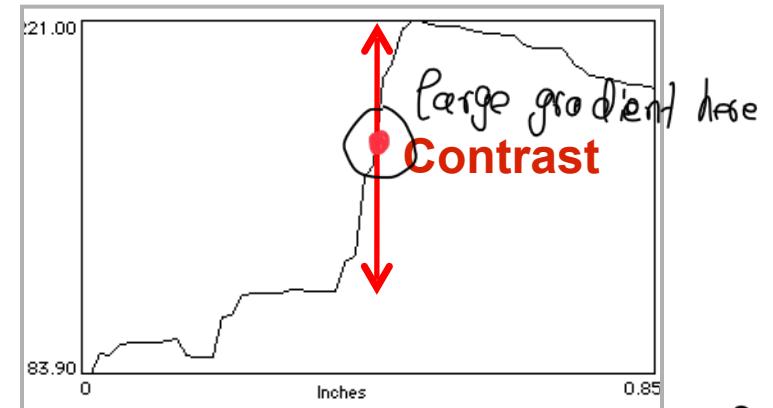
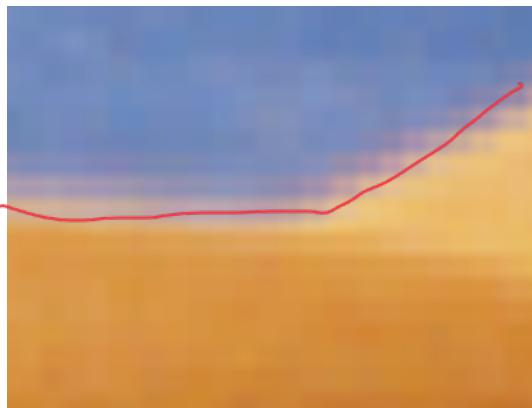
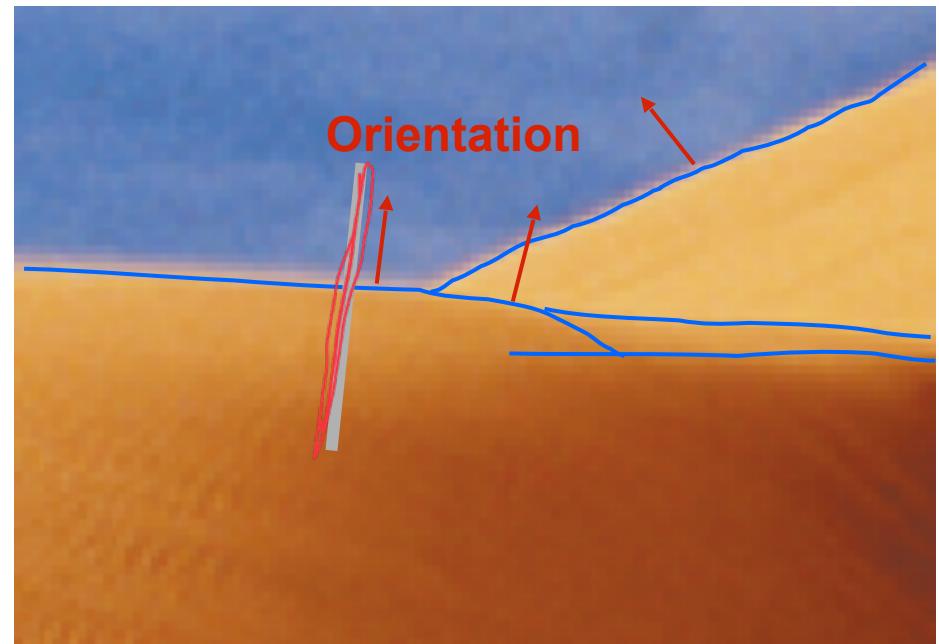
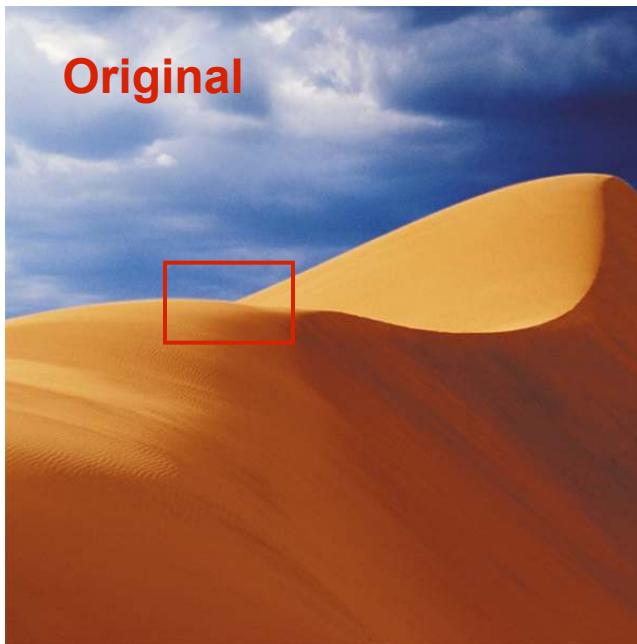
2nd Derivative $f''(x)$



zero crossing
of 2nd derivative

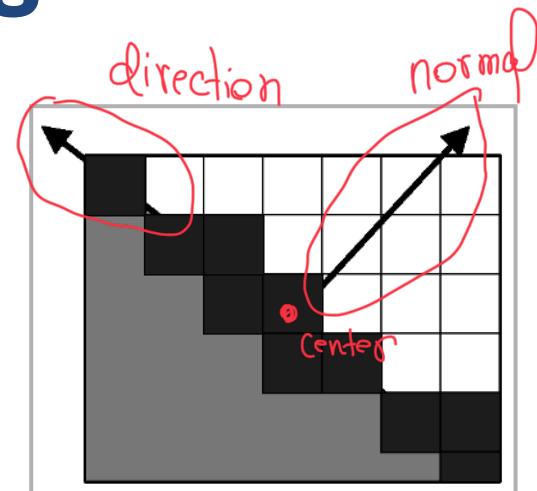


Edge Properties



Edge is located at a place where the slope in the direction of normal to a edge is largest

Edge Descriptors



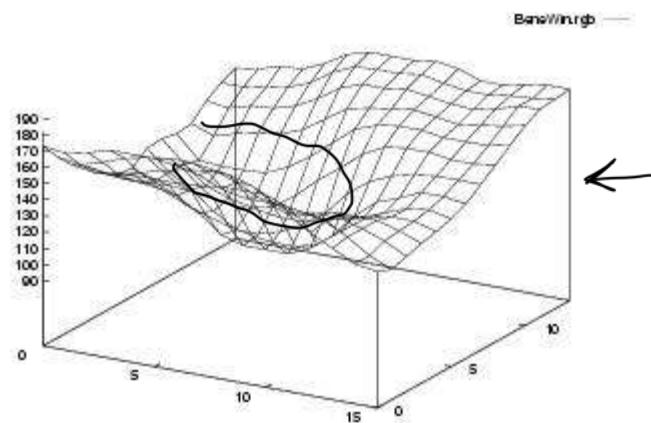
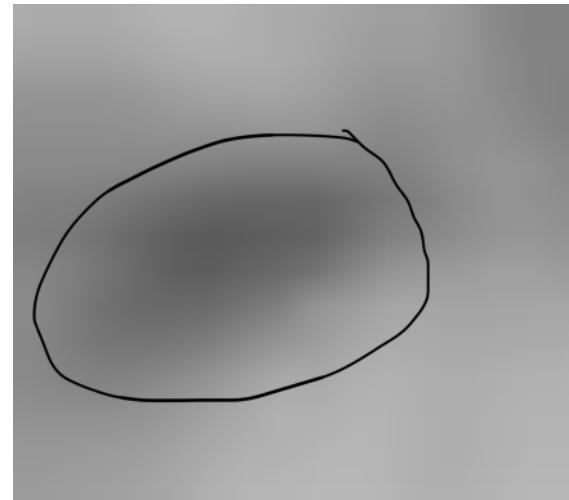
- Edge Normal:
 - Unit vector in the direction of maximum intensity change
- Edge Direction:
 - Unit vector perpendicular to the edge normal
- Edge position or center
 - Image location at which edge is located
- Edge Strength
 - Speed of intensity variation across the edge.

Derivative of gray
edge normal^{level} direction
in

Images as 3-D Surfaces



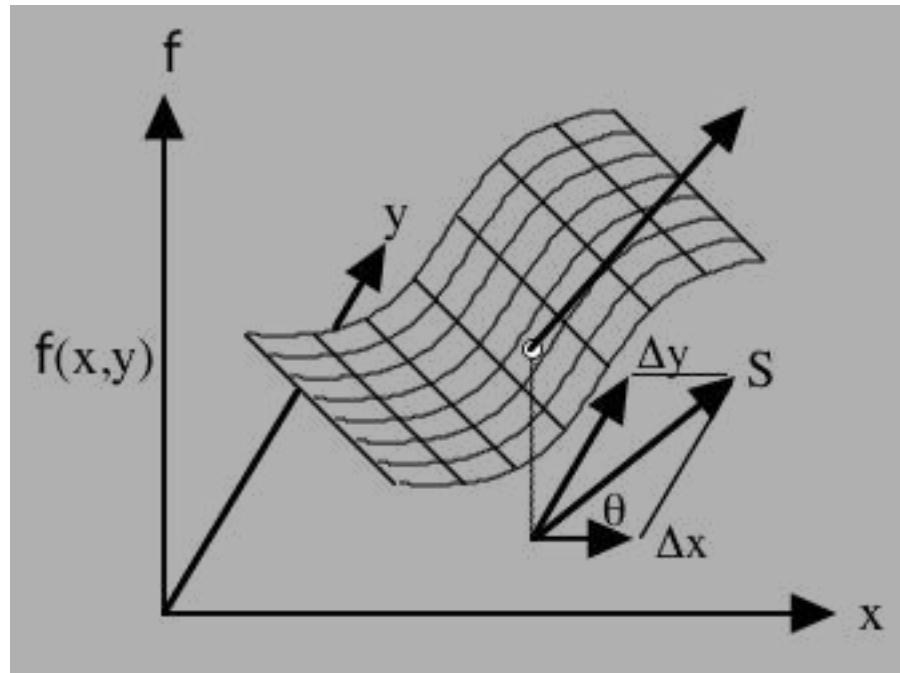
Q& Q Landscape



highest slopes in
gray-level Landscape

elevation \rightarrow gray-level
for pixel (x,y)

Geometric Interpretation



Since $I(x,y)$ is not a continuous function:

1. Locally fit a smooth surface.
2. Compute its derivatives.

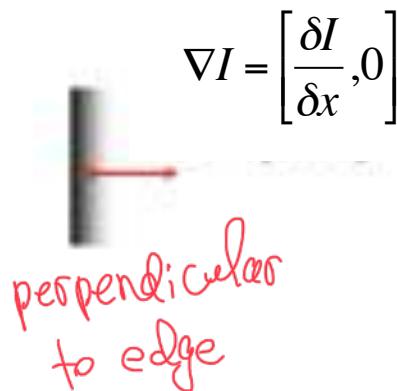
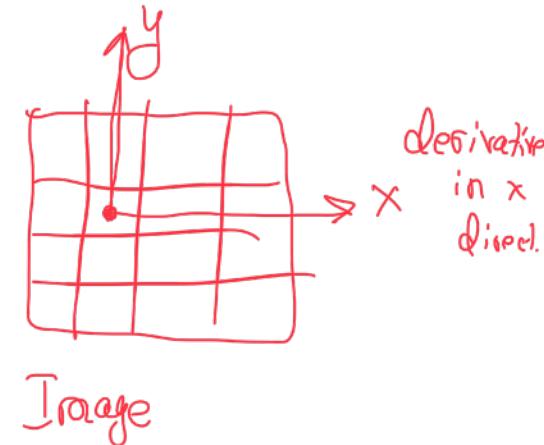
*and find where derivatives
high ↳ dealing
with edges*

Image Gradient

The gradient of an image

$$\nabla I = \left[\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y} \right]$$

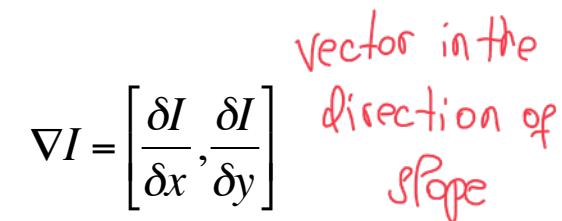
points in the direction of most rapid change in intensity.



$$\nabla I = \left[\frac{\delta I}{\delta x}, 0 \right]$$

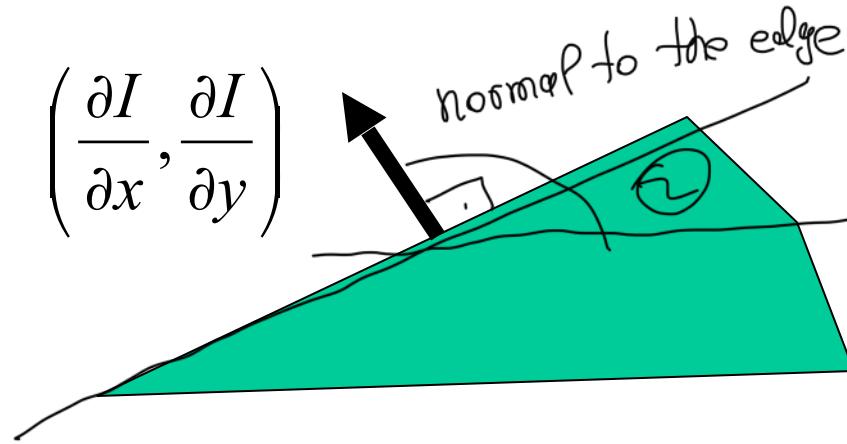


$$\nabla I = \left[0, \frac{\delta I}{\delta y} \right]$$



$$\nabla I = \left[\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y} \right]$$

Magnitude And Orientation

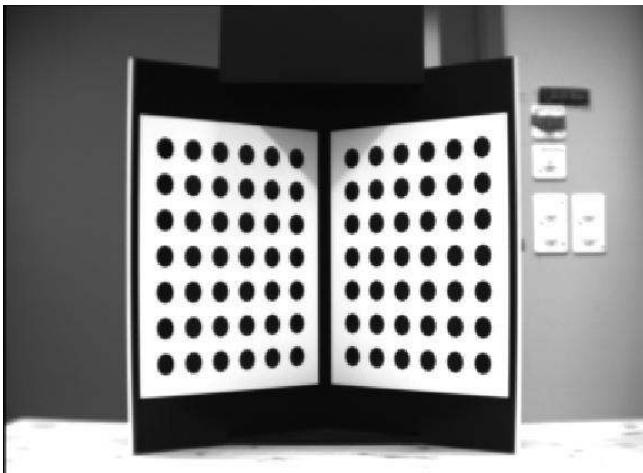


Measure of contrast : $G = \sqrt{\frac{\partial I^2}{\partial x} + \frac{\partial I^2}{\partial y}}$

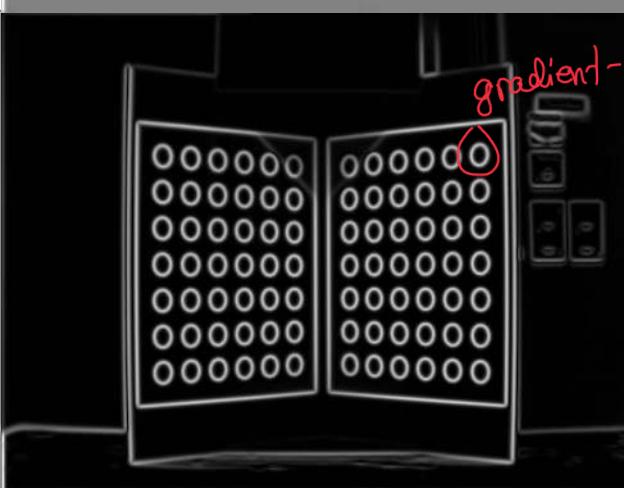
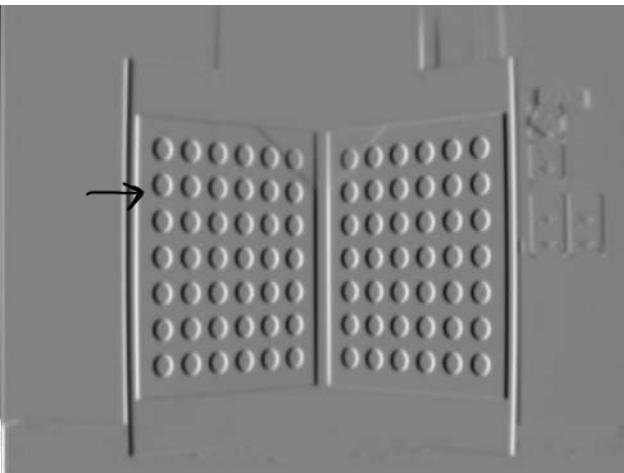
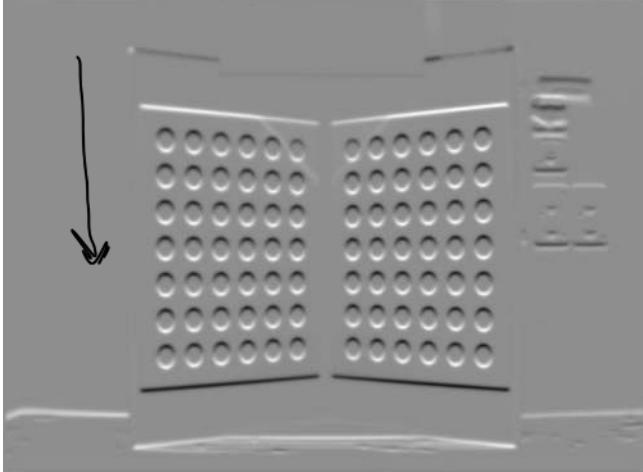
Edge orientation : $\theta = \arctan\left(\frac{\partial I}{\partial y}, \frac{\partial I}{\partial x}\right)$

Gradient Images

I



$$\frac{\partial I}{\partial y}$$



black $\rightarrow (-)$
gray $\rightarrow 0$
white $\rightarrow (+)$

$$\frac{\partial I}{\partial x}$$

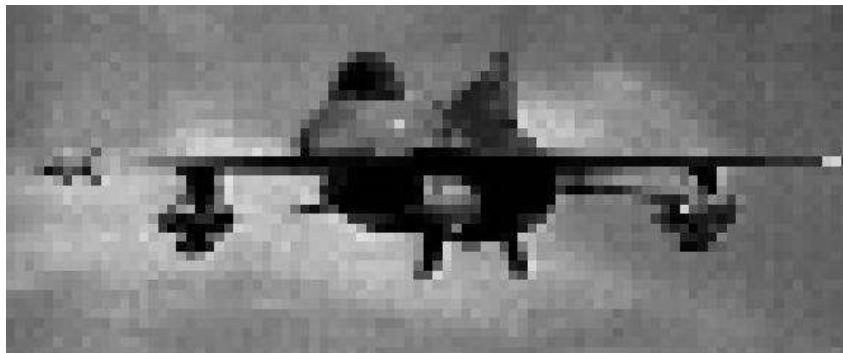
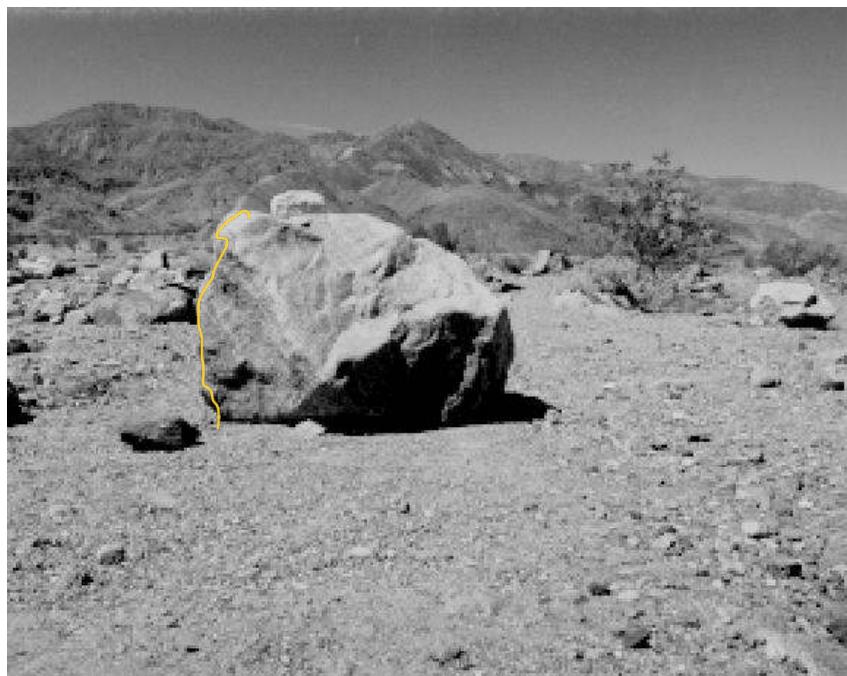
gradient - almost constant

$$\sqrt{\frac{\partial I^2}{\partial x} + \frac{\partial I^2}{\partial y}}$$

invariant

The gradient magnitude is unaffected by orientation

Real Images



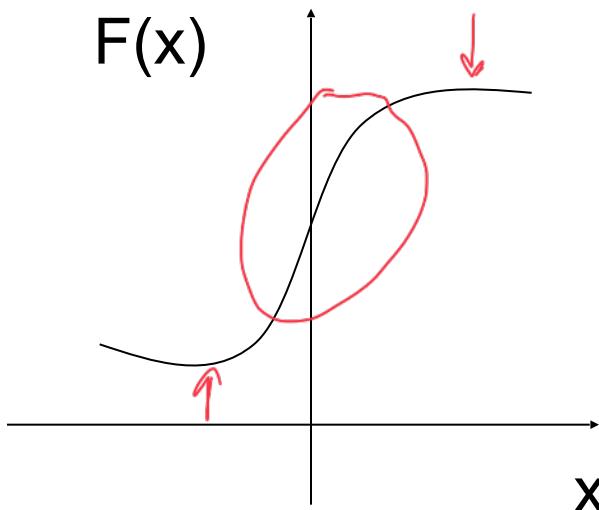
... but not directly usable in most real-world images.

Edge Operators

How do we compute gradients?

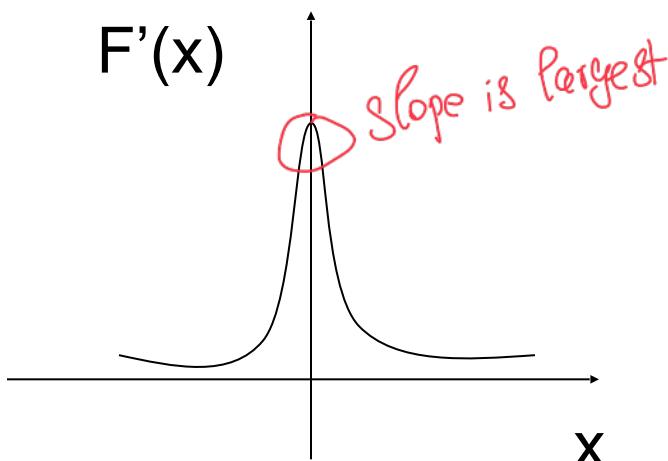
- Difference Operators
 - Convolution Operators
 - Trained Detectors
 - Deep Nets
- ↗ explicit evaluations

Gradient Methods



gray-level goes from low level
to high level

Edge = Sharp variation

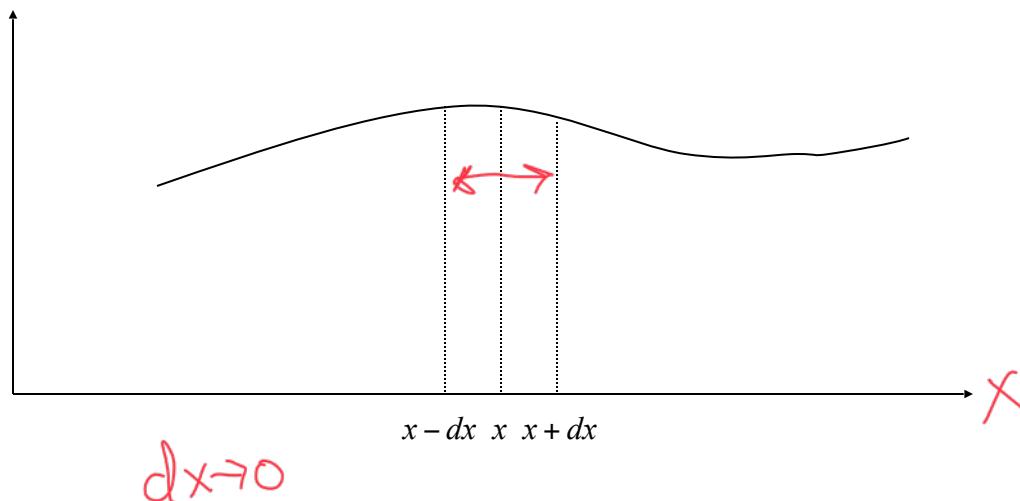


Large first derivative

1D Finite Differences

Digital images \rightarrow 1D
=

In one dimension:



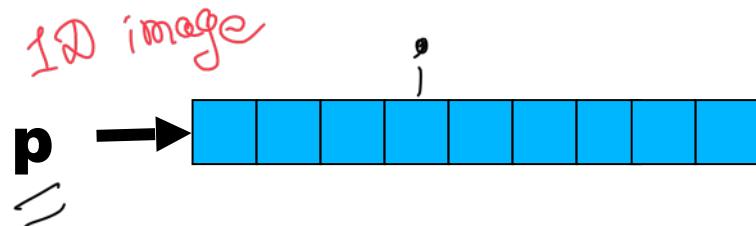
$$\frac{df}{dx} \approx \frac{f(x+dx) - f(x)}{dx} \approx \frac{f(x+dx) - f(x-dx)}{2dx}$$

Approximate

$$\frac{d^2f}{dx^2} \approx \frac{f(x+dx) - 2f(x) + f(x-dx)}{dx^2}$$

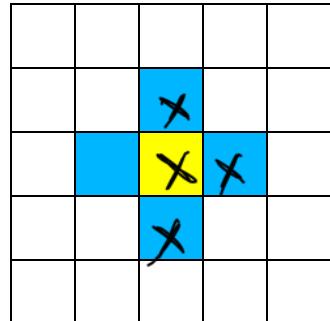
Coding 1D Finite Differences

Line stored as an array:



- for i in range(n-1):
 $\underline{q[i]} = \underline{(p[i+1]-p[i])}$ derivative
- for i in range(1,n-1):
 $q[i] = (p[i+1]-p[i-1])/2$ *other version
of derivative*
 $\partial x=1$
- $q = (p[2:]-p[:-2])/2$ much faster, no loops

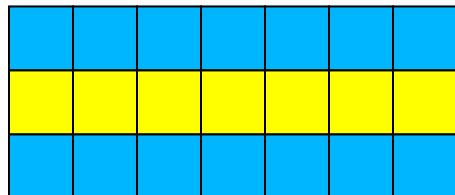
2D Finite Differences



$$\frac{\partial f}{\partial x} \approx \frac{f(x + dx, y) - f(x, y)}{dx} \approx \frac{f(x + dx, y) - f(x - dx, y)}{2dx}$$

$$\frac{\partial f}{\partial y} \approx \frac{f(x, y + dy) - f(x, y)}{dy} \approx \frac{f(x, y + dy) - f(x, y - dy)}{2dy}$$

Coding 2D Finite Differences



Python

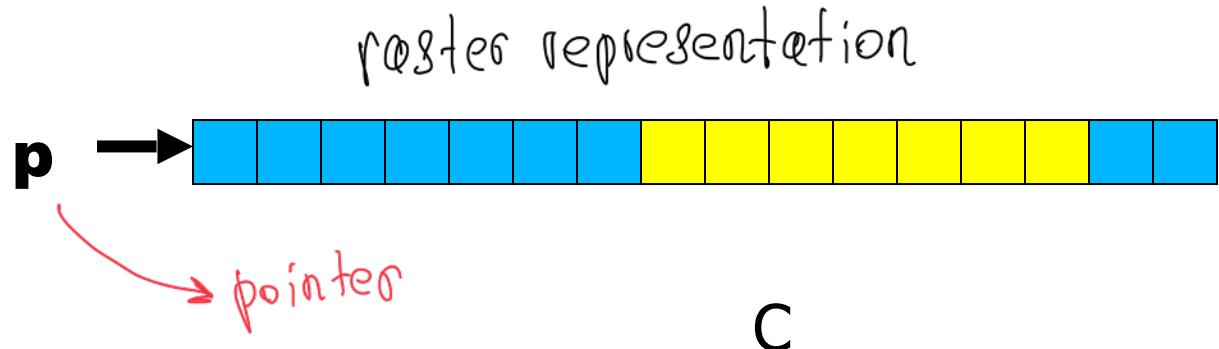


Image stored as a 2D array:

- $dx = p[1:,:] - p[:-1,:]$ difference between immediate neighbors
- $dy = p[:,1:] - p[:,:-1]$

$x+dx, x-dx$

- $dx = (p[2:,:] - p[:-2,:])/2$
- $dy = (p[:,2:] - p[:,:-2])/2$

Image stored in raster format:

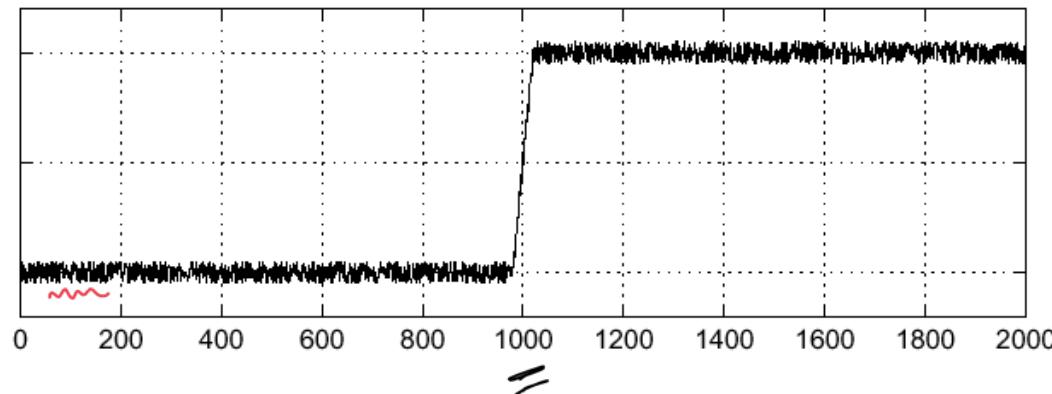
```
{  
    int i;  
    for(i=0;i<xdim;i++){  
        dx[i] = p[i+1] - p[i];  
        dy[i] = p[i+xdim] - p[i];  
    }  
}
```

pixel immediately below i

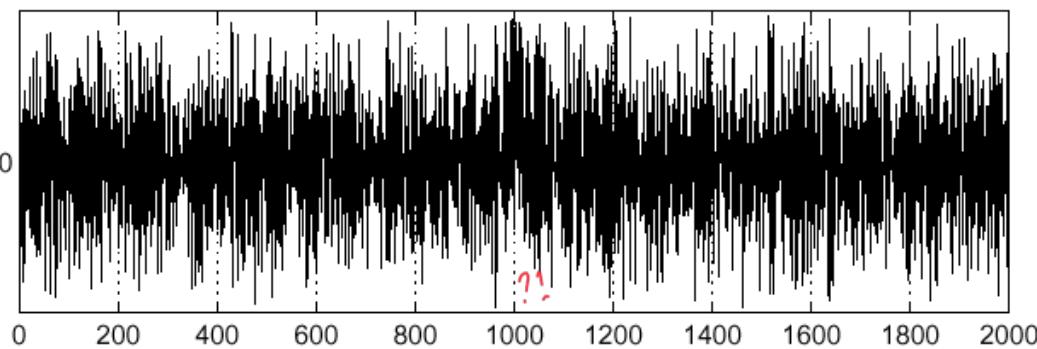
- Only 1D array accesses
 - No multiplications
- Can be exploited to increase speed.

Noise in 1D

Consider a single row or column of the image:



$$\frac{d}{dx} f(x)_0$$



noise magnified

noisy derivatives

Fourier Interpretation

$f \xrightarrow{\quad} F$: Fourier transform

Function	Fourier Transform
$\frac{df}{dx}(x)$	$uF(u) \quad \stackrel{\uparrow u \Rightarrow \text{freq}}{=} \quad \text{frequency domain}$
$\frac{\delta f}{\delta x}(x, y)$	$uF(u, v)$
$\frac{\delta f}{\delta y}(x, y)$	$vF(u, v)$

noise = bunch of freq's

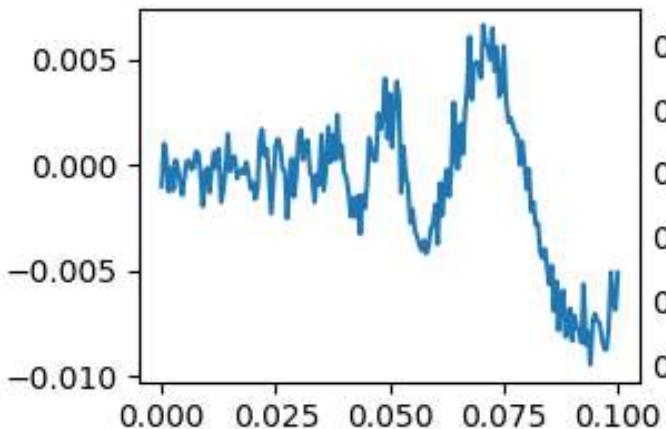
→ Differentiating emphasizes high frequencies
and therefore noise!

large $u, v \Rightarrow \uparrow \text{frequency}$

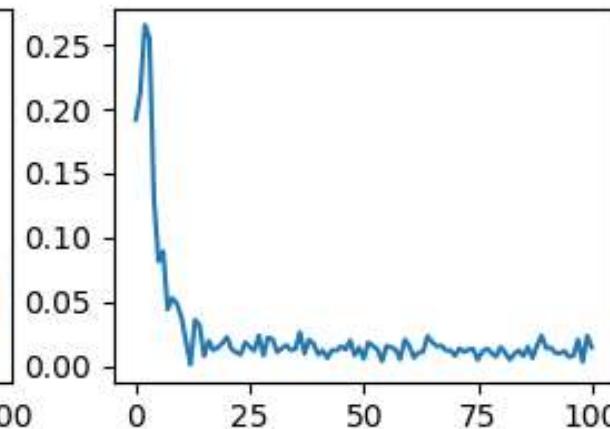
$$f(x) = x^2 \sin(1/x)$$

→ continuous
→ not diff'ble at zero

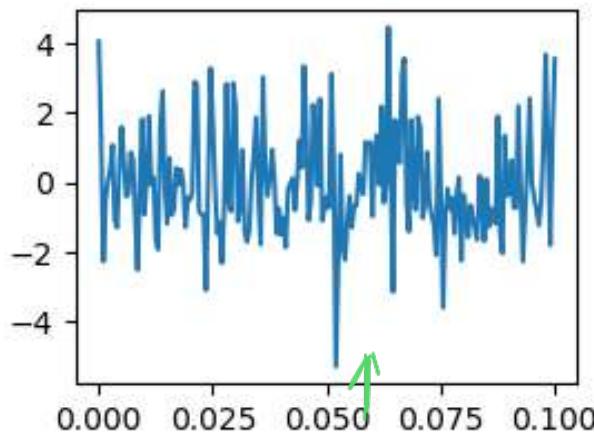
f



F



$\frac{df}{dx}$



Original function
+
Noise

Fourier transform

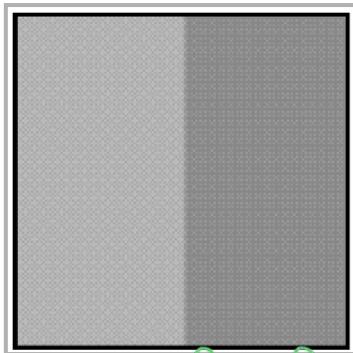
adding noise makes the derivative unrecognizable

adding noise to function
 $(\hat{u}F)$

right
gray
dark

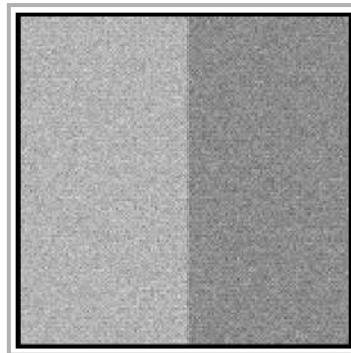
Noise in 2D

Ideal step edge

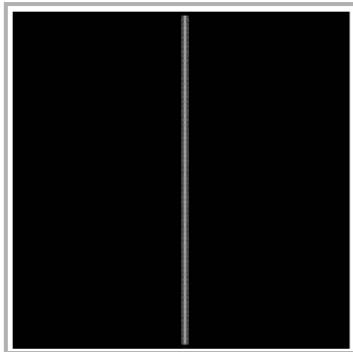
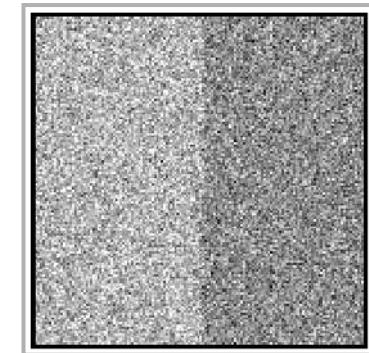
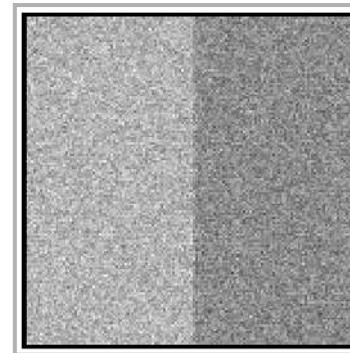


gray-level

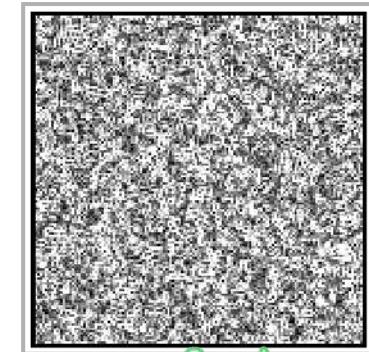
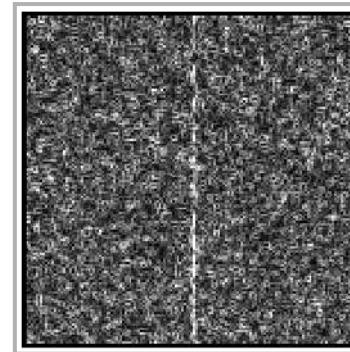
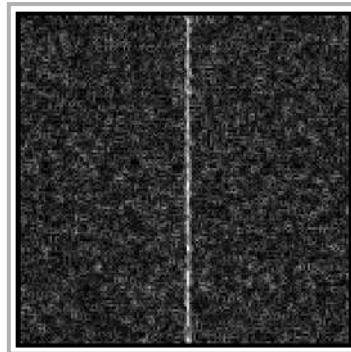
Step edge + noise



Real images



Increasing noise level



gradient
image



As the amount of noise increases, the derivatives stop being meaningful.

Removing Noise

Problem:

- High frequencies and differentiation do not mix well.

magnifies high freq's

Solution:

- Suppress high frequencies by
 - using the Discrete Fourier Transform.

Discrete Fourier Transform

$$F(\mu, \nu) = \frac{1}{\sqrt{M * N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-2i\pi(\mu x/M + \nu y/N)}$$

Sum over all pixels in 2D image

$$f(x, y) = \frac{1}{\sqrt{M * N}} \sum_{\mu=0}^{M-1} \sum_{\nu=0}^{N-1} F(\mu, \nu) e^{+2i\pi(\mu x/M + \nu y/N)}$$

Inverse Fourier
Transform

The DFT is the discrete equivalent of the 2D Fourier transform:

- The 2D function f is written as a sum of sinusoids.
- The DFT of f convolved with g is the product of their DFTs.

$$\mathcal{DFT}[f * g] = \mathcal{DFT}(f) \cdot \mathcal{DFT}(g)$$

Fourier Basis Element



Sinusoidal waves

$$e^{ix} = \cos(x) + i \sin(x)$$

Real part of

$$e^{+2i\pi(ux+vy)}$$

where

- $\sqrt{u^2 + v^2}$ represents the frequency,
- $\text{atan}(v, u)$ represents the orientation.

of Sinusoidal

waves

Fourier Basis Element



Real part of

$$e^{+2i\pi(ux+vy)}$$

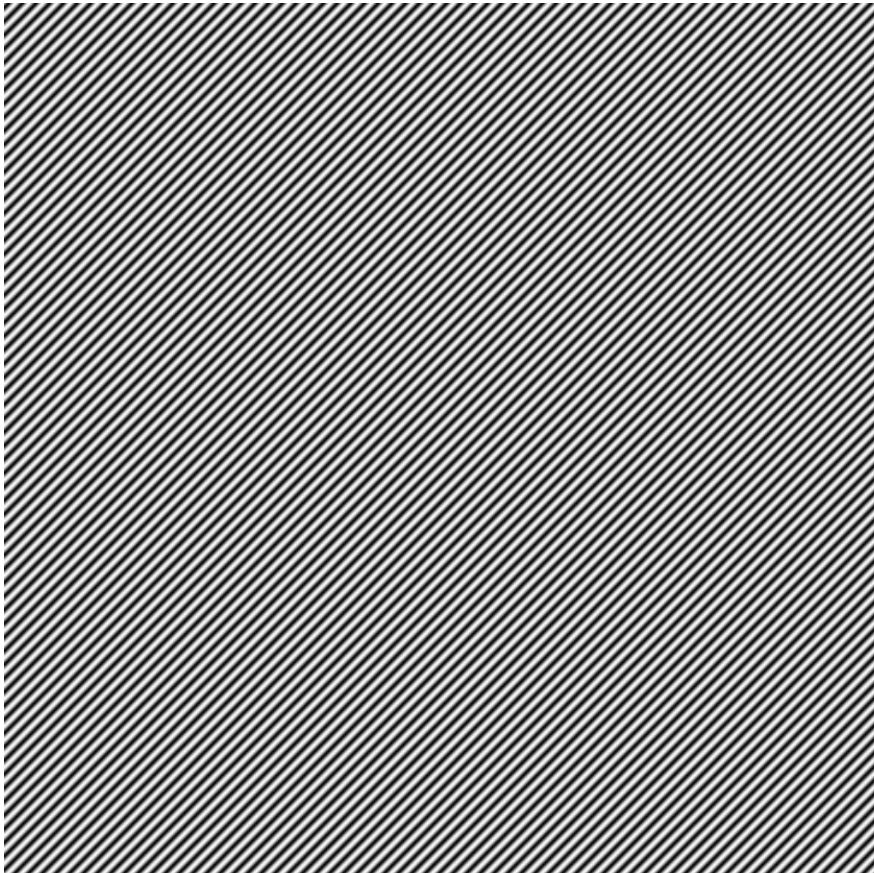
where

- $\sqrt{u^2 + v^2}$ is larger than before.

frequency ↑

orientation changed

Fourier Basis Element



Real part of

$$e^{+2i\pi(ux+vy)}$$

where

- $\sqrt{u^2 + v^2}$ is larger still.

Truncated Inverse DFT

$$F(\mu, \nu) = \frac{1}{\sqrt{M * N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-2i\pi(\mu x/M + \nu y/N)}$$

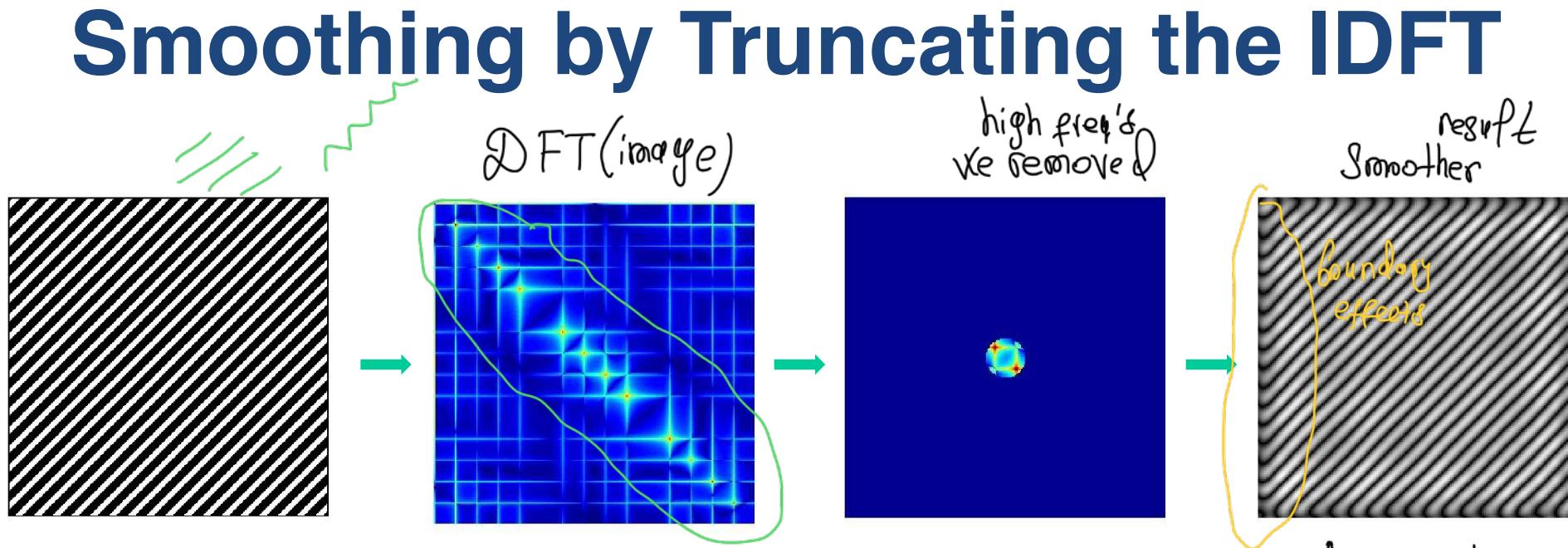
~~$$f(x, y) = \frac{1}{\sqrt{M * N}} \sum_{\mu=0}^{M-1} \sum_{\nu=0}^{N-1} F(\mu, \nu) e^{+2i\pi(\mu x/M + \nu y/N)}$$~~

$$f(x, y) = \frac{1}{\sqrt{M * N}} \sum_{\mu^2 + \nu^2 < T} F(\mu, \nu) e^{+2i\pi(\mu x/M + \nu y/N)}$$

getting rid of high freq's
T is a hand-specified threshold.

- The sinusoids corresponding to $\mu^2 + \nu^2 \geq T$ depict high frequencies.
- Removing them amounts to removing high-frequencies.

Smoothing by Truncating the IDFT



Rotated stripes:

- Dominant diagonal structures
- Discretization produces additional harmonics

—> Removing higher frequencies and reconstructing yields a smoothed image.

Removing Noise

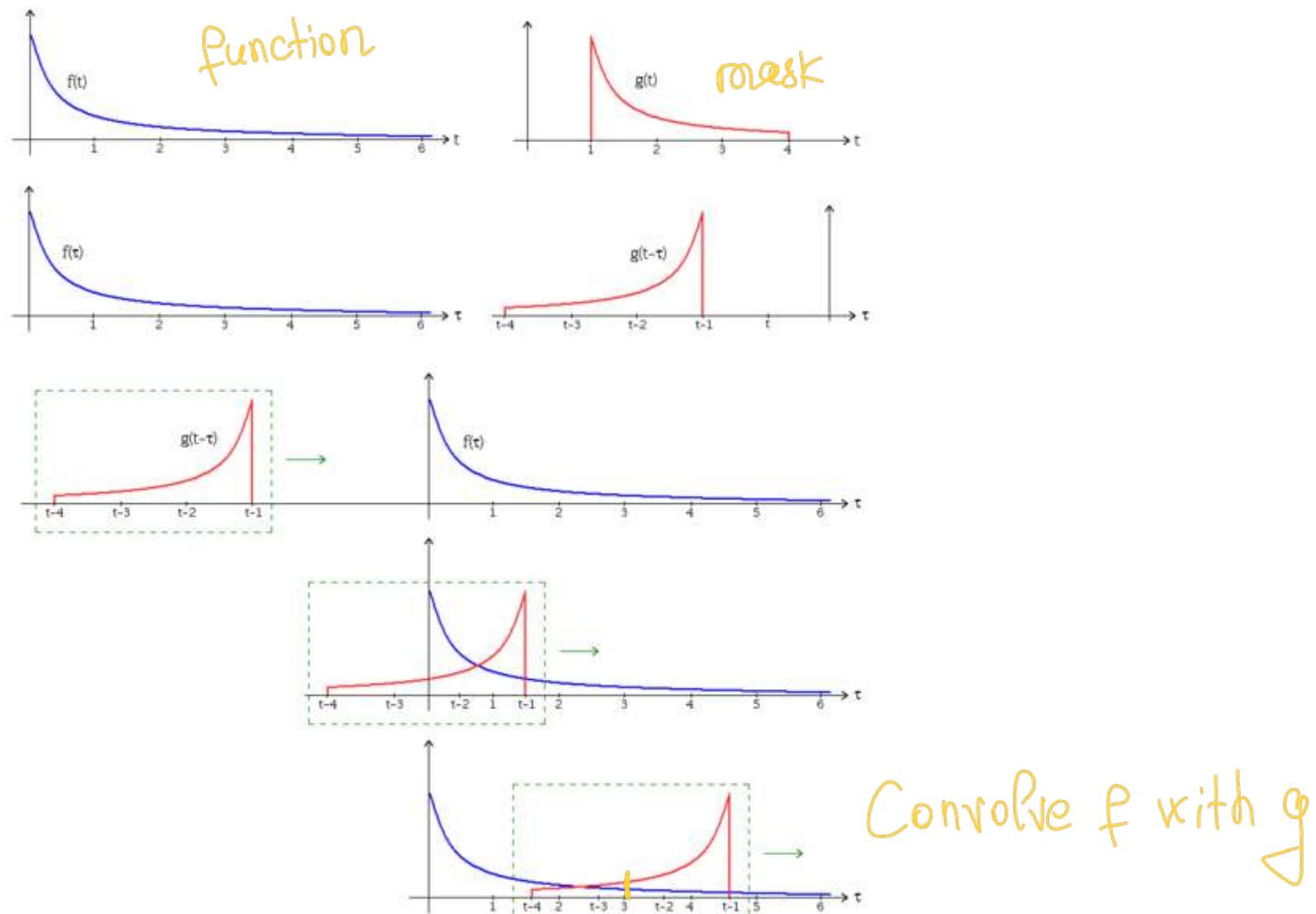
Problem:

- High frequencies and differentiation do not mix well.

Solution:

- Suppress high frequencies by
 - using the Discrete Fourier Transform,
 - convolving with a low-pass filter.

1D Convolution

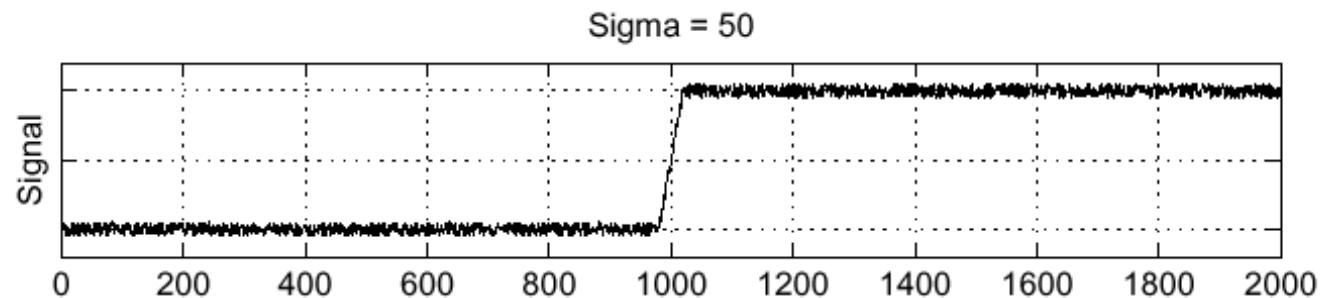


$$g * f(t) = \int_{\tau} g(t - \tau) f(\tau) d\tau$$

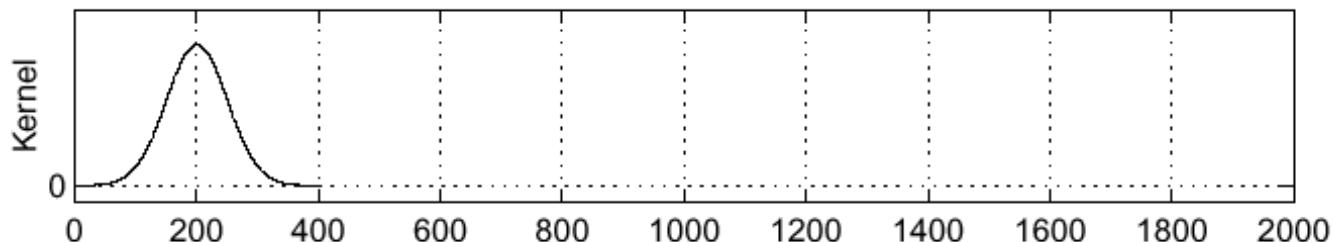
Weighted sum

Smooth Before Differentiating

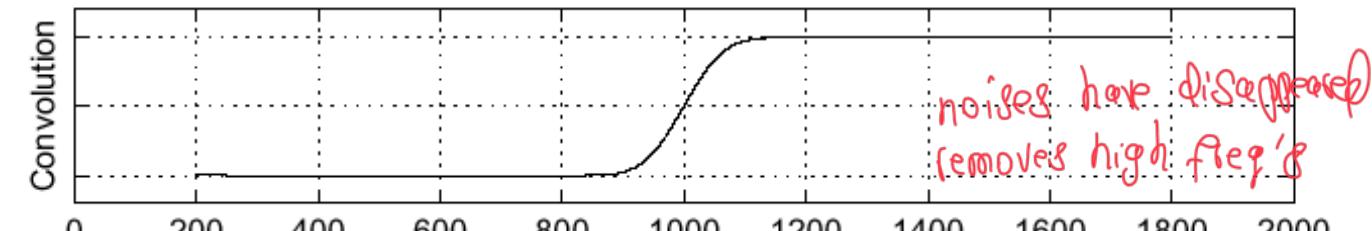
f



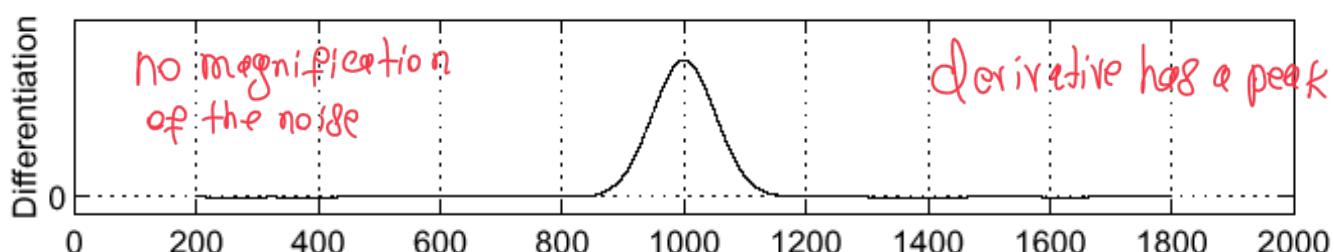
g



$g * f$



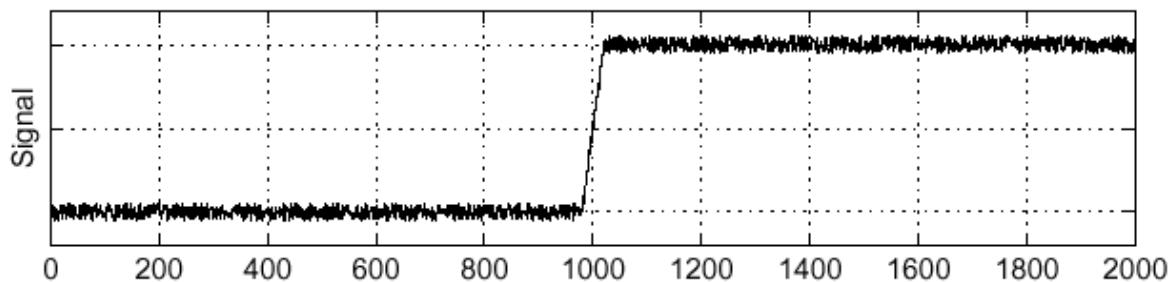
$\frac{\partial}{\partial x}(g * f)$



Simultaneously Smooth and Differentiate

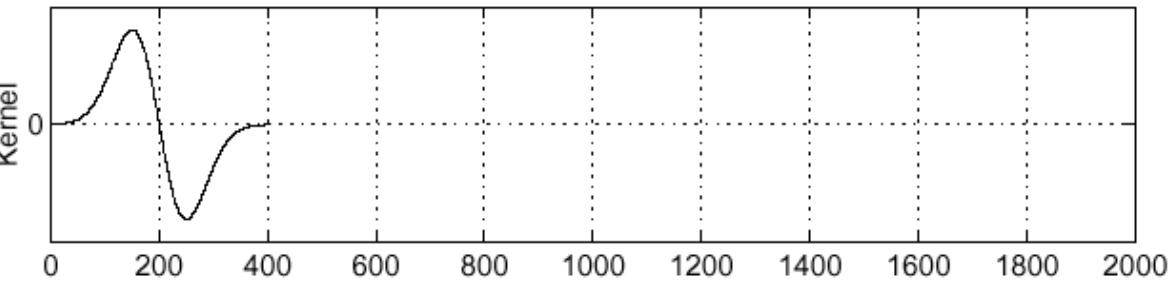
f

Sigma = 50



$$\frac{\partial g}{\partial x}$$

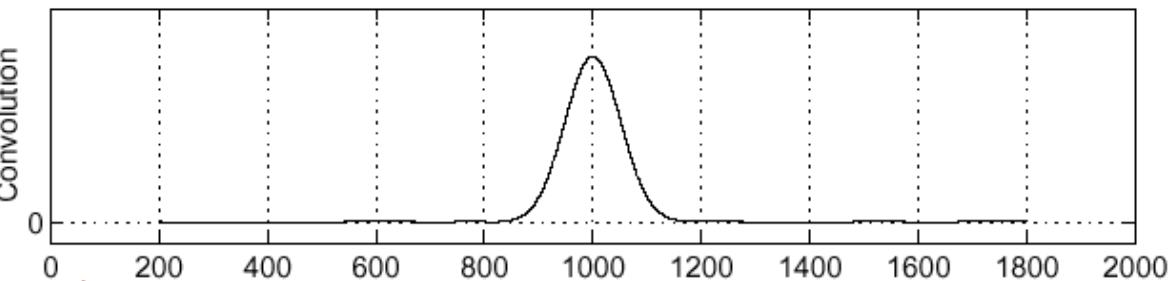
Kernel



11

$$\frac{\partial}{\partial x} (g * f) = \frac{\partial g}{\partial x} * f$$

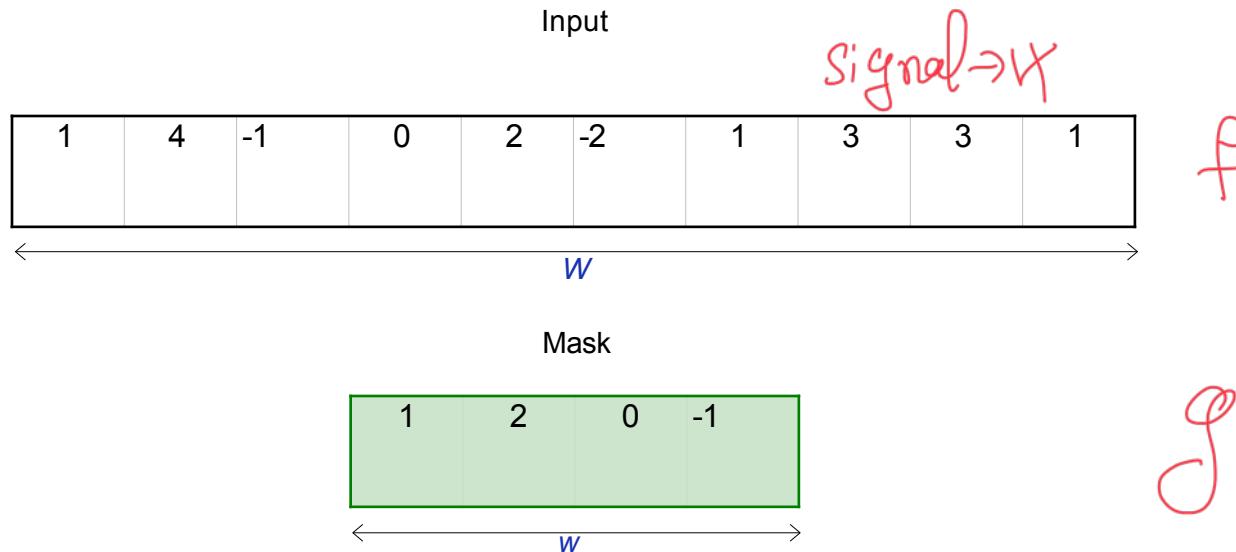
Convolution



differentiation and smoothing
at the same time

--> Faster because dg/dx can be precomputed.

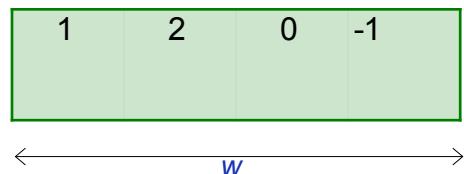
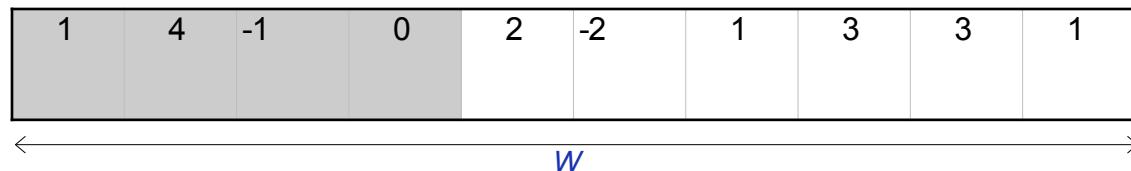
Discrete 1D Convolution



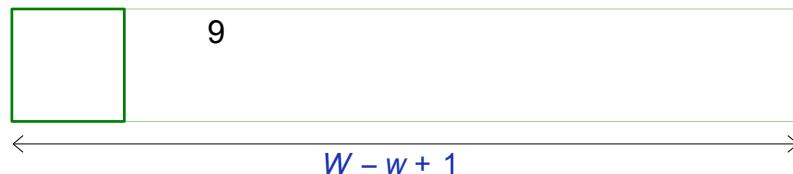
Convolve with \mathcal{G}

Discrete 1D Convolution

Input

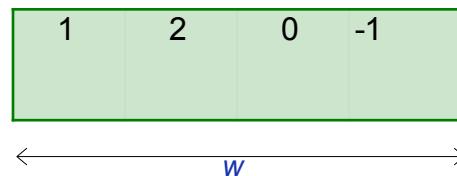
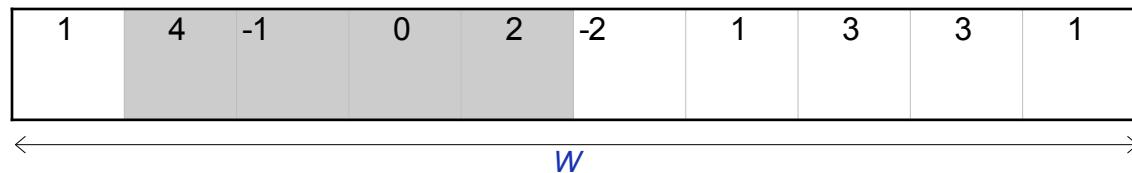


Output



Discrete 1D Convolution

Input

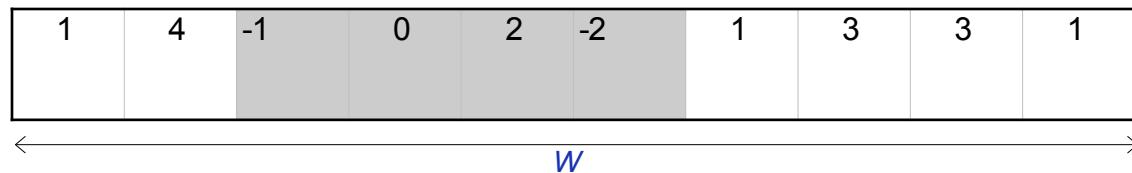


Output

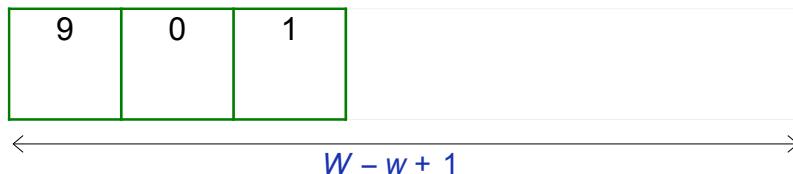


Discrete 1D Convolution

Input

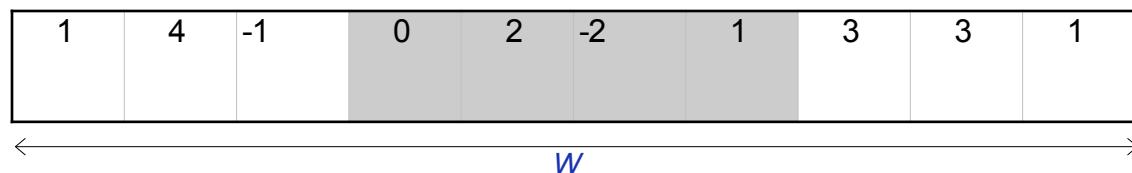


Output

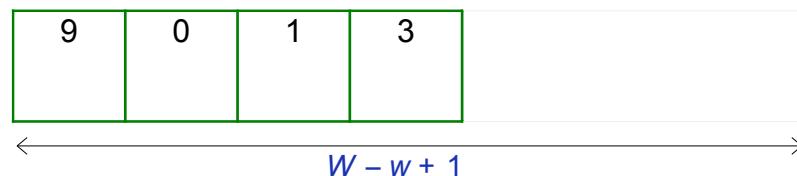


Discrete 1D Convolution

Input

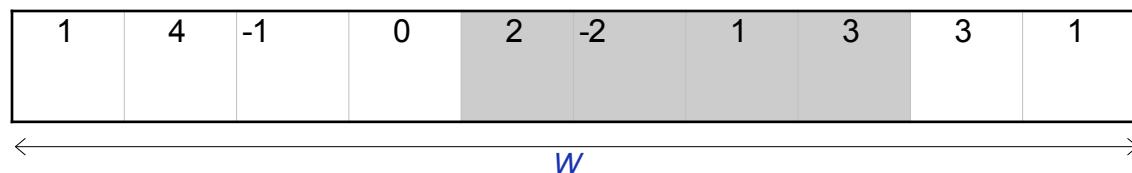


Output

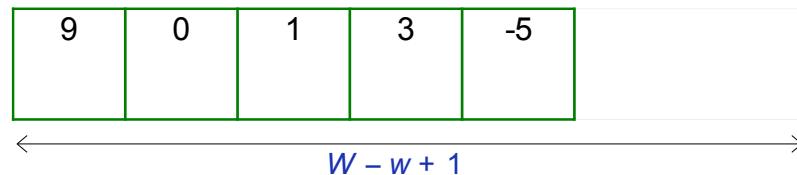


Discrete 1D Convolution

Input

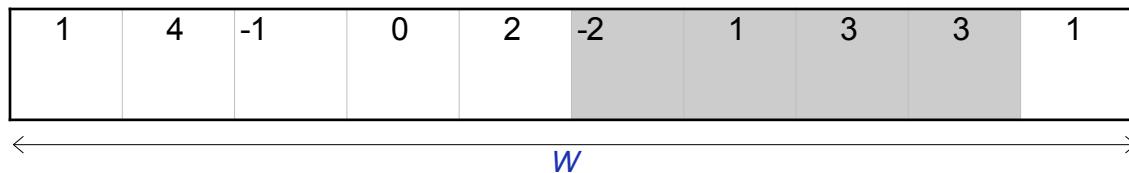


Output

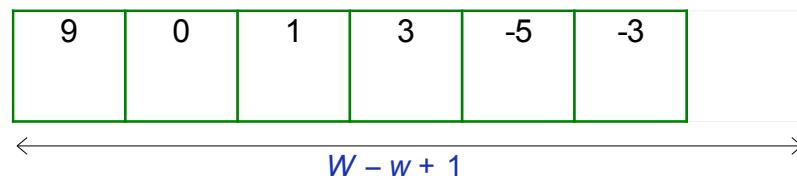


Discrete 1D Convolution

Input

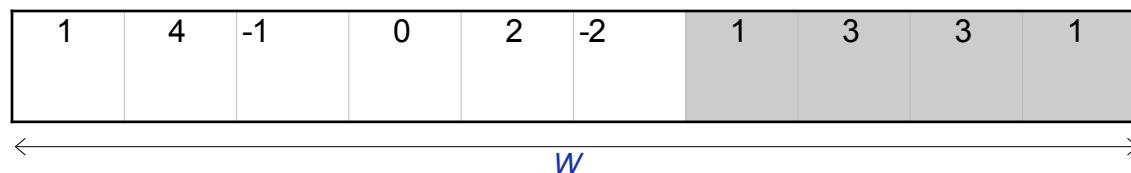


Output

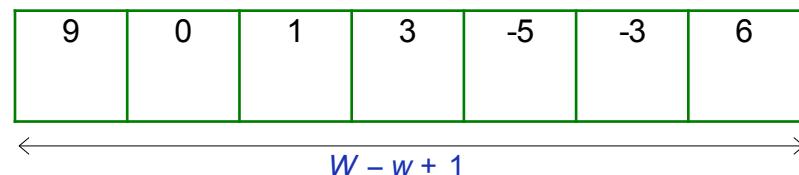


Discrete 1D Convolution

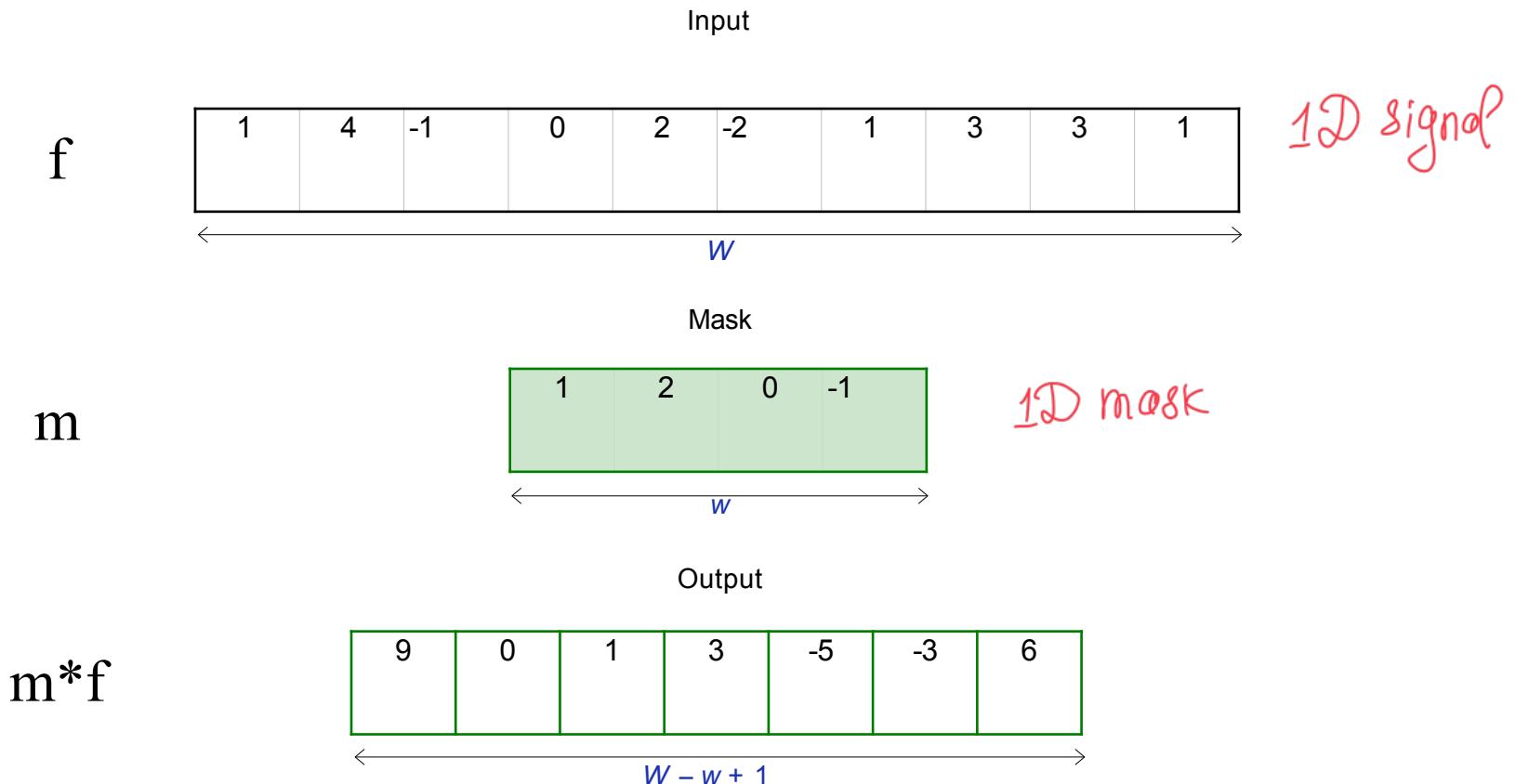
Input



Output



Discrete 1D Convolution

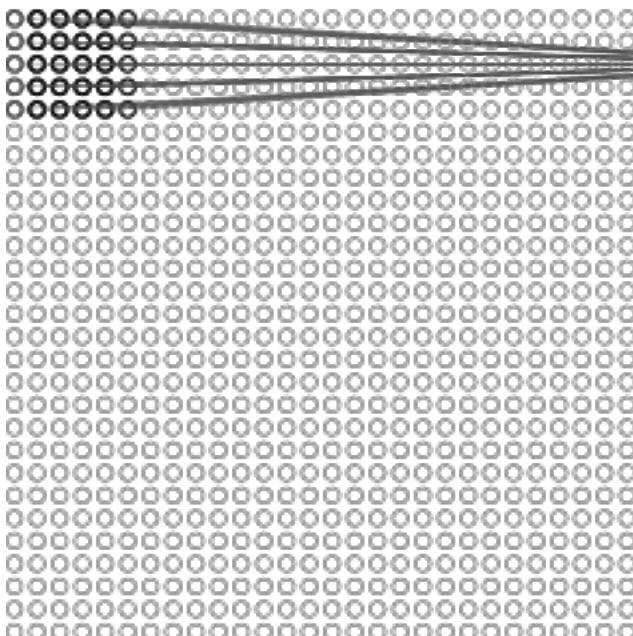


$$m * f(x) = \sum_{i=0}^w m(i)f(x - i)$$

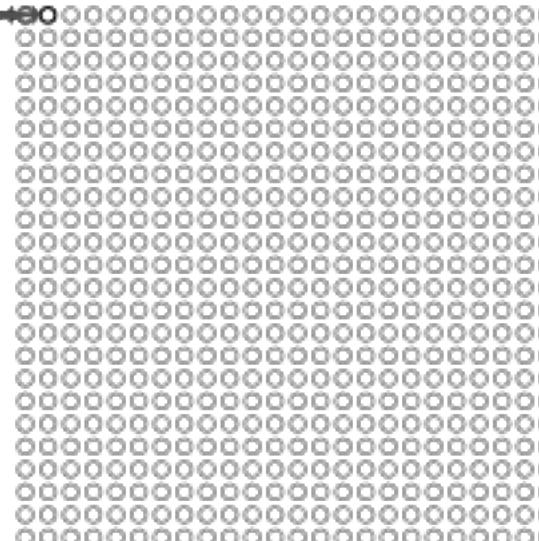
instead of integral,
we've sum

Discrete 2D Convolution

Input image: f



Convolved image: $m * * f$



Convolution mask m , also known as a *kernel*.

$$\begin{bmatrix} m_{11} & \dots & m_{1w} \\ \dots & \dots & \dots \\ m_{w1} & \dots & m_{ww} \end{bmatrix}$$

$\sqrt{w} \times \sqrt{w}$ size

$$m * * f(x, y) = \sum_{i=0}^w \sum_{j=0}^w m(i, j) f(x - i, y - j)$$

Replace (x, y) pixel \rightarrow sum of weighted values of mask and f

Differentiation As Convolution

Convolving with

$$\begin{bmatrix} \text{1D mask} \\ -1,1 \end{bmatrix} \rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x+dx, y) - f(x, y)}{dx}$$

$$\begin{bmatrix} -0.5, 0, 0.5 \end{bmatrix} \rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x+dx, y) - f(x-dx, y)}{2dx}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} \rightarrow \frac{\partial f}{\partial y} \approx \frac{f(x, y+dy) - f(x, y)}{dy}$$

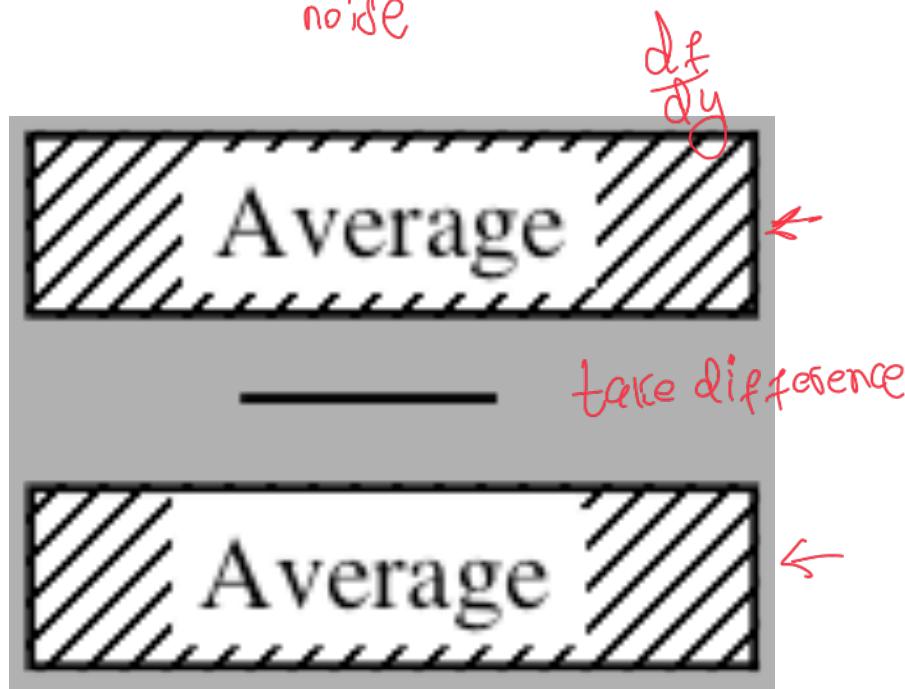
$$\begin{bmatrix} -0.5 \\ 0 \\ 0.5 \end{bmatrix} \rightarrow \frac{\partial f}{\partial y} \approx \frac{f(x, y+dy) - f(x, y-dy)}{2dy}$$

→ to kipp higher freq's

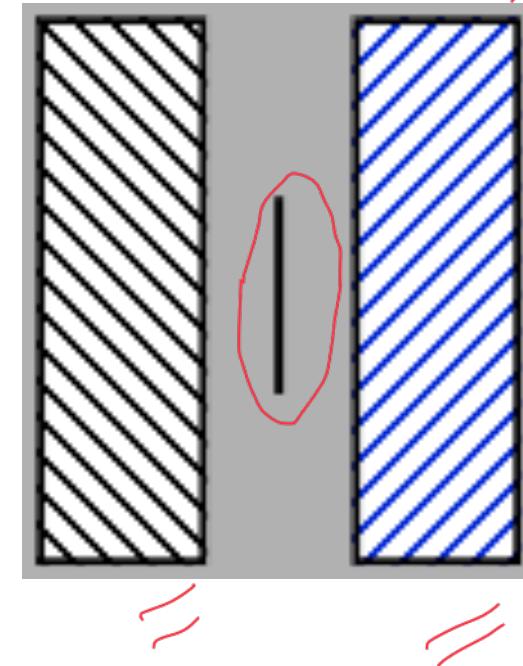
→ Use wider masks to add some smoothing

Smoothing and Differentiating

↳ to get rid of
noise



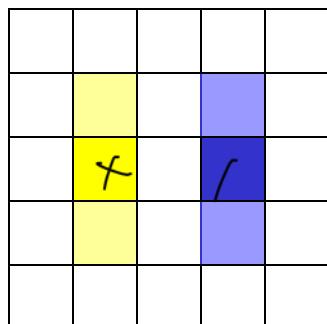
Computing $\frac{df}{dx}$



Compute the difference of averages on either side of the central pixel.

3X3 Masks

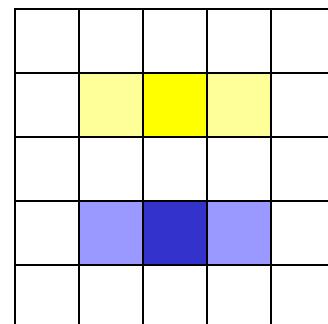
x derivative



$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Prewitt operator

y derivative

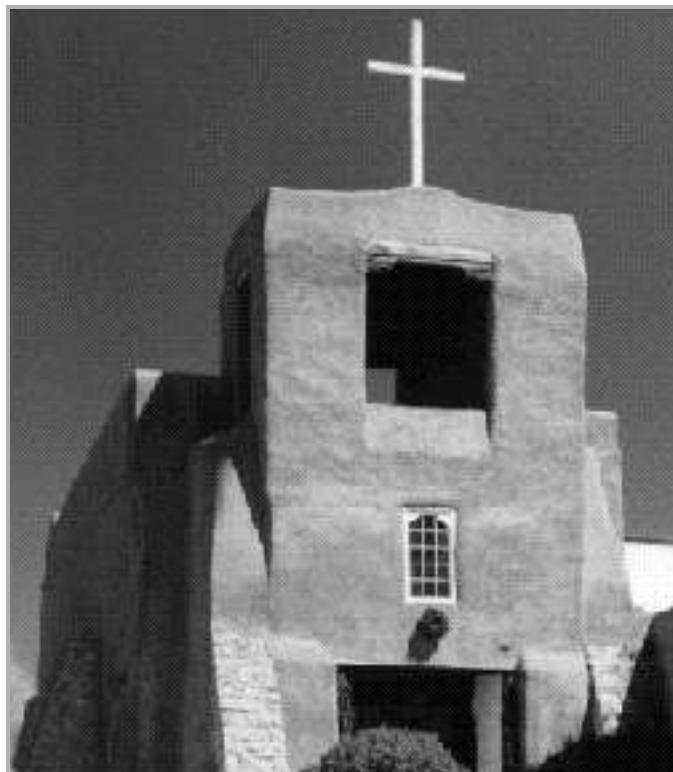


$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

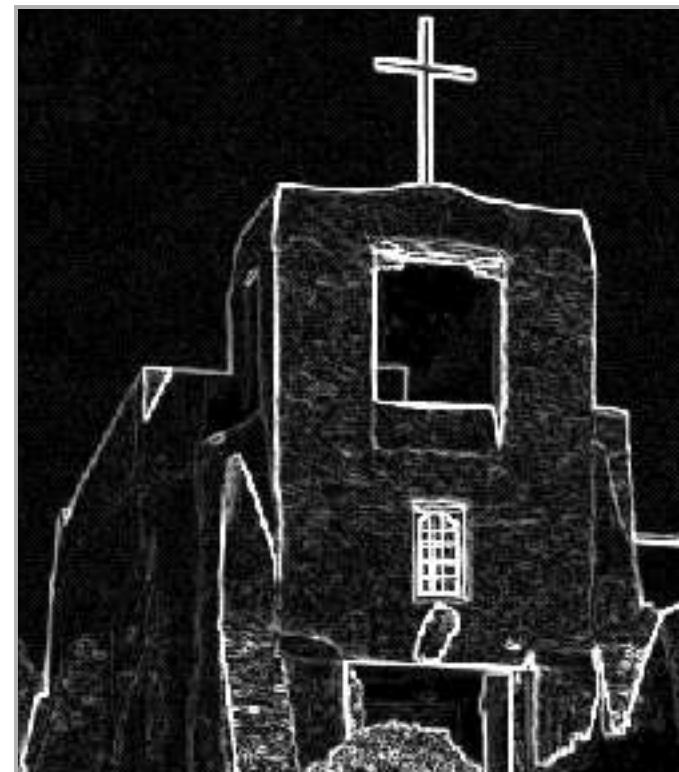
Sobel operator

move weight

Prewitt Example

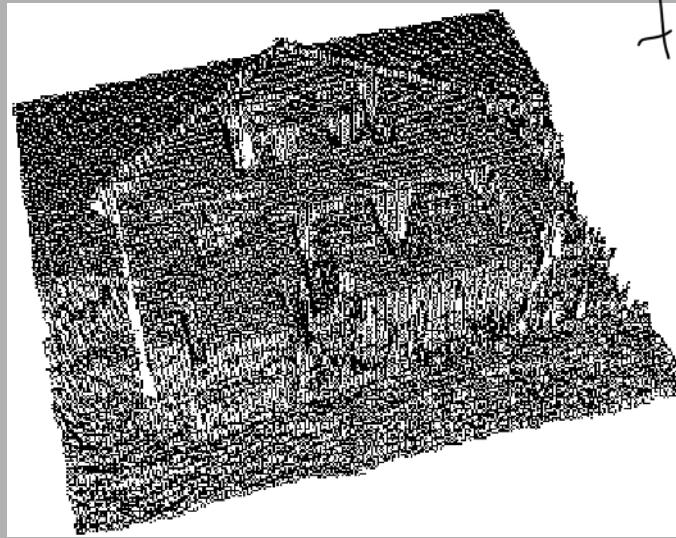
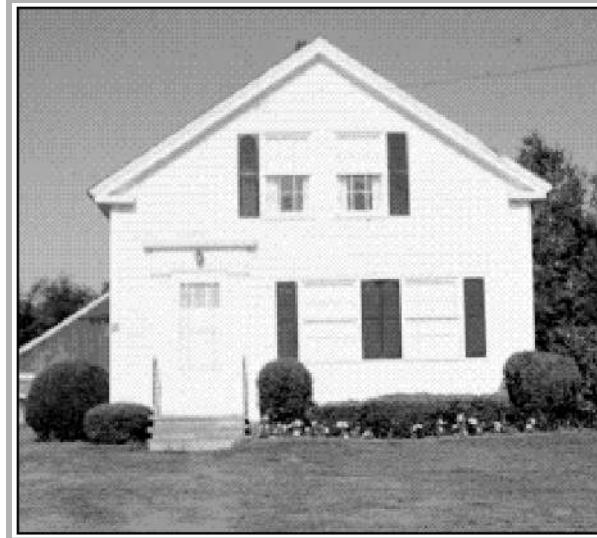


Santa Fe Mission

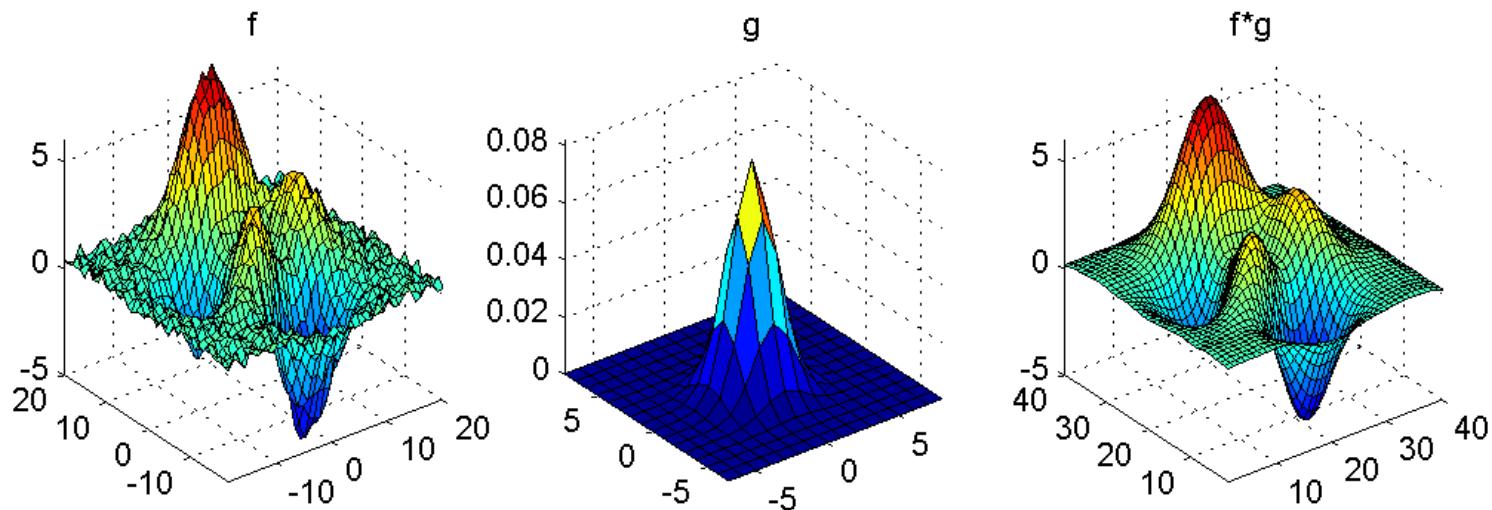


Gradient Image

Sobel Example



Gaussian Smoothing

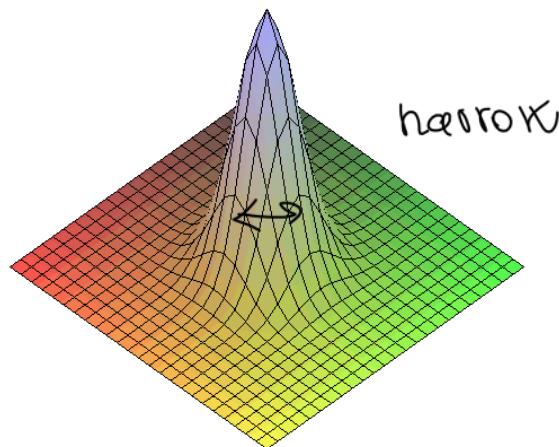


- More principled way to eliminate high frequency noise.
More justifiable
- Is fast because the kernel is
 - small, *fast computation*
 - separable.

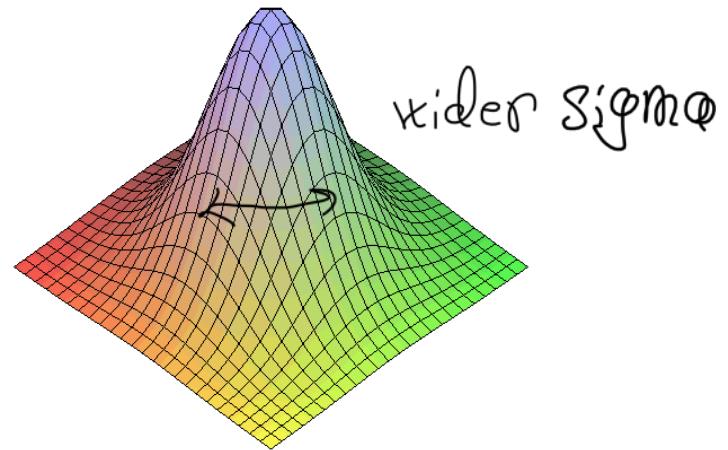
"smoother" mask

DFT \rightarrow IDFT *fast*

Gaussian Functions



narrow



wider sigma

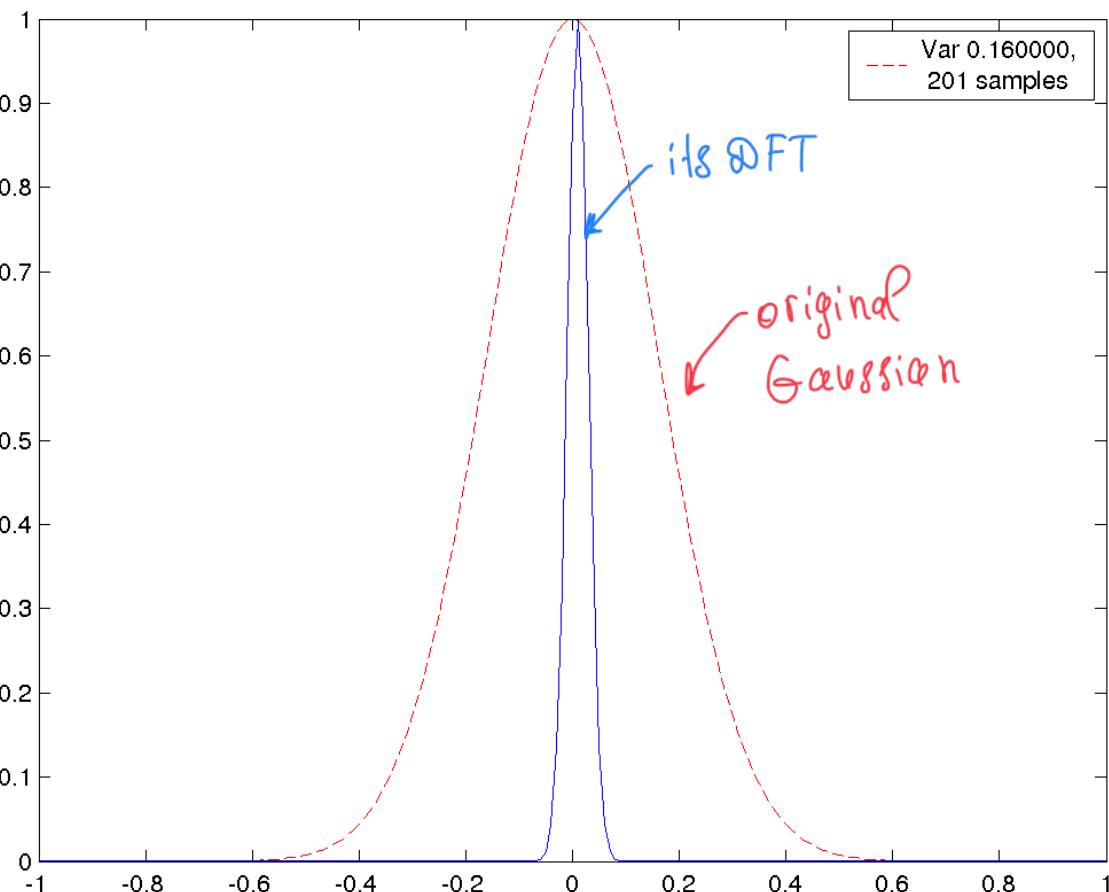
$$\sigma = 1$$

$$g_2(x, y) = \frac{1}{2\pi\sigma^2} \exp(-(x^2 + y^2)/2\sigma^2)$$

\hookrightarrow variance (width)
 $x^2 + y^2 > 6^2 \Rightarrow \exp(\dots)$ becomes super smooth
we ignore it

- The integral is always 1.0
- The larger σ , the broader the Gaussian is.
- As σ approaches 0, the Gaussian approximates a Dirac function.

DFT of a Gaussian



- The DFT of a Gaussian is a Gaussian.
- It has finite support.
- Its width is inversely proportional to that of the original Gaussian.

(OG) (DFT)
Wider → narrower
width width

Gaussians as Low-Pass Filters

Support of a function → set of pts where func
is not zero, or closure of that set

- The Fourier transform of a convolution is the product of their Fourier transforms: $\mathcal{F}(g * f) = \mathcal{F}(g)\mathcal{F}(f)$.
- If g is a Gaussian, so is $\mathcal{F}(g)$. *and gaussian*
- Furthermore if g is broad, the support of $\mathcal{F}(g)$ is small. \longrightarrow narrow
- So is the support of $\mathcal{F}(g * f)$. g -broad $\Rightarrow \mathcal{F}(g * f)$ - g small (broad)
- There are no more high-frequencies in $g * f$.

→ Convolving with a Gaussian suppresses the high frequencies.

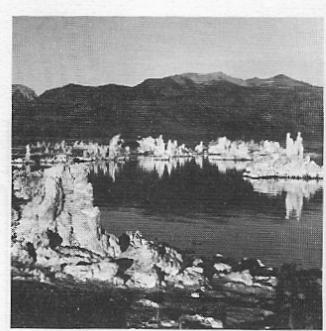
You can reconstruct $g * f$ without high freq's

Gaussian Smoothed Images

Convolving with gaussian σ

(high freq's)
Lose information

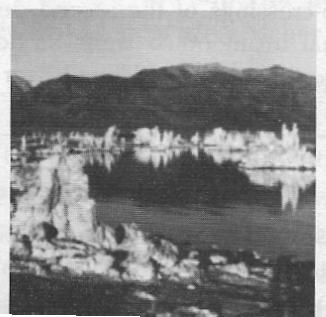
$\sigma = 1$



$\sigma = 2$



$\sigma = 4$



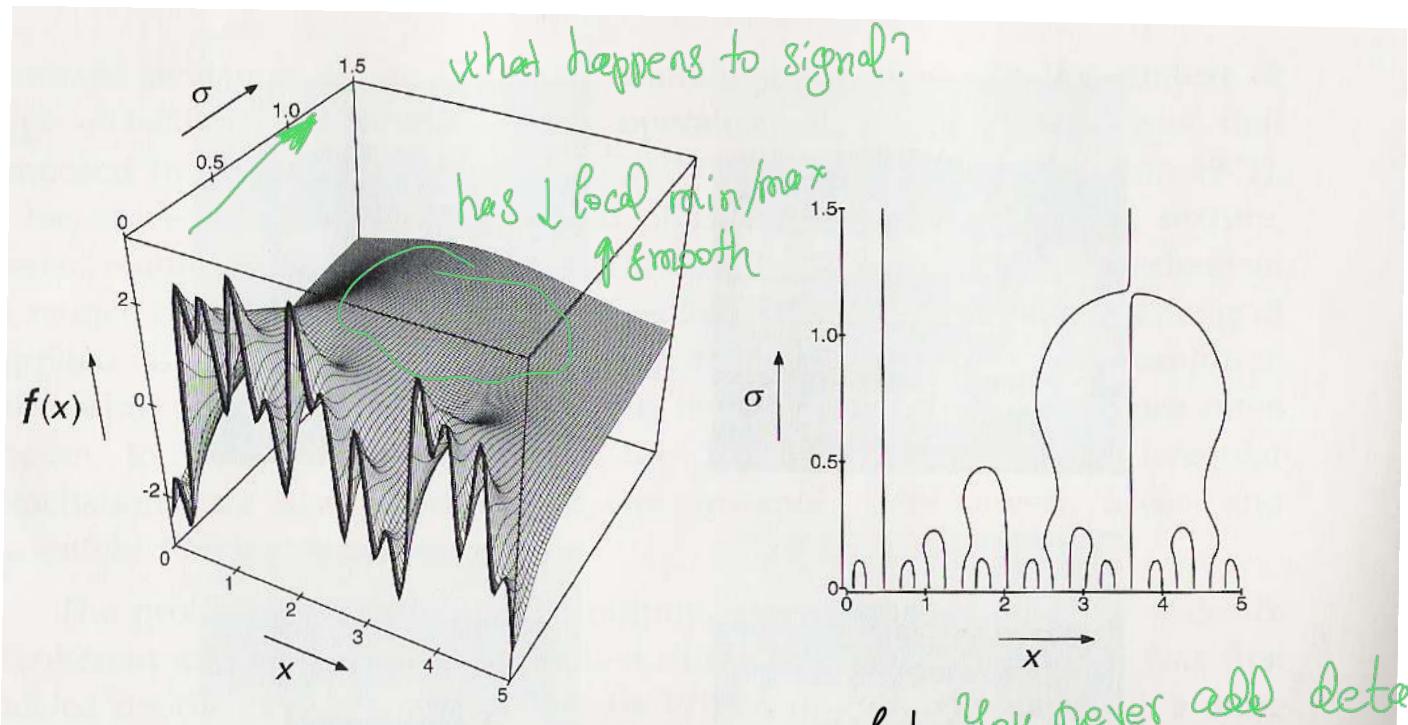
Original image

Blurred image

No right value
for $\sigma \sim 1$ details, resolution, ...

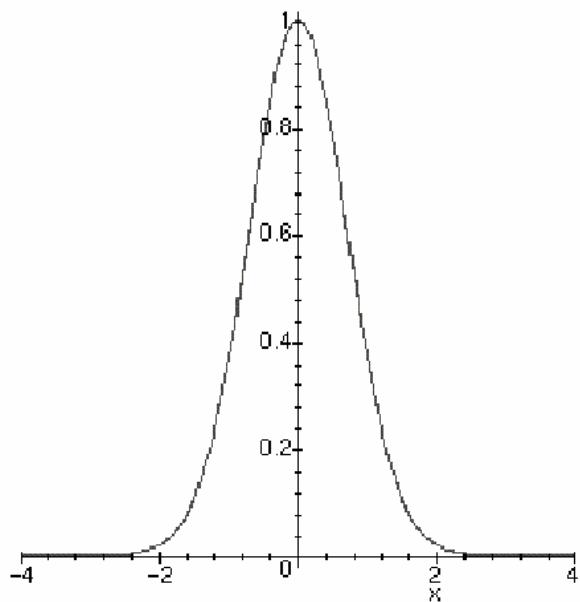
Lose noise details

Scale Space



Increasing scale (σ) removes high frequencies (details) but never adds artifacts.

Separability



1D-Gaussian

$$g_1(x) = \frac{1}{\sqrt{\pi}\sigma} \exp(-x^2 / \sigma^2)$$

2D-Gaussian

$$g_2(x, y) = g_1(x)g_1(y)$$

No need for explicit 2D-Gaussian Convolution

$$\iint_{uv} g_2(u, v) f(x-u, y-v) du dv = \int_u g_1(u) \left(\int_v g_1(v) f(x-u, y-v) dv \right) du$$

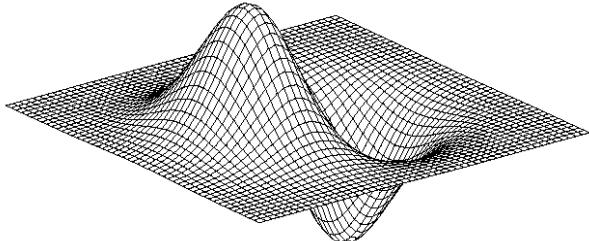
$$= \int_v g_1(v) \left(\int_u g_1(u) f(x-u, y-v) du \right) dv$$

2D-convolutions by Gaussian =
= $2 \times$ 1D convolution by Gaussian

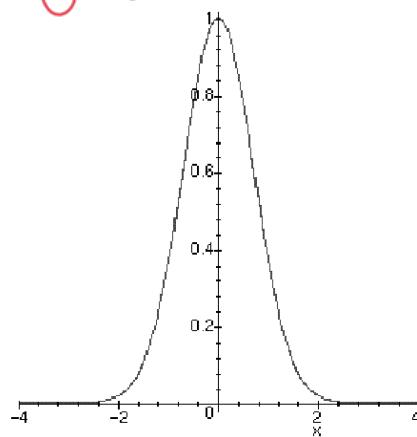
→ 2D convolutions are never required. Smoothing can be achieved by successive 1D convolutions, which is faster.

Continuous Gaussian Derivatives

$$g_2(x,y) = g_1(x) g_1(y)$$



=



x

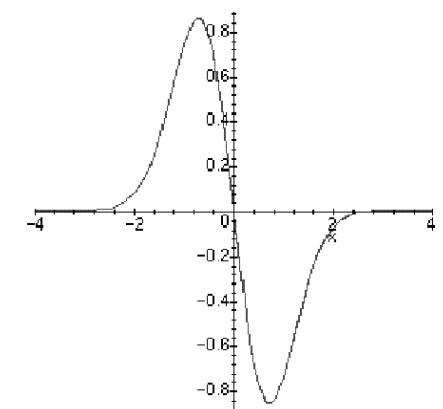


Image derivatives computed by convolving
with the derivative of a Gaussian:

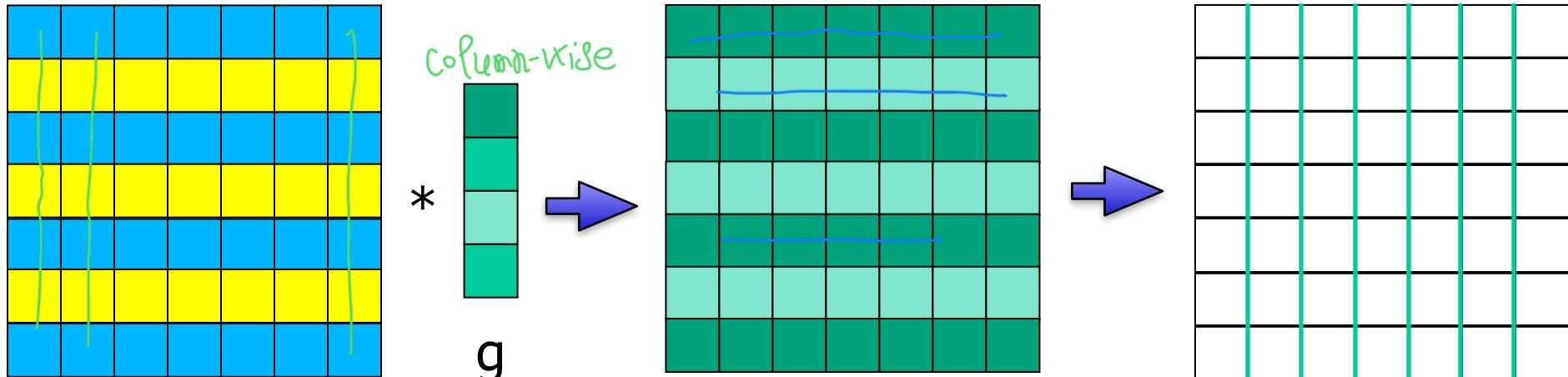
$$\frac{\partial}{\partial x} \iint_{u,v} g_2(u,v) f(x-u, y-v) du dv = \int_u g_1'(u) \left(\int_v g_1(v) f(x-u, y-v) dv \right) du$$

smoothed version *with Gaussian*
with its derivative

$$\frac{\partial}{\partial y} \iint_{u,v} g_2(u,v) f(x-u, y-v) du dv = \int_v g_1'(v) \left(\int_u g_1(u) f(x-u, y-v) du \right) dv$$

Discrete Gaussian Derivatives

Compute x-derivative



Sigma=1:
pre-computed

$g : 0.000070 \quad 0.010332 \quad 0.207532 \quad 0.564131 \quad 0.207532 \quad 0.010332 \quad 0.000070 \rightarrow \text{centered mask Pen} = 5 \checkmark$

$g' : 0.000418 \quad 0.041330 \quad 0.415065 \quad 0.000000 \quad -0.415065 \quad -0.041330 \quad -0.000418$

independently convolve with lines

Sigma=2: ↑ support, ↑ Pen of mask g

$g : 0.005167 \quad 0.029735 \quad 0.103784 \quad 0.219712 \quad 0.282115 \quad 0.219712 \quad 0.103784 \quad 0.029735 \quad 0.005167$

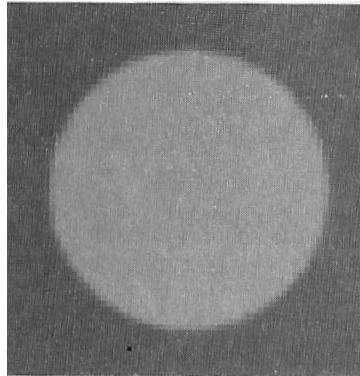
$g' : 0.010334 \quad 0.044602 \quad 0.103784 \quad 0.109856 \quad 0.000000 \quad -0.109856 \quad -0.103784 \quad -0.044602 \quad -0.010334$

fast, efficient *

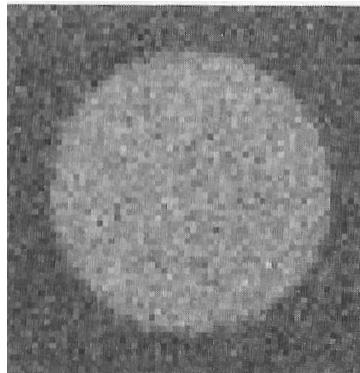
→ Only requires 1D convolutions with relatively small masks.

Increasing Sigma

Input Images



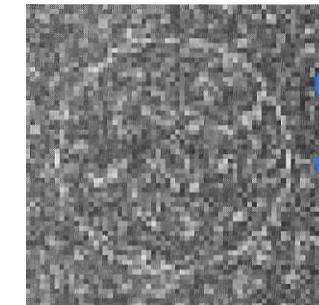
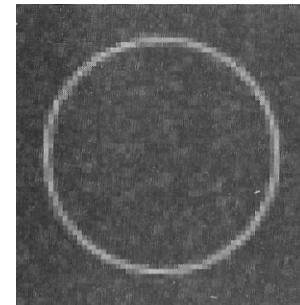
No Noise



Noise Added

No noise: As you ↑ σ ,
your model Pen ↑ with
more convolution op's
 \Rightarrow fatter $\sigma=1$
contours
Polarization \rightarrow not good

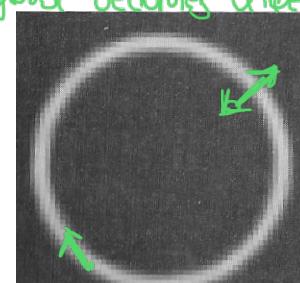
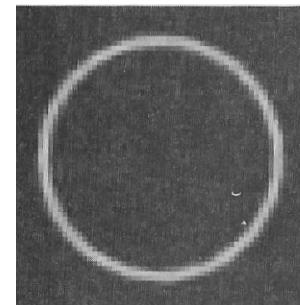
Gradient Images



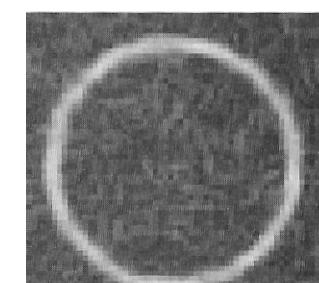
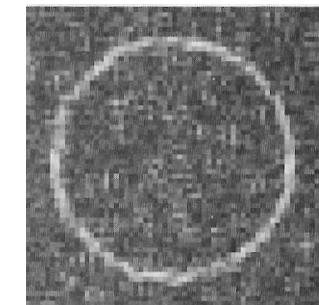
$\sigma=2$

Robustness to noise
vs
Precision of the detection

$\sigma=4$



No Noise

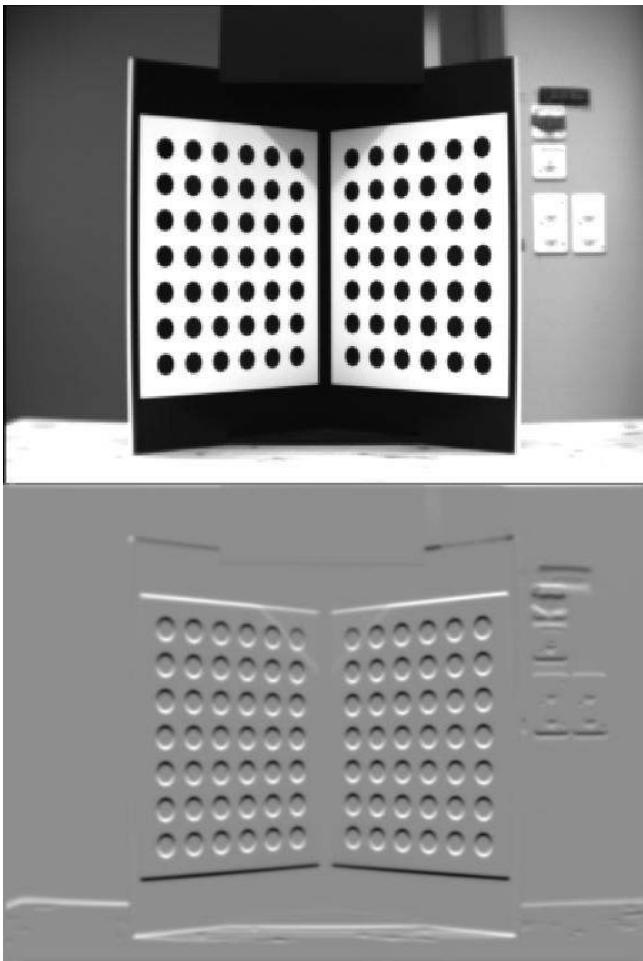


Noise Added

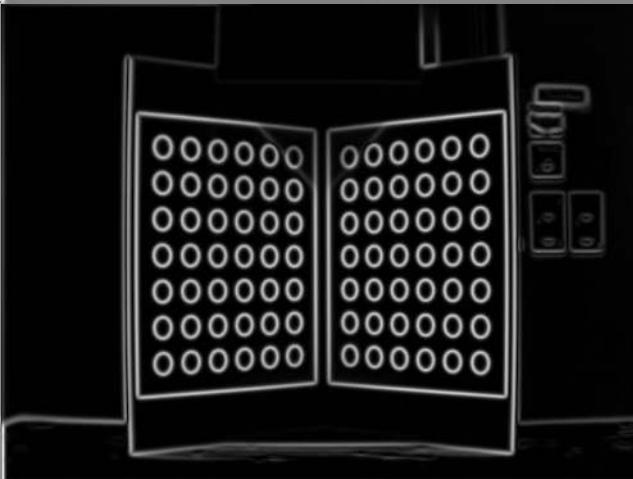
—> Larger sigma values improve robustness but degrade precision.

Derivative Images

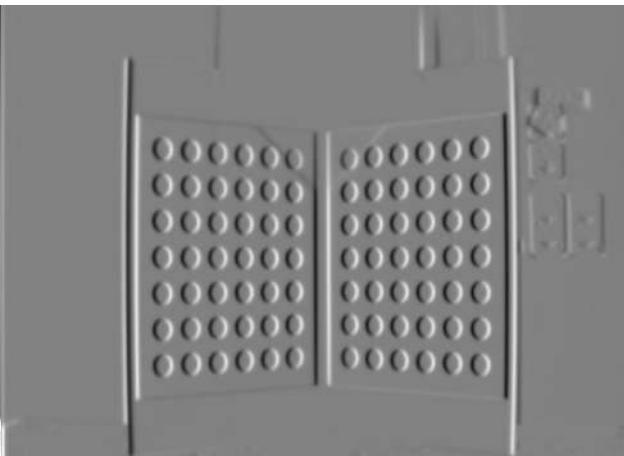
I



$\frac{\partial I}{\partial y}$



$$\frac{\partial I}{\partial x}$$



$$\sqrt{\frac{\partial I^2}{\partial x} + \frac{\partial I^2}{\partial y}}$$

Derivative Images

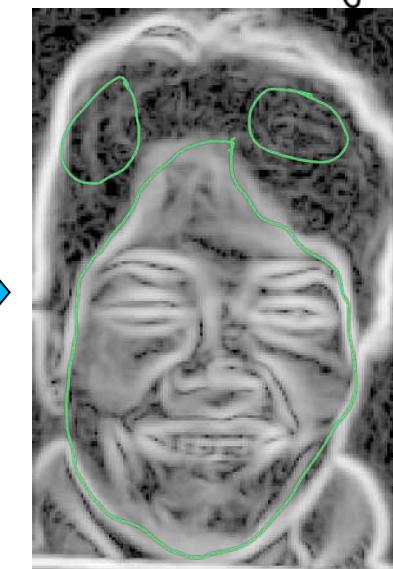


I

$$\frac{\partial I}{\partial x}$$



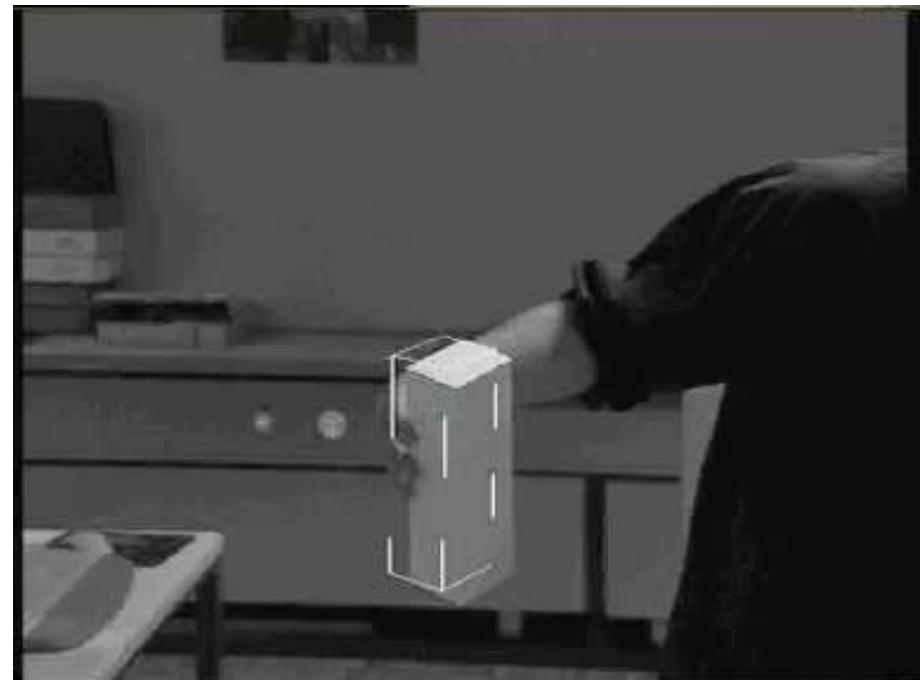
$$\frac{\partial I}{\partial y}$$



$$\sqrt{\frac{\partial I^2}{\partial x} + \frac{\partial I^2}{\partial y}}$$

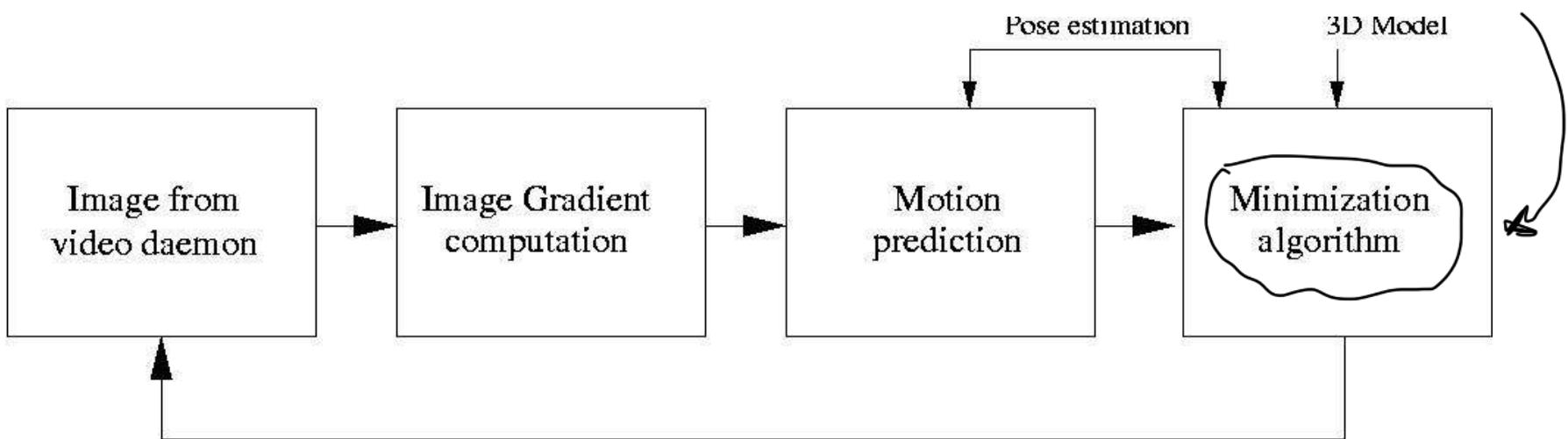
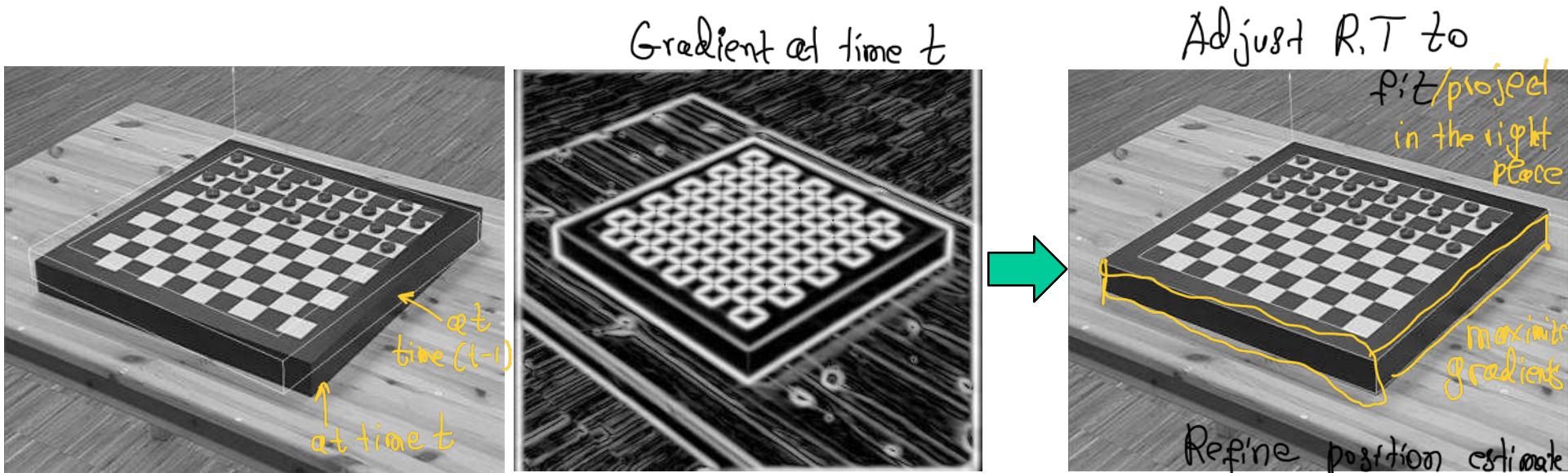
Strong responses ✓
But also unnecessary places ✓
Gradient image

Gradient-Based Tracking



Maximize edge-strength along projection of the 3—D wireframe.

Gradient Maximization



Real-Time Tracking

Results from gradient-computation are not perfect, but they're ^{already} exploitable

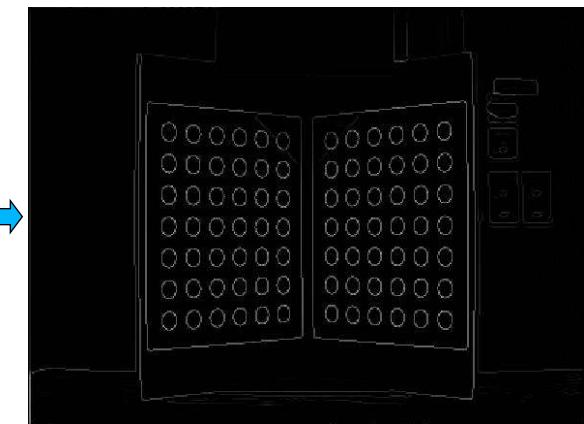
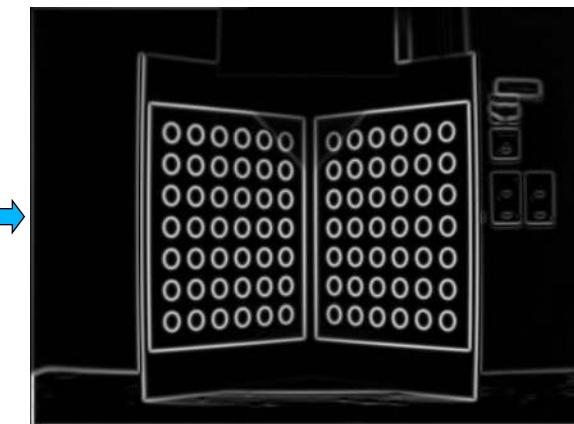
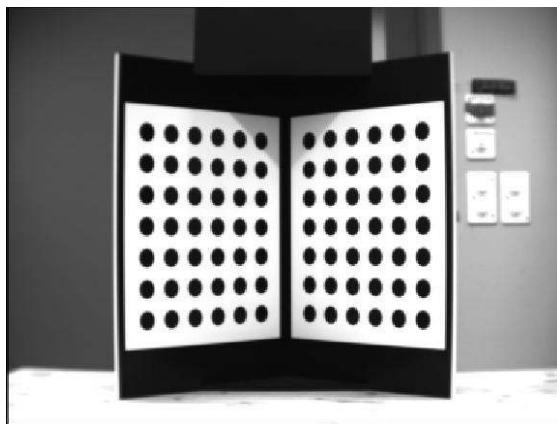


Canny Edge Detector

I

$$\sqrt{\frac{\partial I^2}{\partial x} + \frac{\partial I^2}{\partial y}}$$

Thinned gradient image
^{Thick}



1-pixel
Right

Canny Edge Detector



$\left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right]$ at a point (x, y)
Gradient image

Convolution

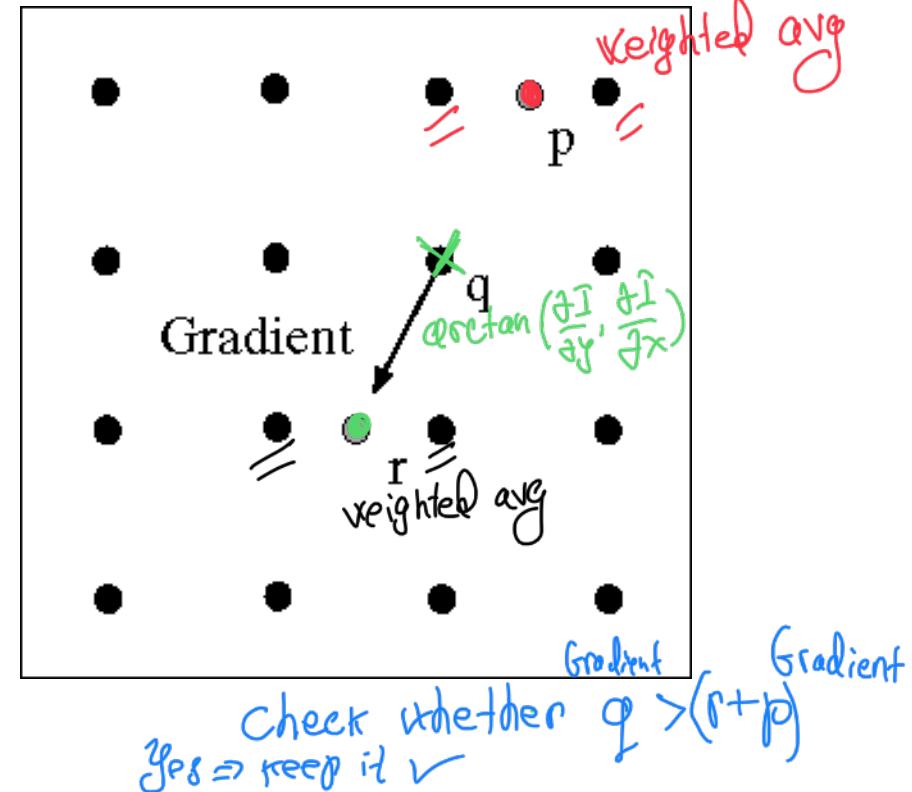
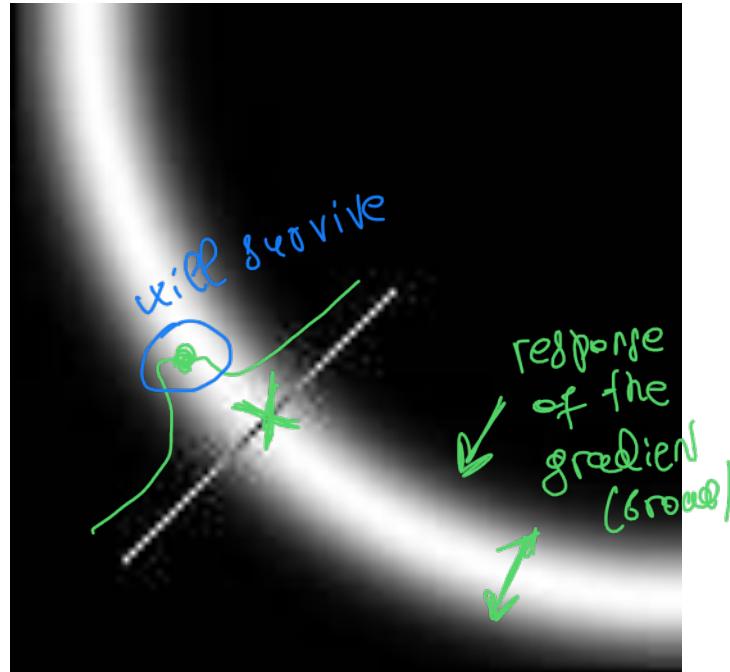
- Gradient strength
- Gradient direction \arctan

Thresholding

Non Maxima Suppression
keep specific pixels
instead of wide π of pixels

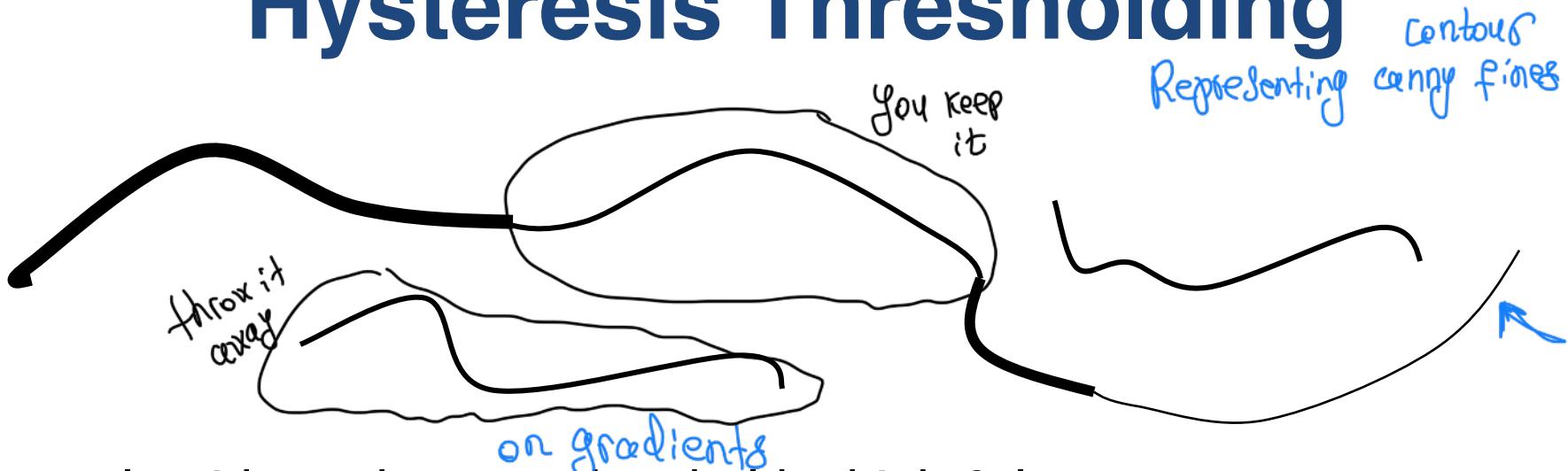
Hysteresis Thresholding

Non-Maxima Suppression



Check if pixel is local maximum along gradient direction, which requires checking interpolated pixels p and r.

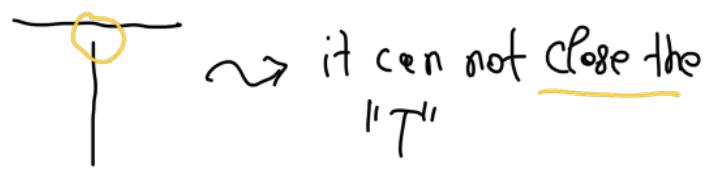
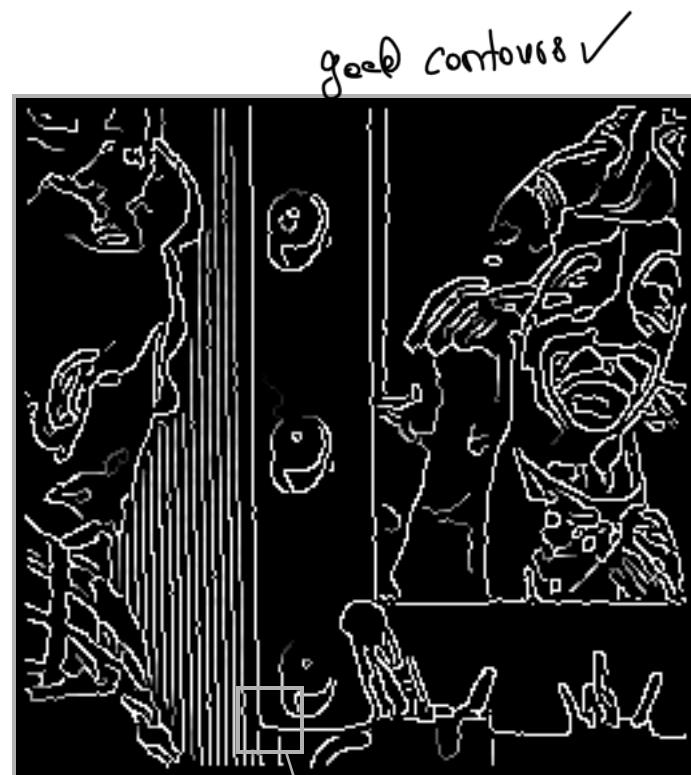
Hysteresis Thresholding



- Algorithm takes two thresholds: high & low
 - A pixel with edge strength above high threshold is an edge.
 - Any pixel with edge strength below low threshold is not.
 - Any pixel above the low threshold and next to an edge is an edge.
- Iteratively label edges
 - Edges grow out from 'strong edges'
 - Iterate until no change in image.

Canny Results

Edge detector



it can not close the
"T"

$\sigma=1$, $T_2=255$, $T_1=1$
high low

'Y' or 'T' junction
problem with
Canny operator

Canny Results



$\sigma=1$, $T_2=255$, $T_1=220$



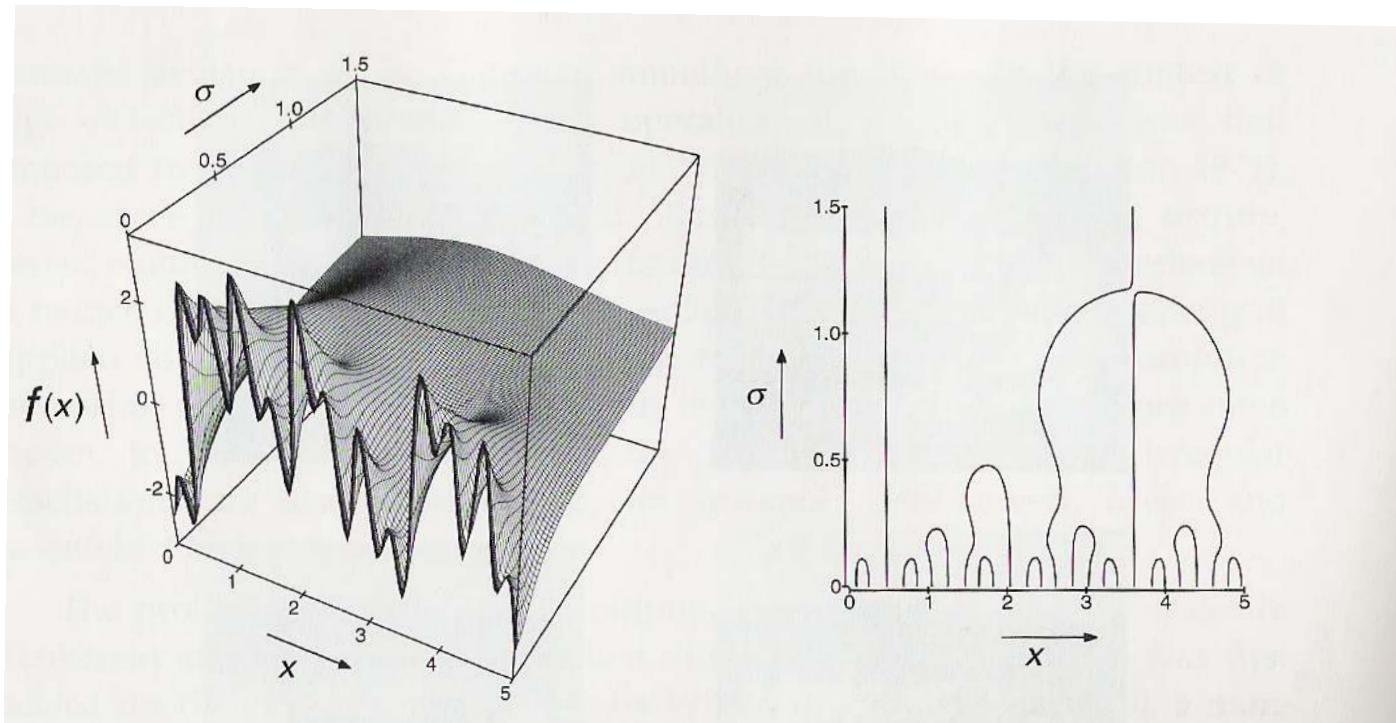
$\sigma=1$, $T_2=128$, $T_1=1$



$\sigma=2$, $T_2=128$, $T_1=1$

$\uparrow 6 \Rightarrow$ gets rid of contours
high freq's
never adds new contours

Scale Space Revisited



Increasing scale (σ) removes details but never adds new ones:

- Edge position may shift.
- Two edges may merge.
- An edge may **not** split into two.

Multiple Scales



$$\sigma = 1$$



$$\sigma = 2$$



$$\sigma = 4$$

→ Choosing the right scale is a difficult semantic problem.

Scale vs Threshold



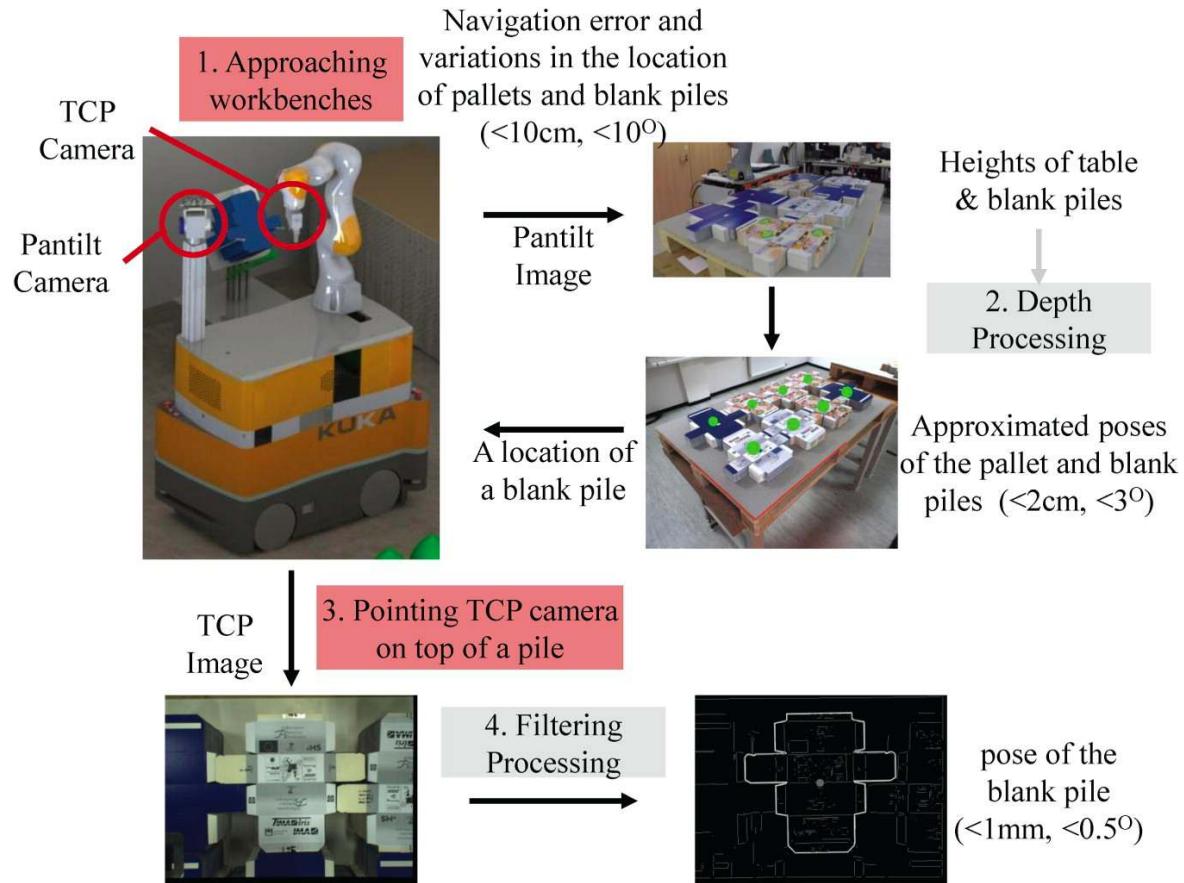
difference
this makes huge

Fine scale
High threshold

Coarse scale
High threshold

Coarse scale
Low threshold

Industrial Application



In industrial environments where the Canny parameters can be properly adjusted:

You can control Pigdlining

- It is fast.
- Does not require training data.

Visual Servoing



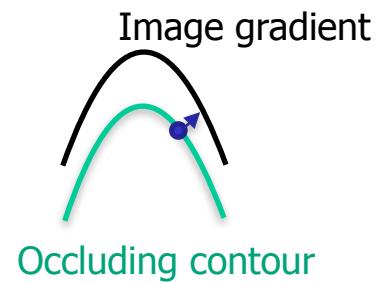
→ A useful tool in our toolbox.

Tracking a Rocket

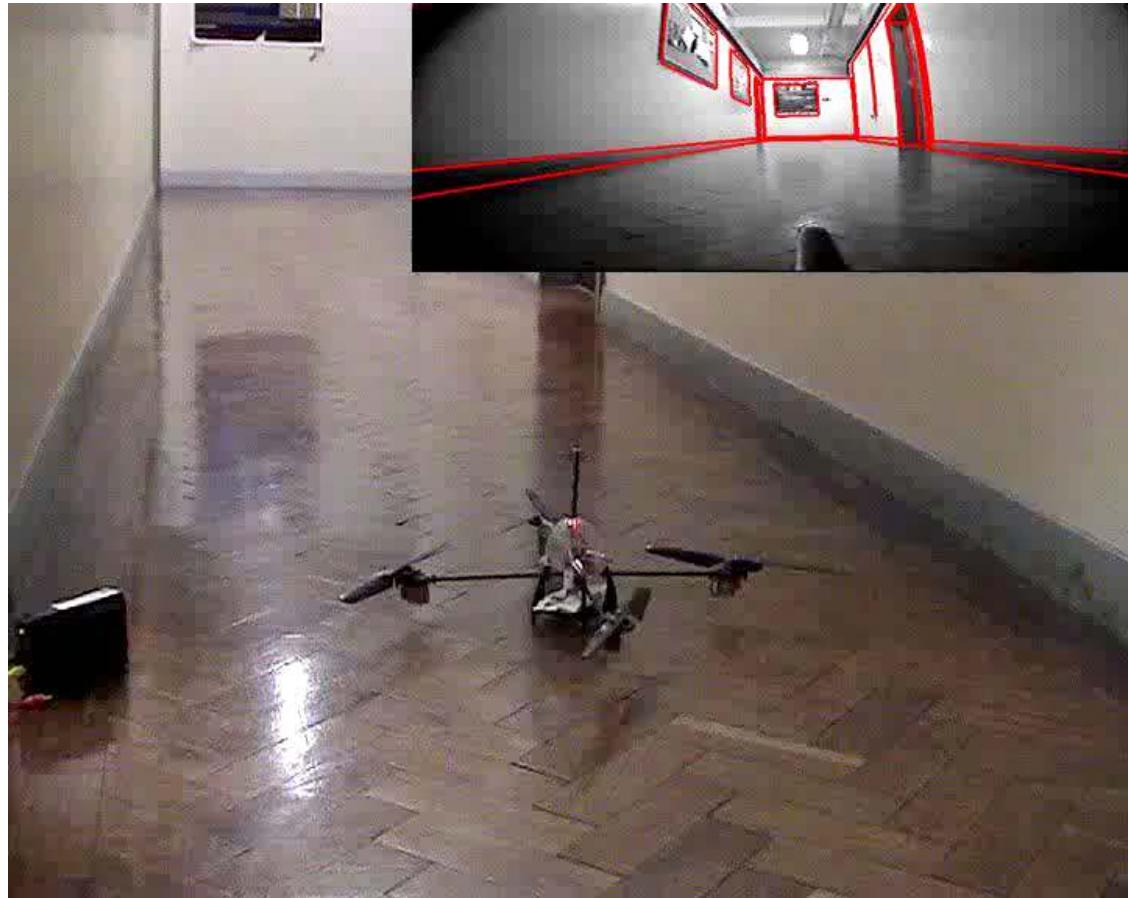


Given an initial pose estimate:

- Find the occluding contours. *Using canny*
- Find closest edge points in the normal direction.
- Re-estimate pose to minimize sum of square distances.
- Iterate until convergence.



Visual Servoing

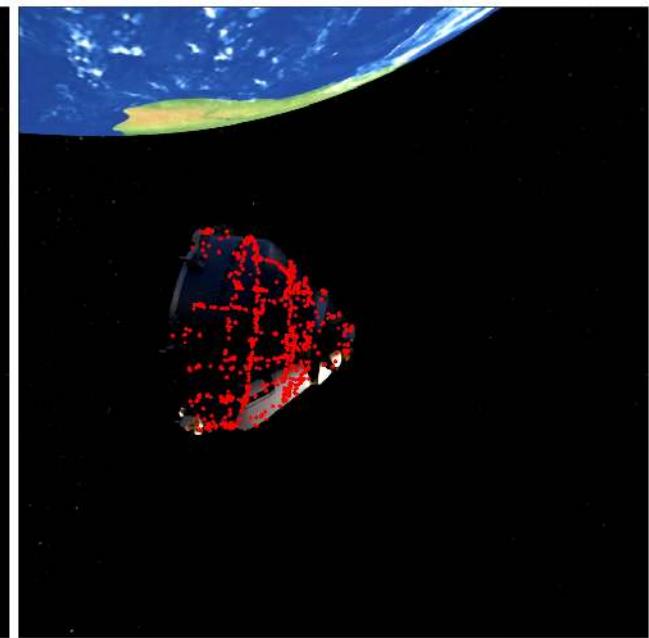
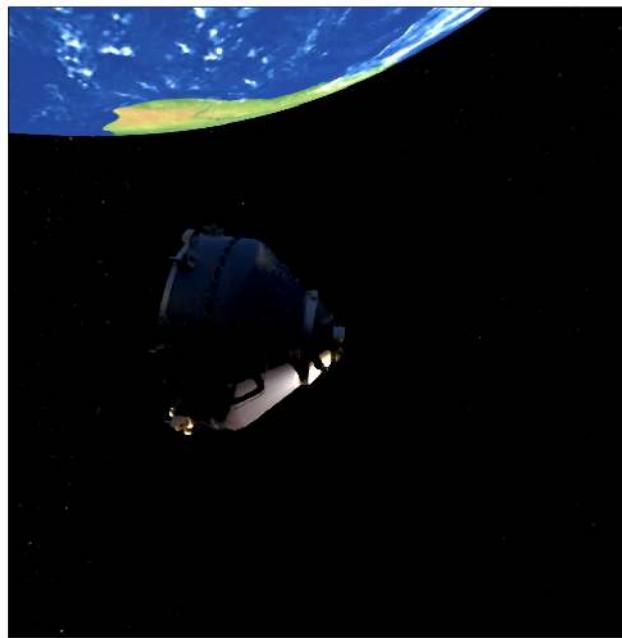


You can set parameters T_1, T_2

Space Cleaning

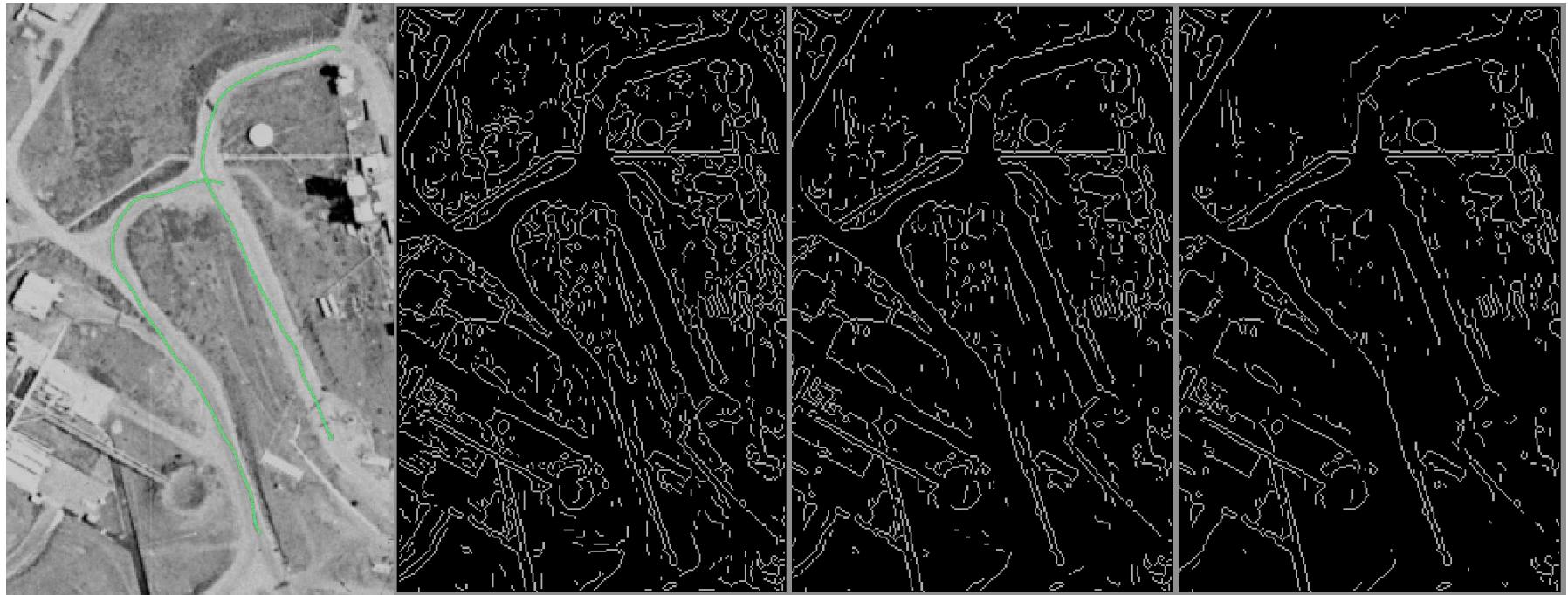


Capturing and deorbiting
a dead satellite.



- A more sophisticated version of this old algorithm will blast off in 2025!
- ESA does not yet trust neural nets for such a mission.

Limitations of the Canny Algorithm



Find contours
of the road

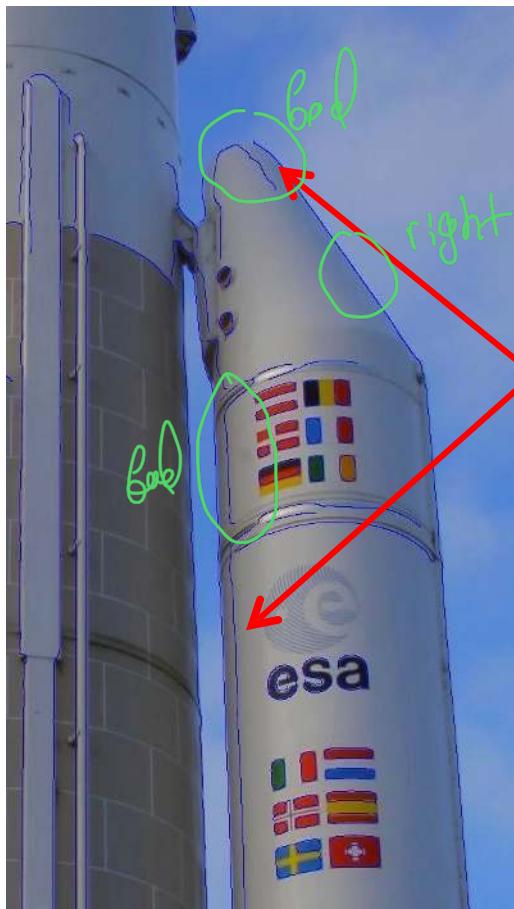
increasing σ

There is no ideal value of σ !

If you want to find edges of the road \Rightarrow no particular σ works

Steep Smooth Shading

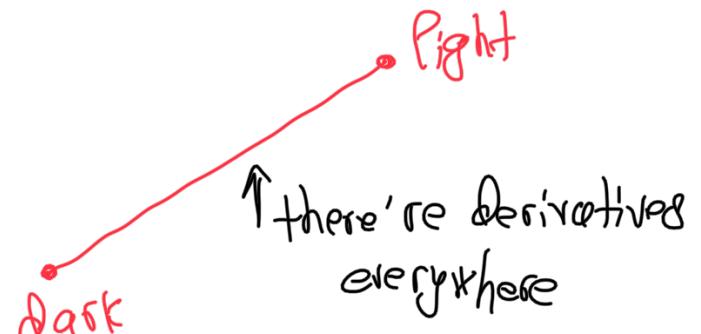
Gradient image



Canny

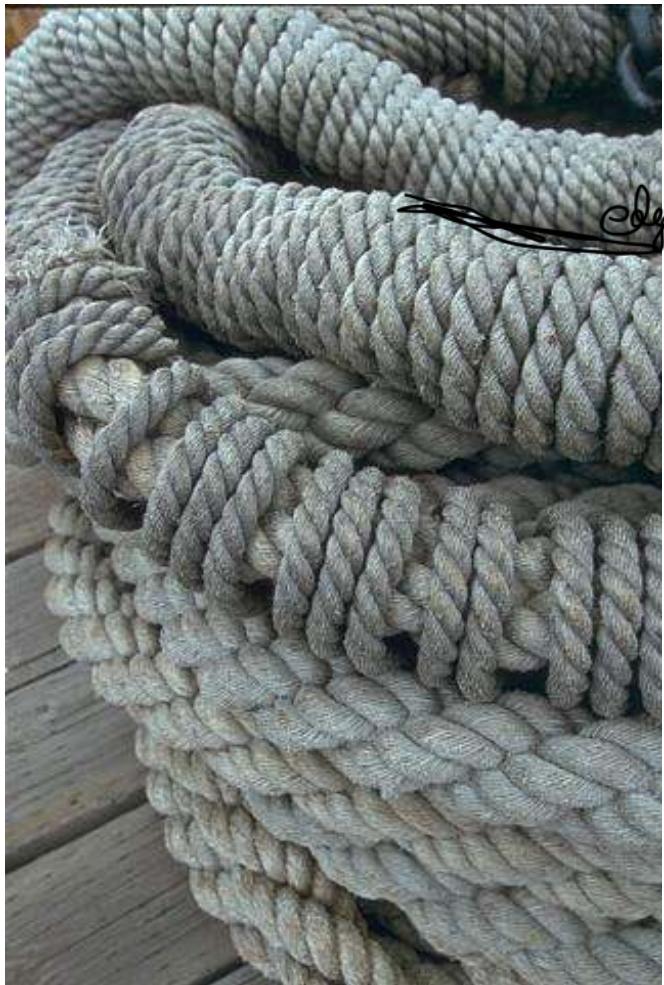
finding
step edges
to find where there
are lots of gradients

- creates large gradients
Transition light → dark continuously
- Rapidly varying gray levels.
 - Large gradients.



→ Shading can produce spurious edges.

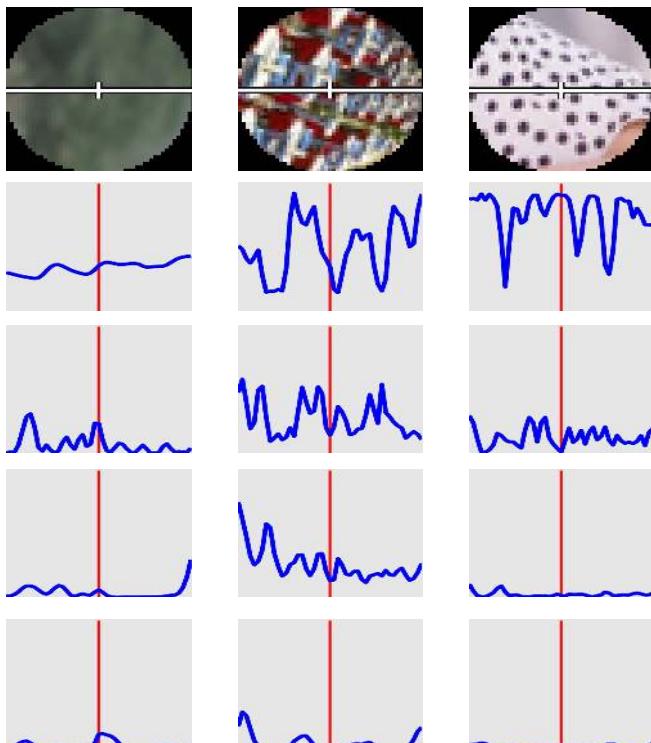
Texture Boundaries



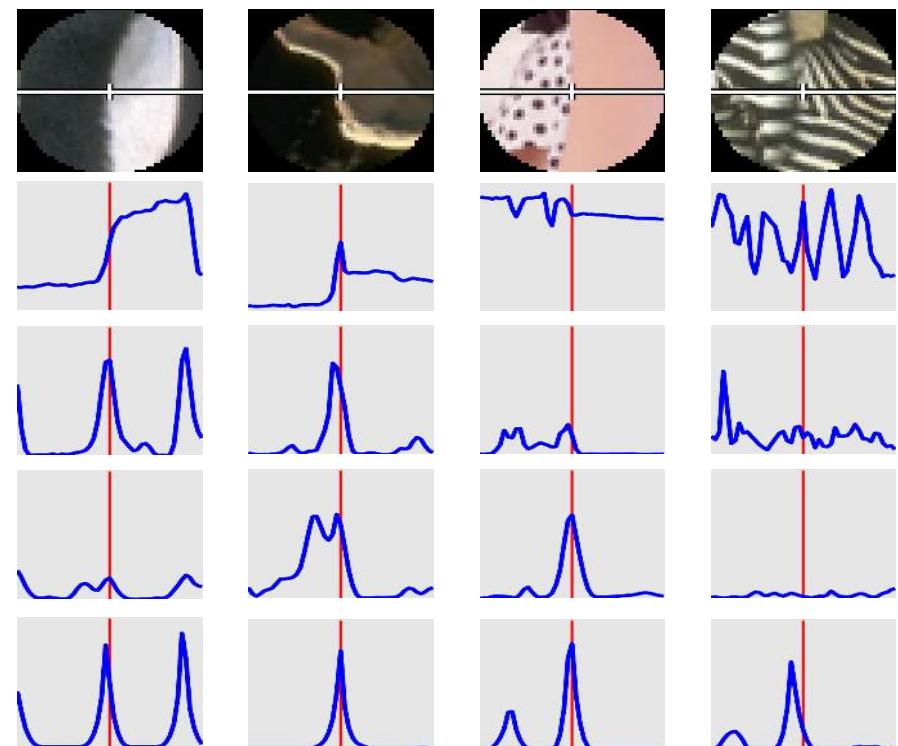
- Not all image contours are characterized by strong contrast.
- Sometimes, textural changes are just as significant.

Different Boundary Types

No contours
Non-boundaries



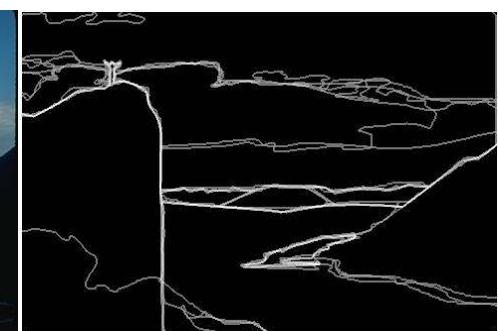
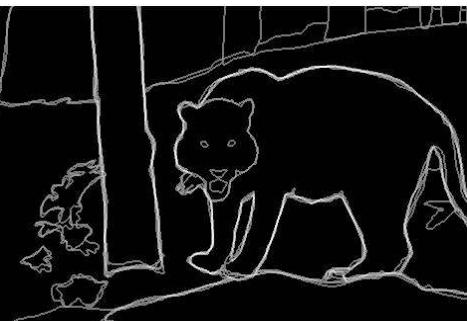
There are
Boundaries



Teach ML model to detect boundaries

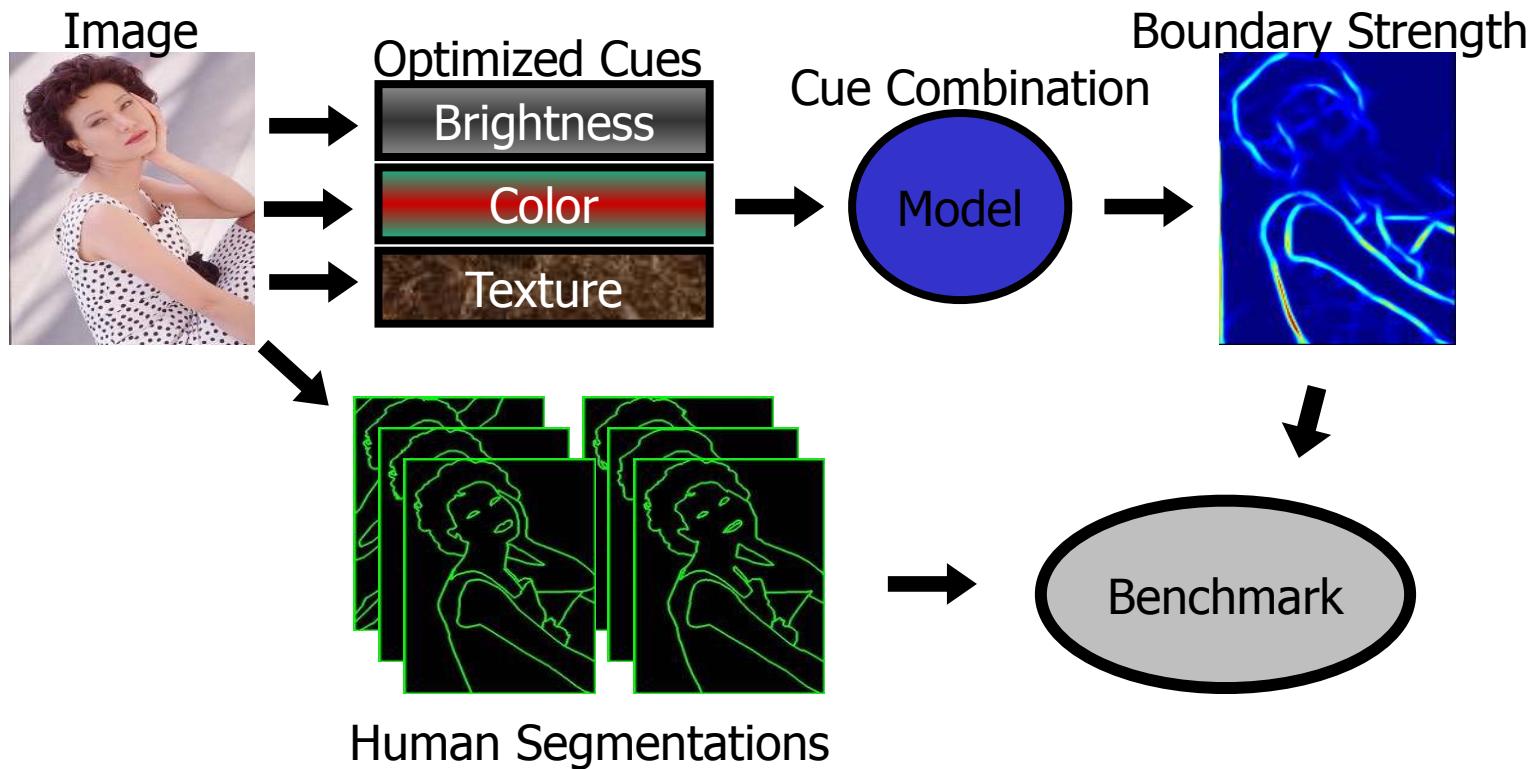
Training Database

Draw contours



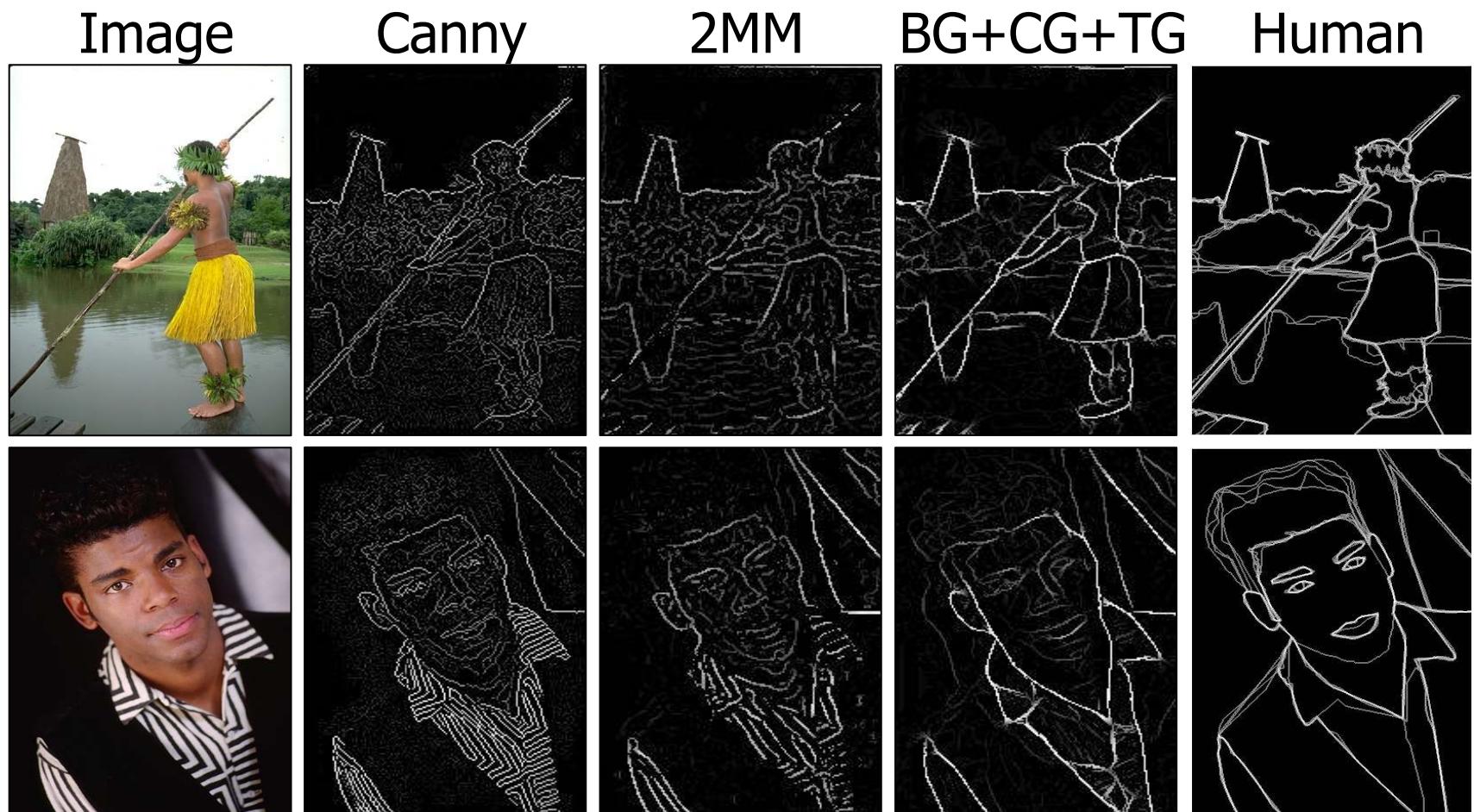
1000 images with 5 to 10 segmentations each.

Machine Learning



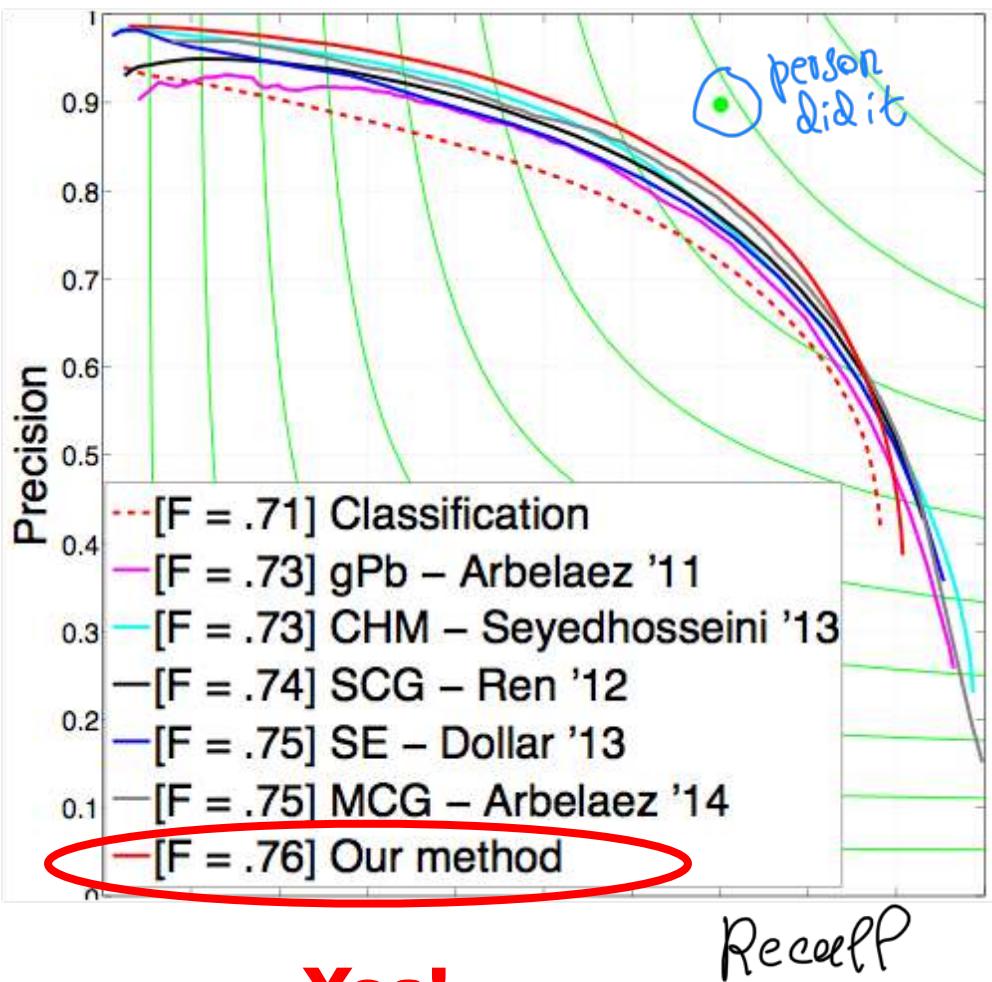
Learn the probability of being a boundary pixel on the basis of a set of features.

Comparative Results

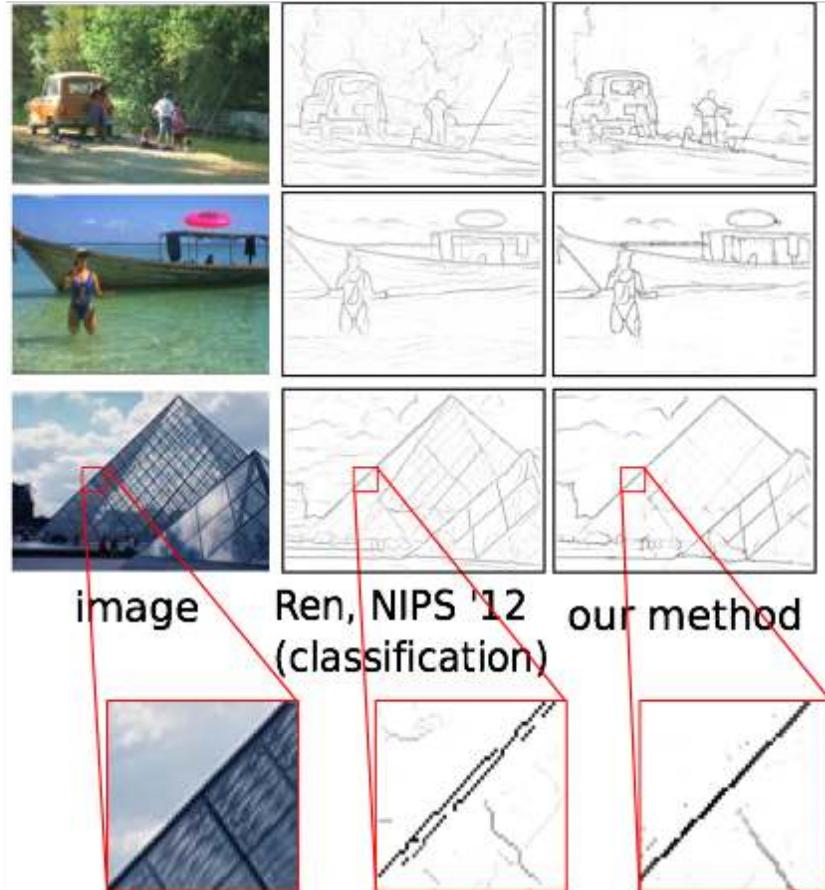


Find contours which person would find

Classification vs Regression

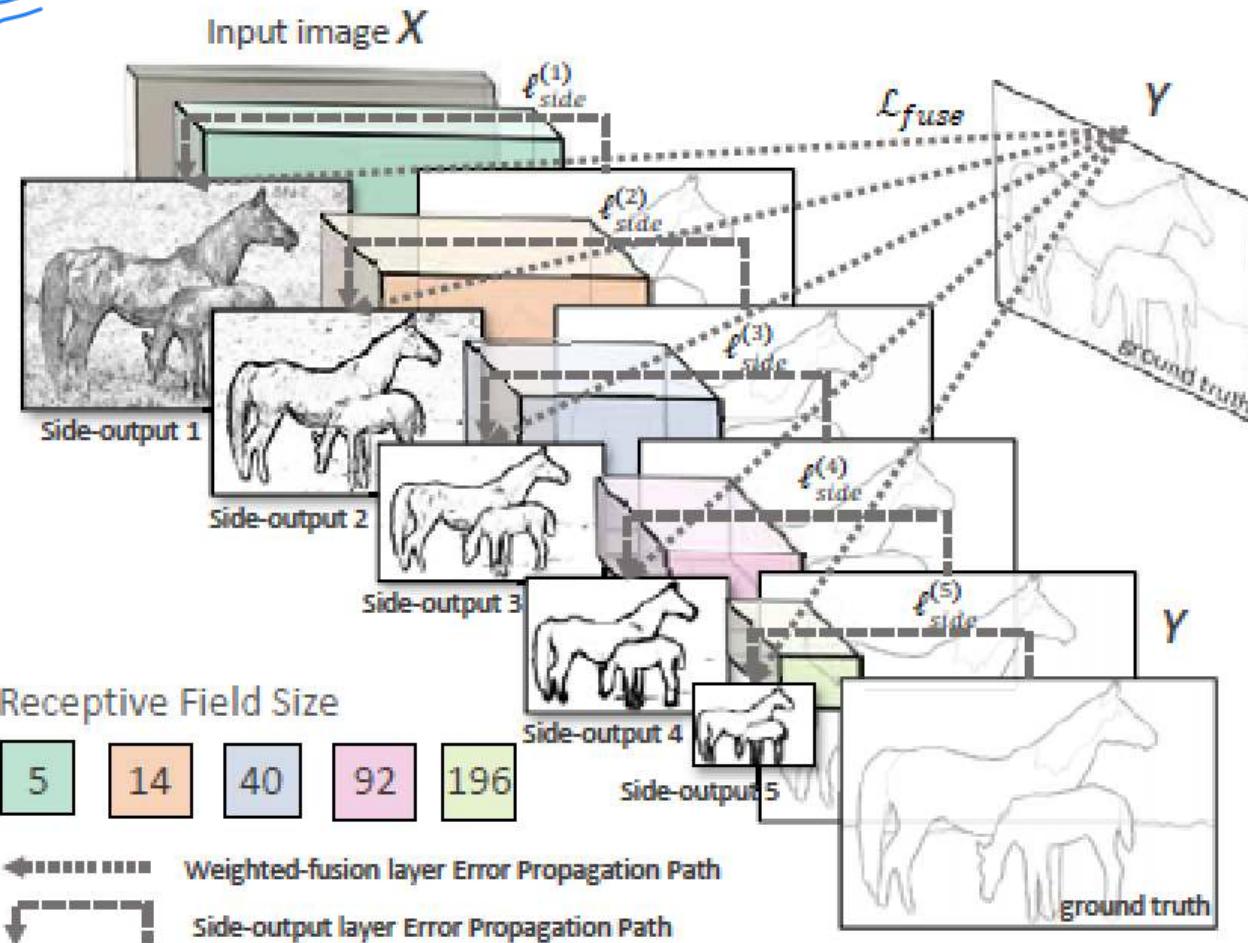


Yes!
human performance → not perfect



Deep Learning

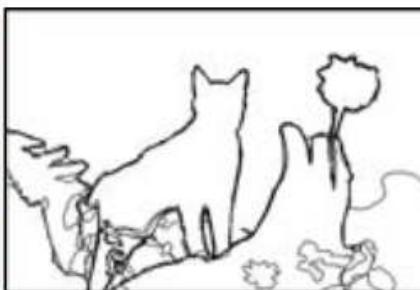
ML on contours \Rightarrow DL these days
=



Deep Learning Vs Canny



(a) original image



(b) ground truth



(c) HED: output

finding relevant contours



(d) HED: side output 2



(e) HED: side output 3



(f) HED: side output 4

Deep network



(g) Canny: $\sigma = 2$



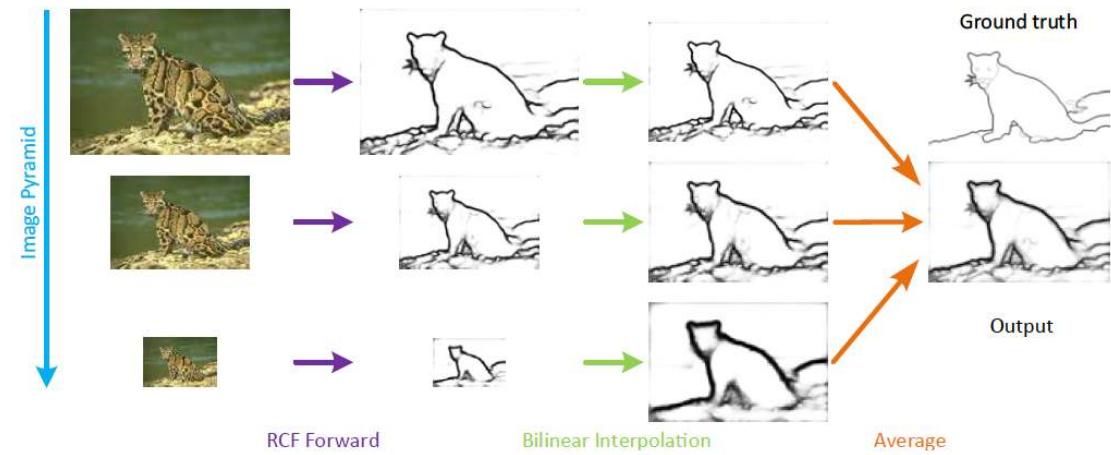
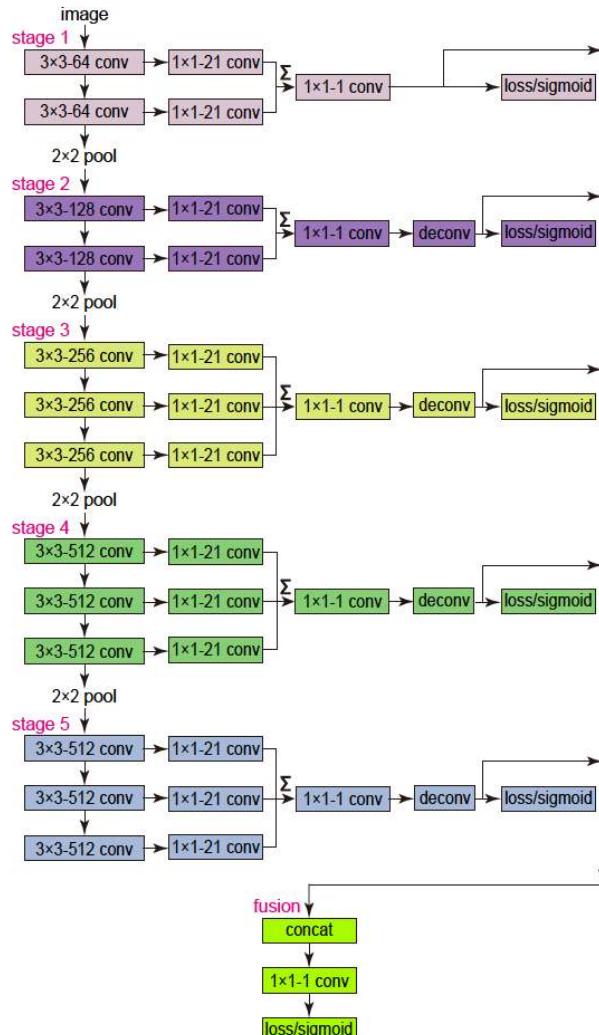
(h) Canny: $\sigma = 4$



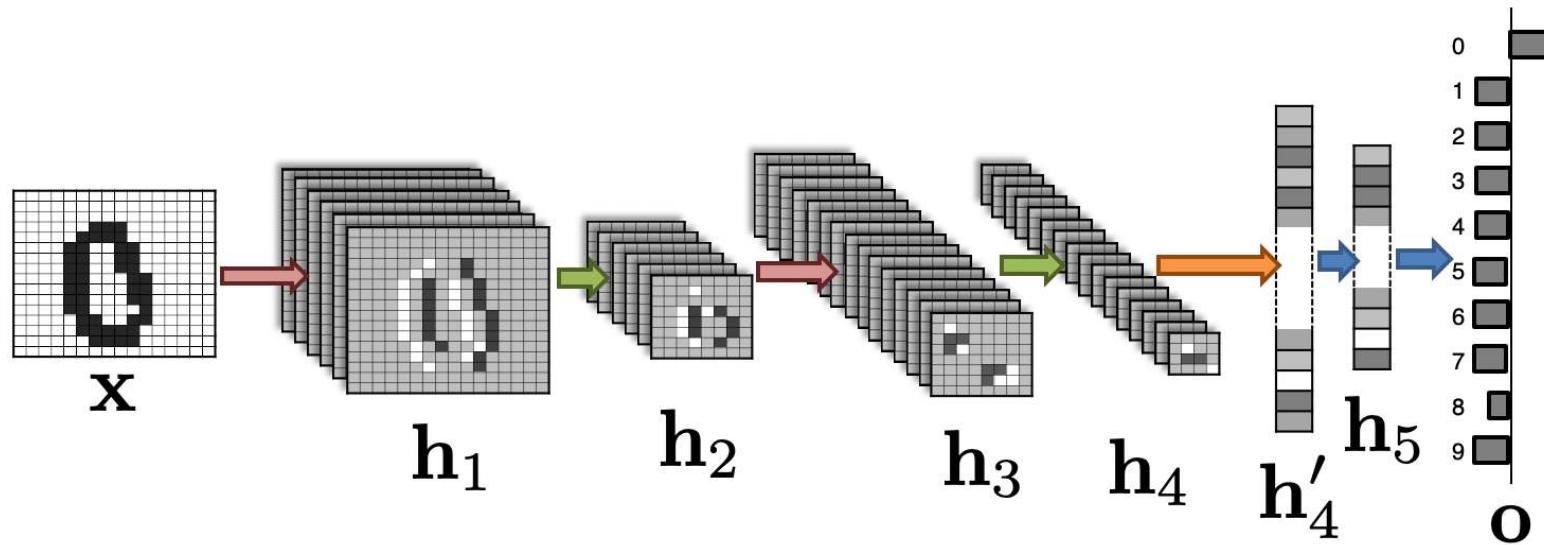
(i) Canny: $\sigma = 8$

finding significant contours \rightarrow hard

Deeper Learning

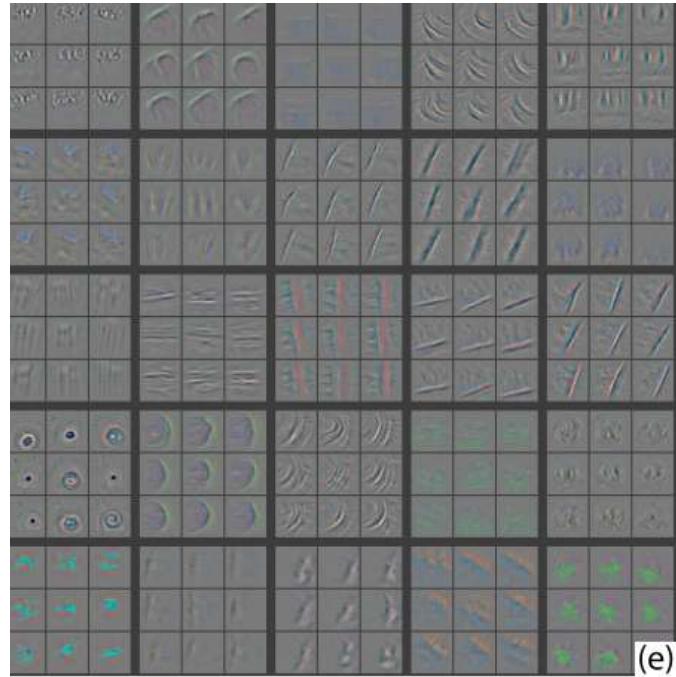
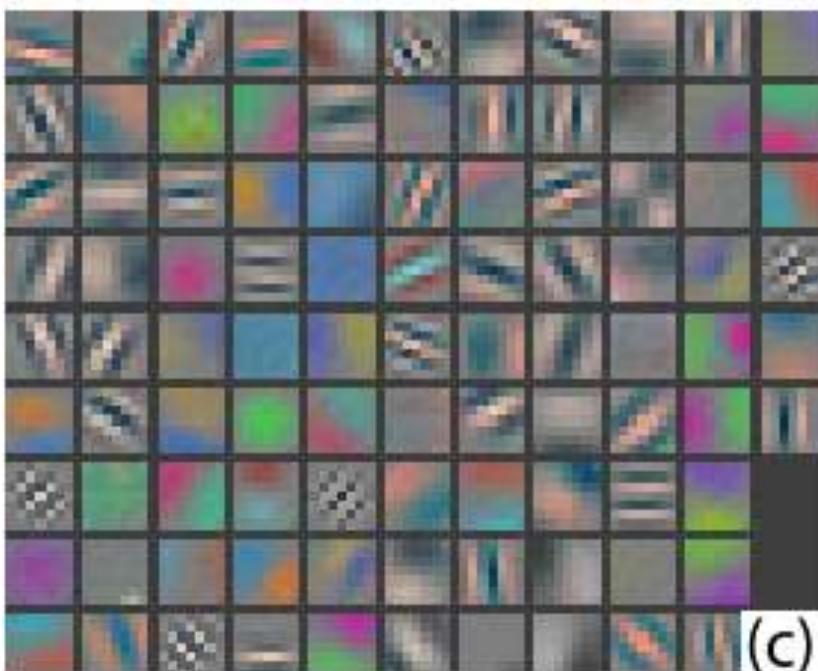


Convolutional Neural Network



- Succession of convolutional and pooling layers.
- Fully connected layers at the end.
→ Will be discussed in more detail in the next lecture.

A Partial Explanation?

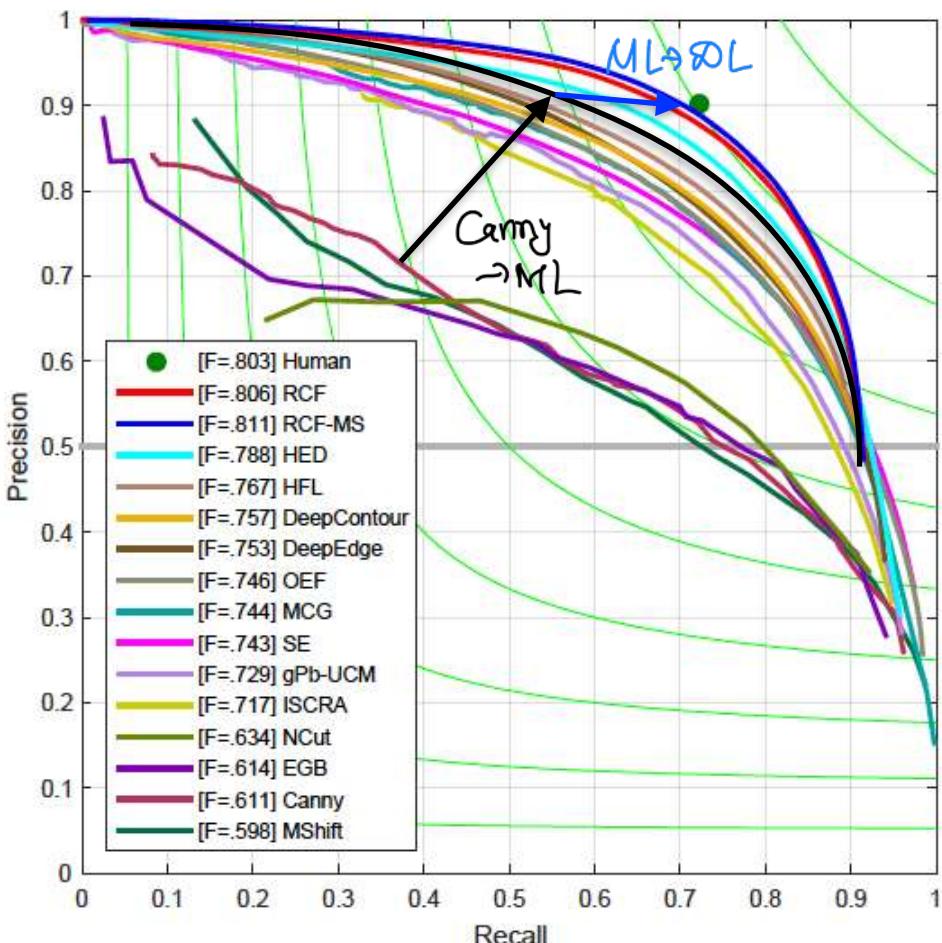


Derivatives of Gaussians ??

First and second layer features of a Convolutional Neural Net:

- They can be understood as performing multiscale filtering.
- The weights and thresholds are chosen by the optimization procedure.

50 Years Of Edge Detection



In rockets, gradual transition dark derivatives are white computed so they get fired everywhere

- Convolution operators respond to steep smooth shading.
- Parametric matchers tend to reject non ideal edges. Too noisy
- Arbitrary thresholds and scale sizes are required. θ, T_1, T_2
- Learning-based methods need exhaustive databases. for ML
Canny survived \rightarrow no training database needed
- There still is work to go from contours to objects.

Canny, PAMI'86 —> Sironi et al. PAMI'15

Sironi et al. PAMI'15 —> Liu et al., CVPR'17

Let us talk about deep networks.