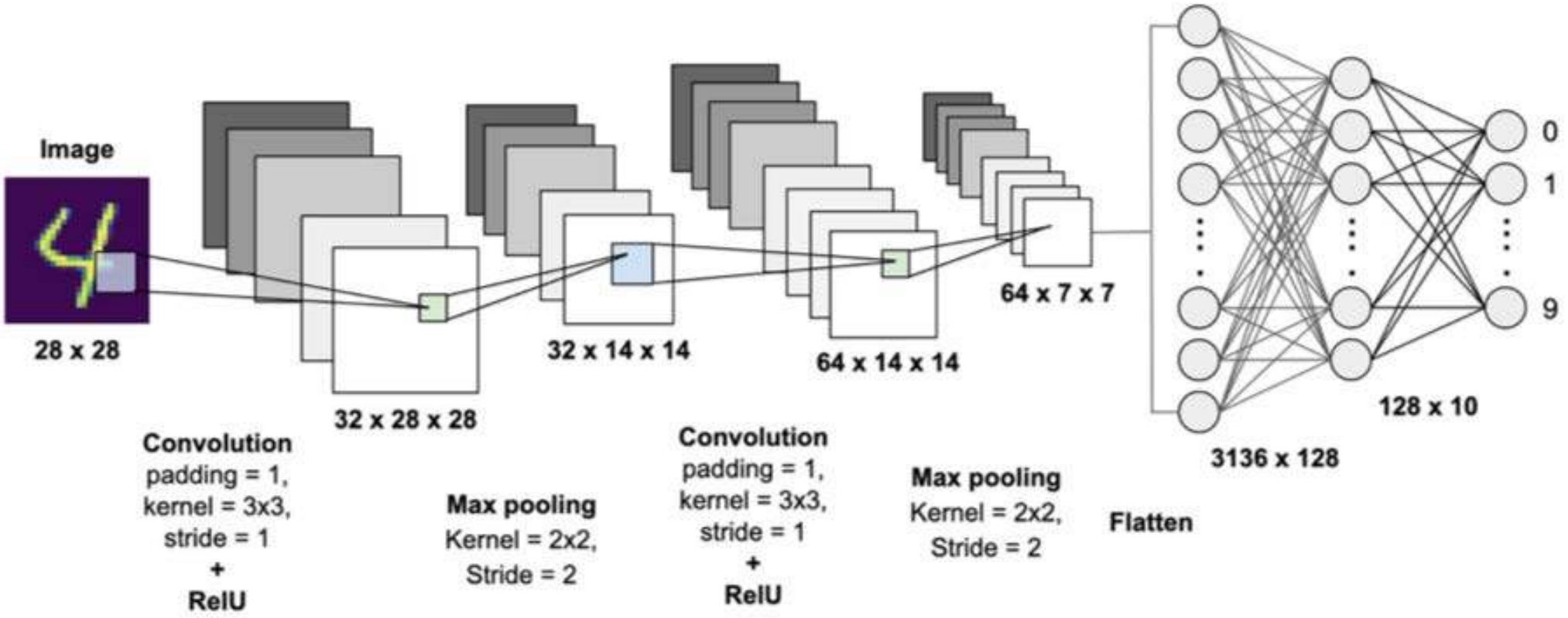
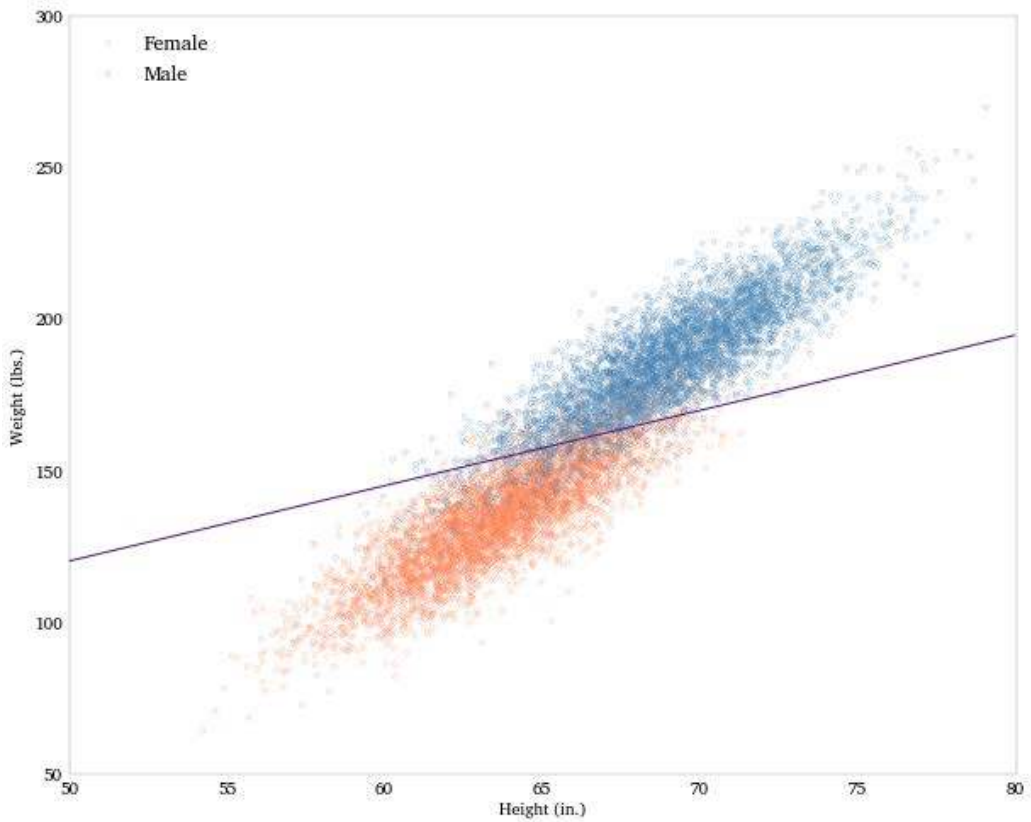
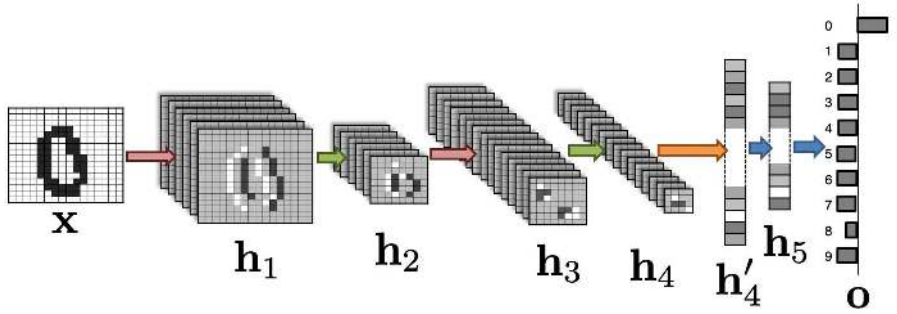
Deep Learning Crash Course

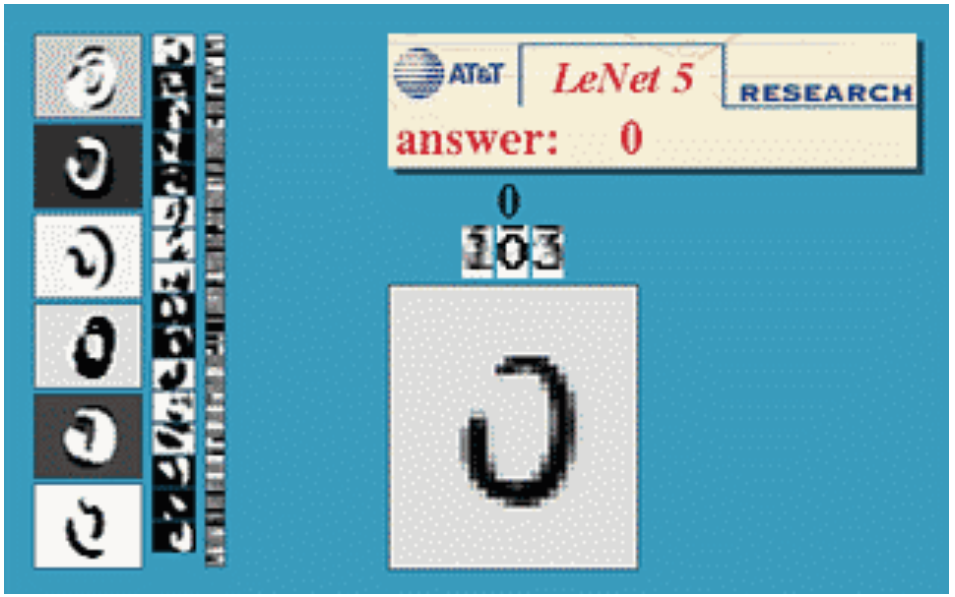


- Single Layer Perceptrons Basis
- Multiple Layer Perceptrons
- Convolutional Neural Nets Most of used

Binary vs Multi-Class Classification



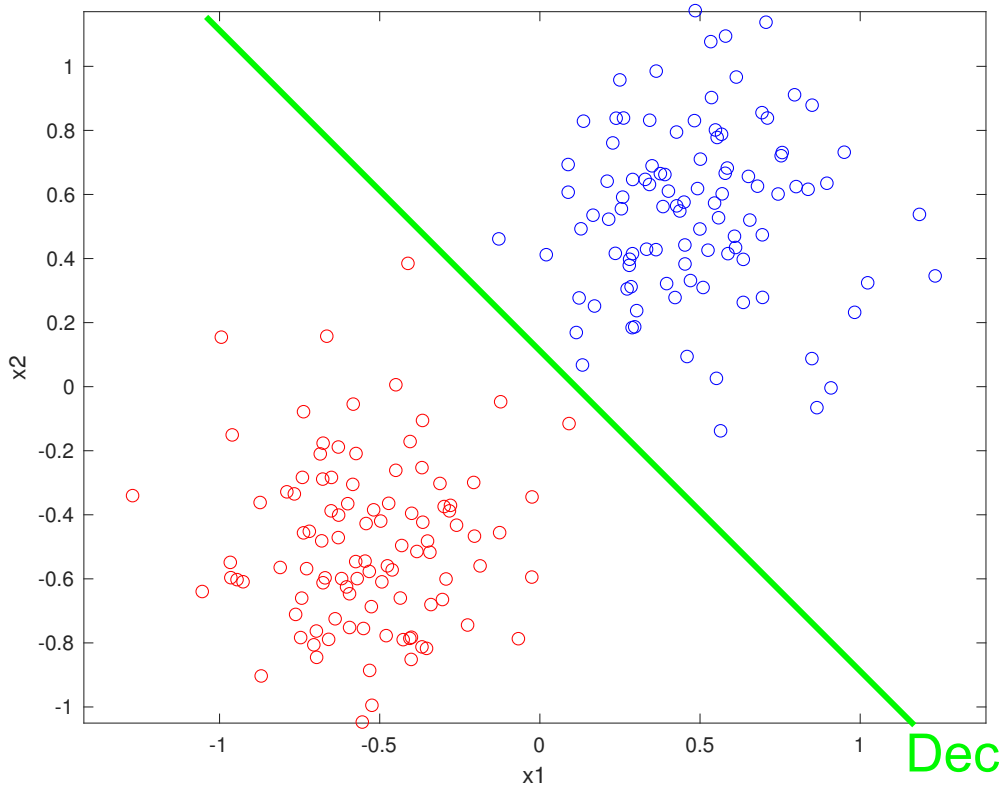




Logistic Regression

LeNet

Linear Binary Classification



General → hyperplane in dim = n-1

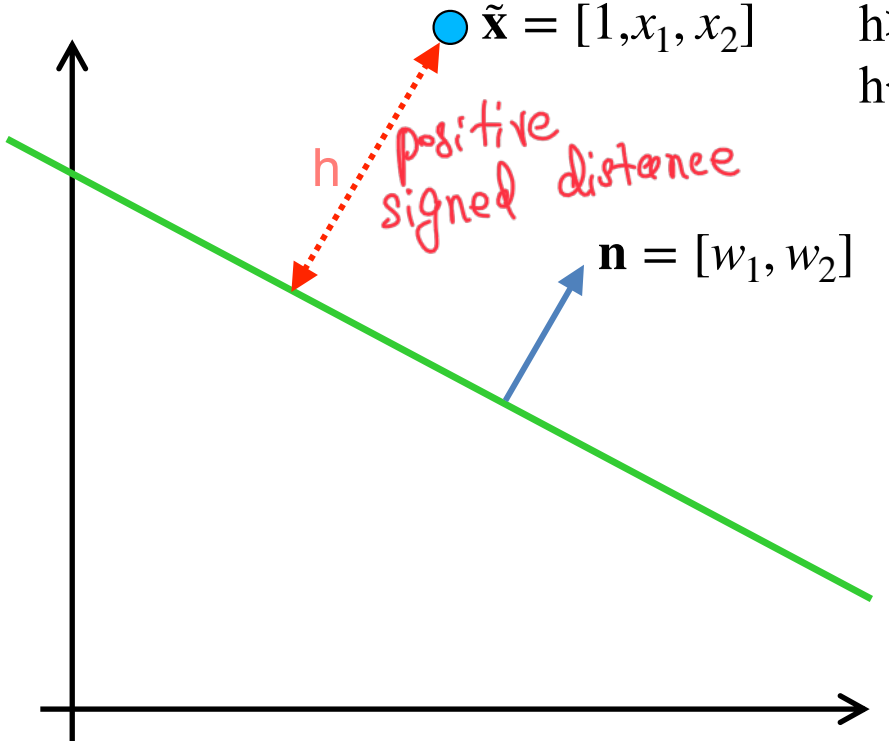
Decision boundary

Two classes shown as different colors:

- The label $y \in \{-1,1\}$ or $y \in \{0,1\}$.
- The samples with label 1 are called positive samples.
- The samples with label -1 or 0 are called negative samples.

Signed Distance

h=0: Point is on the line.
h>0: Point in the normal's direction.
h<0: Point in the other direction.



Notation:

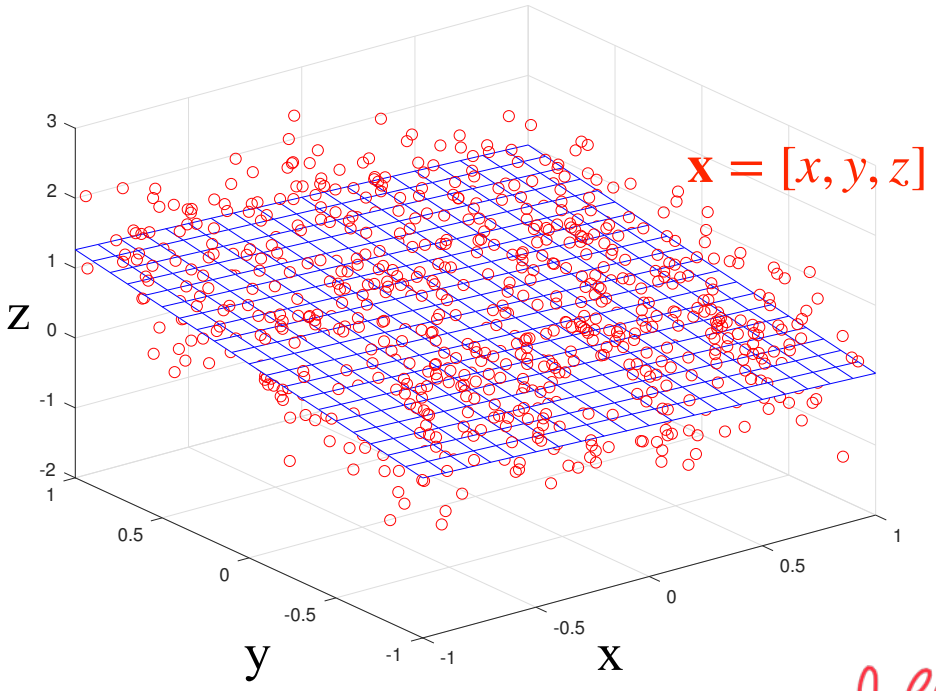
$$\mathbf{x} = [x_1, x_2]$$
$$\tilde{\mathbf{x}} = [1, x_1, x_2]$$

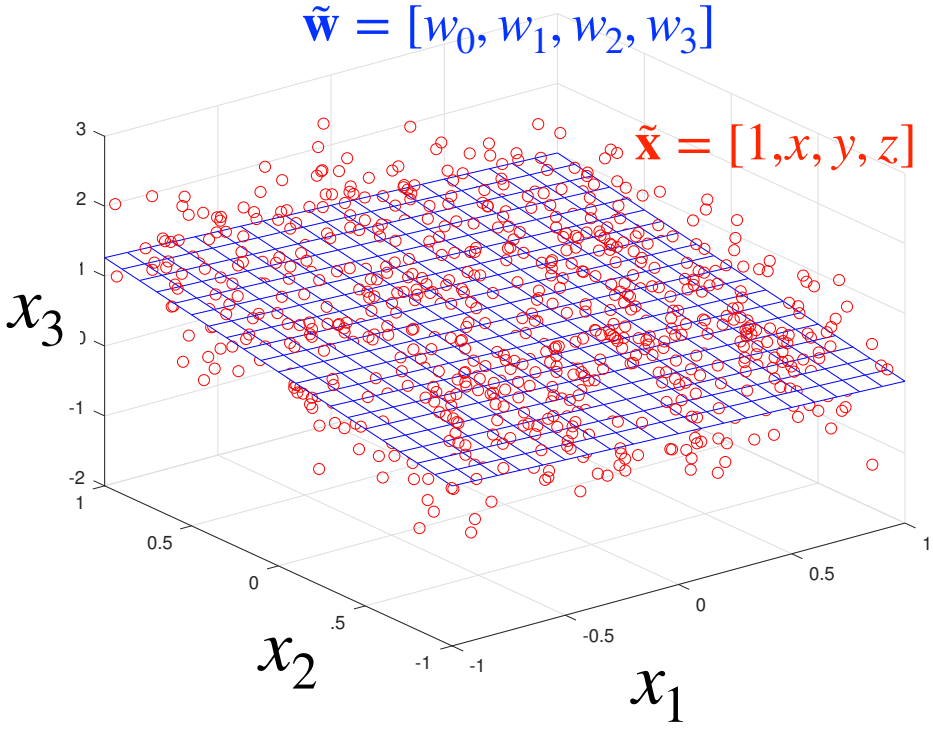
→ represent 2D-point in 3-coordinates

Signed distance:

$$h = w_0 + w_1 x_1 + w_2 x_2$$
$$= \tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}}$$

Signed Distance in 3D





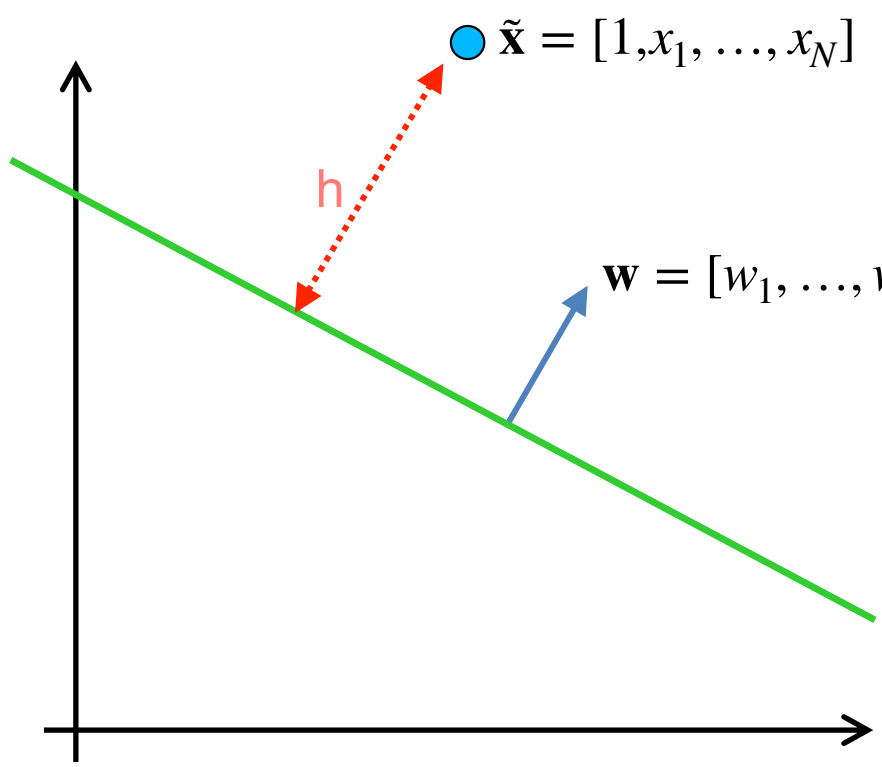
Plane Defined by [a,b,c,d]

$$\mathbf{x} \in R^3, 0 = ax + by + cz + d$$

$$\tilde{\mathbf{x}} \in R^4, \tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}} = 0$$

Signed distance $h = \tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}}$ if $w_1^2 + w_2^2 + w_3^2 = 1$.

Signed Distance in N Dimensions



h=0: Point is on the decision boundary.
h>0: Point on one side.
h<0: Point on the other side.

hyperplane

$\tilde{\mathbf{w}} = [w_0, w_1, \ldots, w_n]$ with $\sum_{i=1}^{N} w_i^2 = 1$

defined by

Notation: $\mathbf{x} = [x_1, \ldots, x_n]$

$\tilde{\mathbf{x}} = [1, x_1, \ldots, x_n]$

Hyperplane: $\mathbf{x} \in R^n, \quad 0 = \tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}}$

$= w_0 + w_1 x_1 + \ldots w_n x_n$

Signed distance: $h = \tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}}$

true in n dimensions

Linear Binary Classification in N Dimensions

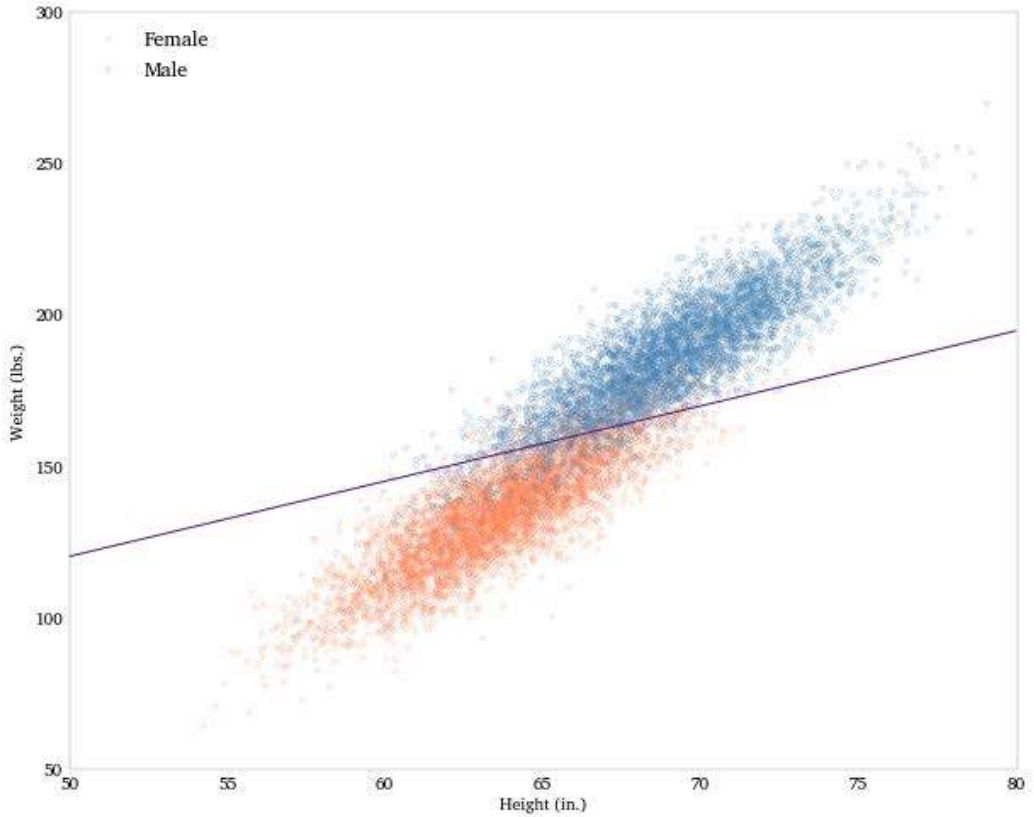
Hyperplane: $\mathbf{x} \in R^N, \tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}} = 0$, with $\tilde{\mathbf{x}} = [1 \,|\, \mathbf{x}]$.

Signed distance: $\tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}}$, with $\tilde{\mathbf{w}} = [w_0 \,|\, \mathbf{w}]$ and $||\mathbf{w}|| = 1$.

Problem statement: Find $\tilde{\mathbf{w}}$ such that Given training set

- for all or most positive samples $\tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}} > 0$,
- for all or most negative samples $\tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}} < 0$.

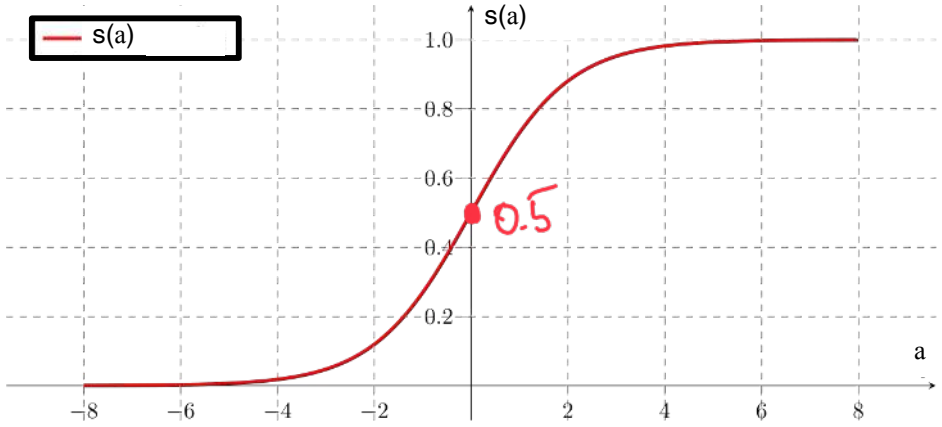
Logistic Regression



$$y(\mathbf{x}; \tilde{\mathbf{w}}) = \sigma(\tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}}) = \frac{1}{1 + \exp(-\tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}})}$$

predictor

signed distance



Given a **training** set $\{(\mathbf{x}_n, t_n)_{1 \le n \le N}\}$ minimize

→ label/target

↳ weight, height

$$BCE = -\sum_n (t_n \ln y(\mathbf{x}_n) + (1 - t_n) \ln(1 - y(\mathbf{x}_n)))$$

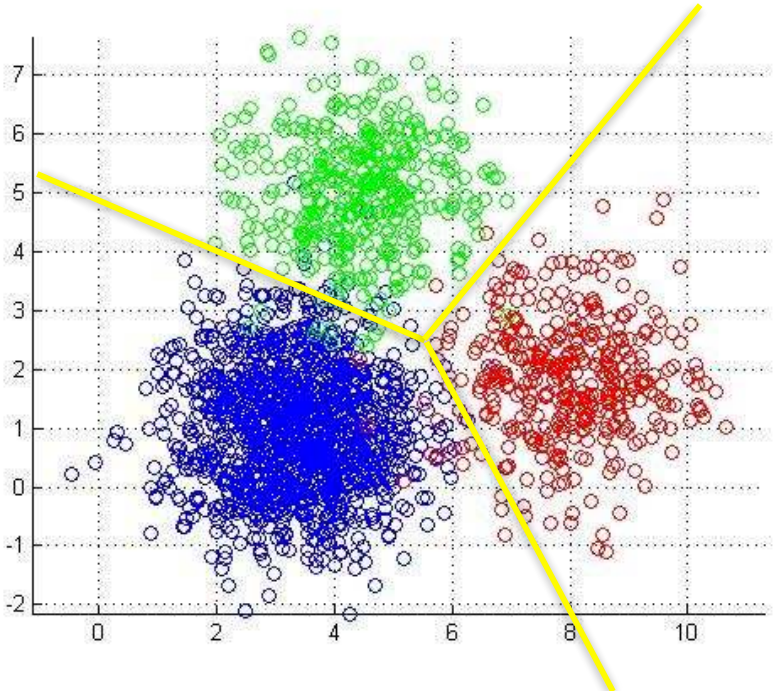
$y(x_n)$ and t_n → encourage to have same signs

with respect to $\tilde{\mathbf{w}}$.

Convex wrt $\tilde{w}$

- When the noise is Gaussian, this is the maximum likelihood solution.
- $y(\mathbf{x}; \tilde{\mathbf{w}})$ can be interpreted at the probability that $\mathbf{x}$ belongs to positive class.

Multi-Class Logistic Regression



$$k = \arg\max_j y_k(\mathbf{x})$$

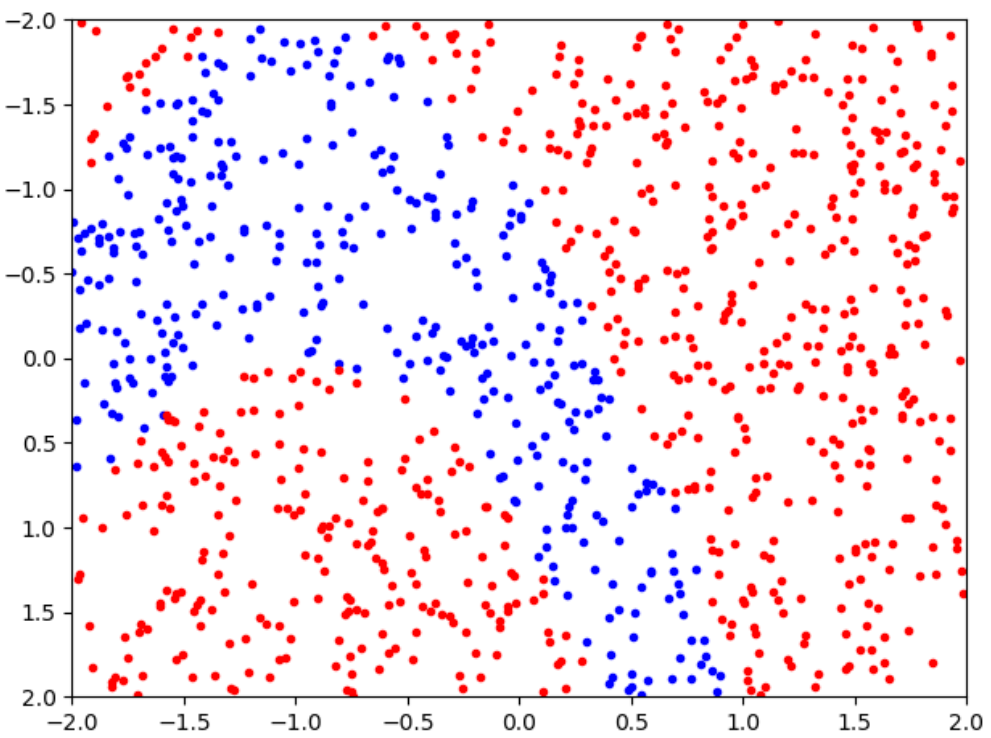
$$\begin{bmatrix} y_1 \\ \vdots \\ y_K \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{w}}_1^T \\ \vdots \\ \tilde{\mathbf{w}}_K^T \end{bmatrix} \tilde{\mathbf{x}}$$

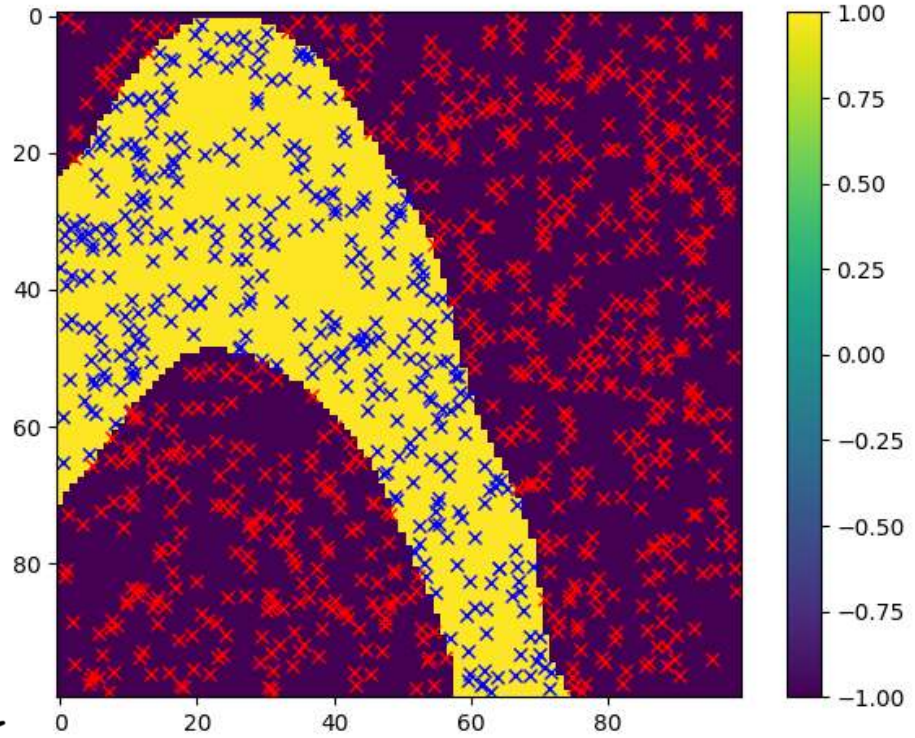
$$k = \arg\max_j y_j$$

- K linear classifiers of the form $y^k(\mathbf{x}) = \sigma(\mathbf{w}_k^T \mathbf{x})$.
- Assign x to class k if $y^k(\mathbf{x}) > y^l(\mathbf{x}) \forall l \neq k$.

- Because the sigmoid function is monotonic, the formulation is almost unchanged.
- Only the objective function being minimized need to be reformulated.

Non Separable Distribution





Positive: $100(x_2 - x_1^2)^2 + (1 - x_1)^2 < 0.5$

Negative: Otherwise

$y(\mathbf{x}; \tilde{\mathbf{w}})$ must be a non-linear function.

↳ parameters of function

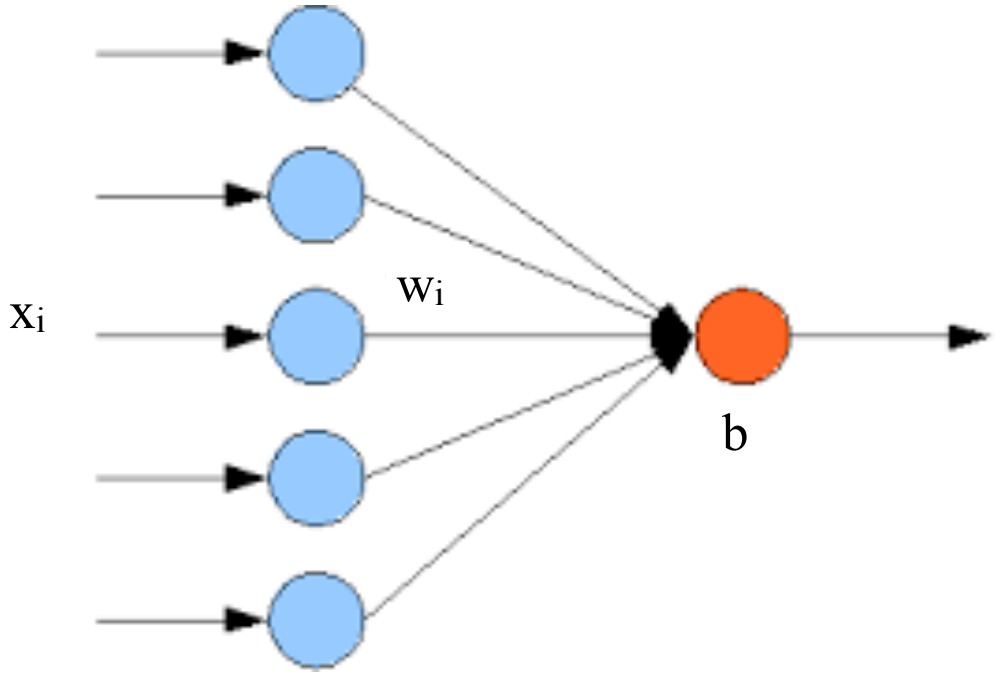
- Logistic regression can handle a few outliers but not a complex non-linear boundary.
- How can we learn a function y such that $y(\mathbf{x}; \tilde{\mathbf{w}})$ is close to 1 for positive samples and close to 0 or -1 for negative ones?

Logistic regression → 1 hyperplane ✓

—> Use LOTS of hyperplanes.

Reformulating Logistic Regression

Shallow networks

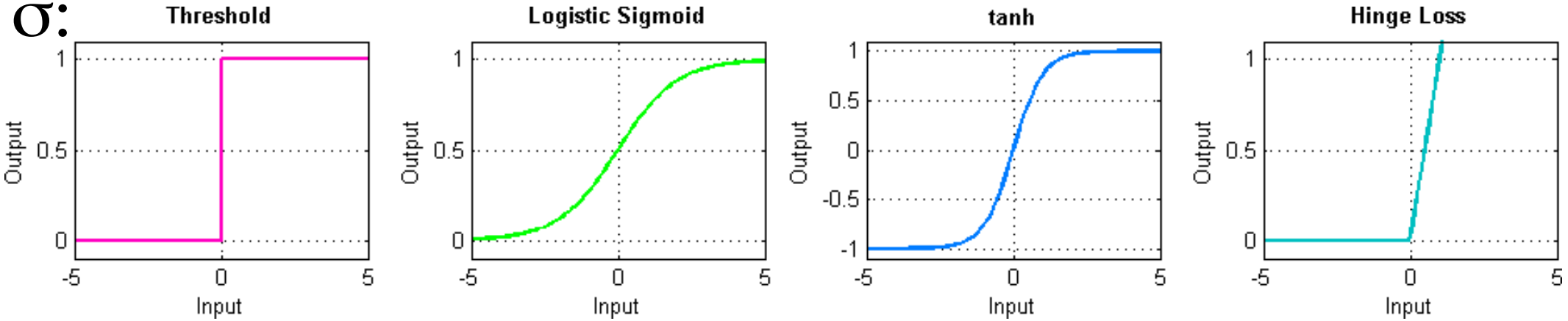


$$y(\mathbf{x}) = \sigma(\mathbf{w} \cdot \mathbf{x} + b)$$

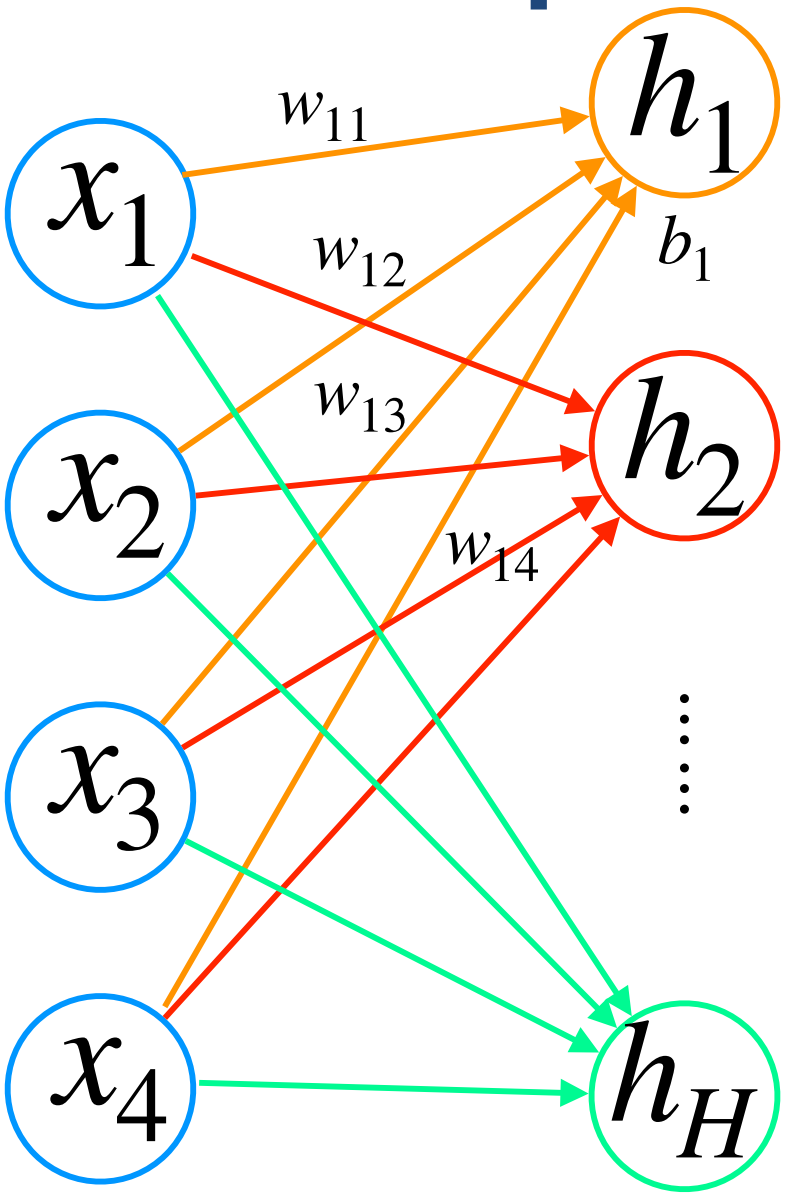
$$\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$$

$$\mathbf{w} = [w_1, w_2, \ldots, w_n]^T$$

σ:



Repeating the Process

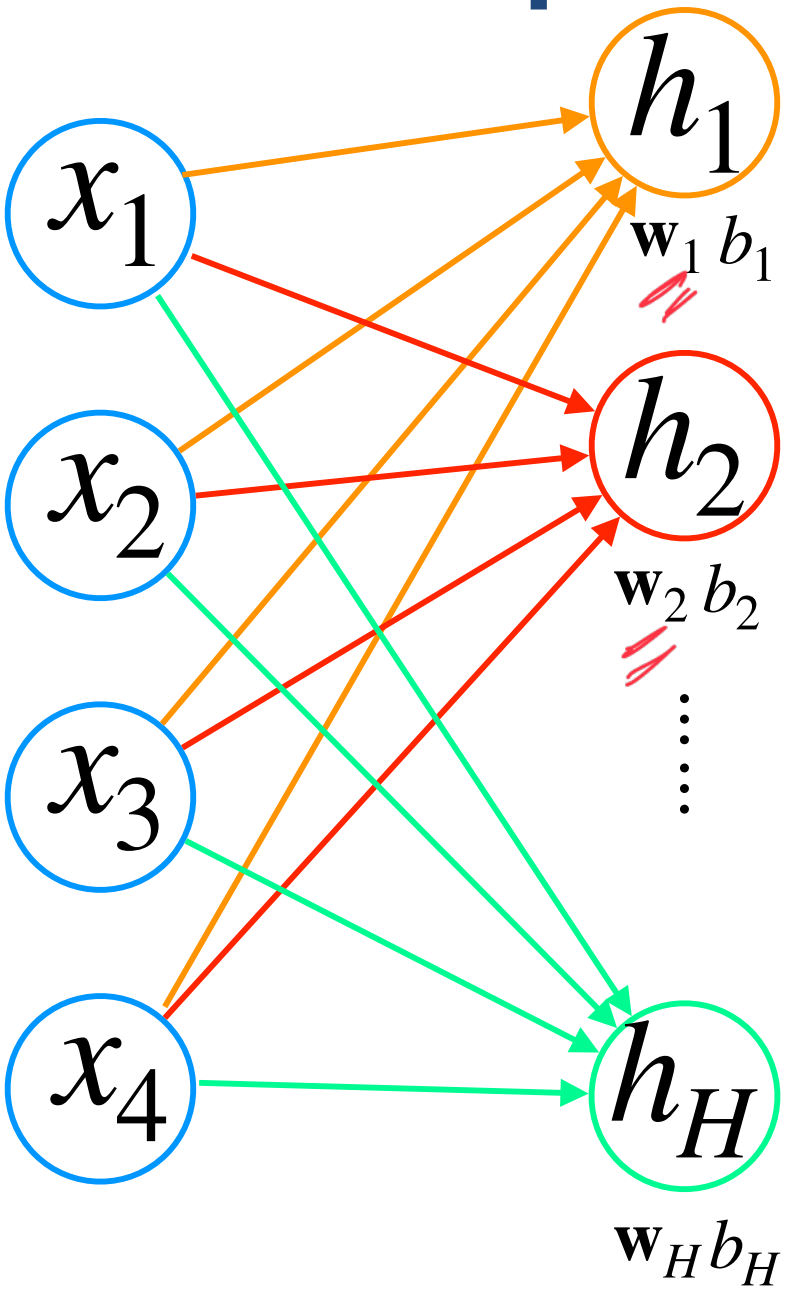


$$h_1 = \sigma(\mathbf{w}_1 \cdot \mathbf{x} + b_1)$$
$$\mathbf{w}_1 = \left[w_{11}, w_{12}, w_{13}, w_{14}\right]^T$$

$$h_2 = \sigma(\mathbf{w}_2 \cdot \mathbf{x} + b_2)$$
$$\mathbf{w}_2 = \left[w_{21}, w_{22}, w_{23}, w_{24}\right]^T$$

$$h_H = \sigma(\mathbf{w}_H \cdot \mathbf{x} + b_H)$$
$$\mathbf{w}_H = \left[w_{H1}, w_{H2}, w_{H3}, w_{H4}\right]^T$$

Repeating the Process



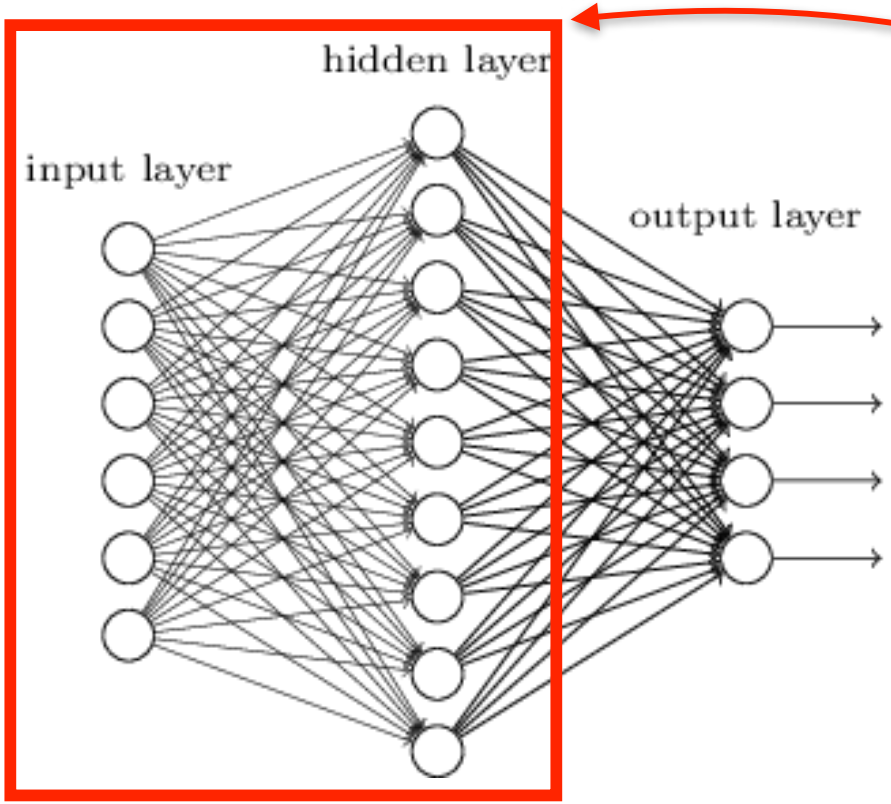
$$\mathbf{h} = \sigma(\mathbf{W}\mathbf{x} + \mathbf{b}) \ ,$$

$$\text{with } \mathbf{W} = \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_H \end{bmatrix}$$

Line in matrix

$$\text{and } \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_H \end{bmatrix} \ .$$

Multi-Layer Perceptron

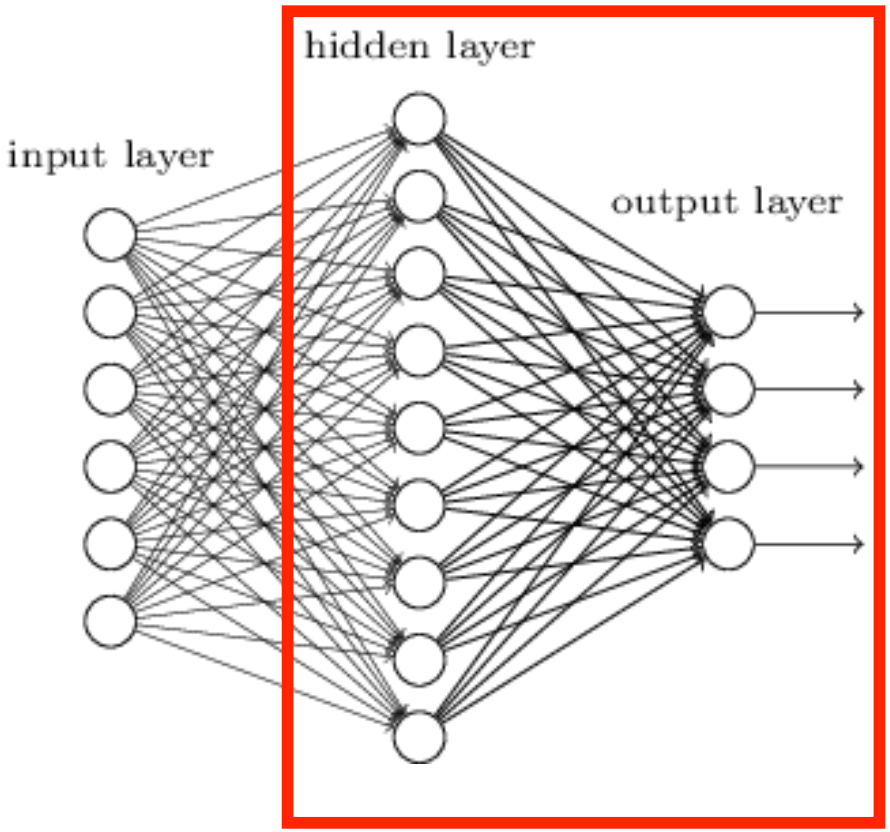


$$\mathbf{h} = \sigma_1(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1)$$

$$\mathbf{y} = \sigma_2(\mathbf{W}_2\mathbf{h} + \mathbf{b}_2)$$

- The process can be repeated several times to create a vector **h**.

Multi-Layer Perceptron



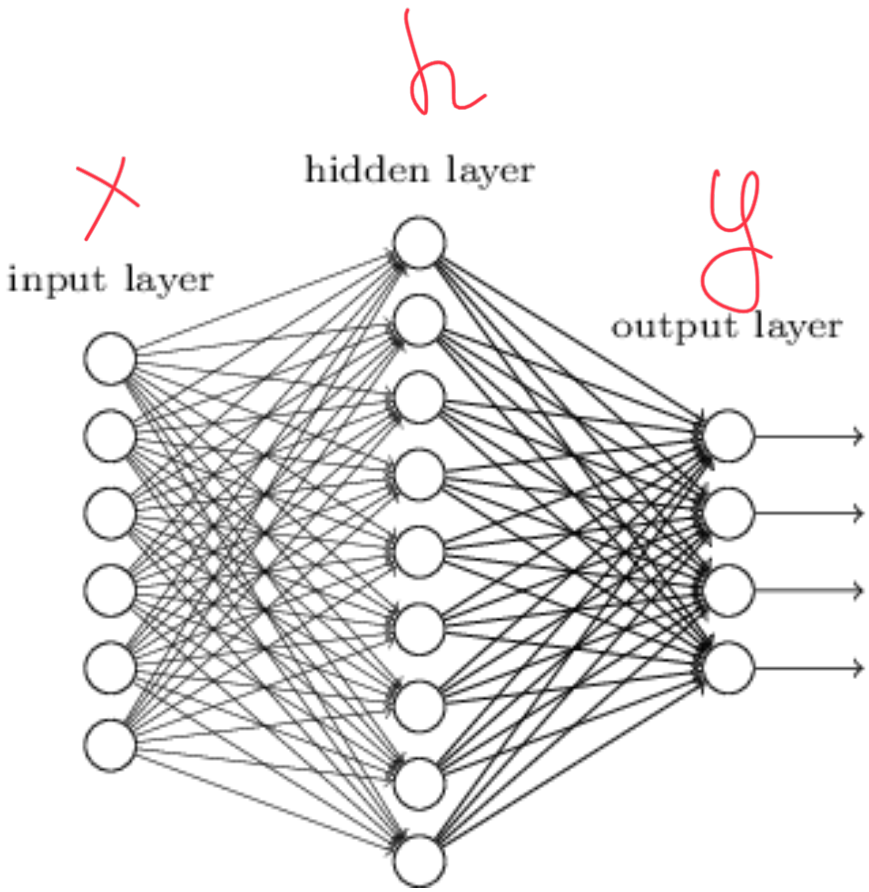
$$\mathbf{h} = \sigma_1(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1)$$

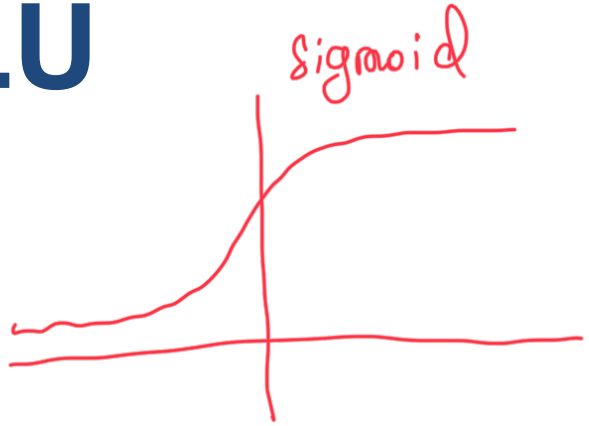
$$\mathbf{y} = \sigma_2(\mathbf{W}_2\mathbf{h} + \mathbf{b}_2)$$

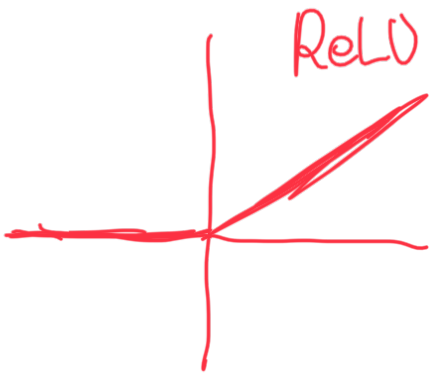
- The process can be repeated several times to create a vector **h**.
- It can then be done again to produce an output **y**.

—> This output is a **differentiable** function of the weights.

ReLU





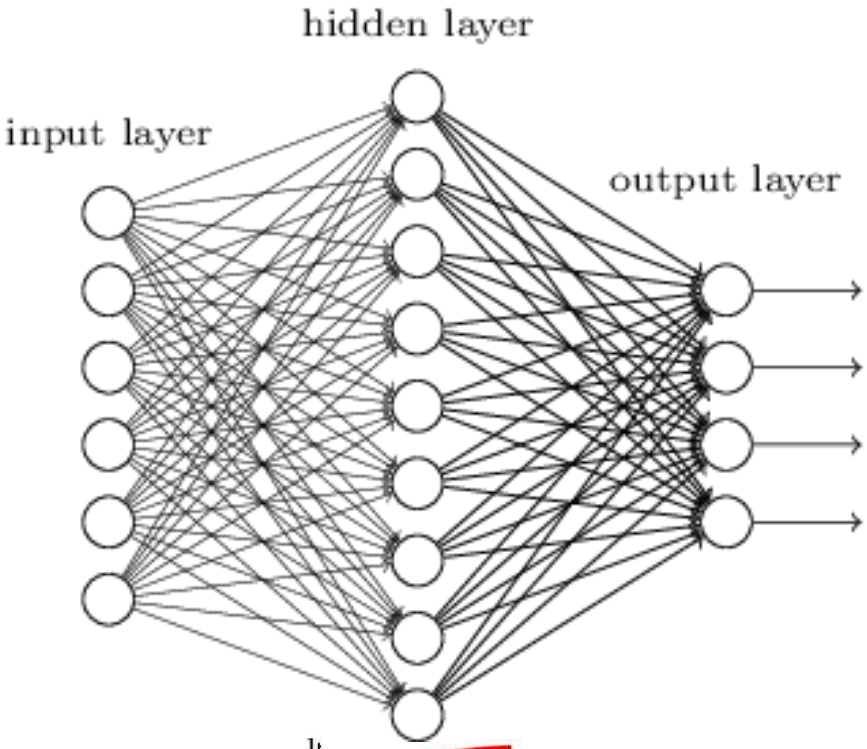


$$\mathbf{h} = \sigma_1(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1)$$
$$\mathbf{y} = \sigma_2(\mathbf{W}_2\mathbf{h} + \mathbf{b}_2)$$

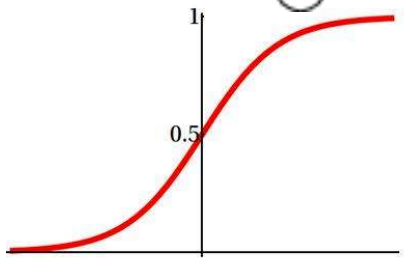
$$\sigma(\mathbf{x}) = \max(0,\mathbf{x})$$

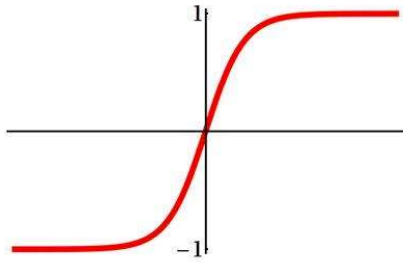
- Each node defines a hyperplane.
- The resulting function is piecewise linear affine and continuous.

Sigmoid and Tanh



$$\mathbf{h} = \sigma(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1)$$
$$\mathbf{y} = \sigma(\mathbf{W}_2\mathbf{h} + \mathbf{b}_2)$$

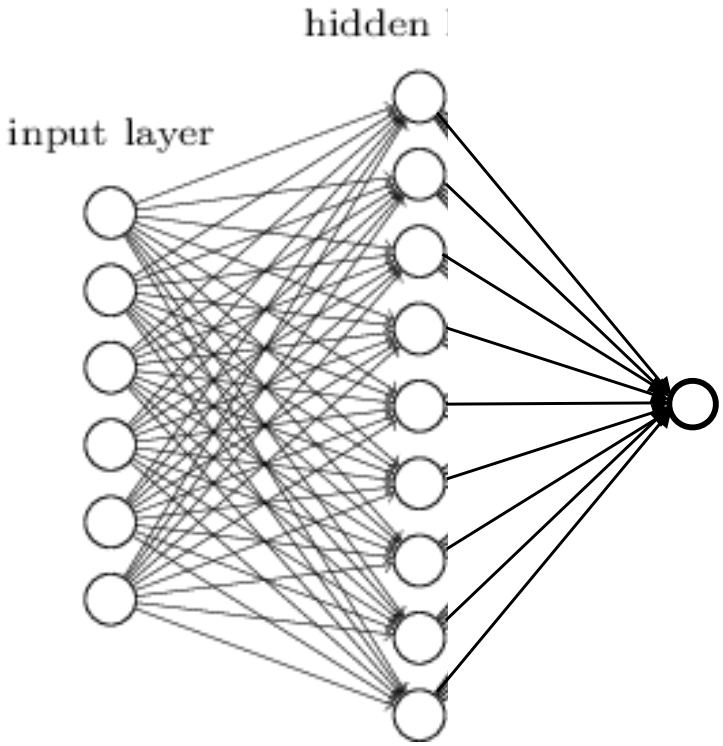




$$\text{sigm:} \quad \sigma(x) = \frac{1}{1+\exp(-x)}$$
$$\tanh: \quad \sigma(x) = \frac{\exp(x)-\exp(-x)}{\exp(x)+\exp(-x)}$$

- Each node defines a hyperplane.
- The resulting function is continuously differentiable.

Binary Case



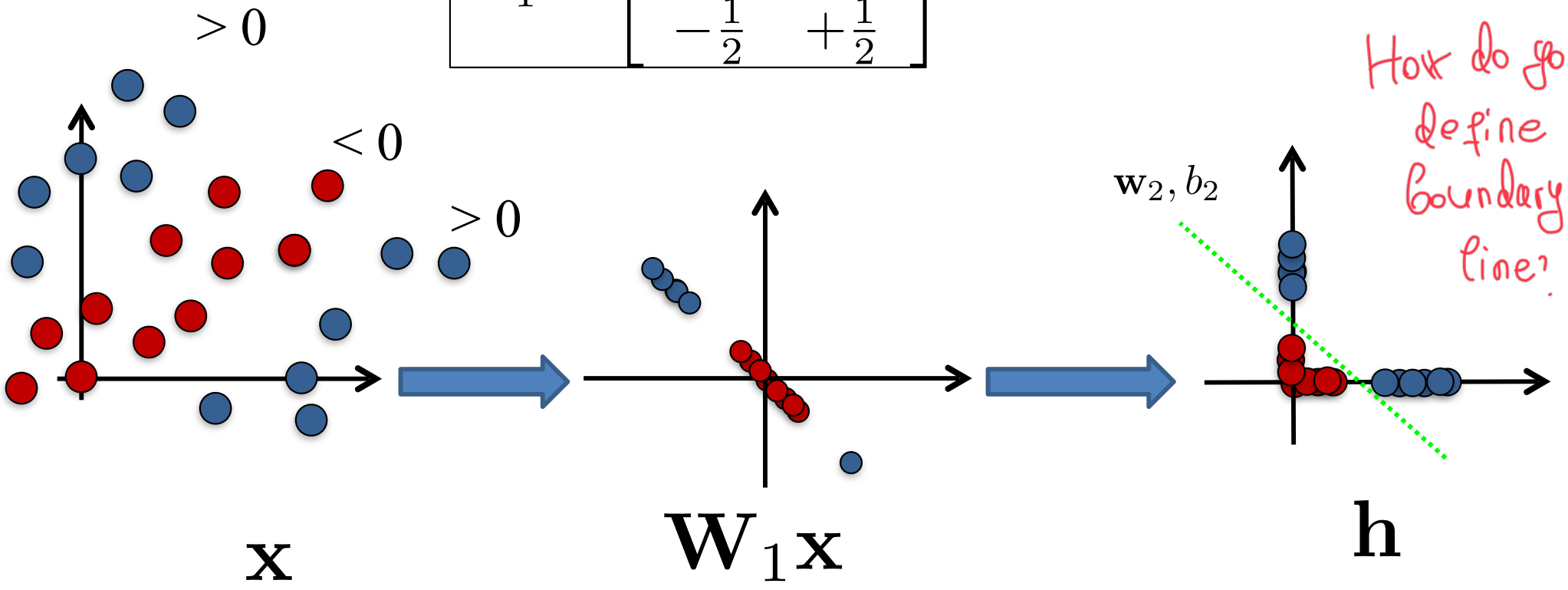
$$\mathbf{h} = \sigma(\mathbf{W}_1\mathbf{x}_n + \mathbf{b}_1)$$

$$y = \sigma(\mathbf{w}_2\mathbf{h} + b_2)$$

In this case w_2 is vector.

ReLu Behavior

$$\mathbf{W}_1 = \begin{bmatrix} +\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & +\frac{1}{2} \end{bmatrix}$$



$$\mathbf{h} = \mathrm{ReLu}(\mathbf{W}_1\mathbf{x})$$

$$\mathbf{y} = \mathbf{w}_2^T\mathbf{h} + b_2$$

Simple perceptrons

Binary Case

- Let the training set be $\{(\mathbf{x}_n, t_n)_{1 \leq n \leq N}\}$ where $t_n \in \{0,1\}$ is the class label and let us consider a neural net with a 1D output.

$t_n \rightarrow$ (-1)/1 man/woman
0/1
$x_n \rightarrow$ weight and height

- We write

$$y_n = \sigma(\mathbf{w}_2(\sigma(\mathbf{W}_1\mathbf{x}_n + \mathbf{b}_1)) + \mathbf{b}_2) \in [0,1] \, .$$

$\mathbf{w}_2$: a vector

diff'ble wrt $\mathbf{W}_1, \mathbf{W}_2$

- We want to minimize the binary cross entropy

$$E(\mathbf{W}_1, \mathbf{w}_2, \mathbf{b}_1, \mathbf{b}_2) = \frac{1}{N} \sum_{n=1}^{N} E_n(\mathbf{W}_1, \mathbf{w}_2, \mathbf{b}_1, \mathbf{b}_2) \, ,$$

$$E_n(\mathbf{W}_1, \mathbf{w}_2, \mathbf{b}_1, \mathbf{b}_2) = -\left(t_n \ln(y_n) + (1 - t_n)\ln(1 - y_n)\right) \, ,$$

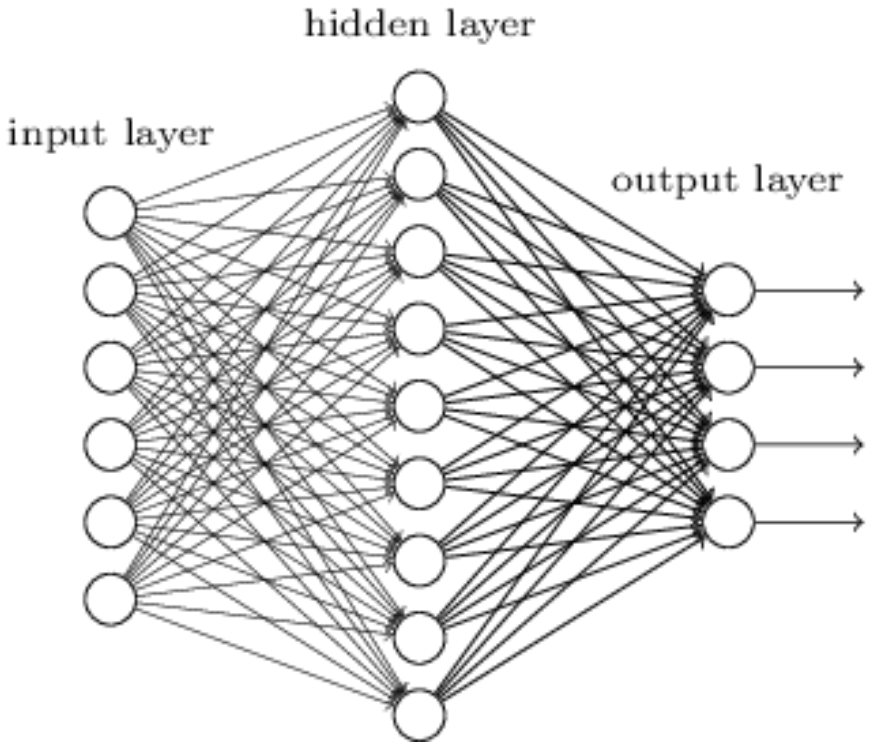
Cross Entropy

with respect to the coefficients of $\mathbf{W}_1$, $\mathbf{w}_2$, $\mathbf{b}_1$, and $\mathbf{b}_2$.

- E can be minimized using a gradient-based technique.

Stochastic-gradient descent

Multi-Class Case



$$\mathbf{h} = \sigma(\mathbf{W}_1\mathbf{x}_n + \mathbf{b}_1)$$

$$y = \sigma(\mathbf{W}_2\mathbf{h} + \mathbf{b}_2)$$

In this case $\mathbf{W}_2$ is a matrix.

Multi-Class Case

Let the training set be $\{(\mathbf{x}_n, [t_n^1, \ldots, t_n^K])_{1 \leq n \leq N}\}$ where $t_n^k \in \{0,1\}$ is the probability that sample $\mathbf{x}_n$ belongs to class k.

- We write

$$\mathbf{y}_n = \sigma(\mathbf{W}_2(\sigma(\mathbf{W}_1\mathbf{x}_n + \mathbf{b}_1)) + \mathbf{b}_2) \in R^K$$

$$p_n^k = \frac{\exp(\mathbf{y}_n[k])}{\sum_j \exp(\mathbf{y}_n[j])}$$

⇒ replace y_n by exp's to turn into prob's

$$\sum_k p_n^k = 1$$

- We minimize the cross entropy

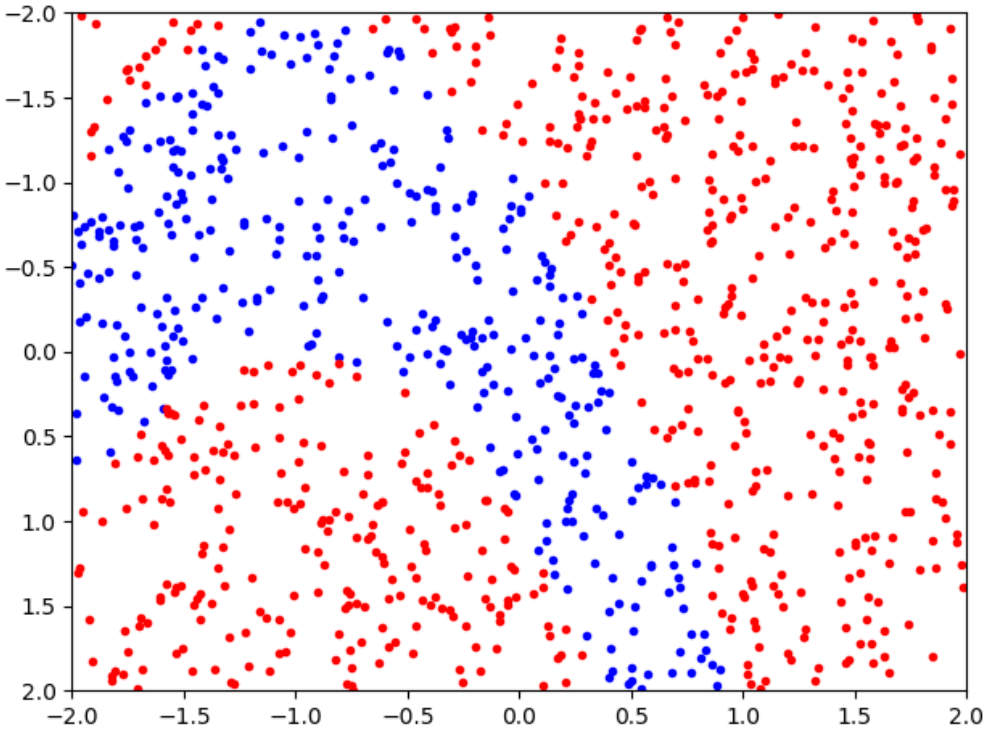
$$E(\mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = \frac{1}{N} \sum_{n=1}^{N} E_n(\mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) ,$$

$$E_n(\mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = -\sum t_n^k \ln(p_n^k) ,$$

↳ Entropy Loss

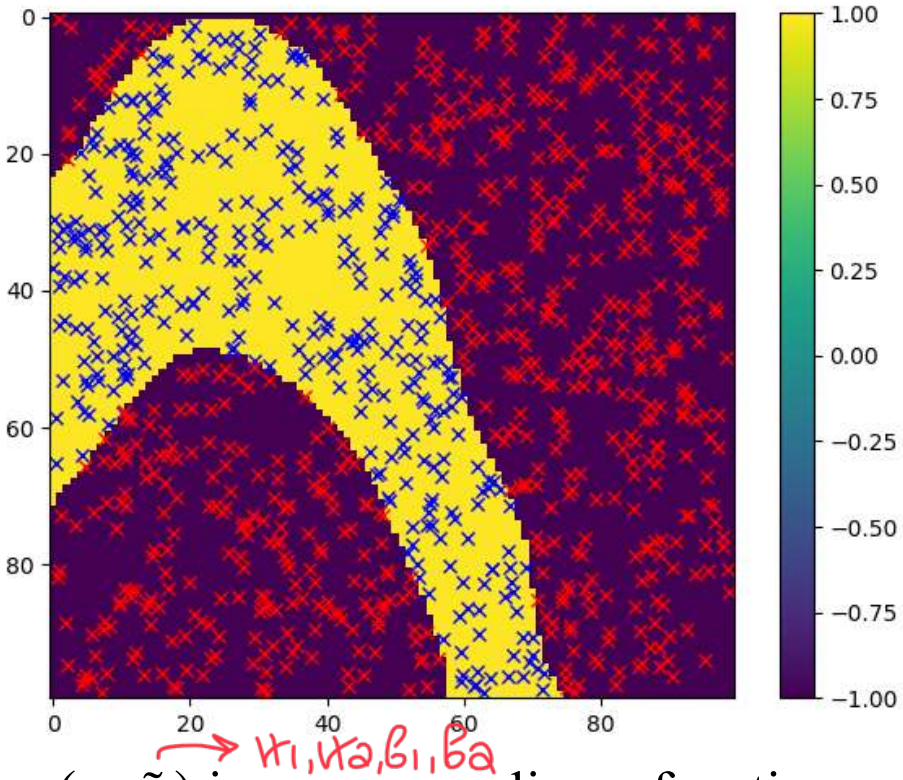
with respect to the coefficients of $\mathbf{W}_1$, $\mathbf{W}_2$, $\mathbf{b}_1$, and $\mathbf{b}_2$.

Non-Linear Binary Classification



Positive: $100(x_2 - x_1^2)^2 + (1 - x_1)^2 < 0.5$

Negative: Otherwise

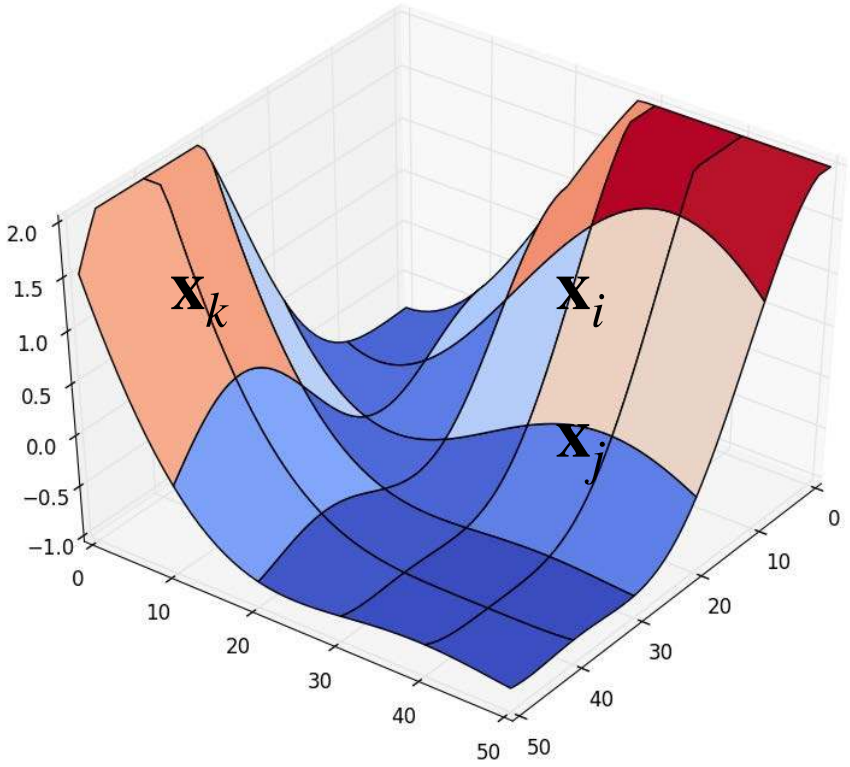


$y(\mathbf{x}; \tilde{\mathbf{w}})$ is now a non-linear function implemented by the network.

Problem statement: Find $\tilde{\mathbf{w}}$ such that

- for all or most positive samples $y(\tilde{\mathbf{x}}; \tilde{\mathbf{w}}) > 0.0$,
- for all or most negative samples $y(\tilde{\mathbf{x}}; \tilde{\mathbf{w}}) < 0.0$.

Non-Linear Regression



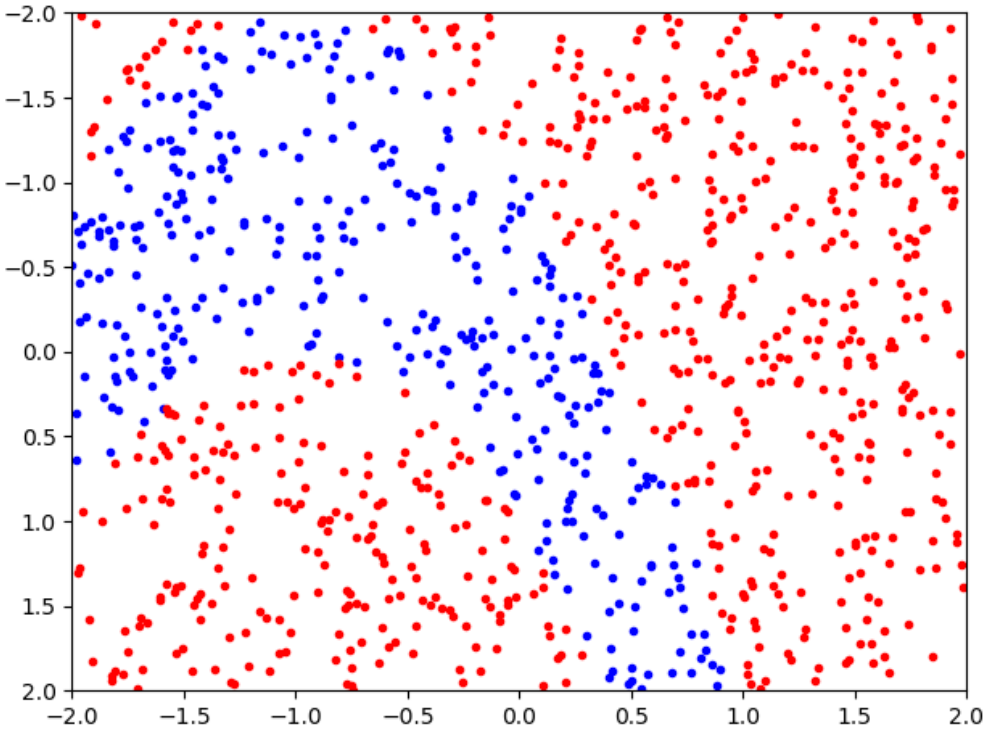
$$z = f(\mathbf{x})$$
$$= 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

Problem statement: Given $(\{\mathbf{x}_1, z_1\}, \ldots, \{\mathbf{x}_n, z_n\})$, minimize

$$\sum_i (z_i - f(\mathbf{x}_i, \tilde{\mathbf{w}})^2)$$

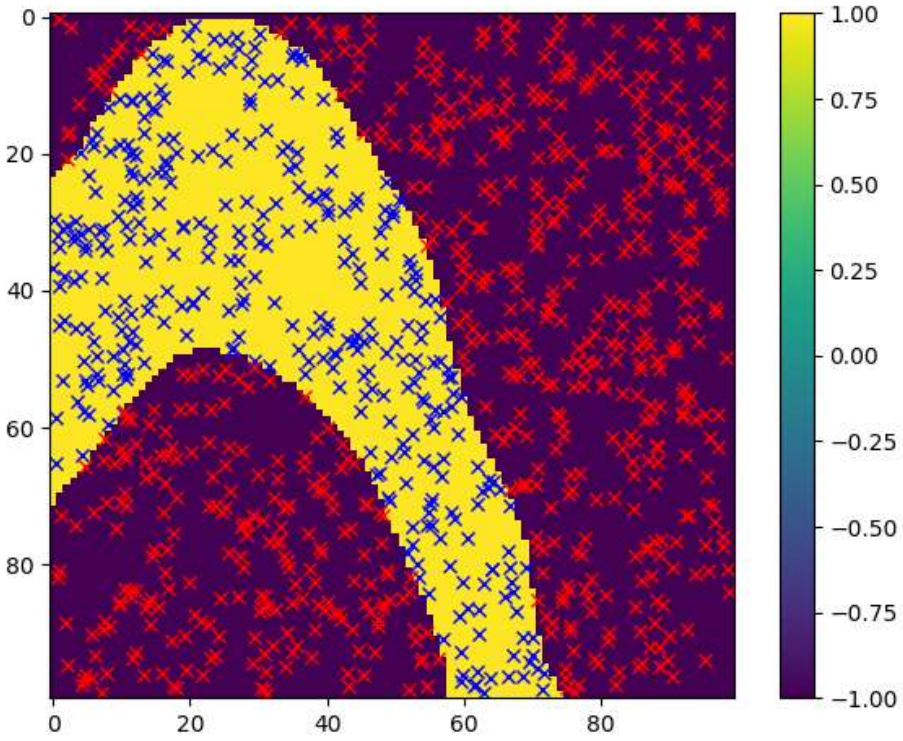
w.r.t. $\tilde{\mathbf{w}}$.

Classification / Regression



Positive: $f(\mathbf{x}) < 0.5$

Negative: Otherwise

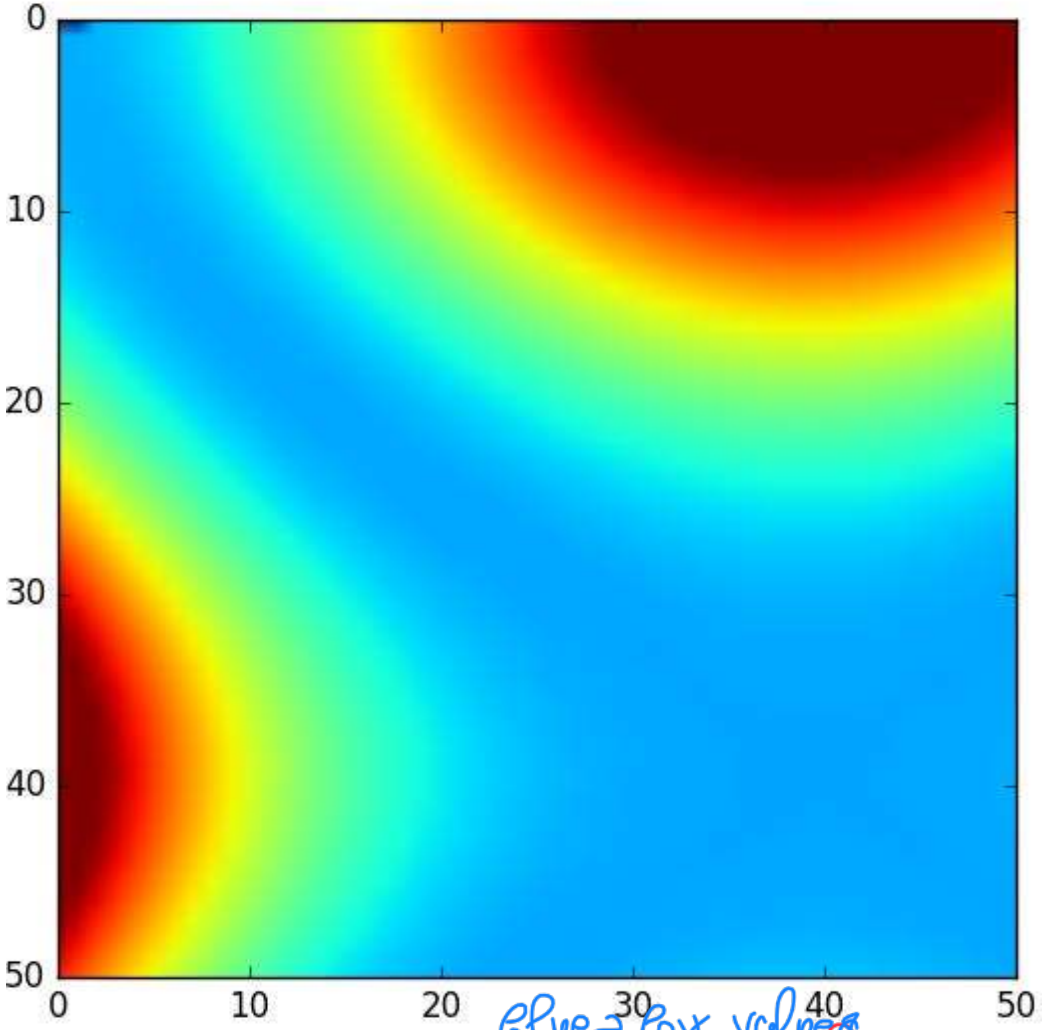


$y(\mathbf{x}; \tilde{\mathbf{w}})$ is now a non-linear function implemented by the network.

Classification can be understood as finding $\tilde{\mathbf{w}}$ such that

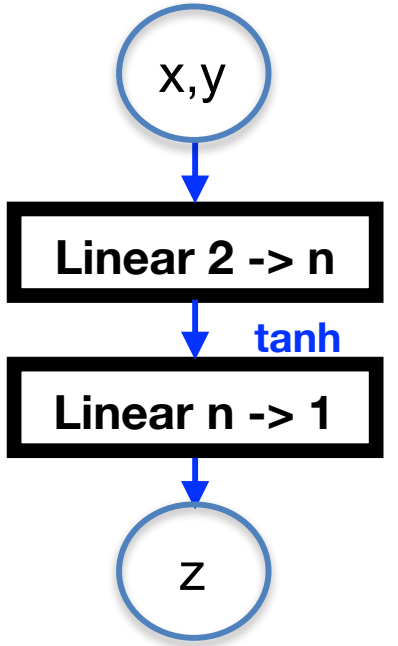
$$y(\mathbf{x}; \tilde{\mathbf{w}}) \approx f(x)$$

Approximating a Surface



blue → low values
red → high values

$$z = f(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$



$$\begin{aligned} z &= f(x, y) \\ &= \mathbf{w}_2 \sigma(\mathbf{W}_1 \begin{bmatrix} x \\ y \end{bmatrix} + \mathbf{b}_1) + b_2 \end{aligned}$$

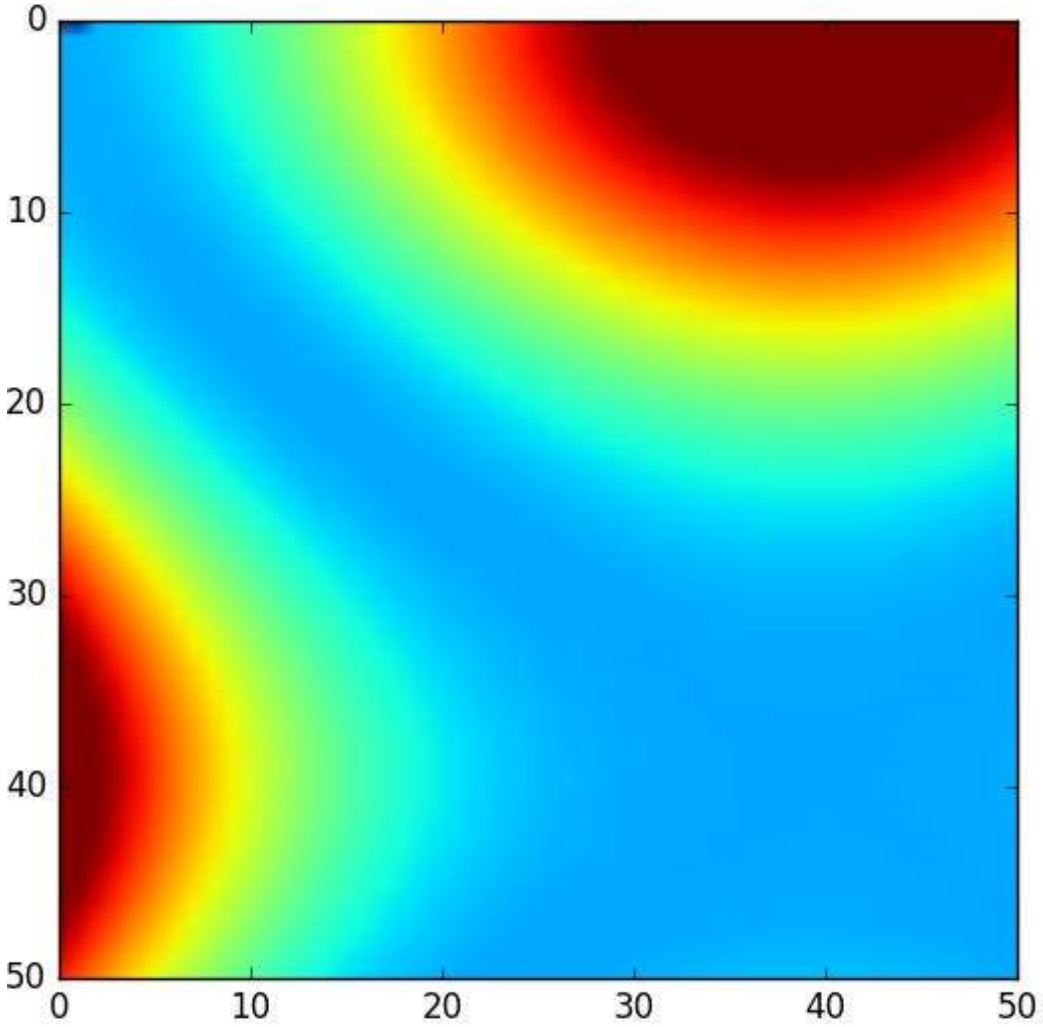
Given $(\{\mathbf{x}_1, z_1\}, \ldots, \{\mathbf{x}_n, z_n\})$, minimize

$$\sum_i (z_i - f(\mathbf{x}_i))^2$$

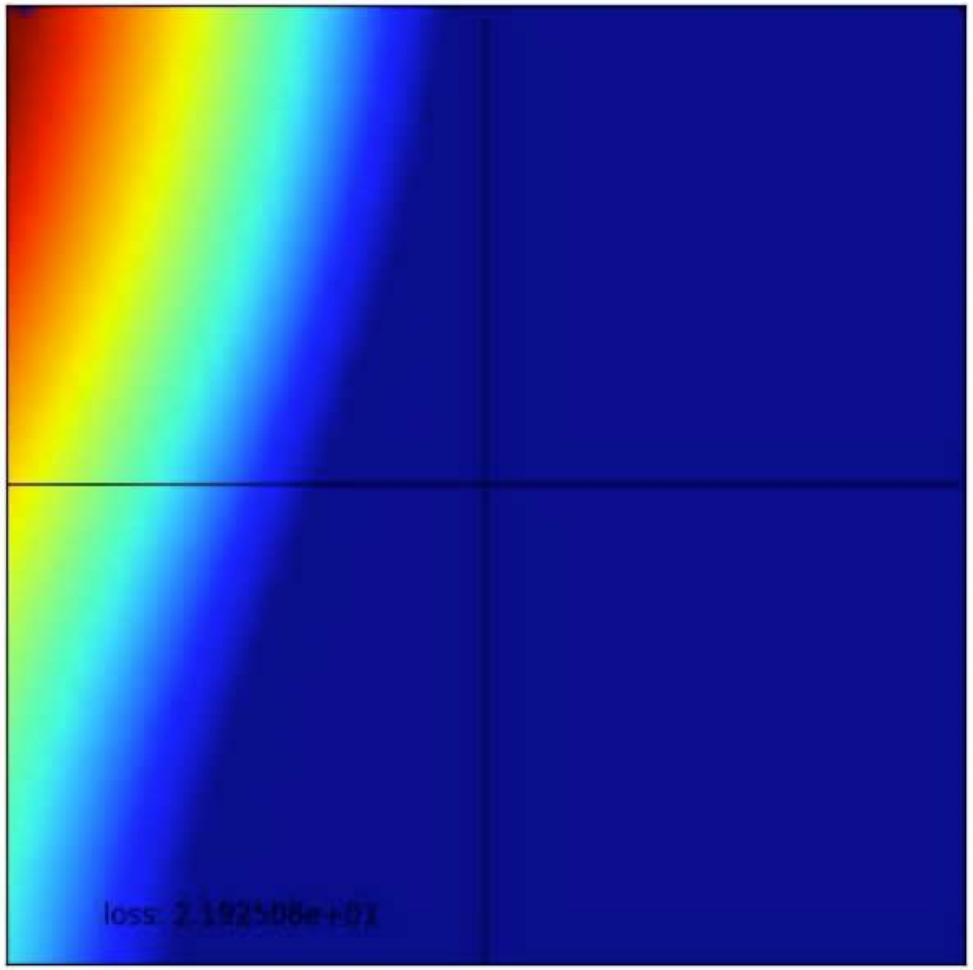
with respect to $\mathbf{W}_1, \mathbf{w}_2, \mathbf{b}_1, b_2$.

Interpolate function $f(x)$

Interpolating a Surface

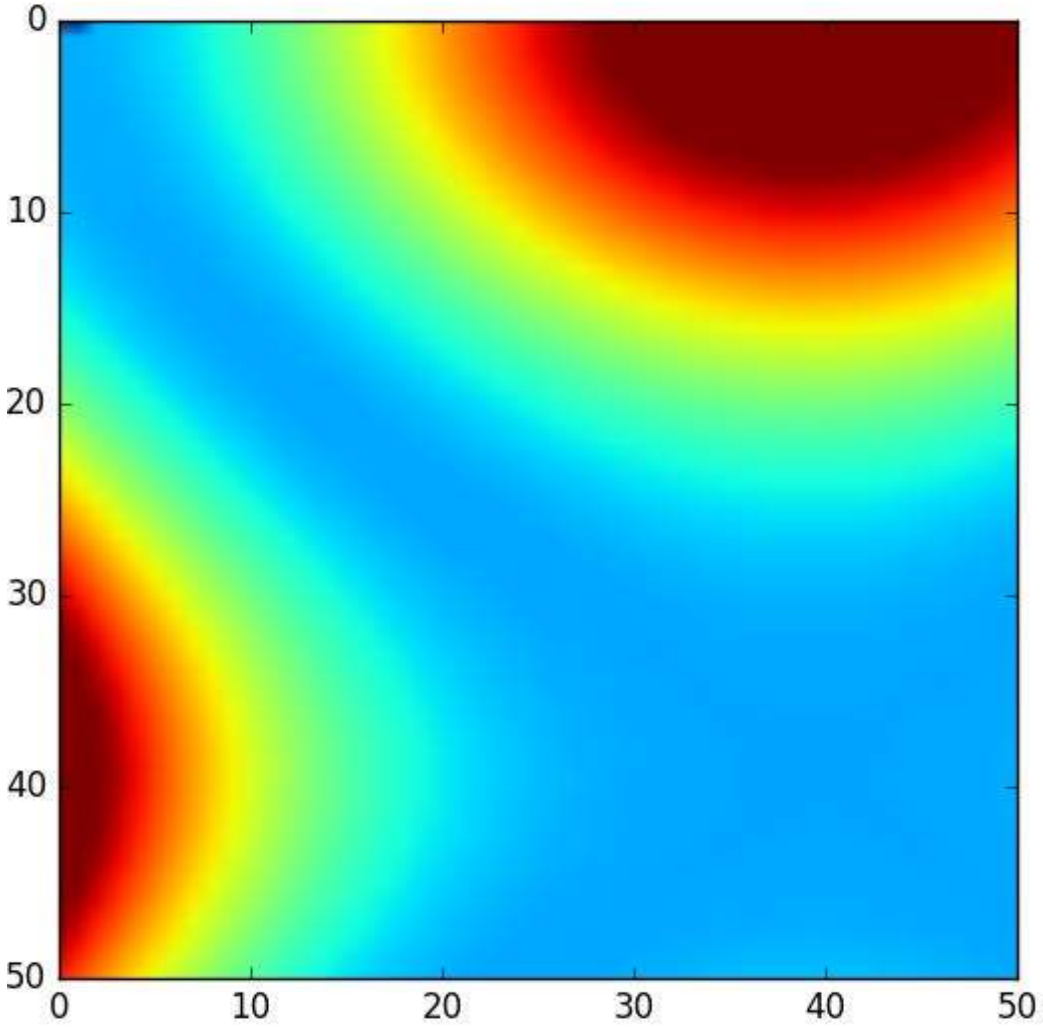


$$z = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

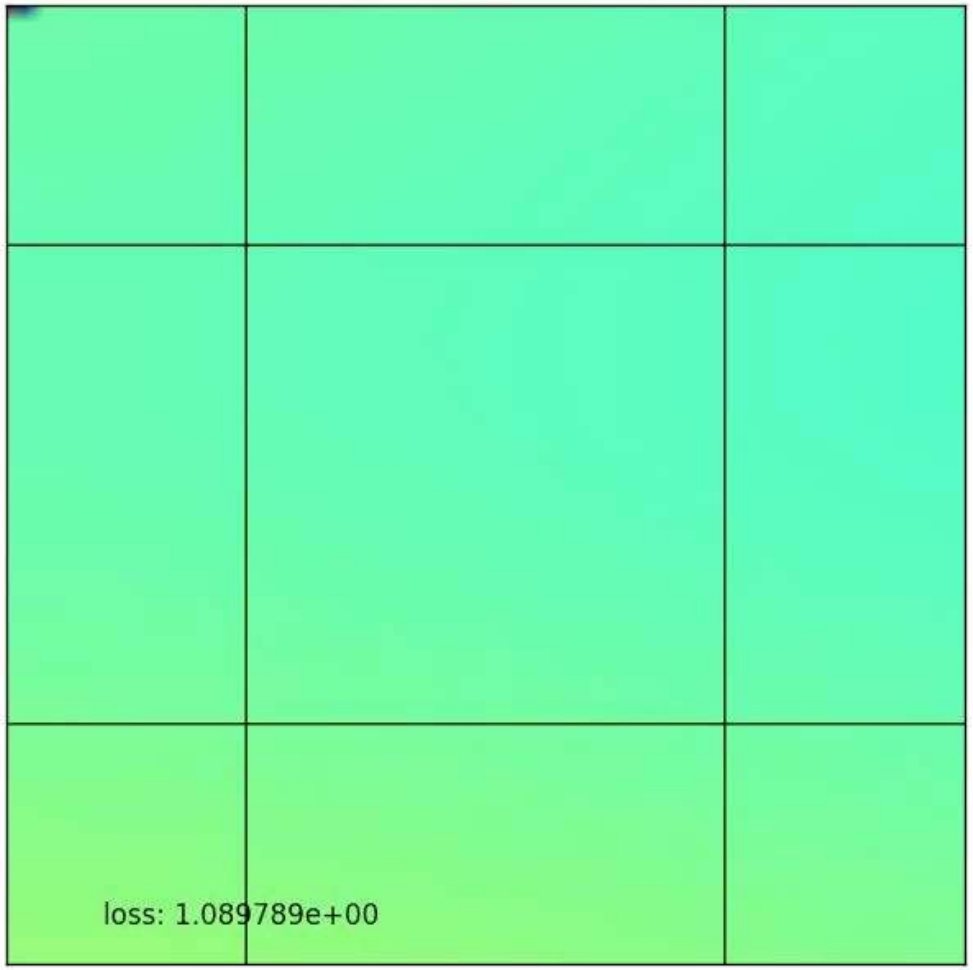


3-node hidden layer

Interpolating a Surface



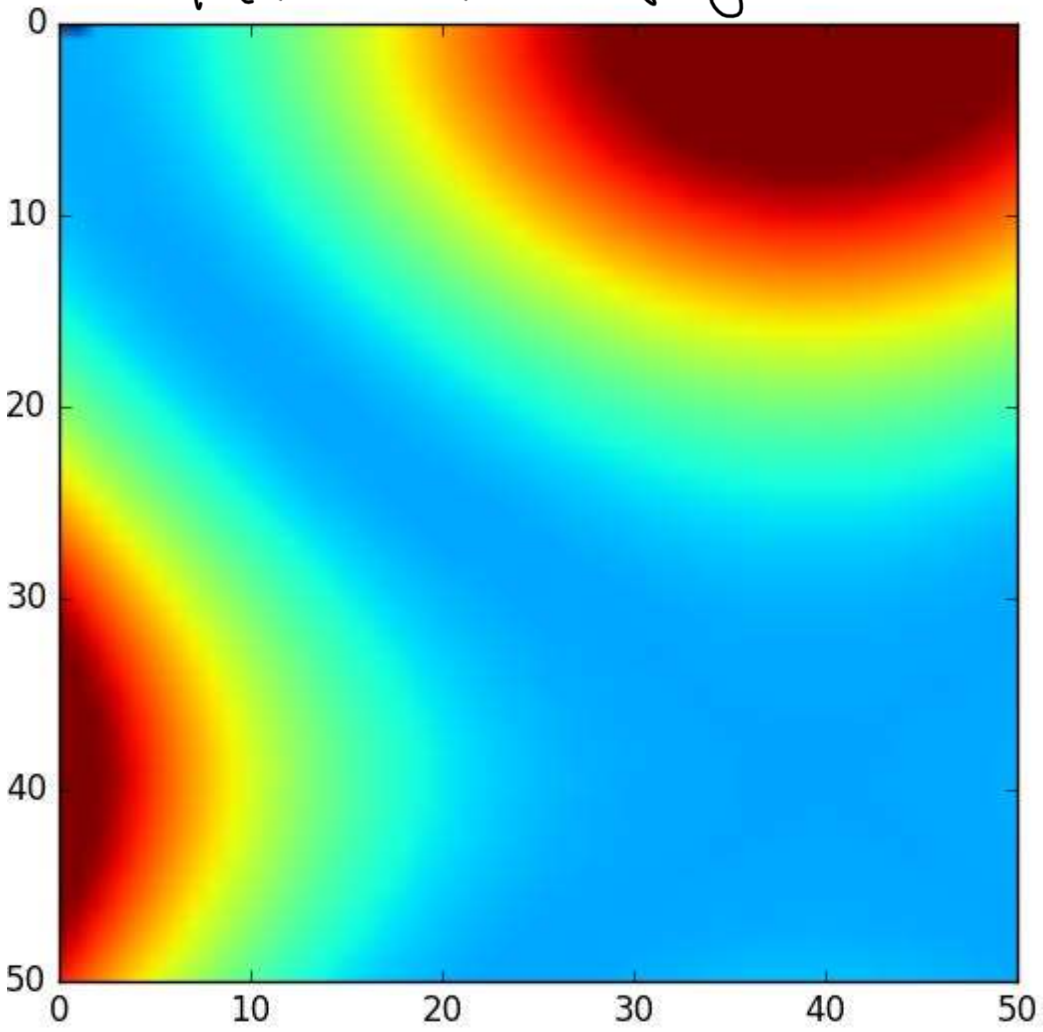
$$z = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$



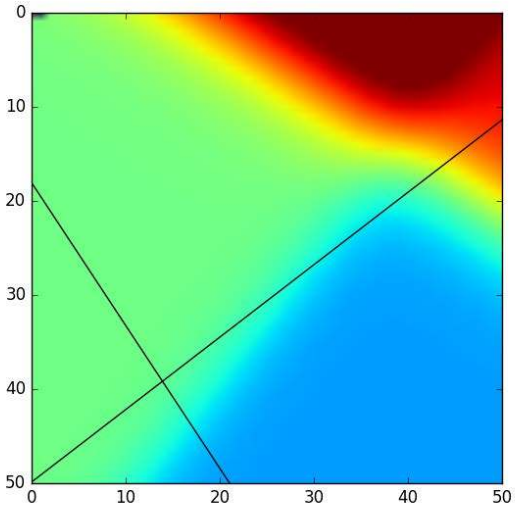
4-node hidden layer

Adding more Nodes

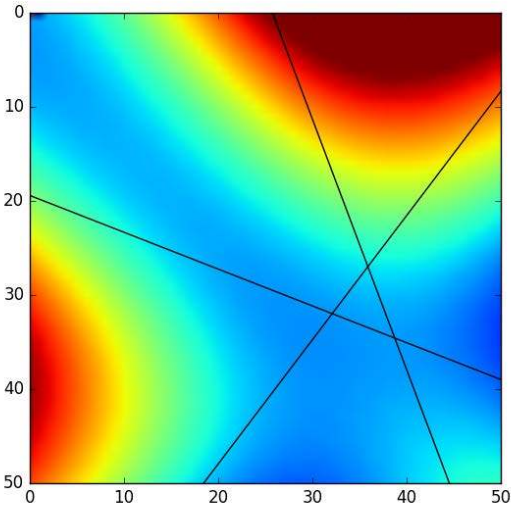
approximate
Function I'm trying to



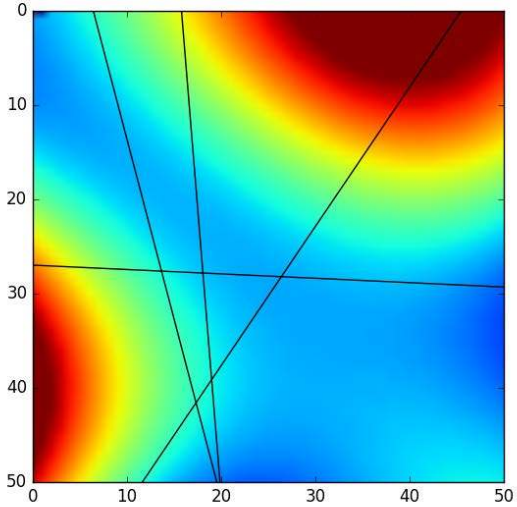
$$z = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$



2 nodes -> loss 3.02e-01



3 nodes -> loss 2.08e-02

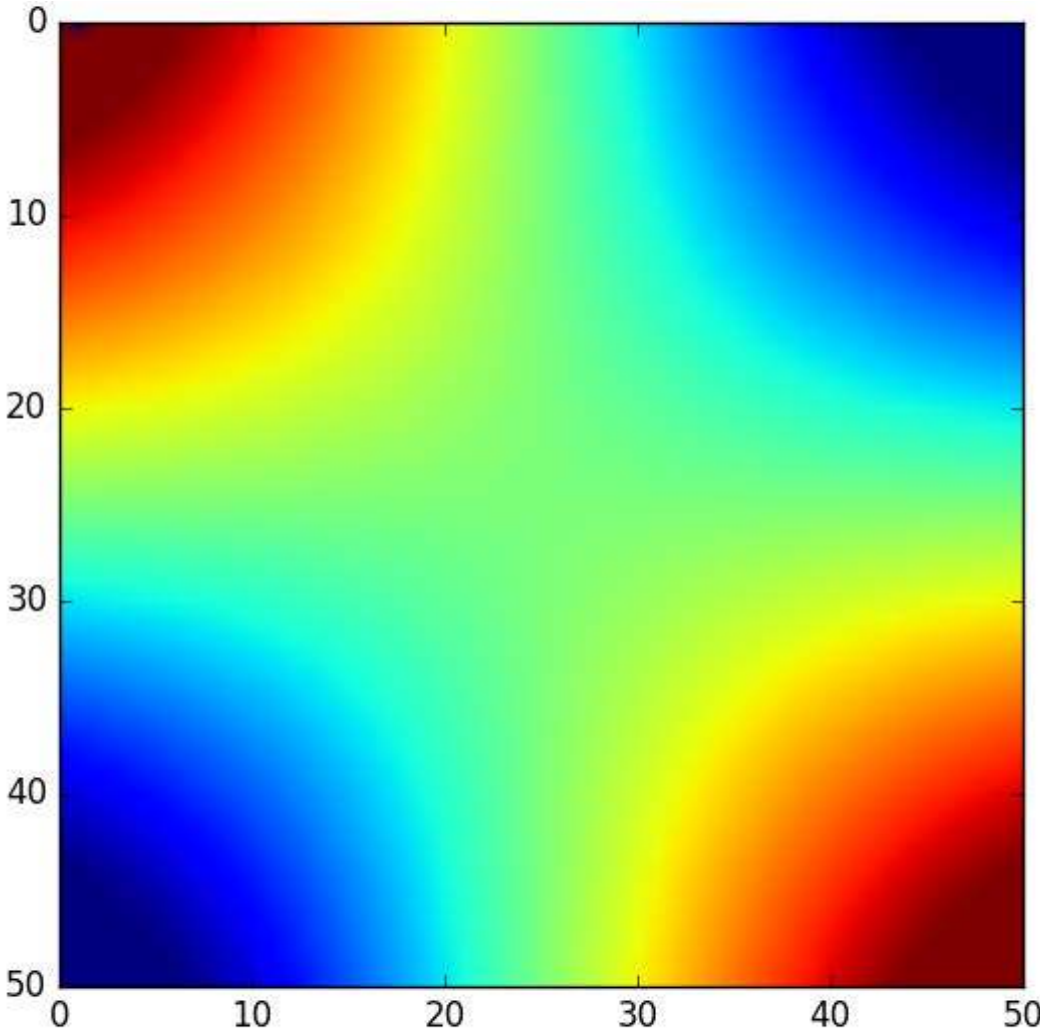


4 nodes -> loss 8.27e-03 ✓

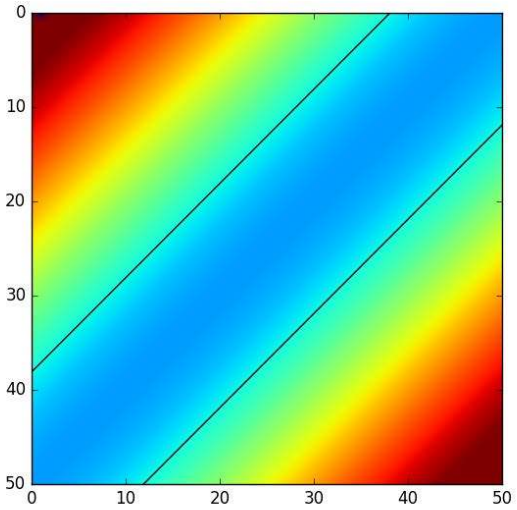
pretty good

Adding more Nodes

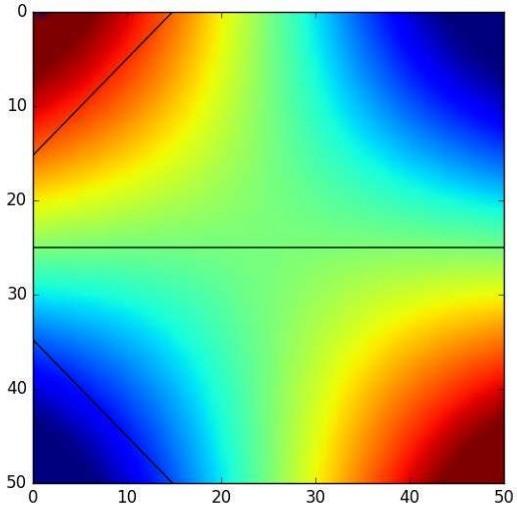
z-values and Have only access to x,y



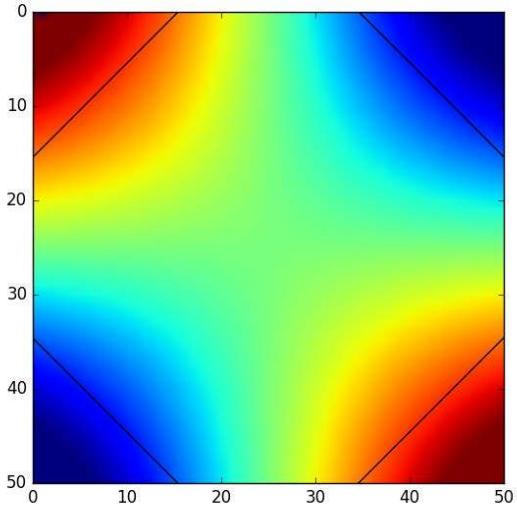
$$z = sin(x)sin(y)$$



2 nodes -> loss 2.61e-01



3 nodes -> loss 2.51e-04



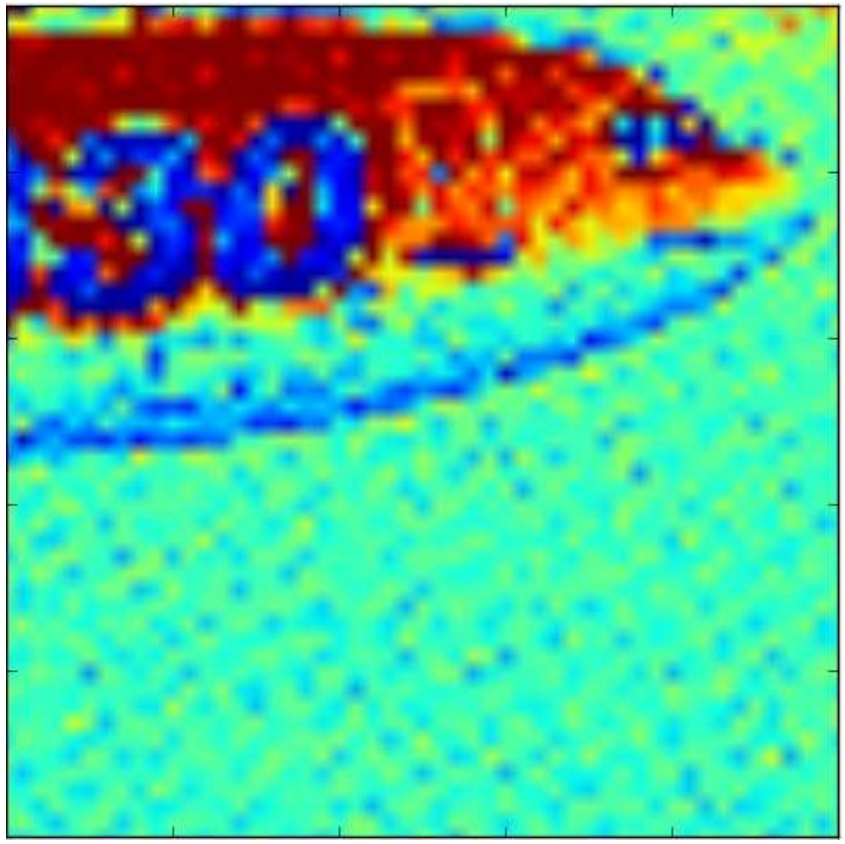
4 nodes -> loss 3.07e-07

Minimizing residue

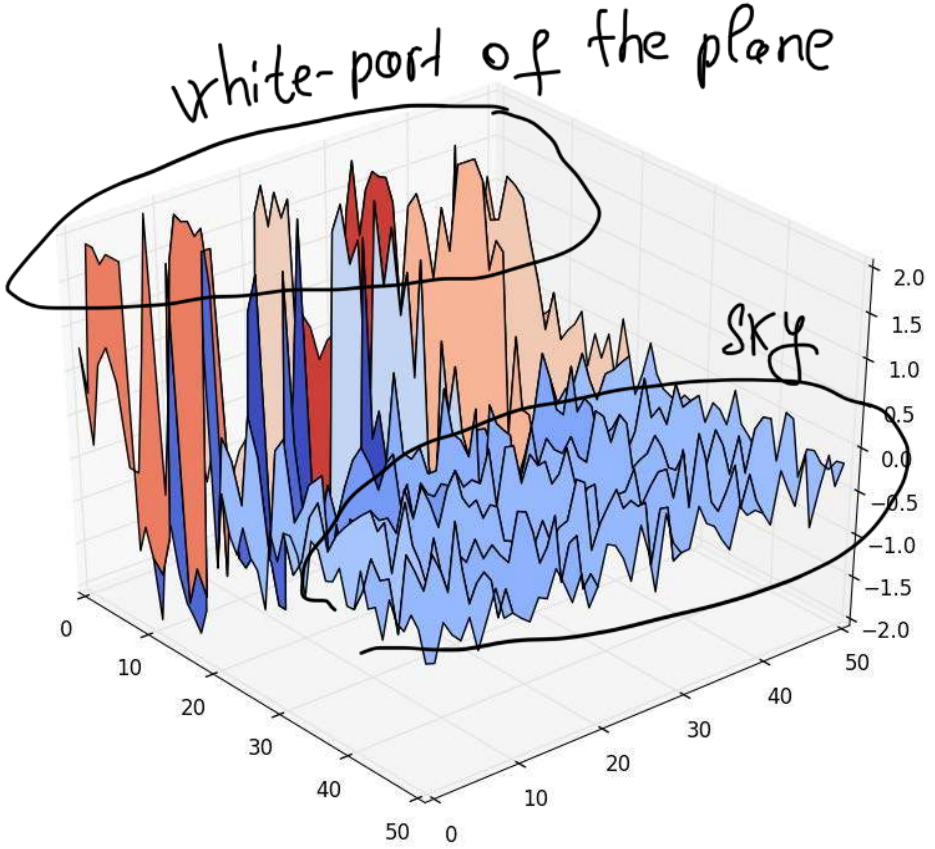
More Complex Surface

Image as a surface

z → gray-level

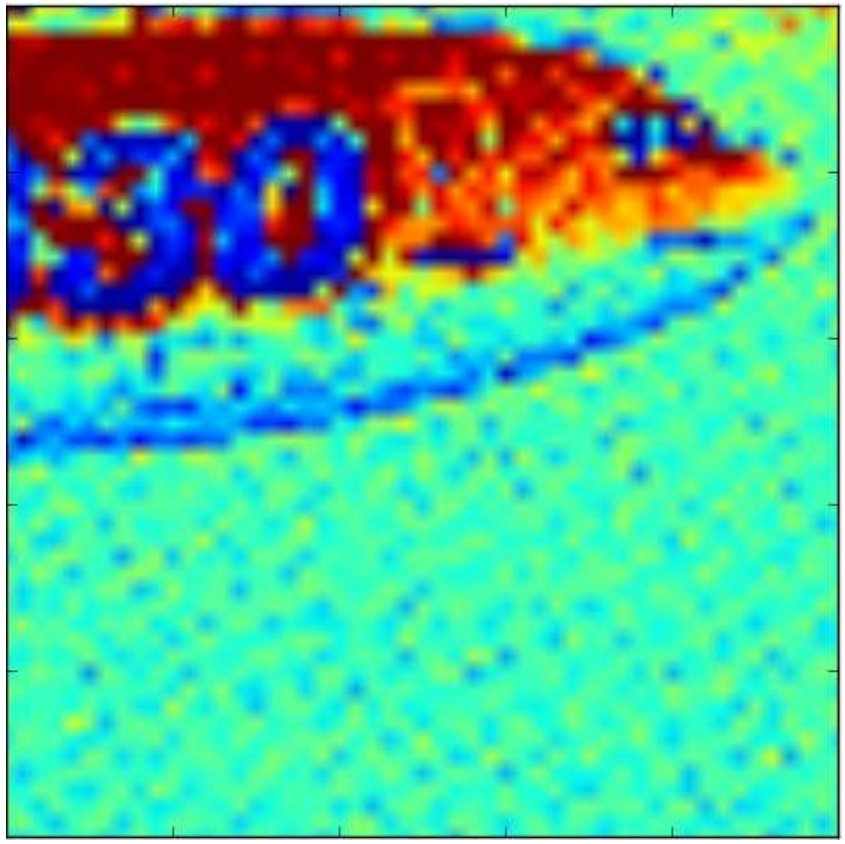


$$I = f(x, y)$$

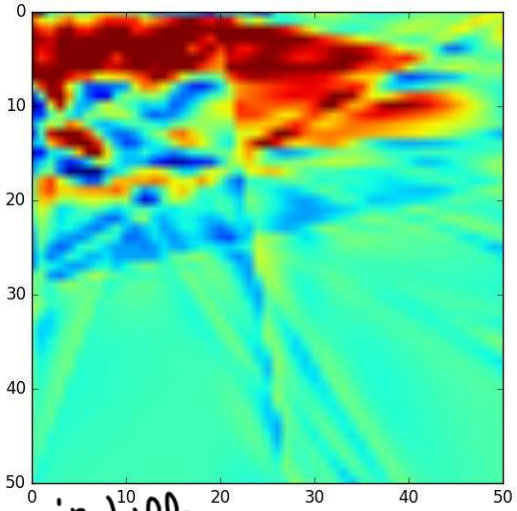


red → high-value
Blue → low-value
Green → intermediate (≈ 128)

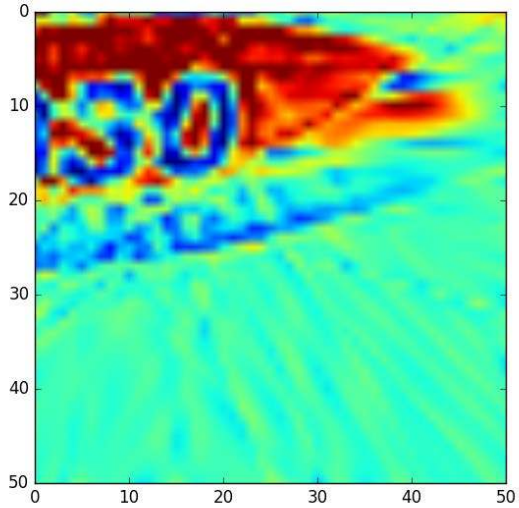
More Complex Surface



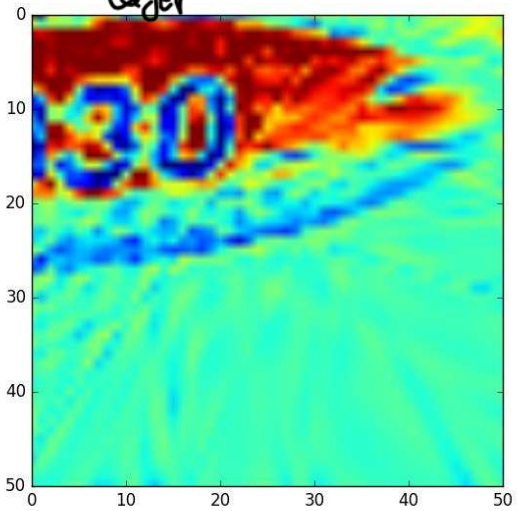
$$I = f(x, y)$$



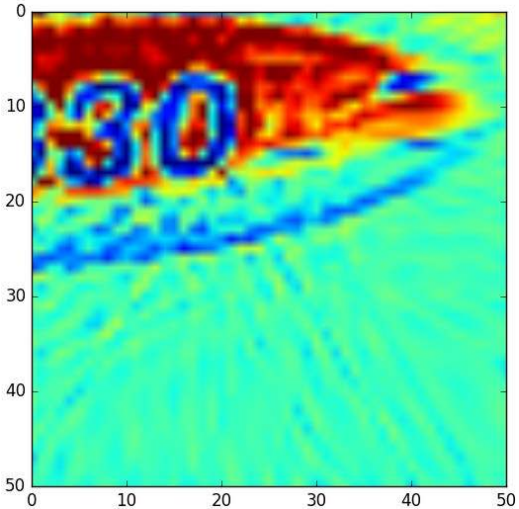
50 nodes -> loss 3.65e-01



100 nodes -> loss 2.50e-01



125 nodes -> loss 2.40e-01



300 nodes -> loss 1.92e-01

approximation

Universal Approximation Theorem

A feedforward network with a linear output layer and at least one hidden layer with any 'squashing' activation function (e.g. logistic sigmoid) can approximate any Borel measurable function (from one finite-dimensional space to another) with any desired nonzero error.

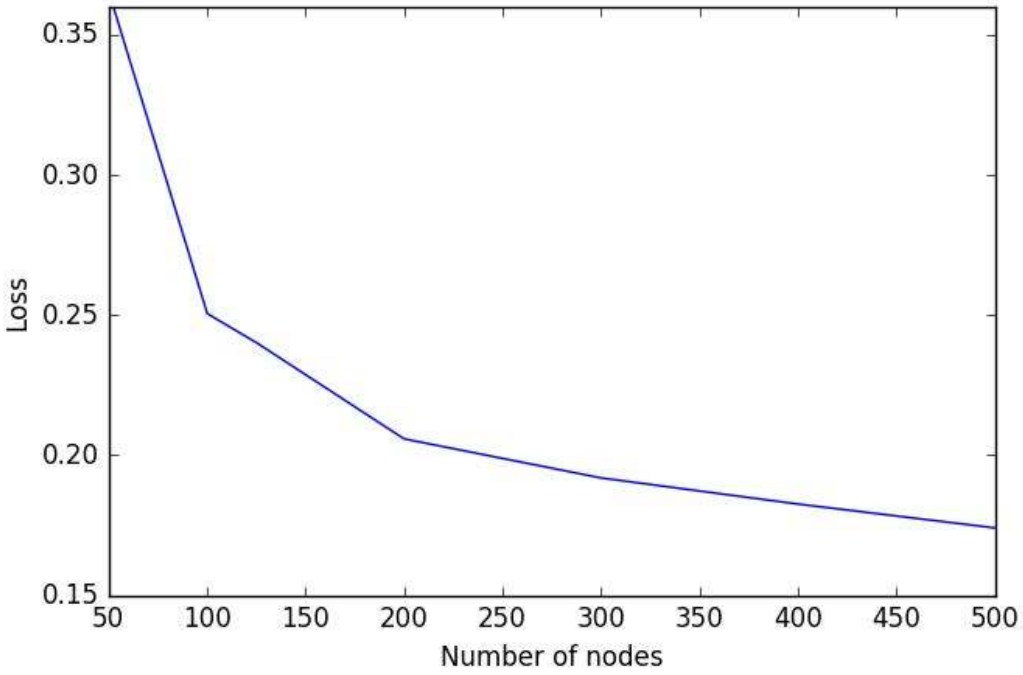
Any continuous function on a closed and bounded set of R^n is Borel-measurable.

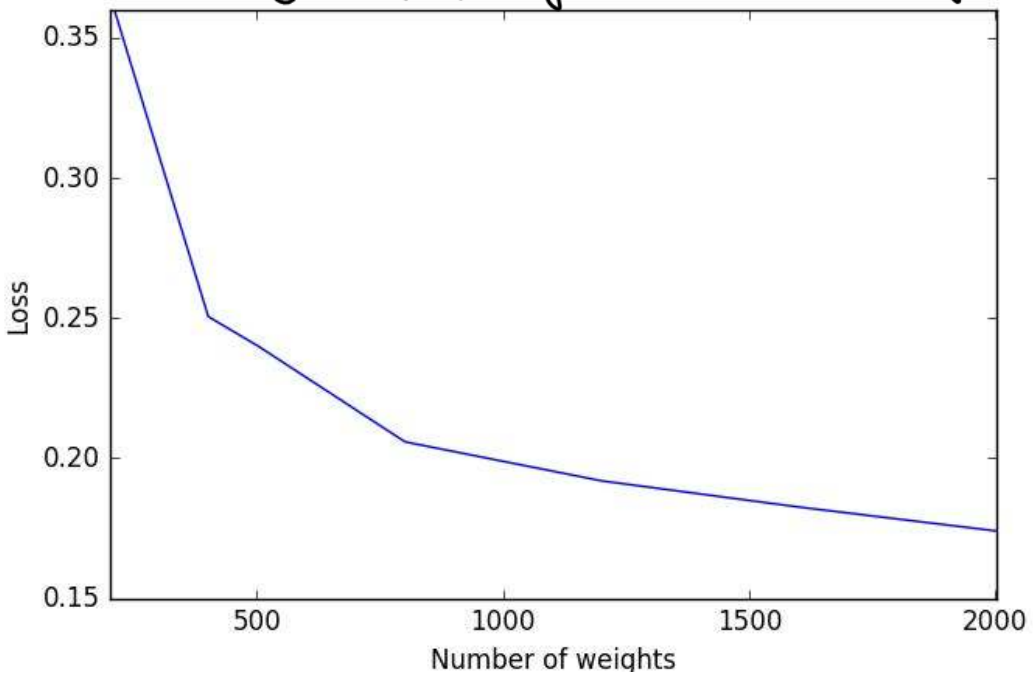
—> In theory, any reasonable function can be approximated by a one-hidden layer network as long as it is continuous.

[Hornik et al, 1989; Cybenko, 1989]

In Practice

nodes ⇒ define matrices ⇒ bunch of weights
(hidden-layer ⇝ extremely large)

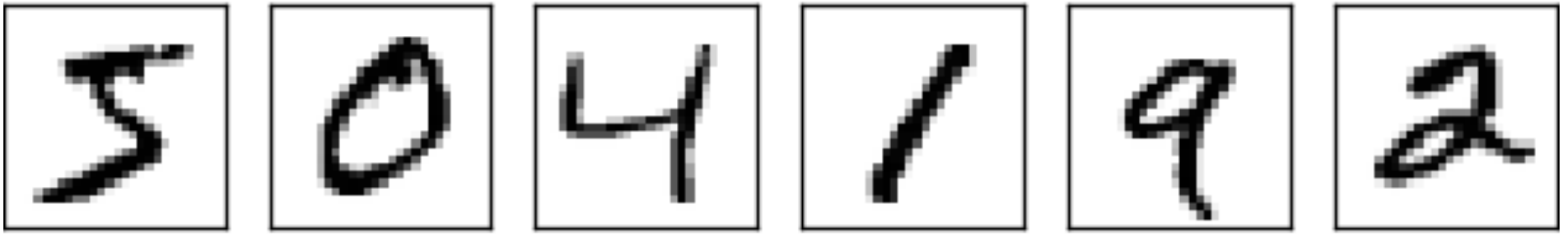




- It may take an exponentially large number of parameters for a good approximation.
- The optimization problem becomes increasingly difficult.

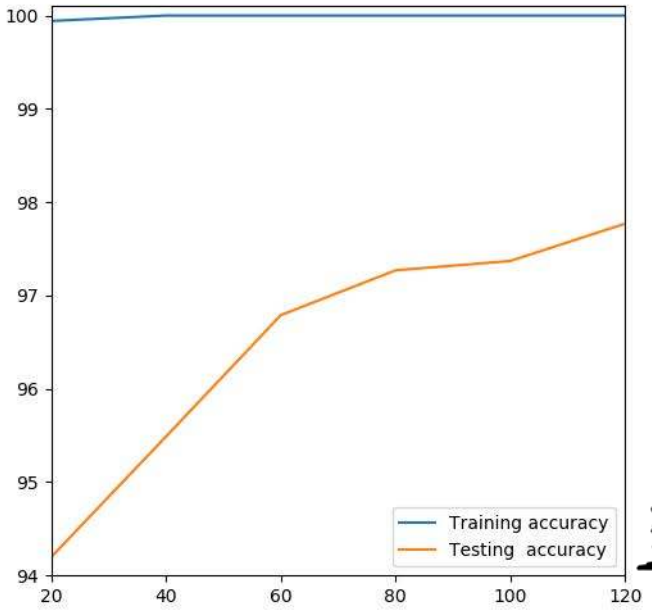
—> The one hidden layer perceptron may not converge to the best solution!

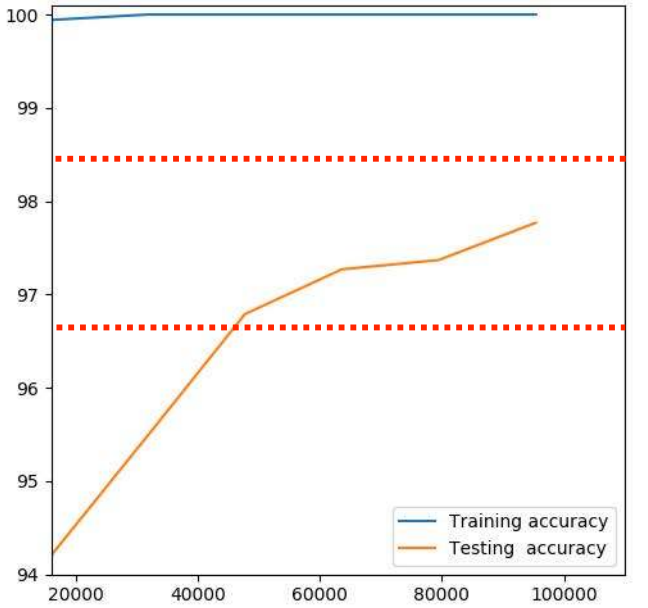
MNIST



- The network takes as input 28x28 images represented as 784D vectors.
- The output is a 10D vector giving the probability of the image representing any of the 10 digits.
- There are 50’000 training pairs of images and the corresponding label, 10’000 validation pairs, and 5’000 testing pairs.

MNIST Results





nIn =784 = 28×28

nOut = 10 ← 0, 1, ... 9

20 < hidden layer size < 120

- MLPs have **many** parameters.
- This has long been a major problem.

—> Was eventually solved by using GPUs.

SVM: 98.6

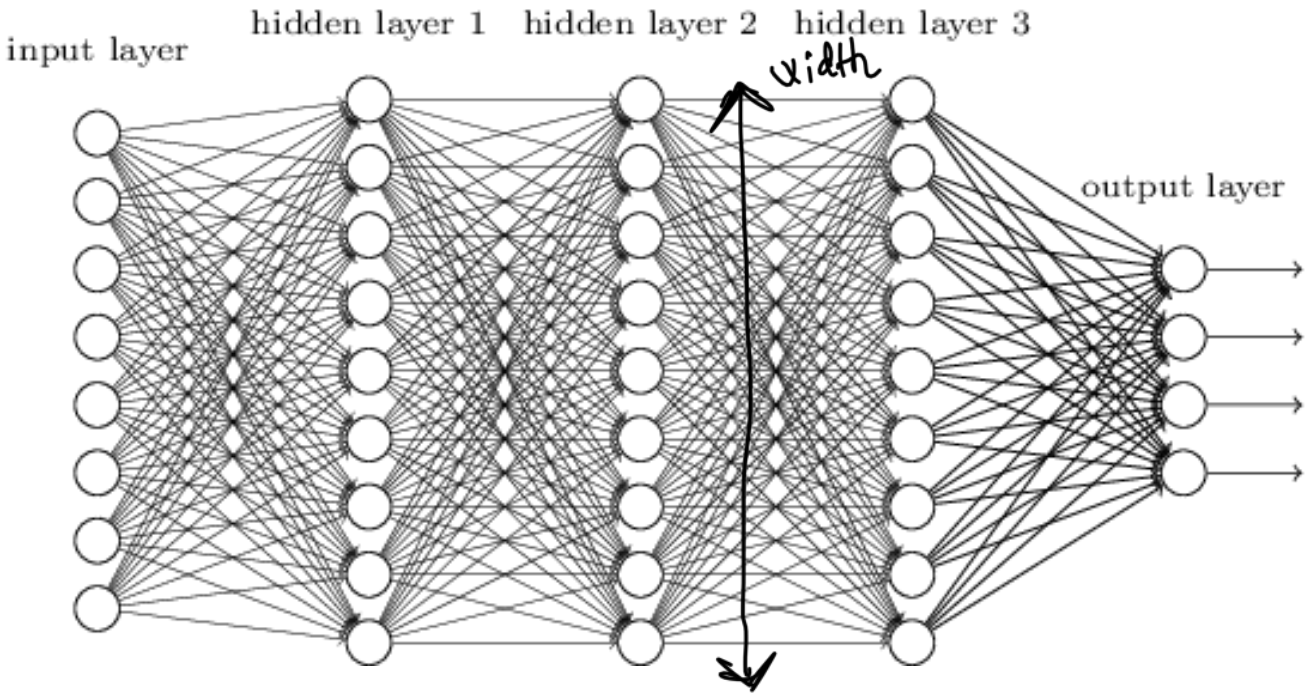
Knn: 96.8

machines

→ support-vector

- Around 2005, SVMs were often felt to be superior to neural nets.
- This is no longer the case ….

Deep Learning



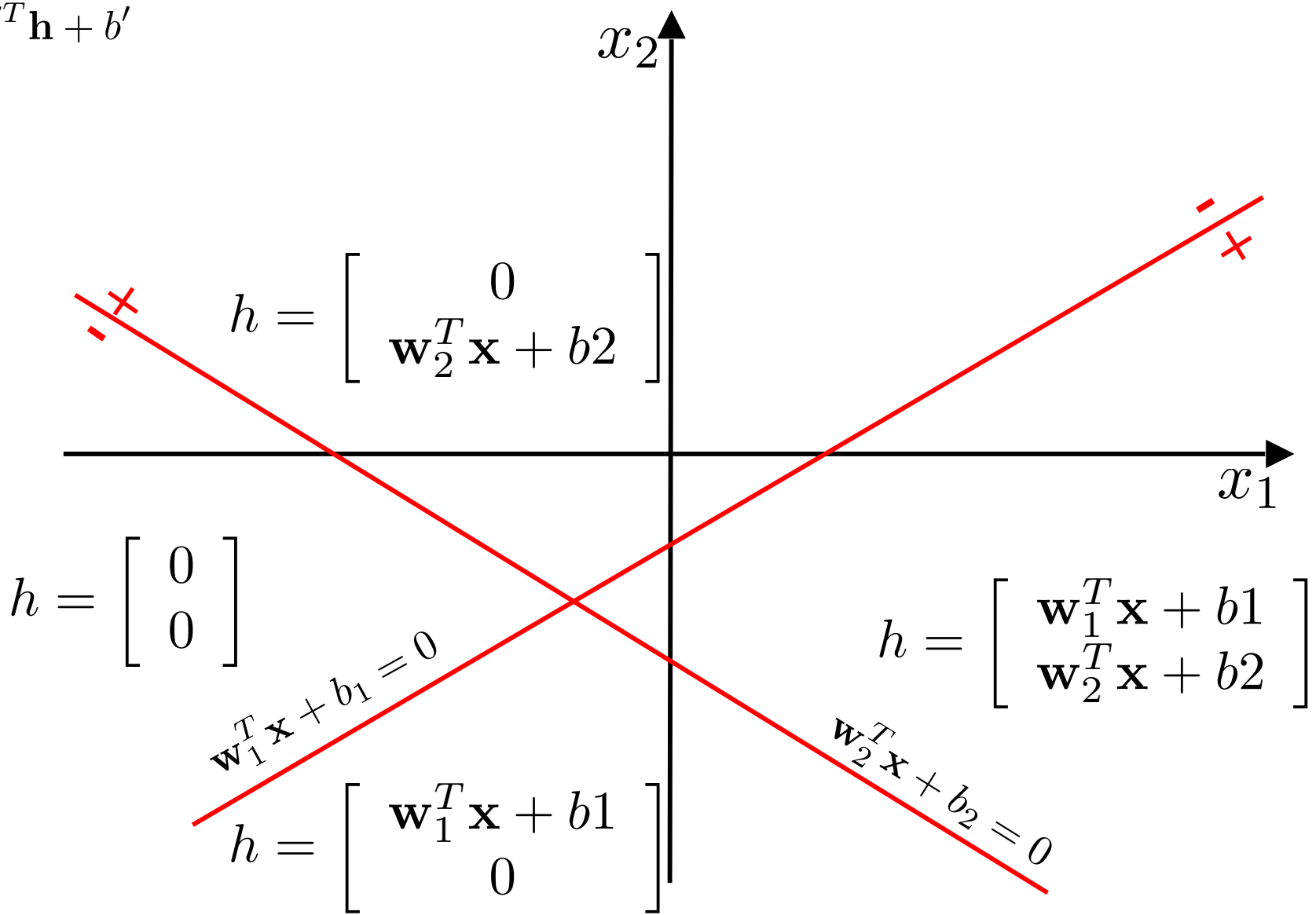
$$\begin{aligned} \mathbf{h}_1 &= \sigma_1(\mathbf{W}_1\mathbf{x}_1 + \mathbf{b}_1) \\ \mathbf{h}_2 &= \sigma_2(\mathbf{W}_2\mathbf{h}_2 + \mathbf{b}_2) \\ &\dots \\ \mathbf{y} &= \sigma_n(\mathbf{W}_n\mathbf{h}_n + \mathbf{b}_n) \end{aligned}$$

- The descriptive power of the net increases with the number of layers.
- In the case of a 1D signal, it is roughly proportional to $\prod_n W_n$ where w_n is the width of layer n.

One Layer: Two Hyperplanes

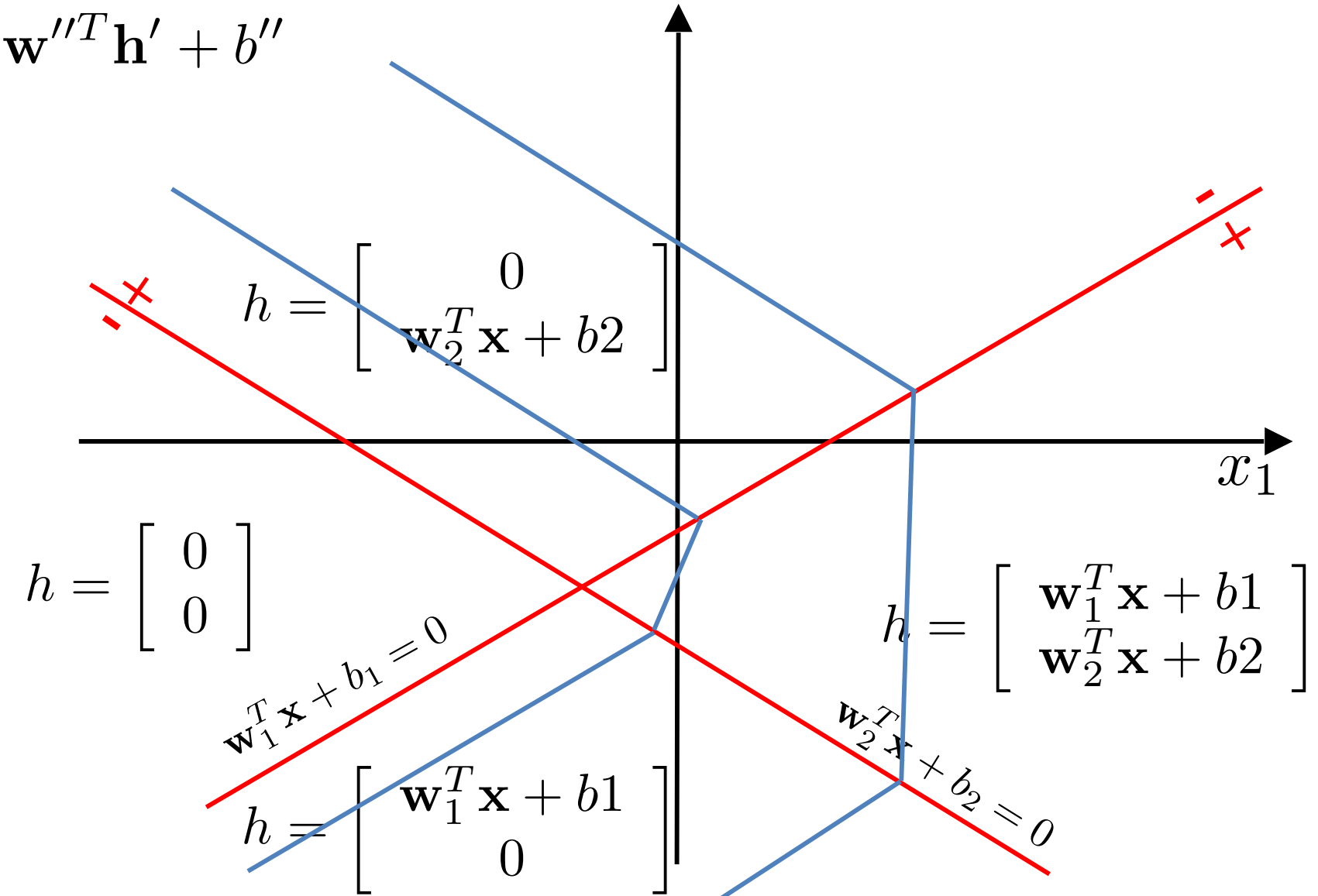
$$\mathbf{h} = \max(\mathbf{W}\mathbf{x} + \mathbf{b}, 0) \text{ with } \mathbf{W} = \begin{bmatrix} \mathbf{w}_1^T \\ \mathbf{w}_2^T \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$y = \mathbf{w}'^T \mathbf{h} + b'$$

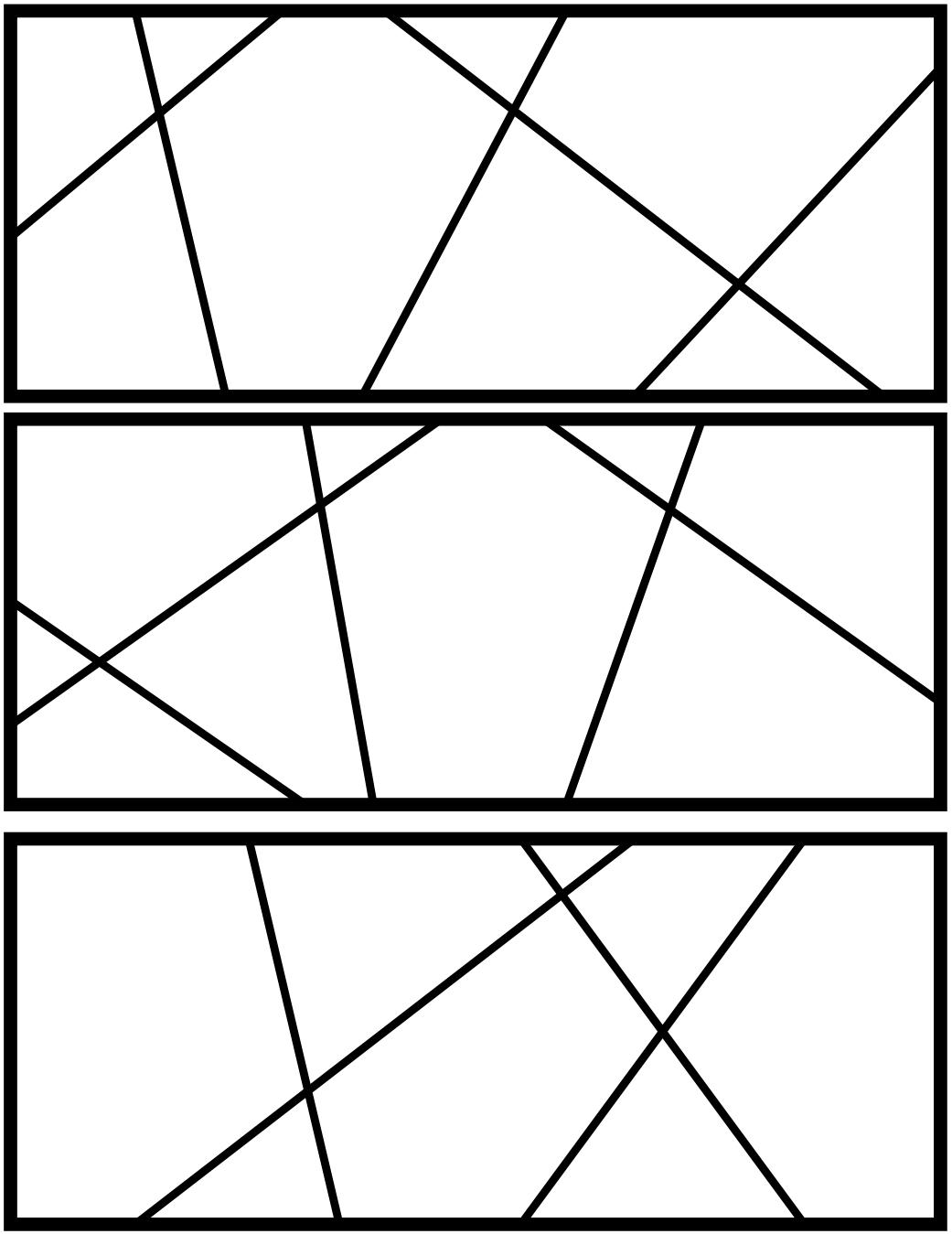


Two Layers: Two Hyperplanes

$$\mathbf{h} = \max(\mathbf{W}\mathbf{x} + \mathbf{b}, 0)$$
$$\mathbf{h}' = \max(\mathbf{W}'\mathbf{h} + \mathbf{b}', 0)$$
$$y = \mathbf{w}''^T\mathbf{h}' + b''$$

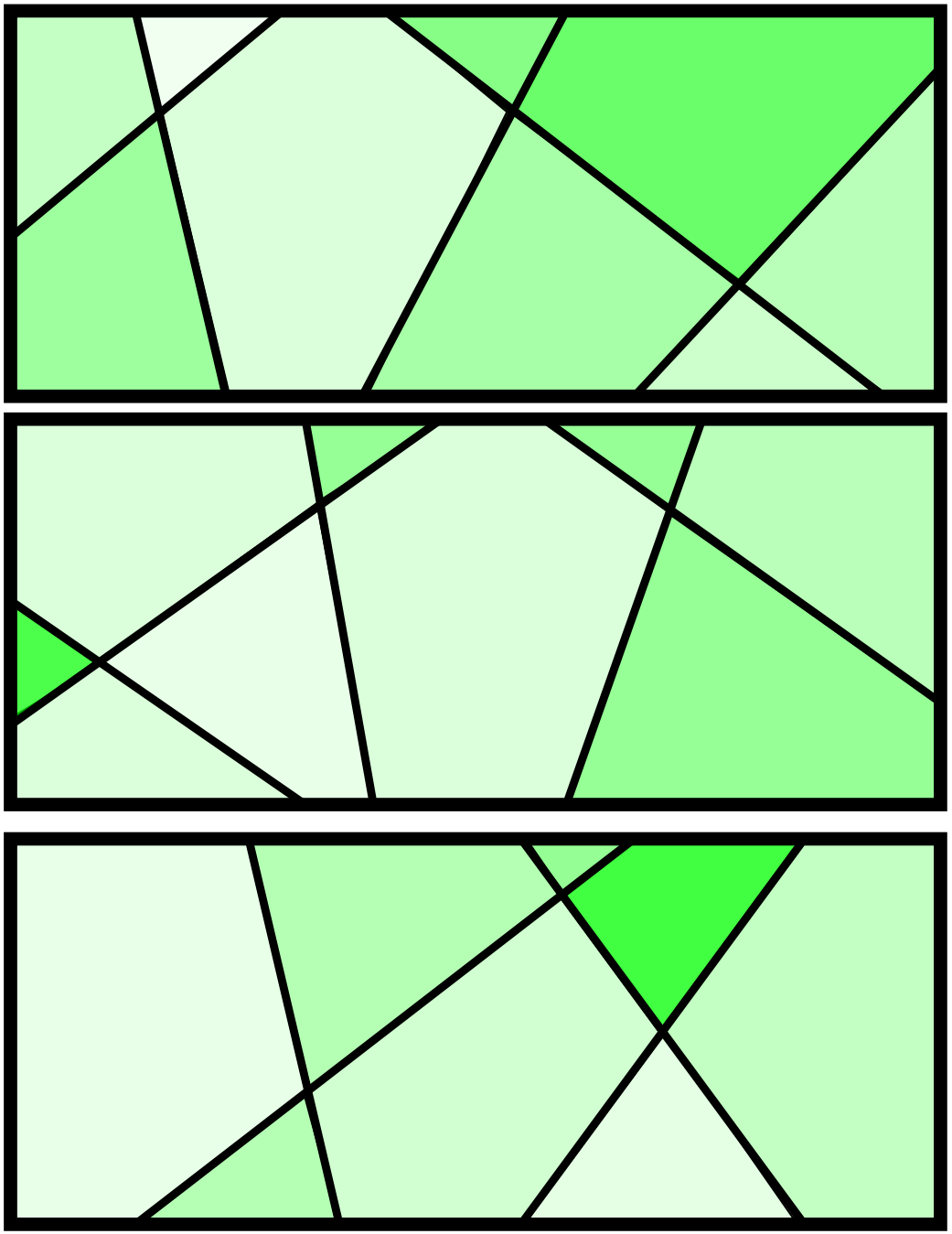


Graphical Interpretation



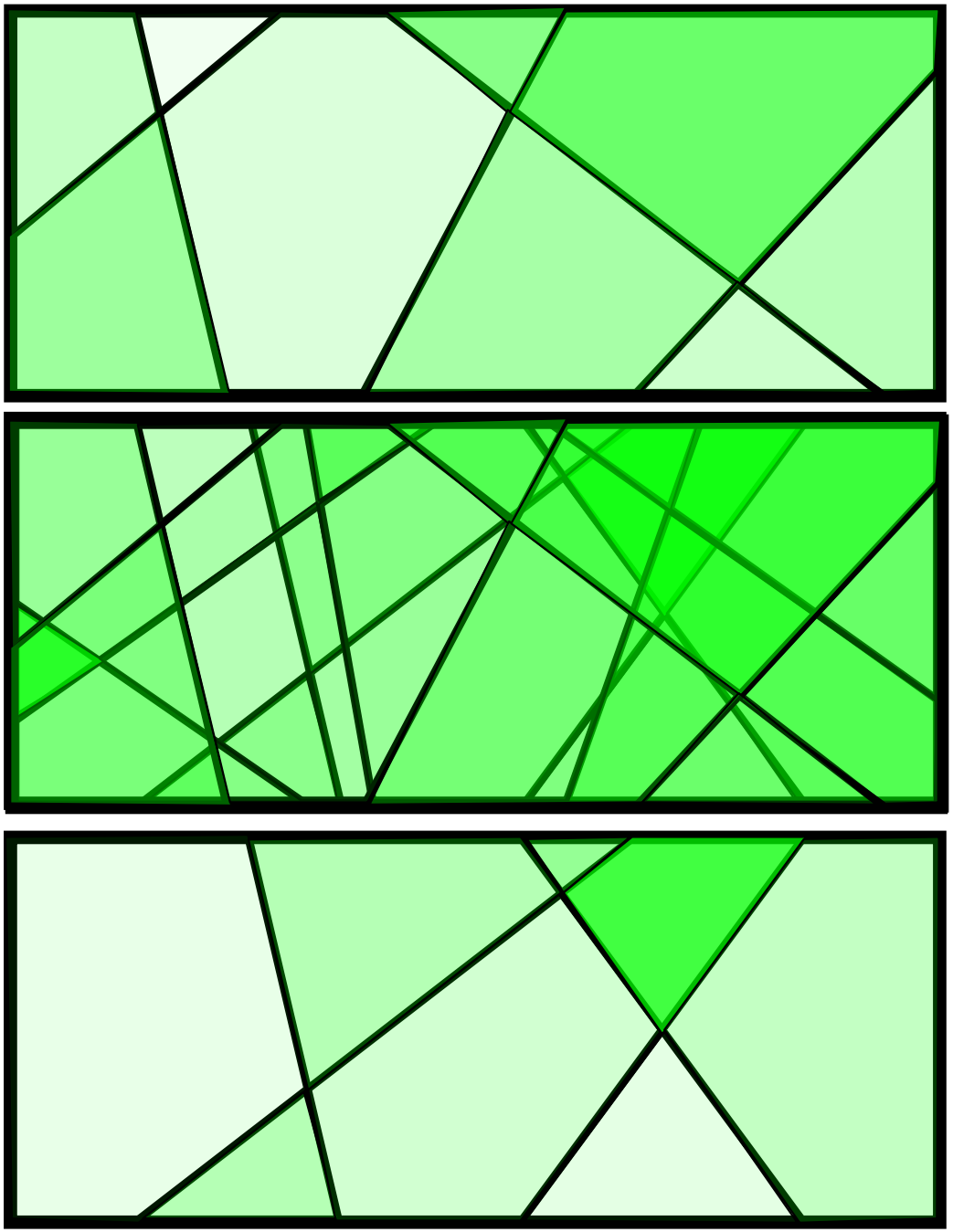
Hyperplanes at every level of the network split the space.

Graphical Interpretation



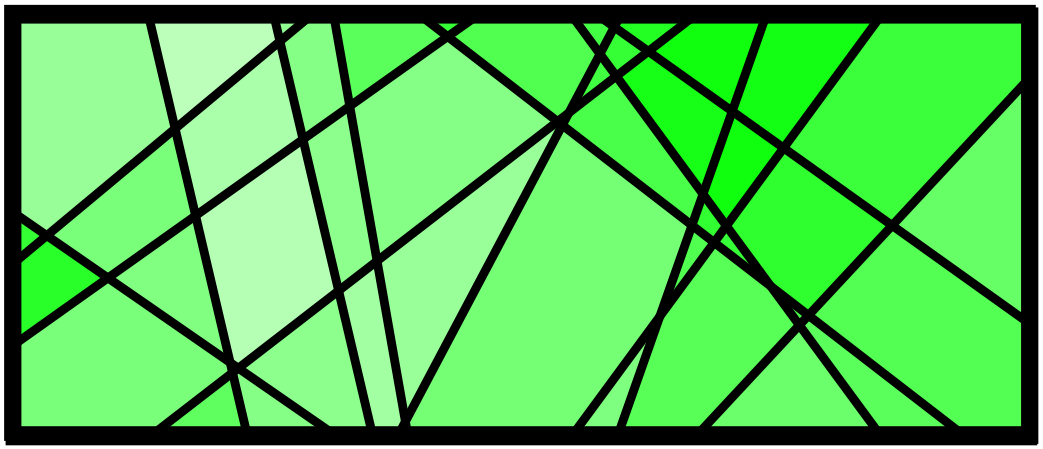
Hyperplanes at every level of the network split the space.

Graphical Interpretation



The splits are combined by the hierarchical nature of the tree.

Graphical Interpretation



The splits are combined due to the sequential nature of the network.

Multi Layer Perceptrons

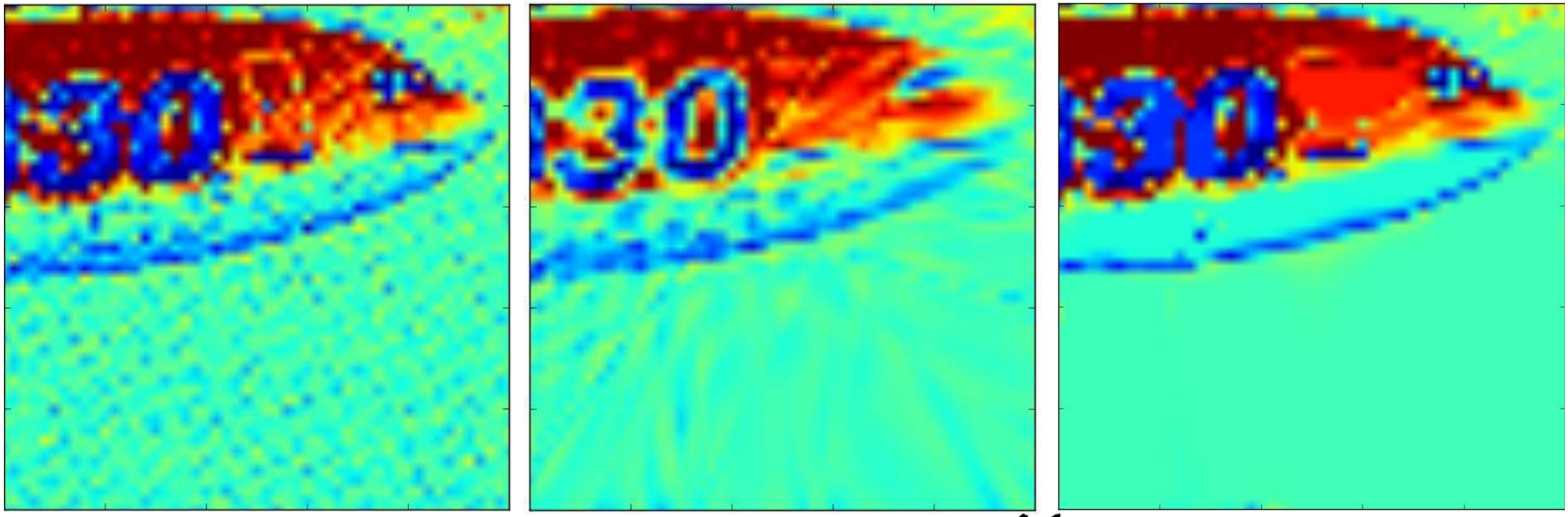
The function learned by an MLP using the ReLU, Sigmoid, or Tanh operators is:

- piecewise affine or smooth;
- continuous because it is a composition of continuous functions.

Each region created by a layer is split into smaller regions:

- Their boundaries are correlated in a complex way.
- Their descriptive power is larger than shallow networks for the same number of parameters.

Second Layer for Approximation



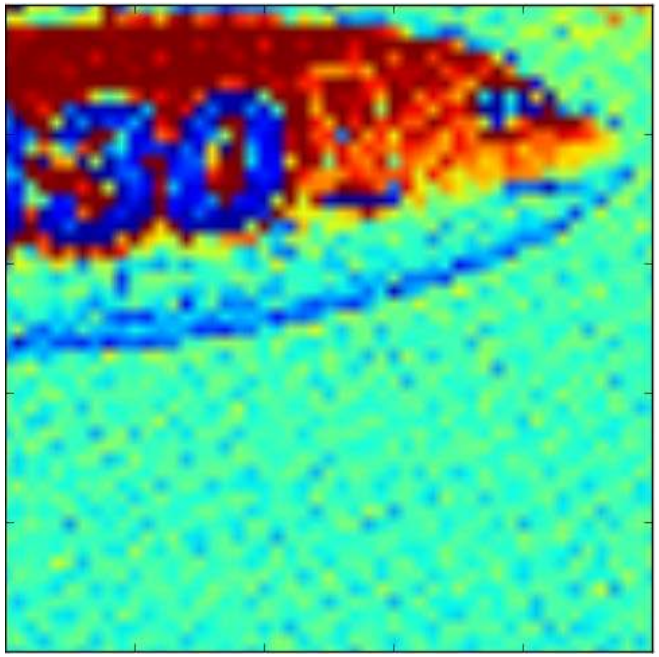
$$I = f(x, y)$$

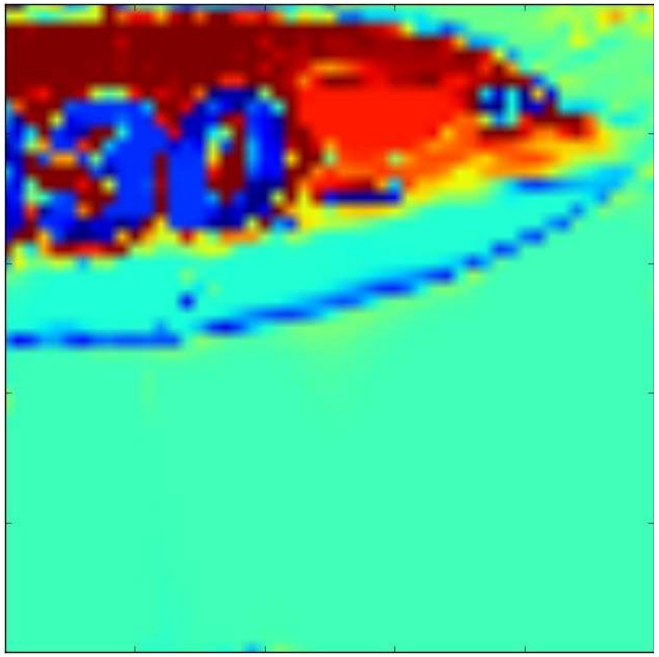
1 Layer: 125 nodes -> loss 2.40e-01 2 Layers: 20 nodes -> loss 8.31e-02

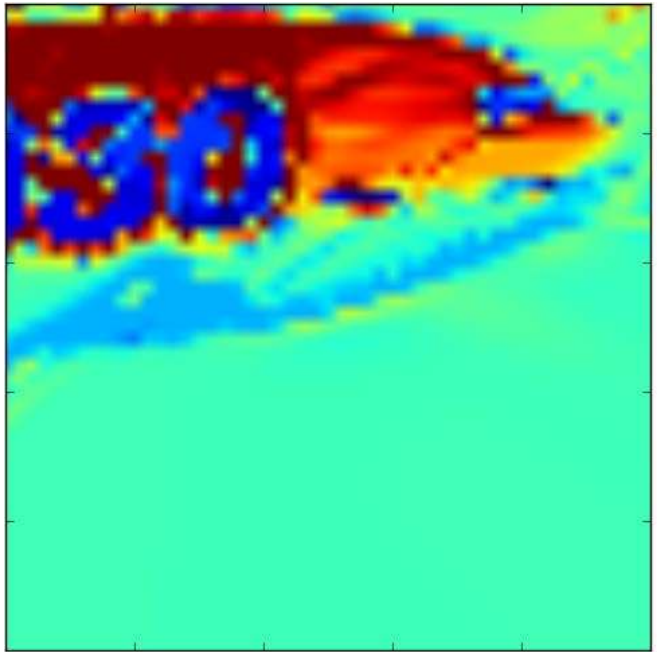
501 weights in both cases

Same # of coefficients

Adding a Third Layer







$$I = f(x, y)$$

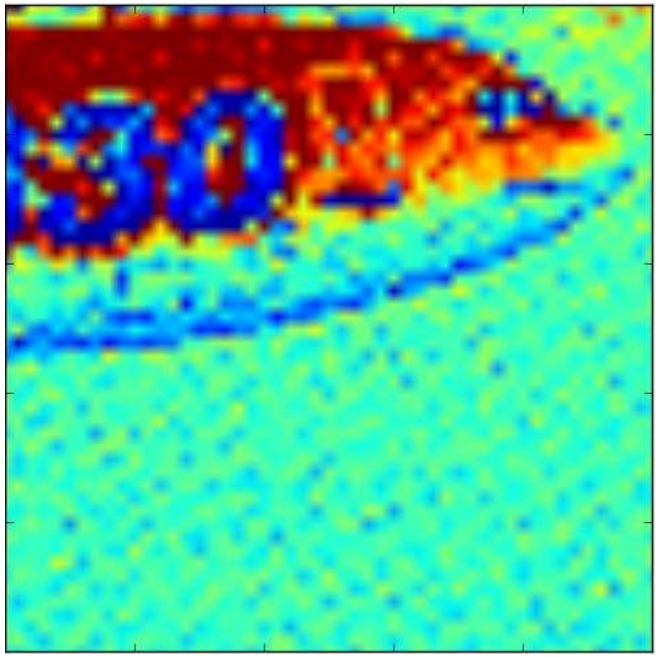
2 Layers: 20 nodes -> loss 8.31e-02

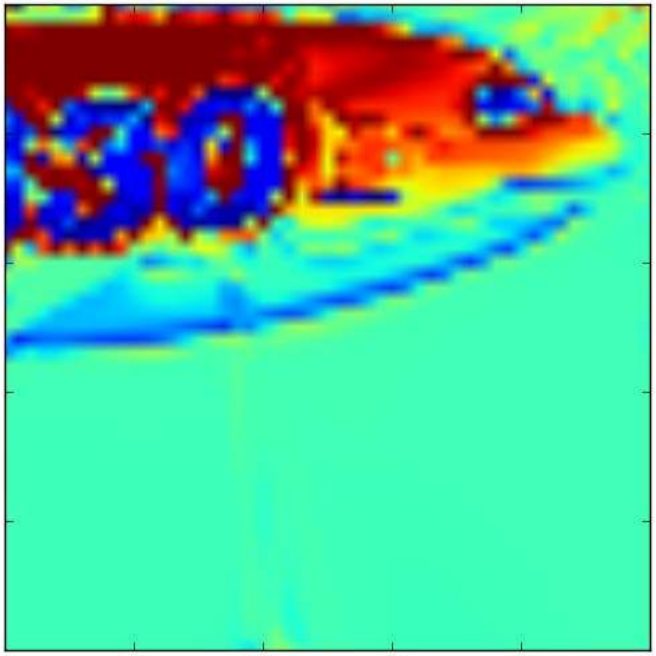
501 weights

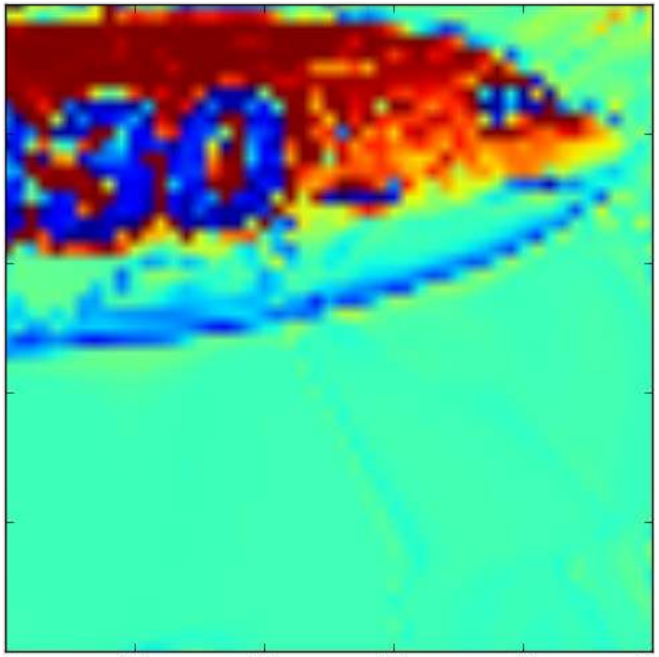
3 Layers: 14 nodes -> loss 7.55e-02

477 weights

Adding a Third Layer







$$I = f(x, y)$$

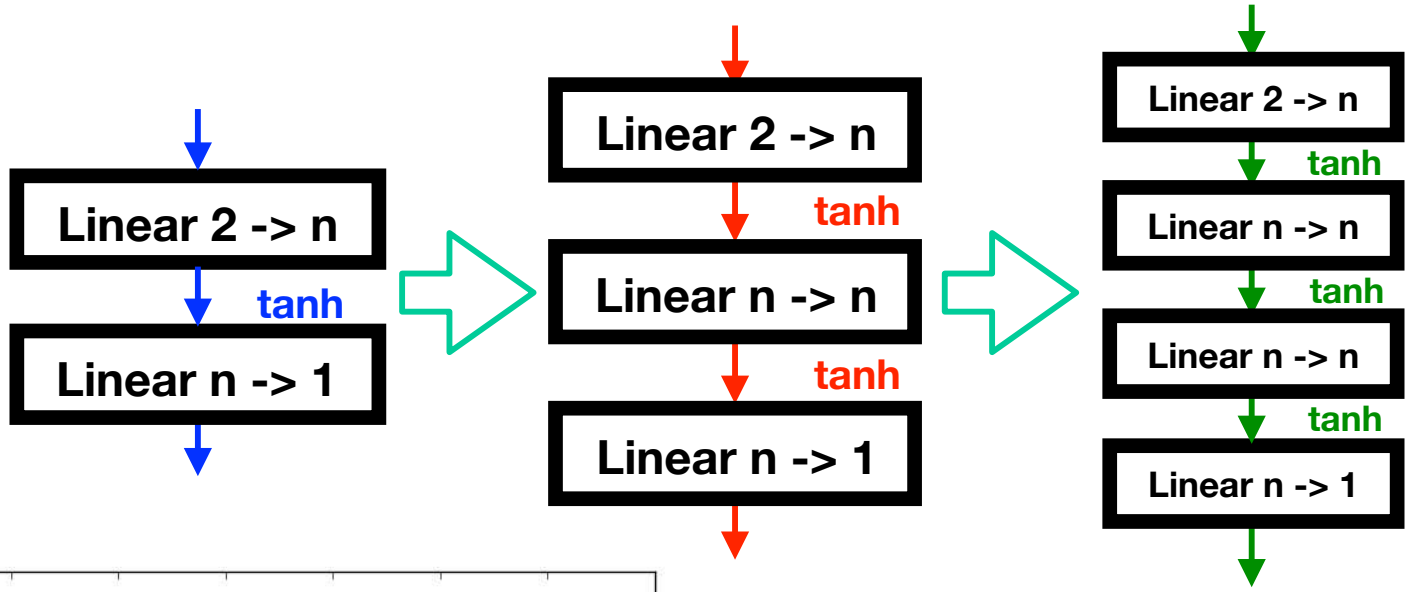
3 Layers: 15 nodes -> loss 5.93e-02

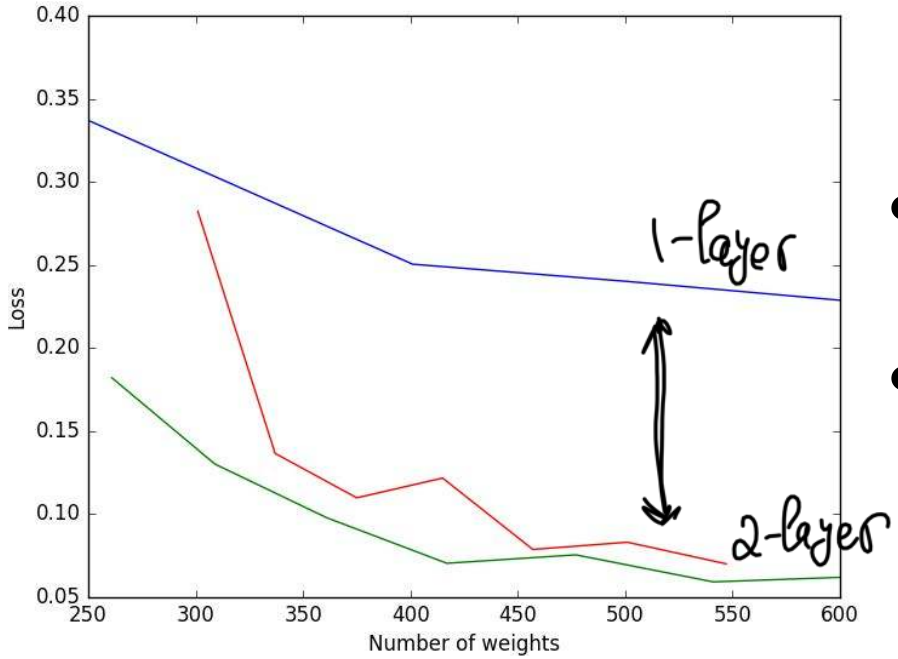
541 weights

3 Layers: 19 nodes -> loss 4.38e-02

837 weights

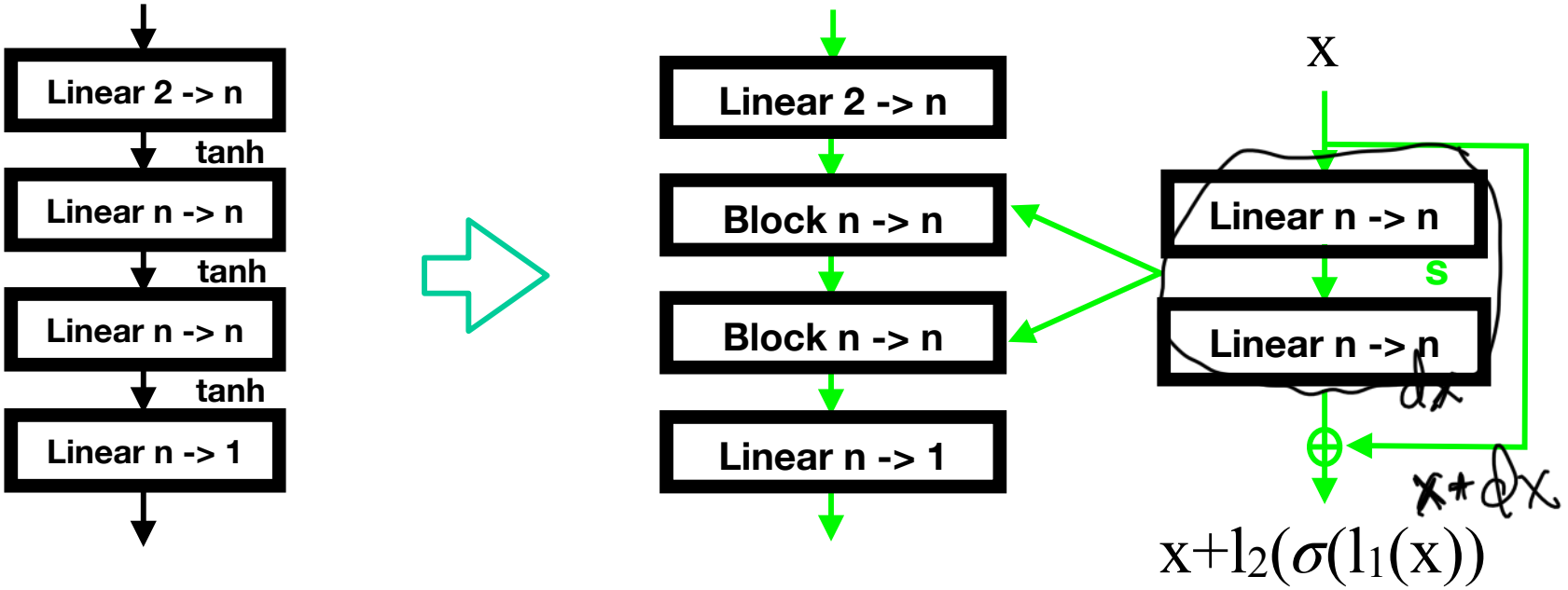
Multi Layer Perceptrons





- Adding layers tends to deliver better convergence properties.
- In current practice, deeper is usually better.

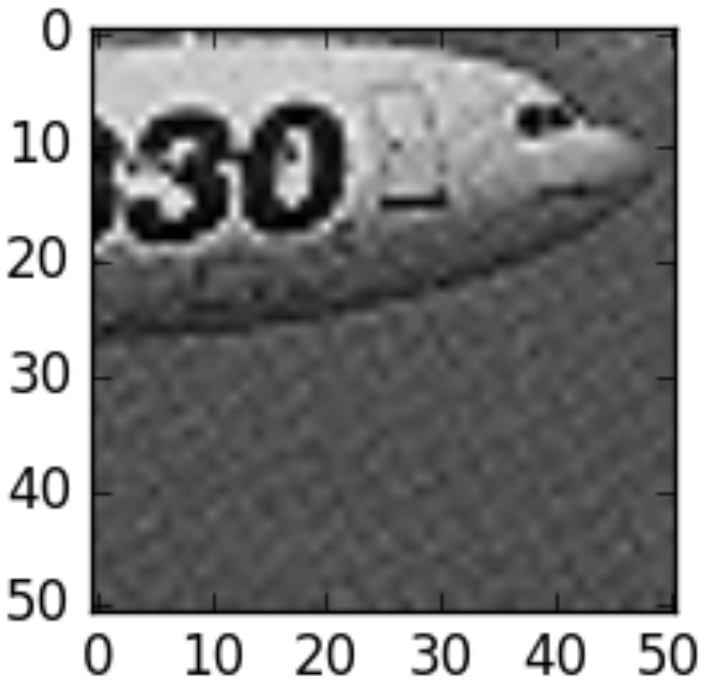
MLP to ResNet



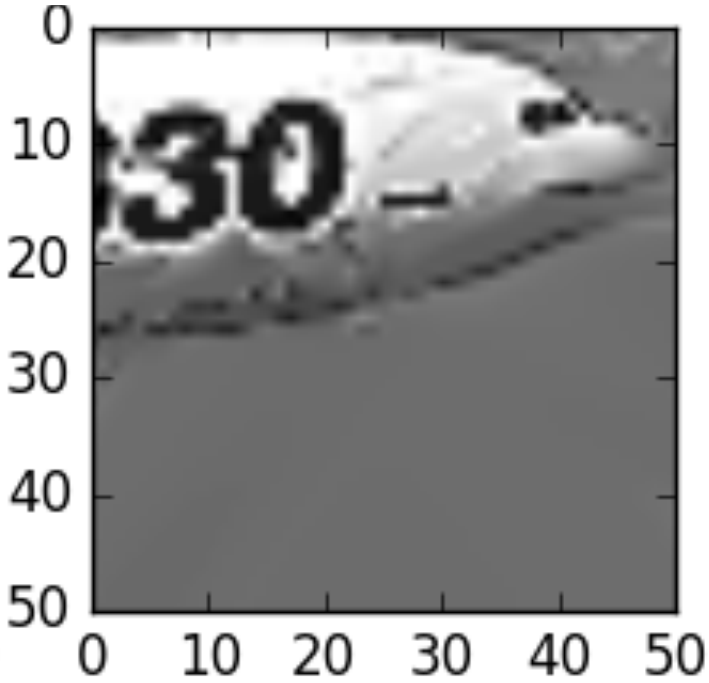
Further improvements in the convergence properties have been obtained by adding a bypass, which allows the final layers to only compute residuals.

Improving the Network

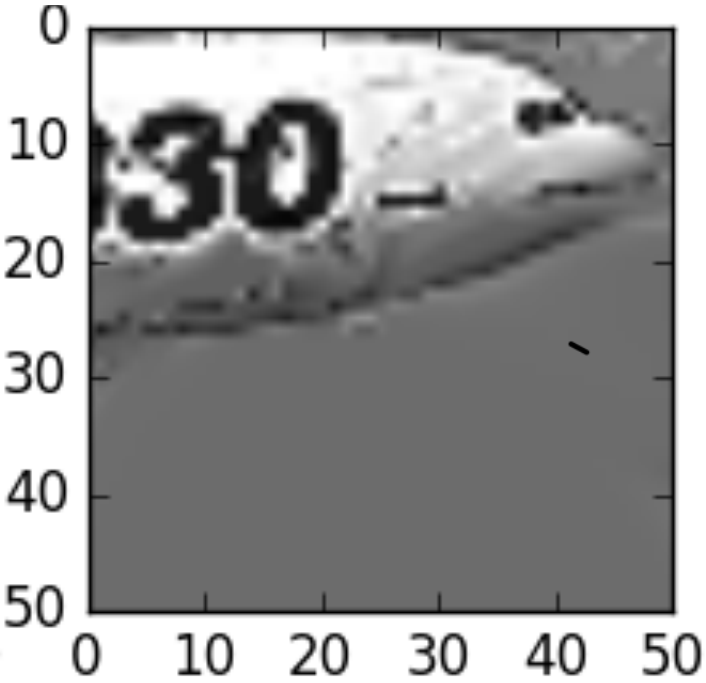
Boosting with ResNet



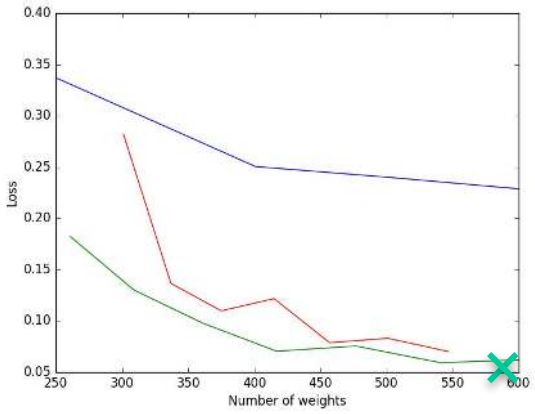
Original 51x51 image:
2601 gray level values.

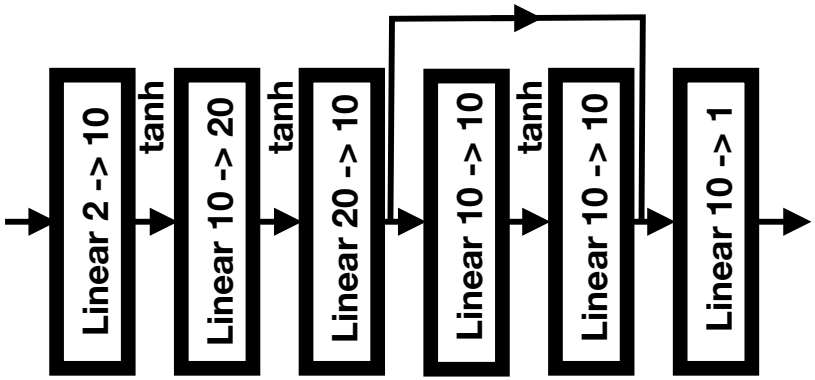


MLP 10/20/10 Interpolation:
471 weights, loss 6.43e-02.

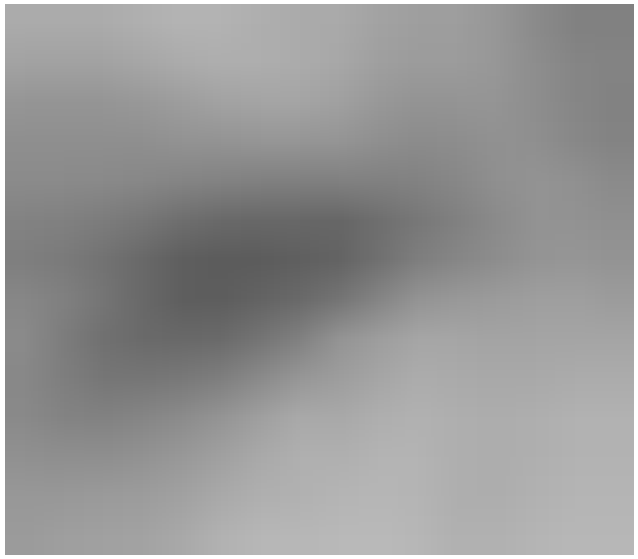


MLP 10/20/10/10 Interpolation:
581 weights, loss 5.30e-2.

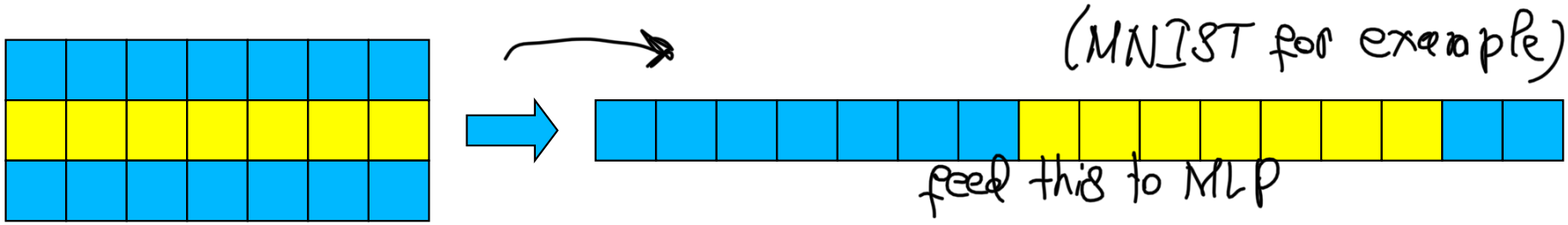




Digital Images

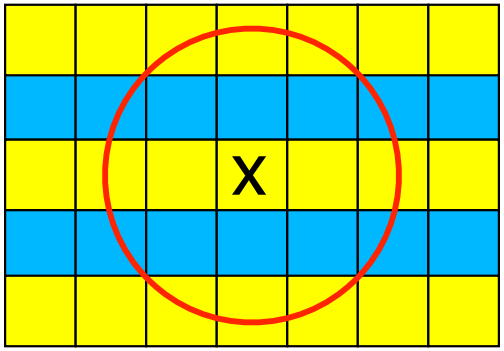


136 134 161 159 163 168 171 173 173 171 166 159 157 155
152 145 136 130 151 149 151 154 158 161 163 163 159 151
145 149 149 145 140 133 145 143 145 145 145 146 148 148
148 143 141 145 145 145 141 136 136 135 135 136 135 133
131 131 129 129 133 136 140 142 142 138 130 128 126 120
115 111 108 106 106 110 120 130 137 142 144 141 129 123
117 109 098 094 094 094 100 110 125 136 141 147 147 145
136 124 116 105 096 096 100 107 116 131 141 147 150 152
152 152 137 124 113 108 105 108 117 129 139 150 157 159
159 157 157 159 135 121 120 120 121 127 136 147 158 163
165 165 163 163 163 166 136 131 135 138 140 145 154 163
166 168 170 168 166 168 170 173 145 143 147 148 152 159
168 173 173 175 173 171 170 173 177 178 151 151 153 156
161 170 176 177 177 179 176 174 174 176 177 179 155 157
161 162 168 176 180 180 180 182 180 175 175 178 180 180



- A MxN image can be represented as an MN vector, in which case neighborhood relationships are lost.
- By contrast, treating it as a 2D array preserves neighborhood relationships.

Image Specificities

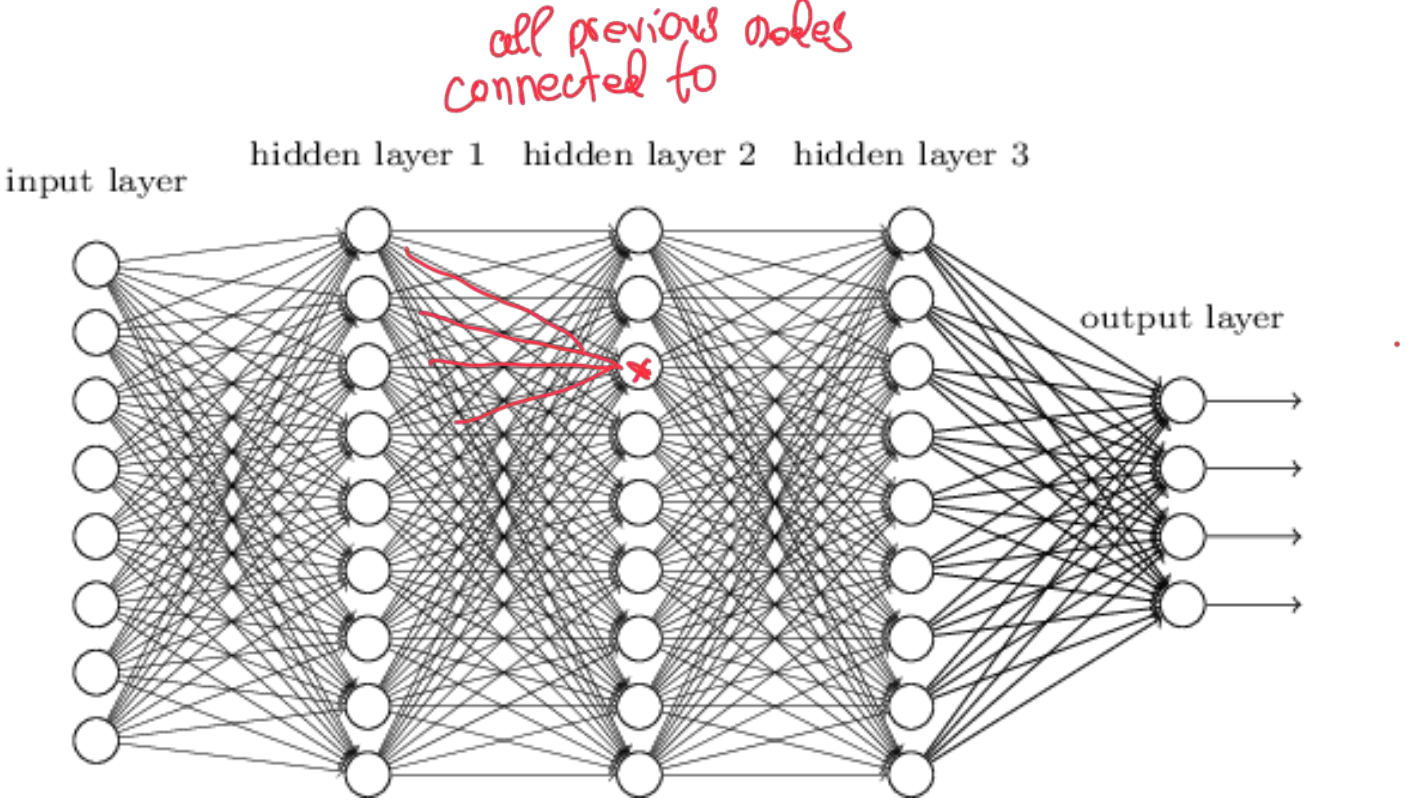


- In a typical image, the values of neighboring pixels tend to be more highly correlated than those of distant ones.
- An image filter should be translation invariant.

input: shifted image → output: gives a shifted filtered image

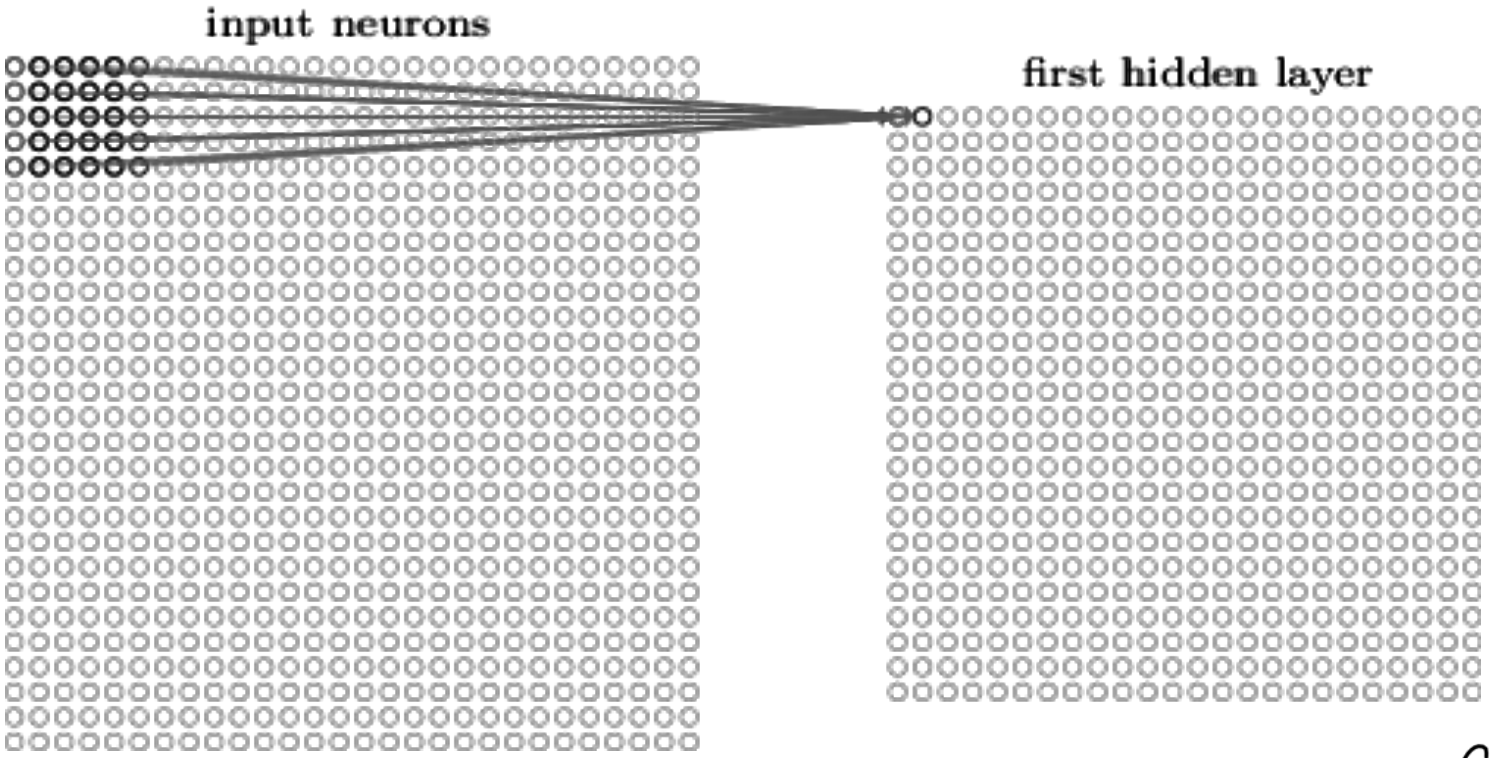
—> These two properties can be exploited to drastically reduce the number of weights required by CNNs using so-called convolutional layers.

Fully Connected Layers



- The descriptive power of the net increases with the number of layers.
- In the case of a 1D signal, it is roughly proportional to $\prod_n W_n$ where W_n represents the width of a layer.

Convolutional Layer

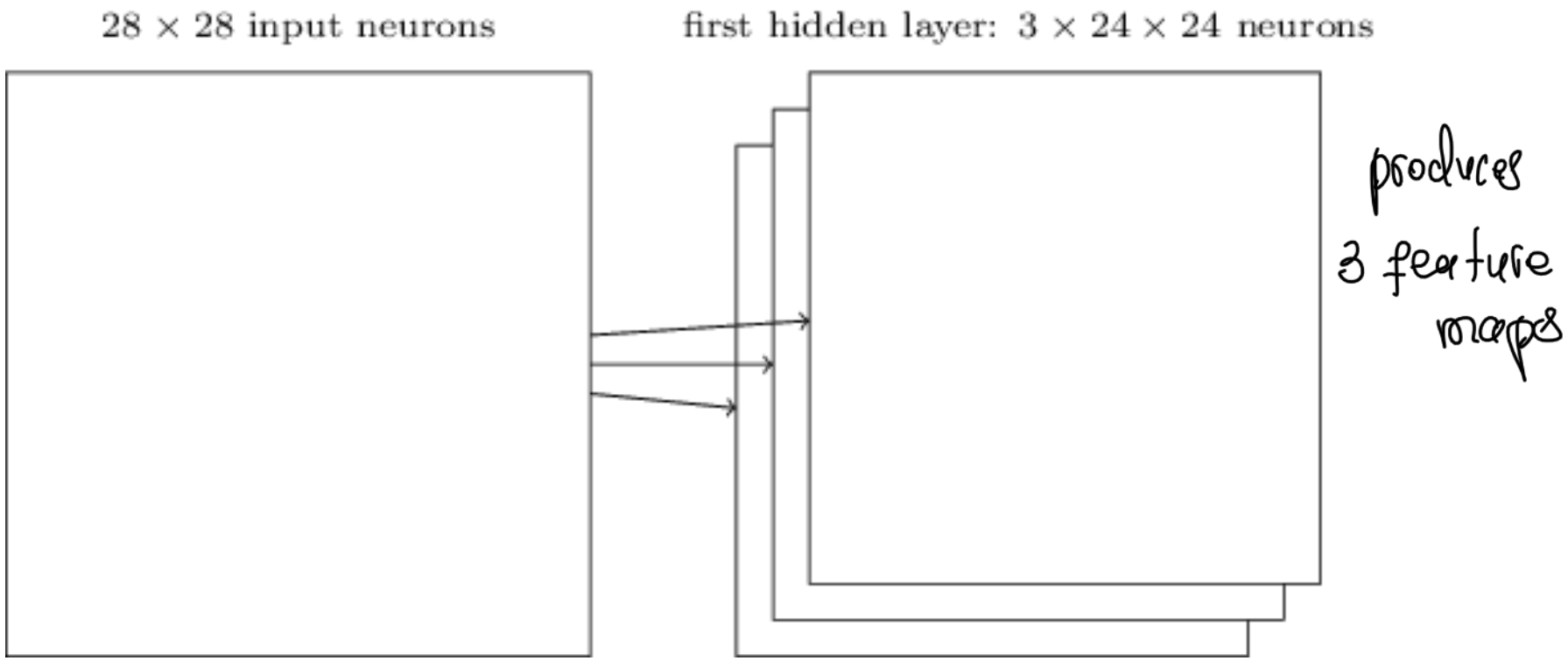


convolution

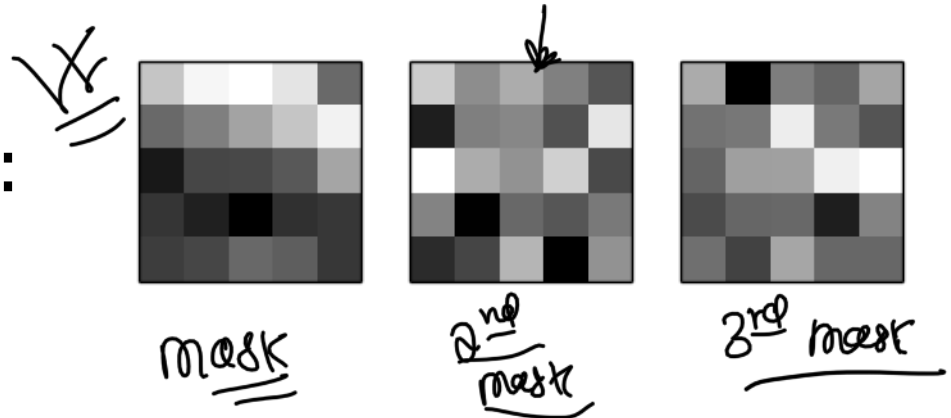
$$\sigma\left(b + \sum_{x=0}^{n_x}\sum_{y=0}^{n_y} w_{x,y} a_{i+x,j+y}\right)$$

Same as convolution layers

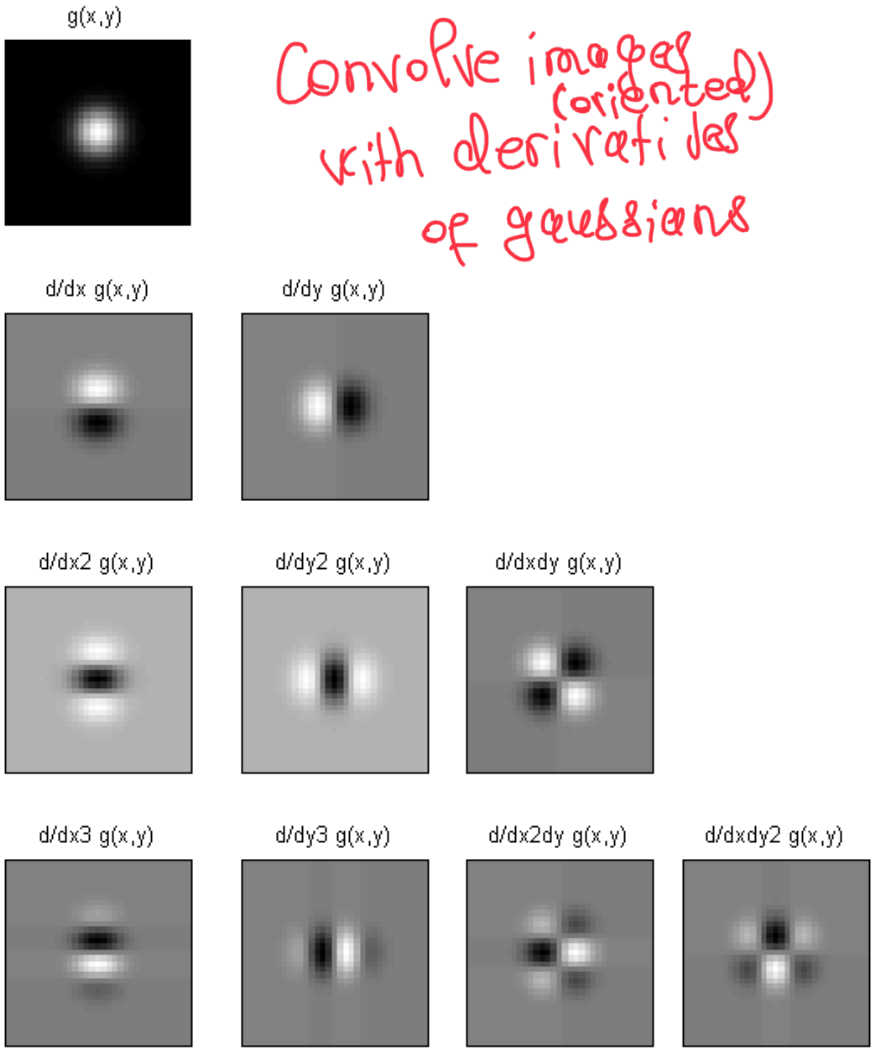
Feature Maps



Filters:



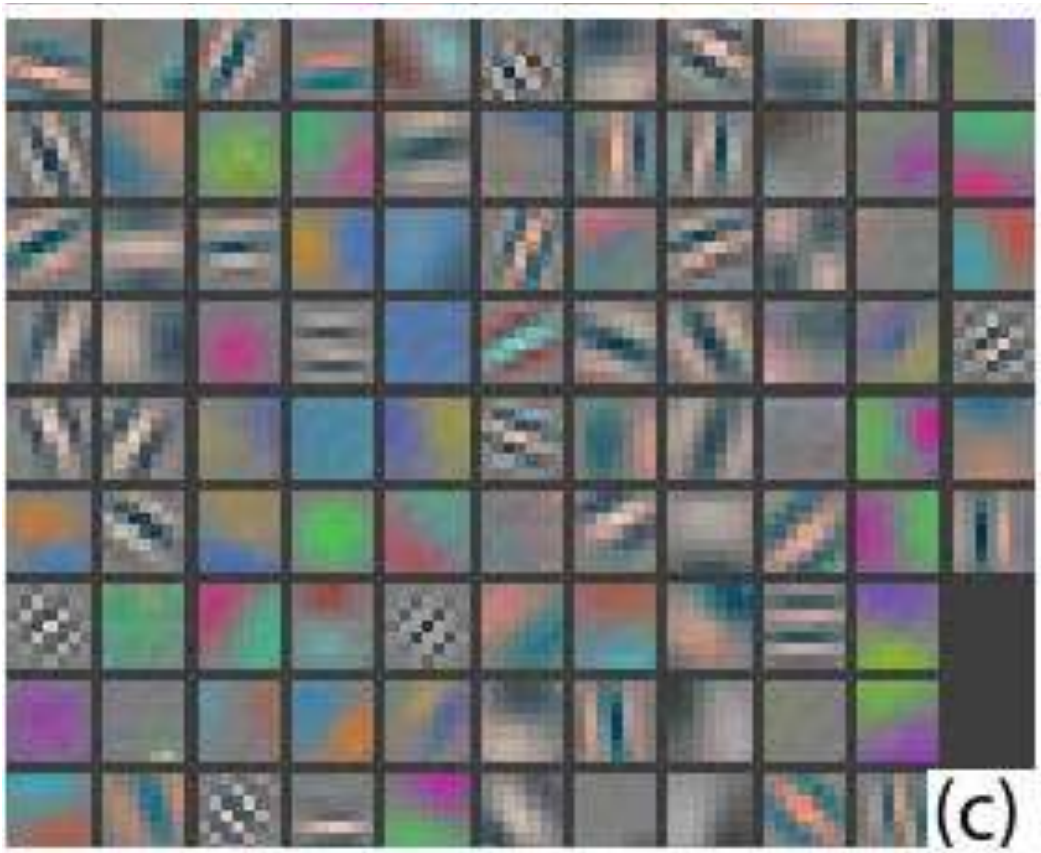
Filters (how to choose them?)



Convolve images (oriented) with derivatives of gaussians

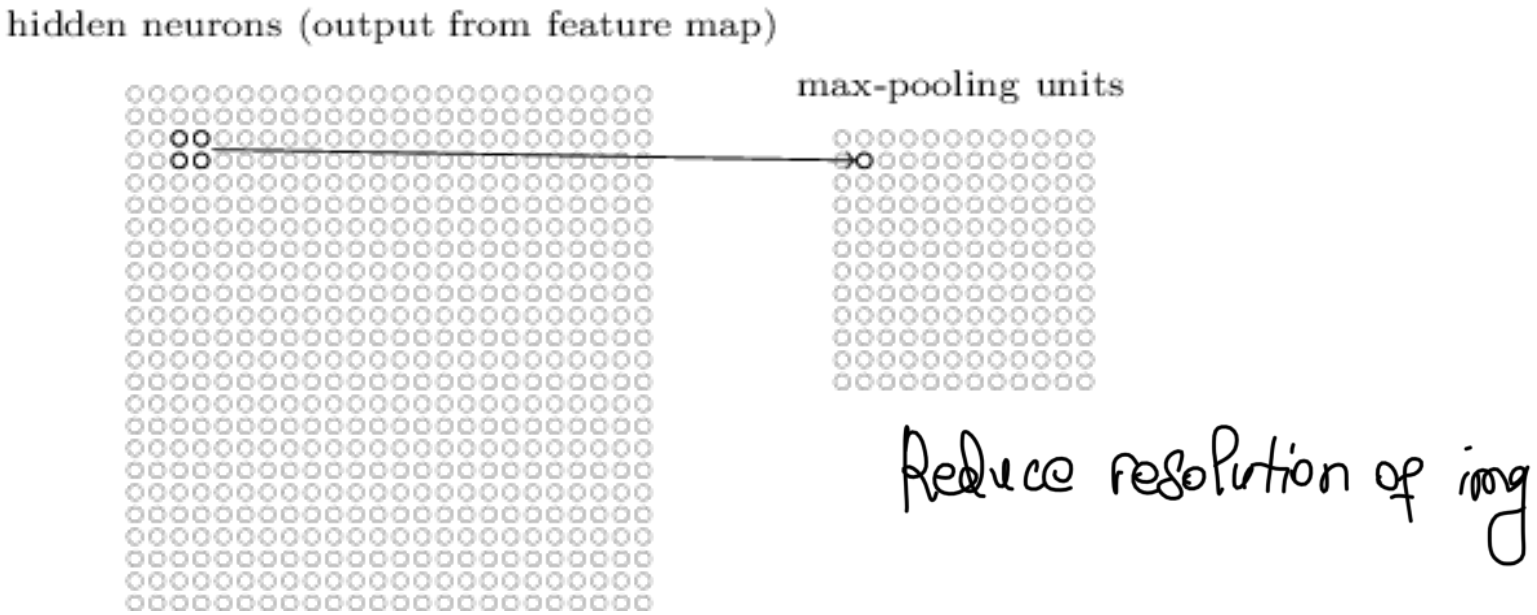
fixed set of filters

Derivatives



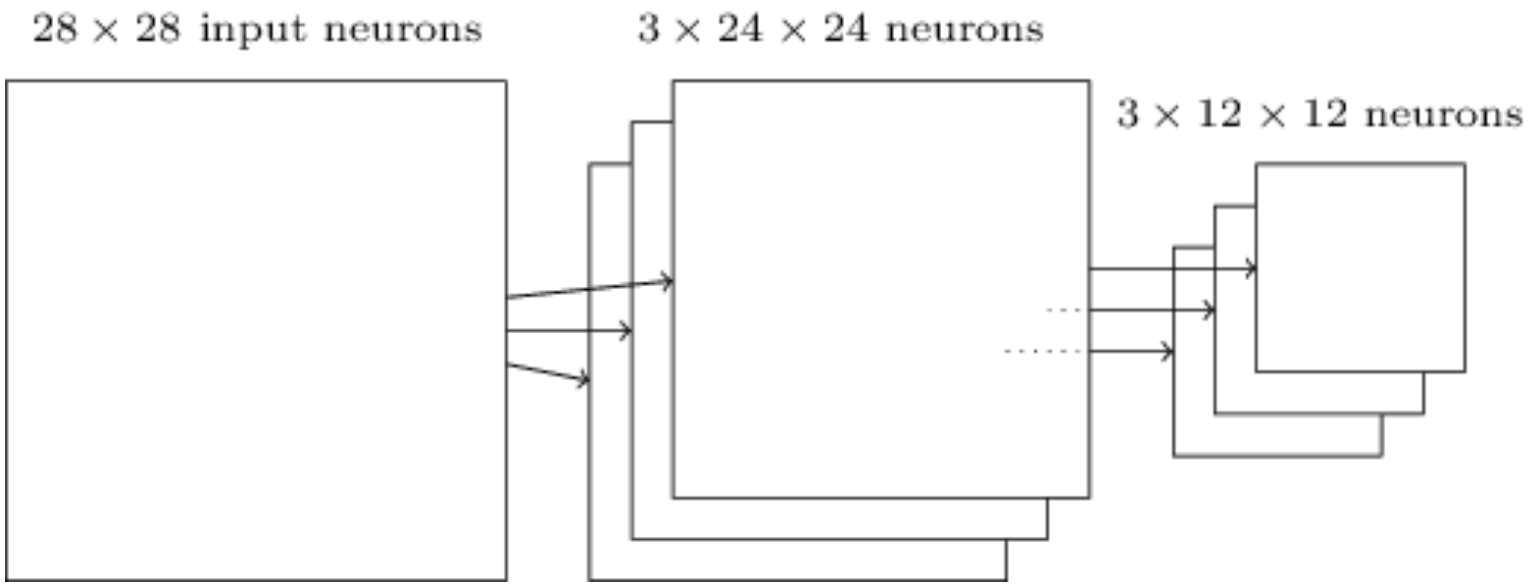
CNN's Learned filters

Pooling Layer



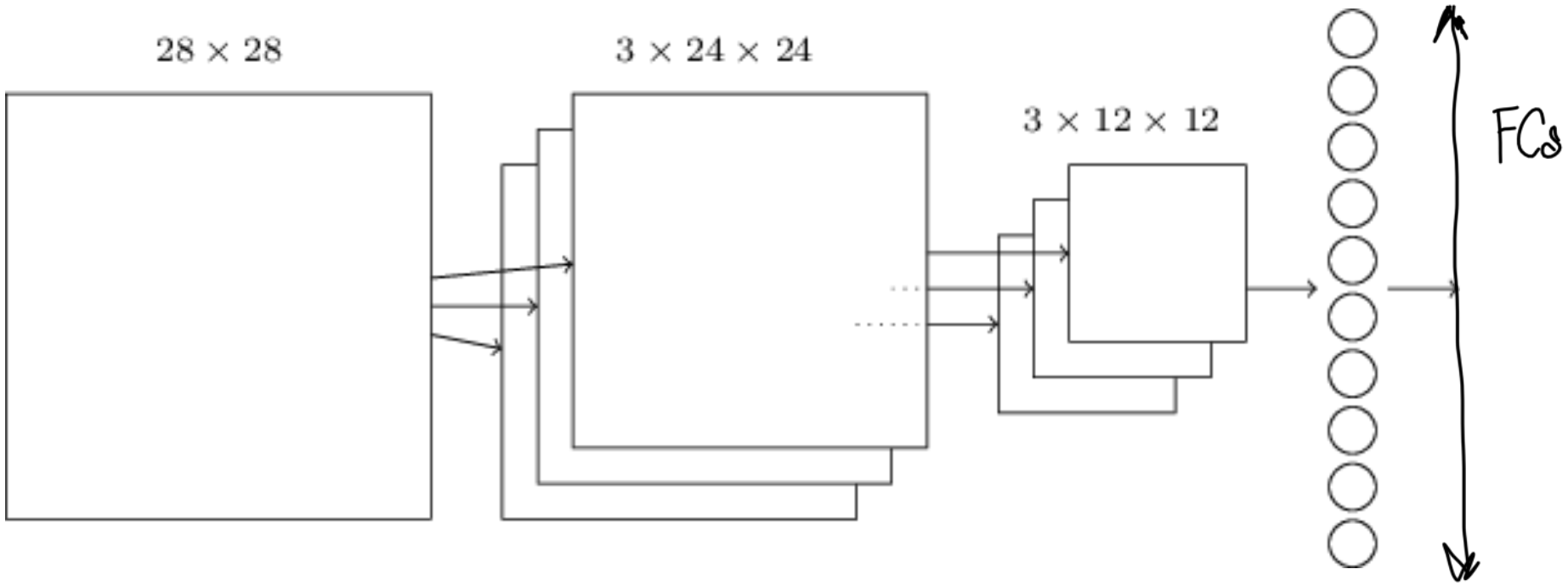
- Reduces the number of inputs by replacing all activations in a neighborhood by a single one.
- Can be thought as asking if a particular feature is present in that neighborhood while ignoring the exact location.

Adding the Pooling Layers



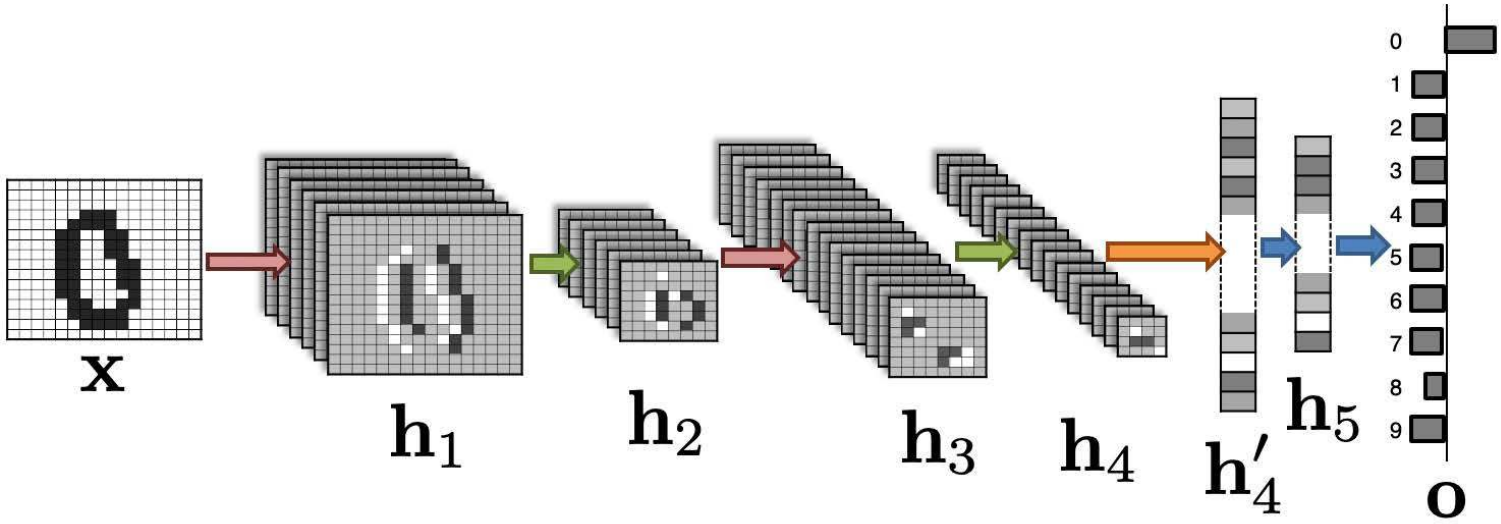
The output size is reduced by the pooling layers.

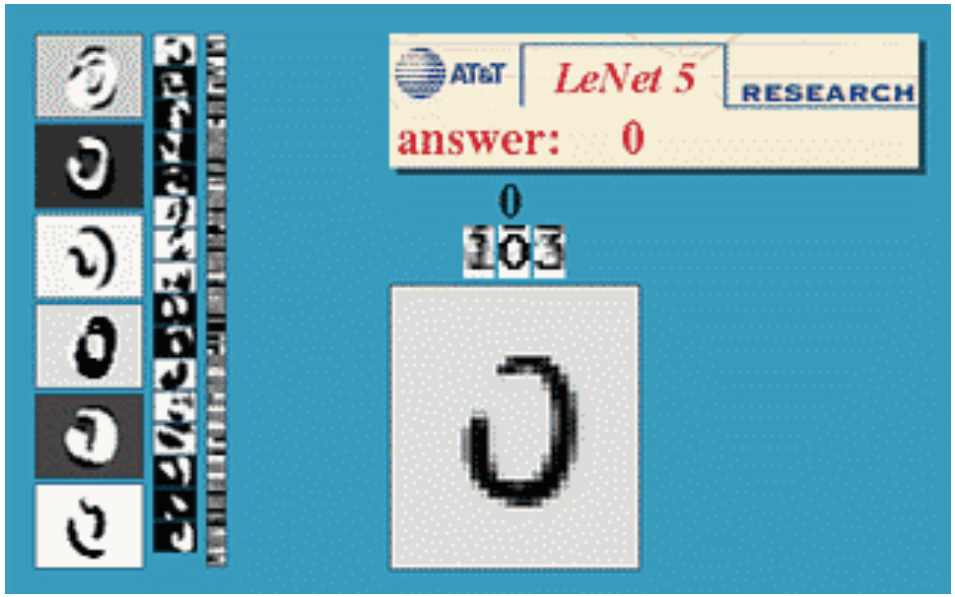
Adding a Fully Connected Layer



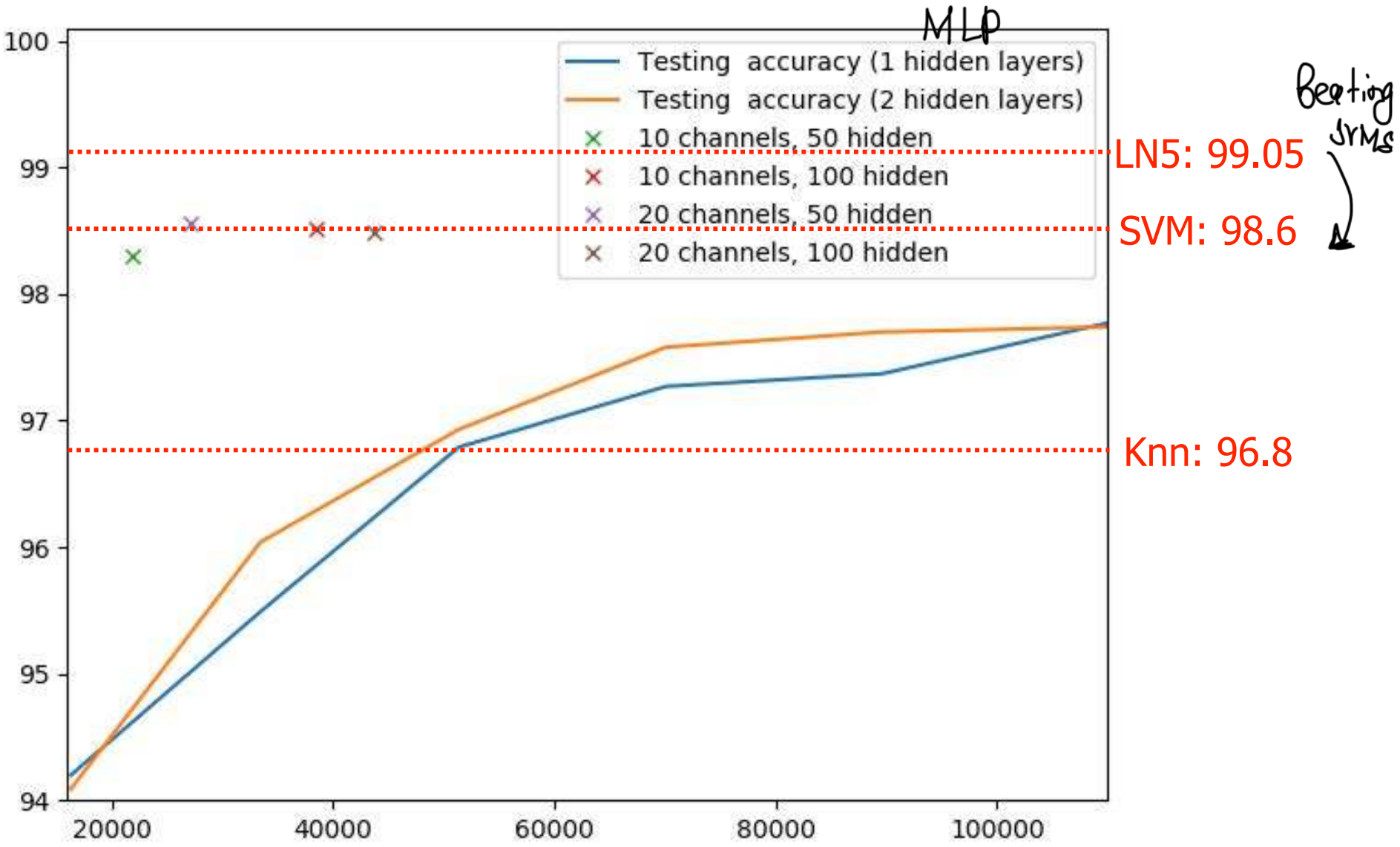
- Each neutron in the final fully connected layer is connected to all neurons in the preceding one.
- Deep architecture with many parameters to learn but still far fewer than an equivalent multilayer perceptron.

LeNet (1989-1999)

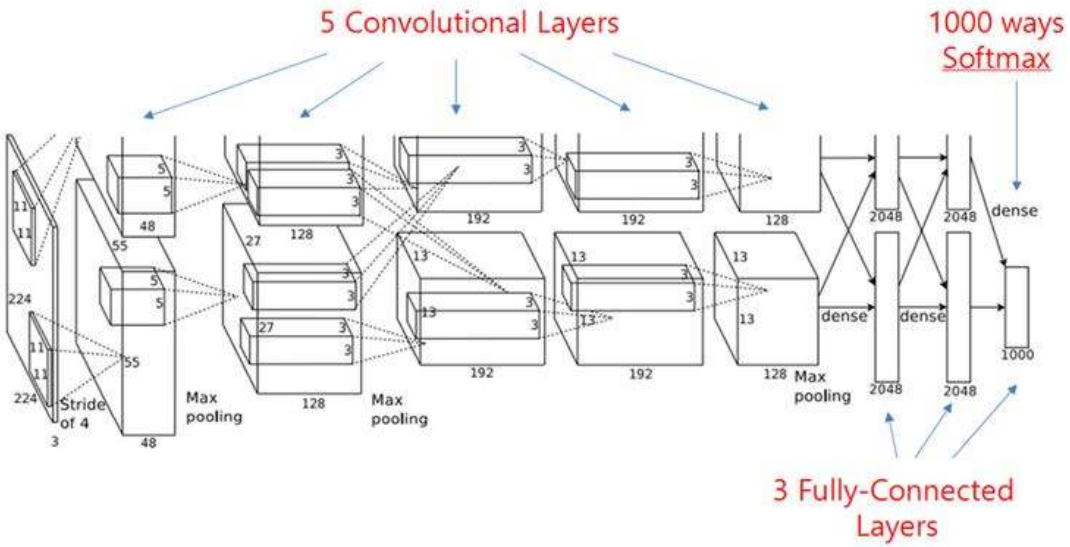




Lenet Results



AlexNet (2012)



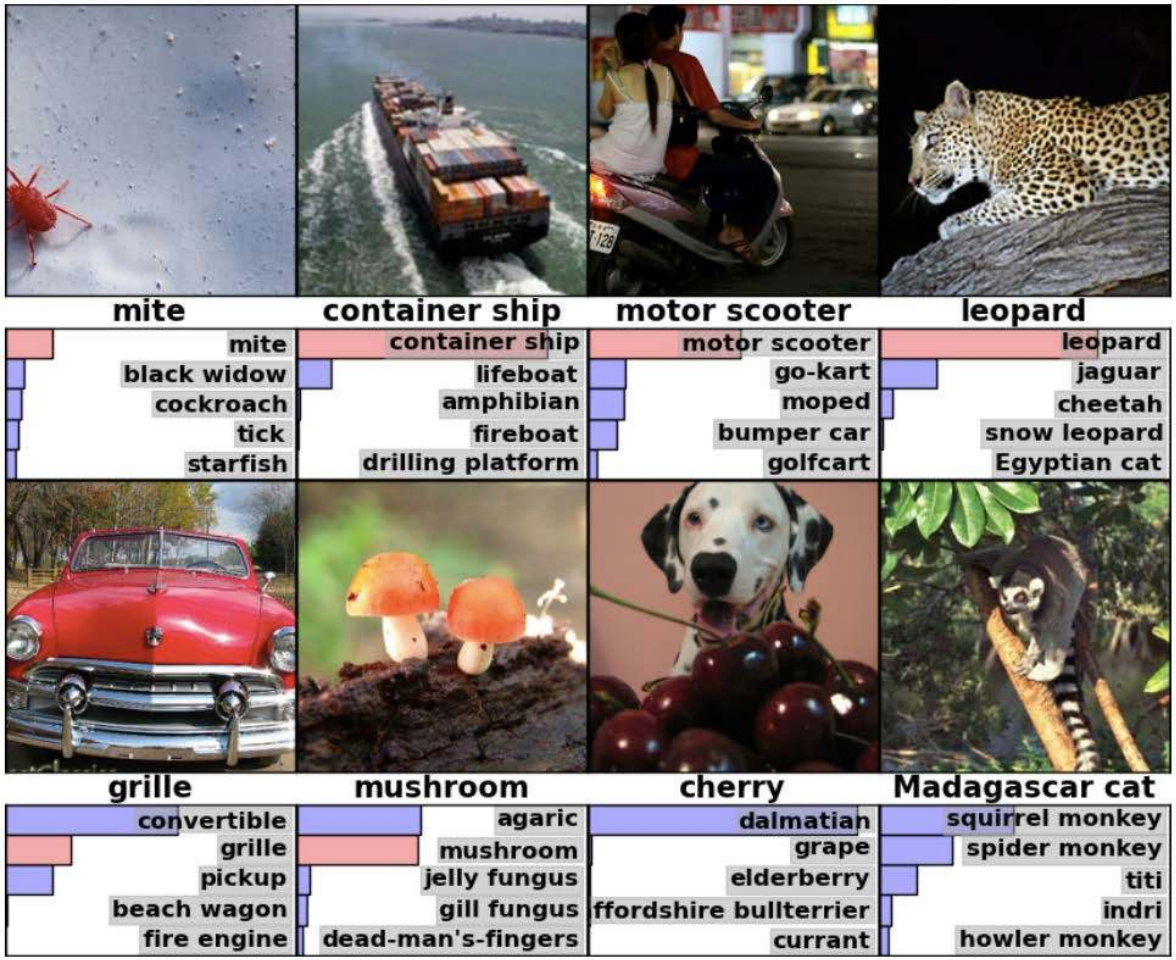
Task: Image classification

Training images: Large Scale Visual Recognition Challenge 2010

Training time: 2 weeks on 2 GPUs

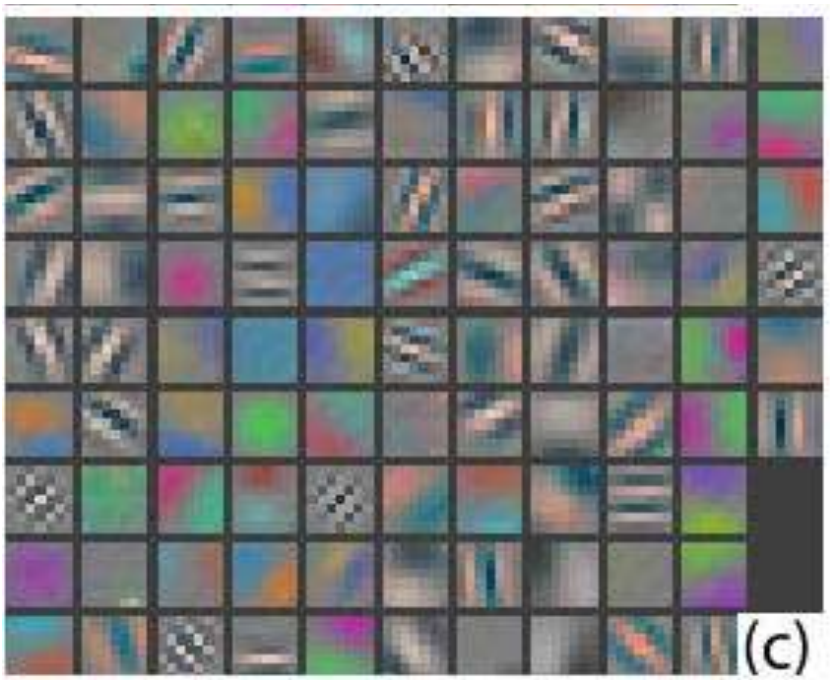
Major Breakthrough: Training large networks has now been shown to be practical!!

AlexNet Results

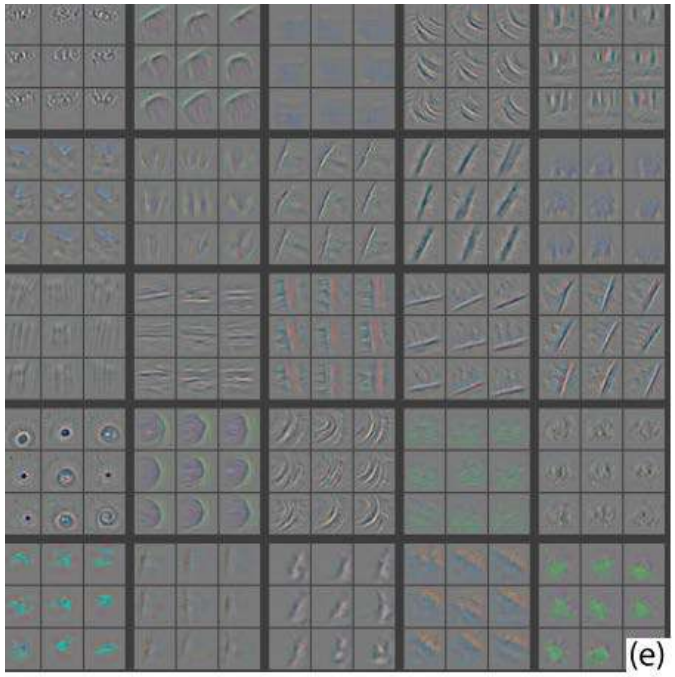


- At the 2012 ImageNet Large Scale Visual Recognition Challenge, AlexNet achieved a top-5 error of 15.3%, more than 10.8% lower than the runner up.
- Since 2015, networks outperform humans on this task.

Feature Maps



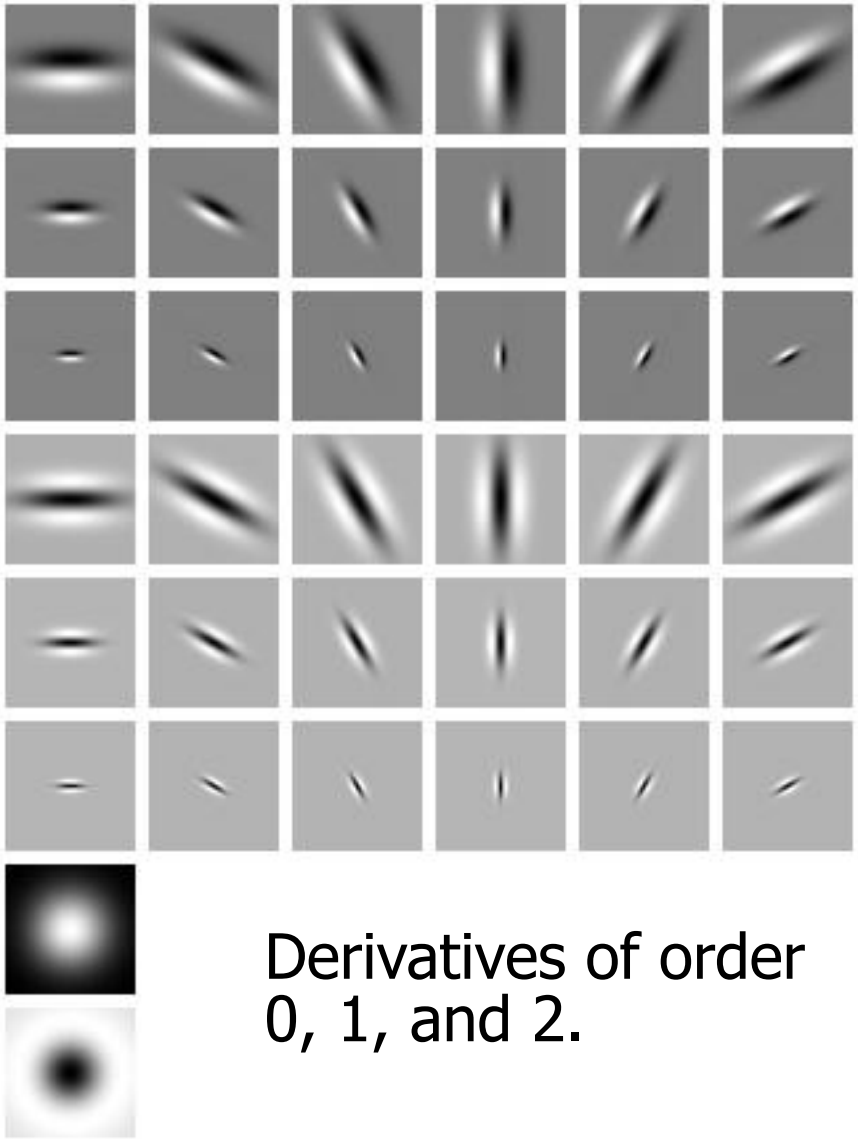
First convolutional layer



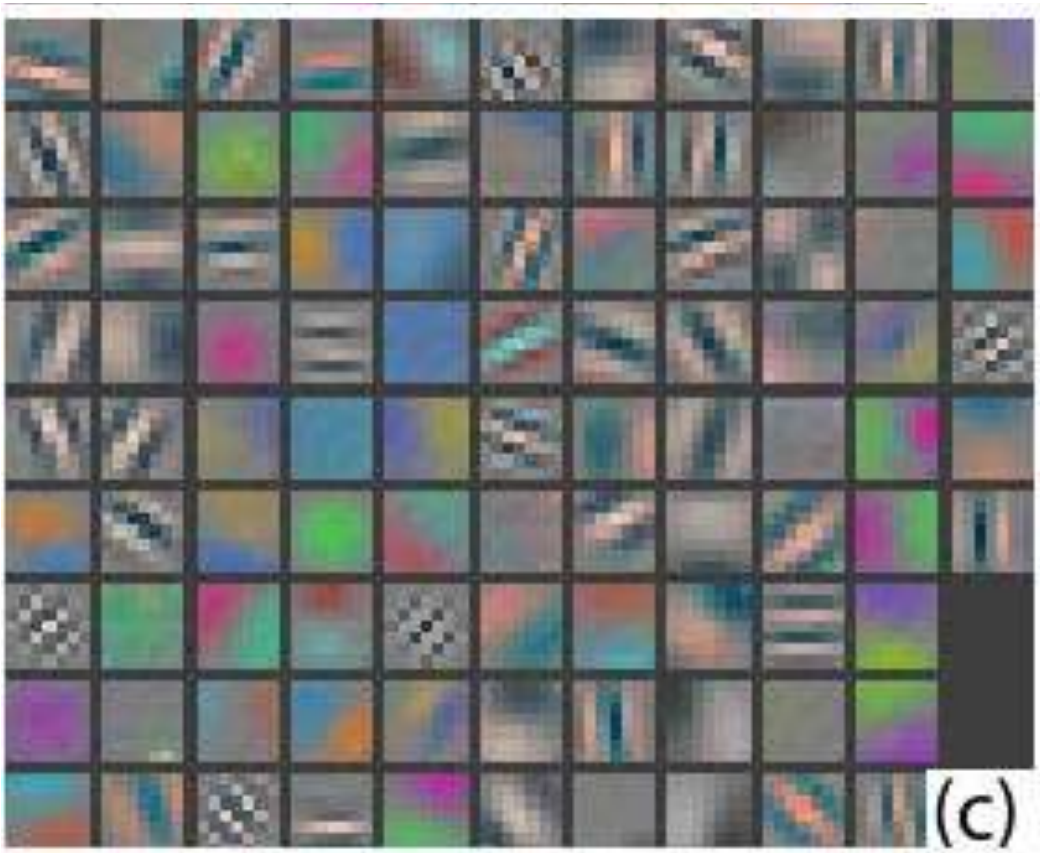
Second convolutional layer

- Some of the convolutional masks are very similar to oriented Gaussian or Gabor filters.
- The trained neural nets compute oriented derivatives, which the brain is also **believed** to do. Training dataset

Filter Banks



Derivatives of order 0, 1, and 2.

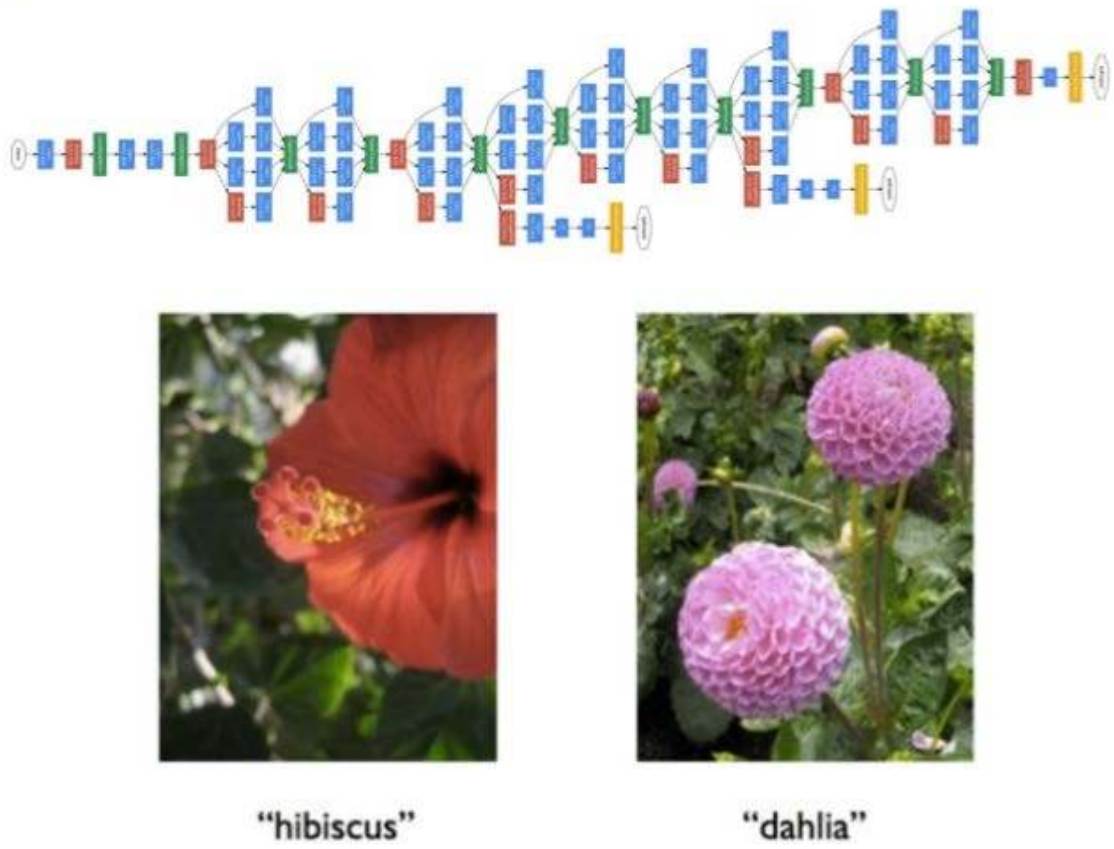


Look carefully similar

Learned

Bigger and Deeper

VGG19, 3 weeks of training.



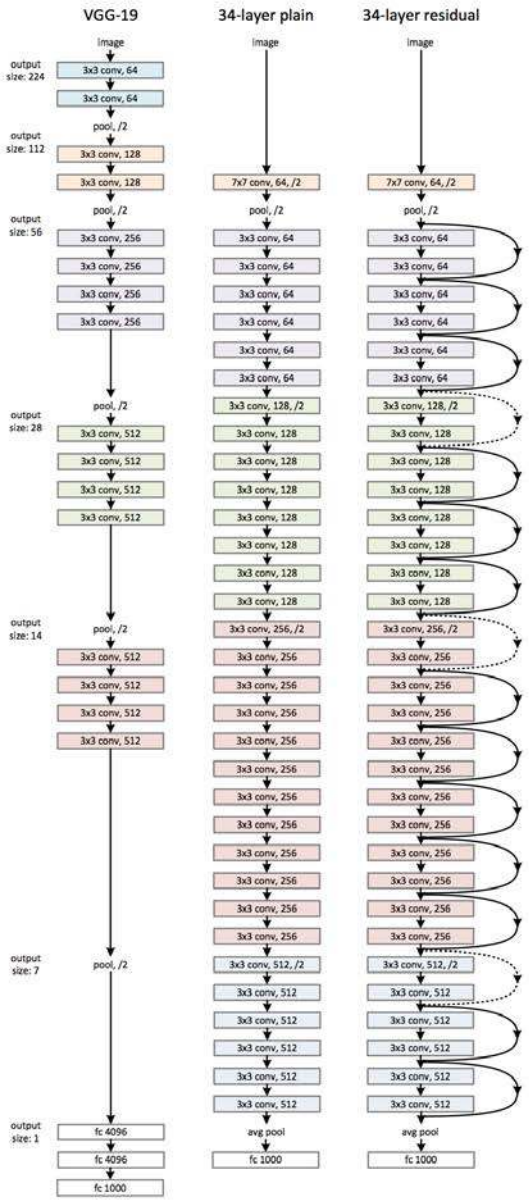
GoogleLeNet.

"It was demonstrated that the representation depth is beneficial for the classification accuracy, and that state-of-the-art performance on the ImageNet challenge dataset can be achieved using a conventional ConvNet architecture."

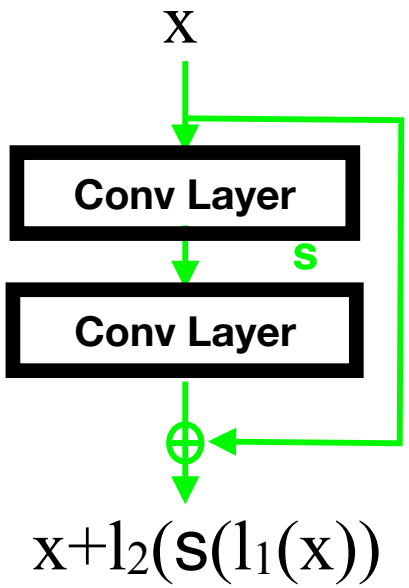
↑ Training DB → Bigger Deeper Nets

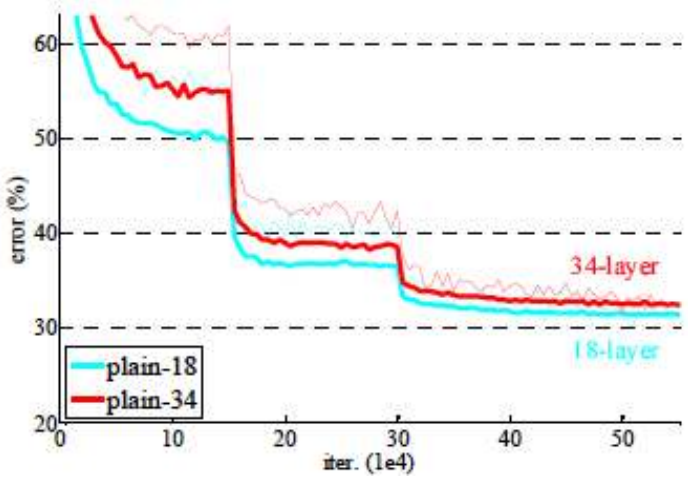
Simonyan & Zisserman, ICLR'15

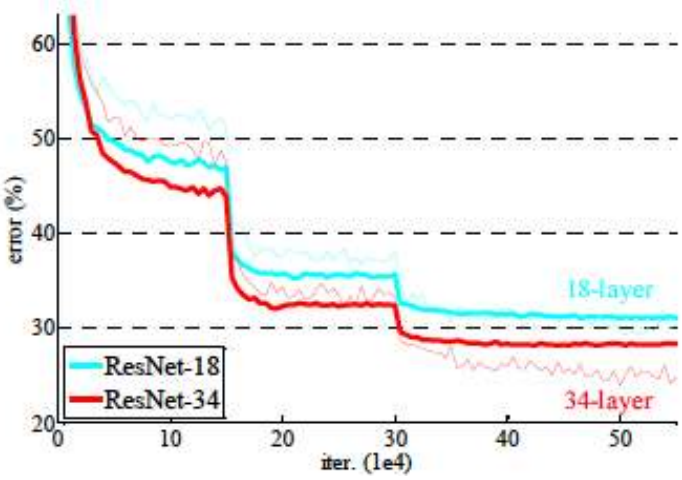
Deeper and Deeper



Resnet

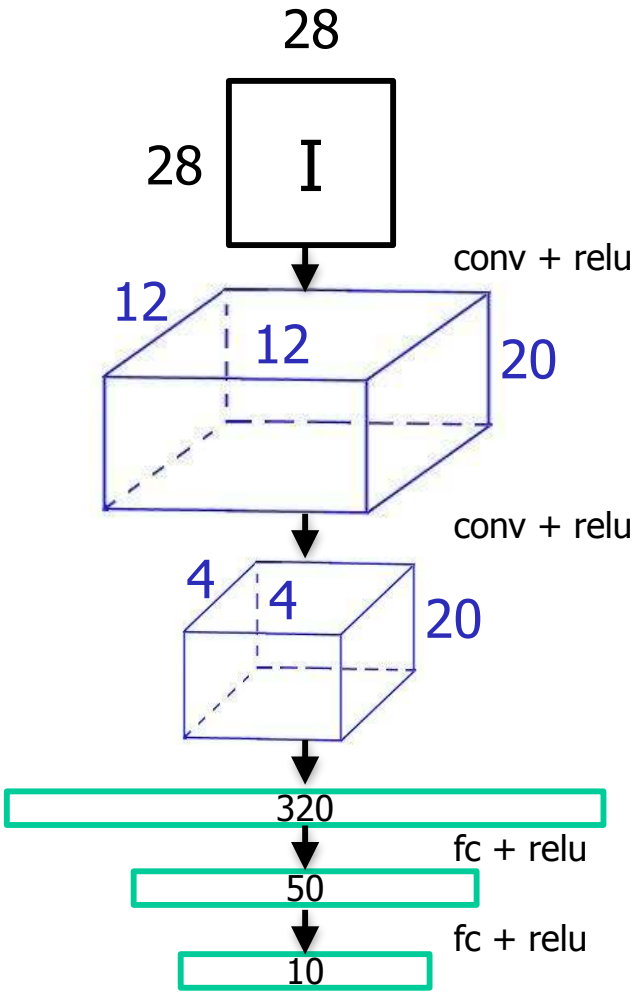






He et al. , CVPR'16

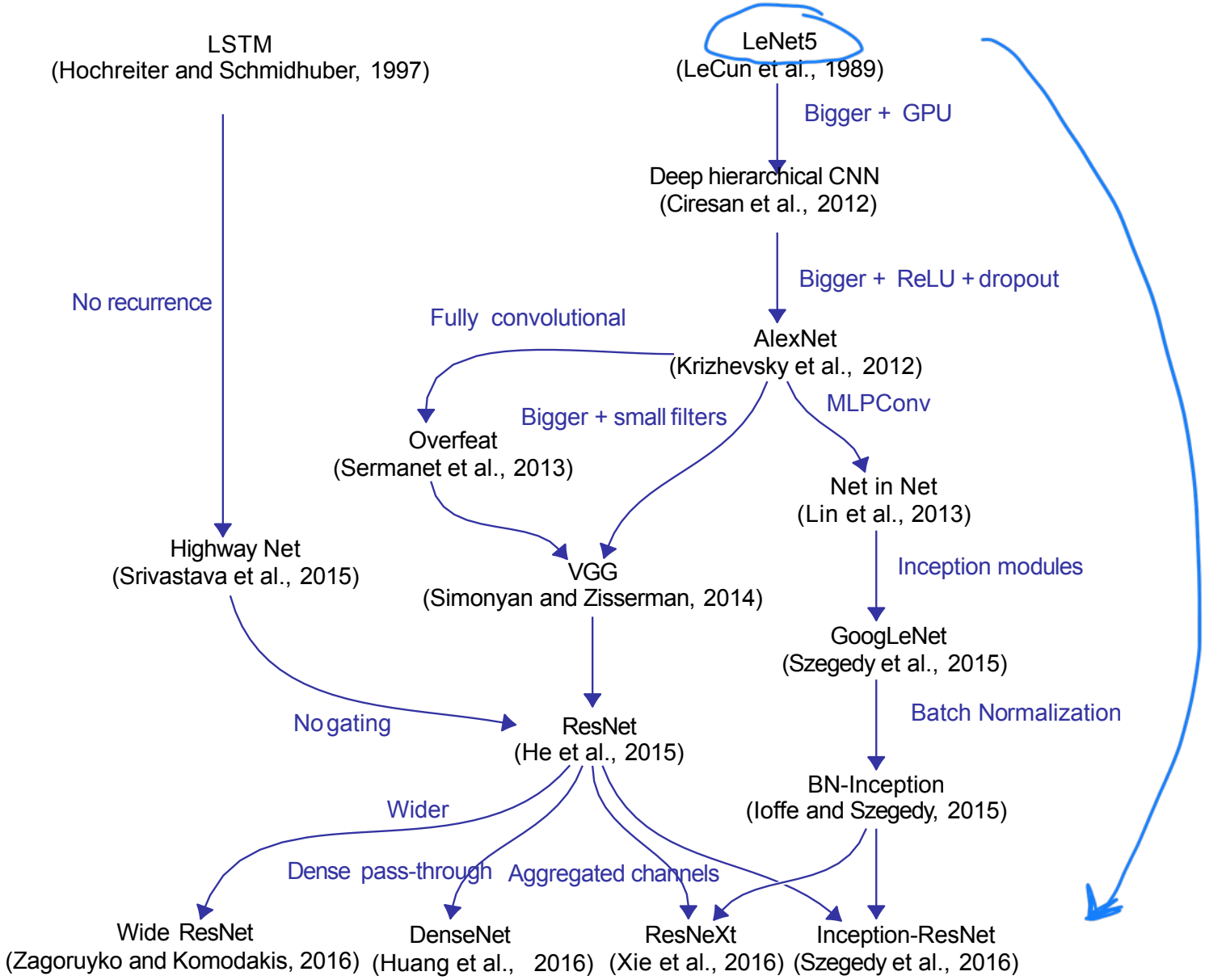
Without Max Pooling



Accuracy	Train	Test
Conv 5x5, stride 1 Max pool 2x3	99.58	98.77
Conv 5x5, stride 2	99.42	98.31
Conv 5x5, stride 1 Conv 3x3, stride 2	99.38	98.57

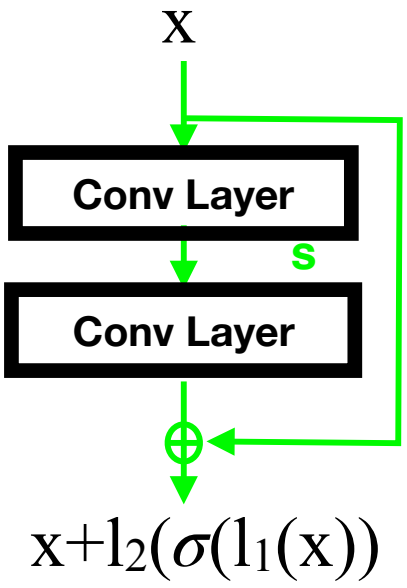
Max pooling can be replaced by Gaussian convolutions with stride > 1 .

Image Classification Taxonomy 1989 —2016

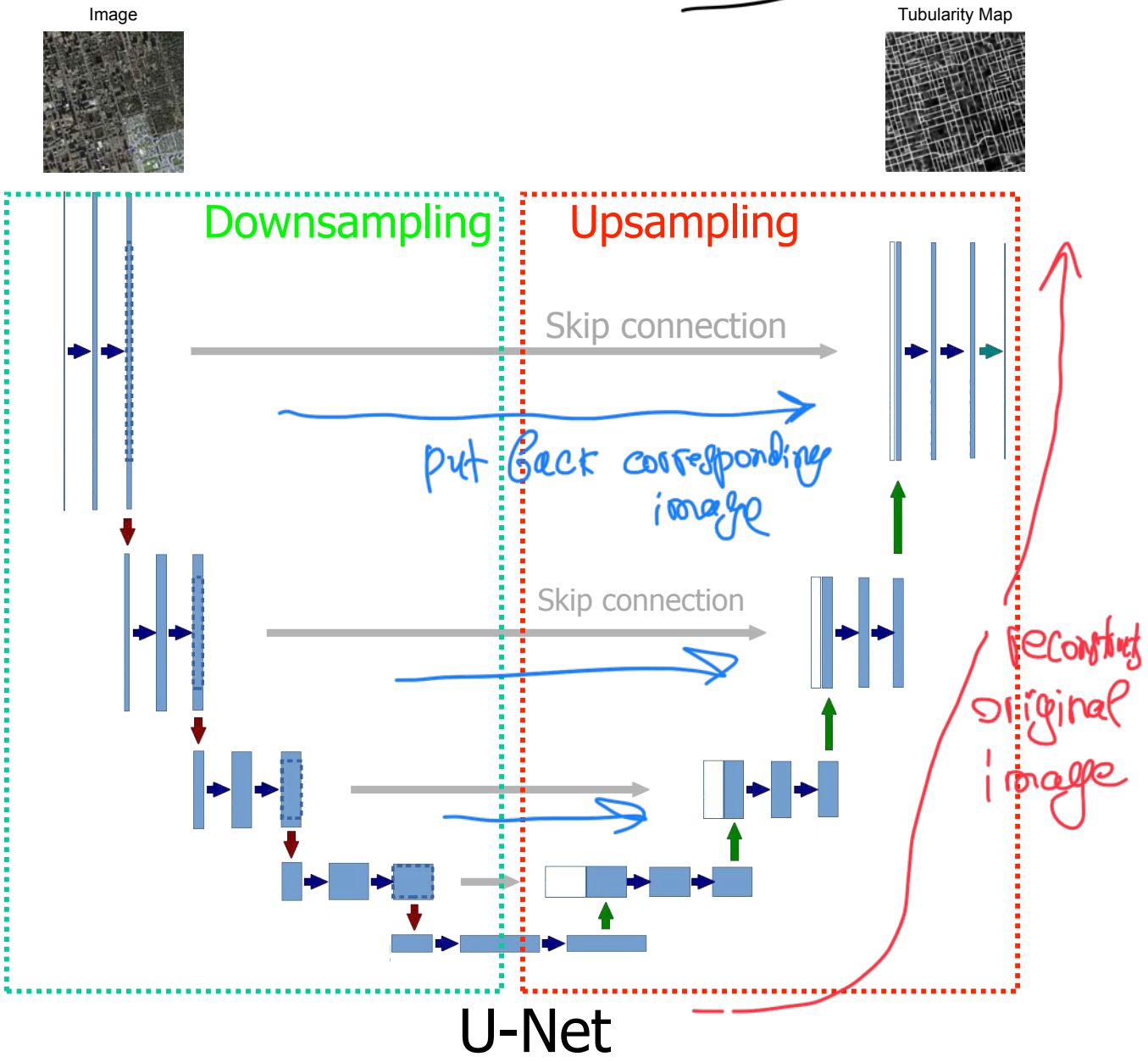


ResNet to U-Net

Image Segmentation



ResNet block

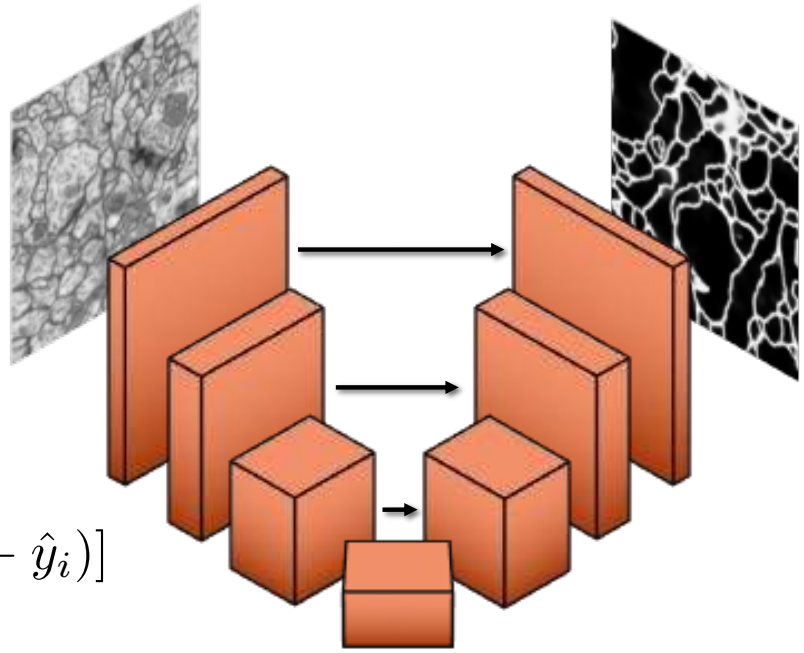


U-Net

—> Add skip connection to produce an output of the same size as the input.

Training a U-Net

Train Encoder-decoder U-Net architecture using binary cross-entropy



Minimize

$$L_{bce}(\mathbf{x}, \mathbf{y}; \mathbf{w}) = -\frac{1}{i}\sum_{1}^{P}[y_{\,\mathrm{i}}\log(\hat{y}_i) + (1-y_i)\log(1-\hat{y}_i)]$$

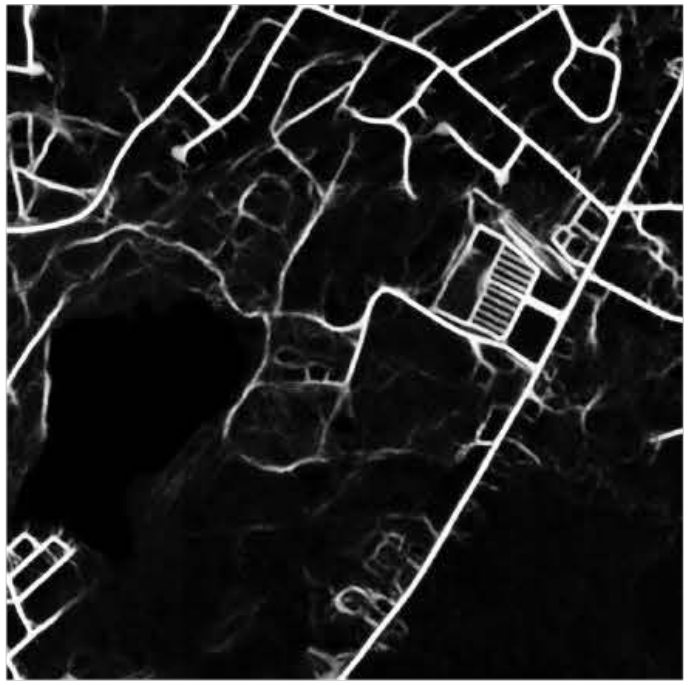
where

- $\hat{\mathbf{y}} = f_{\mathbf{w}}(\mathbf{x})$,
- $\mathbf{x}$ in an input image,
- $\mathbf{y}$ the corresponding ground truth.

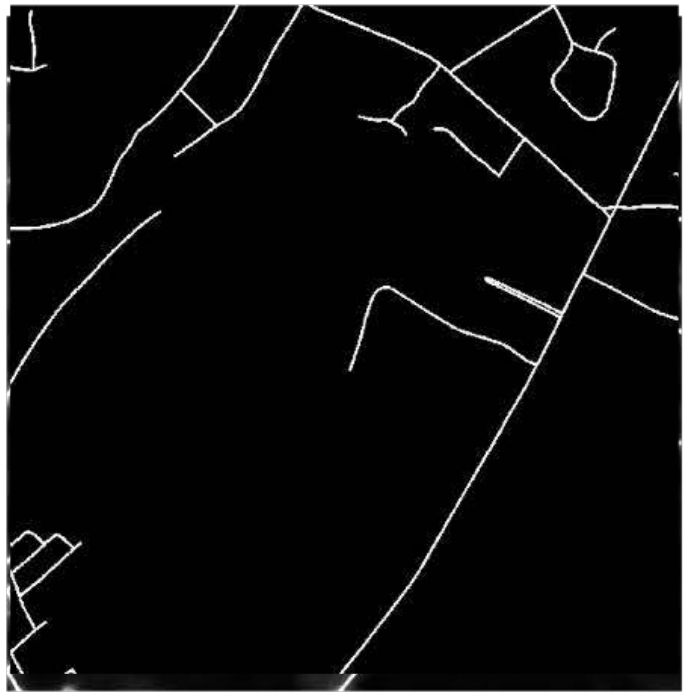
Mosinska et al, CVPR'18.

Network Output

Image



BCE Loss



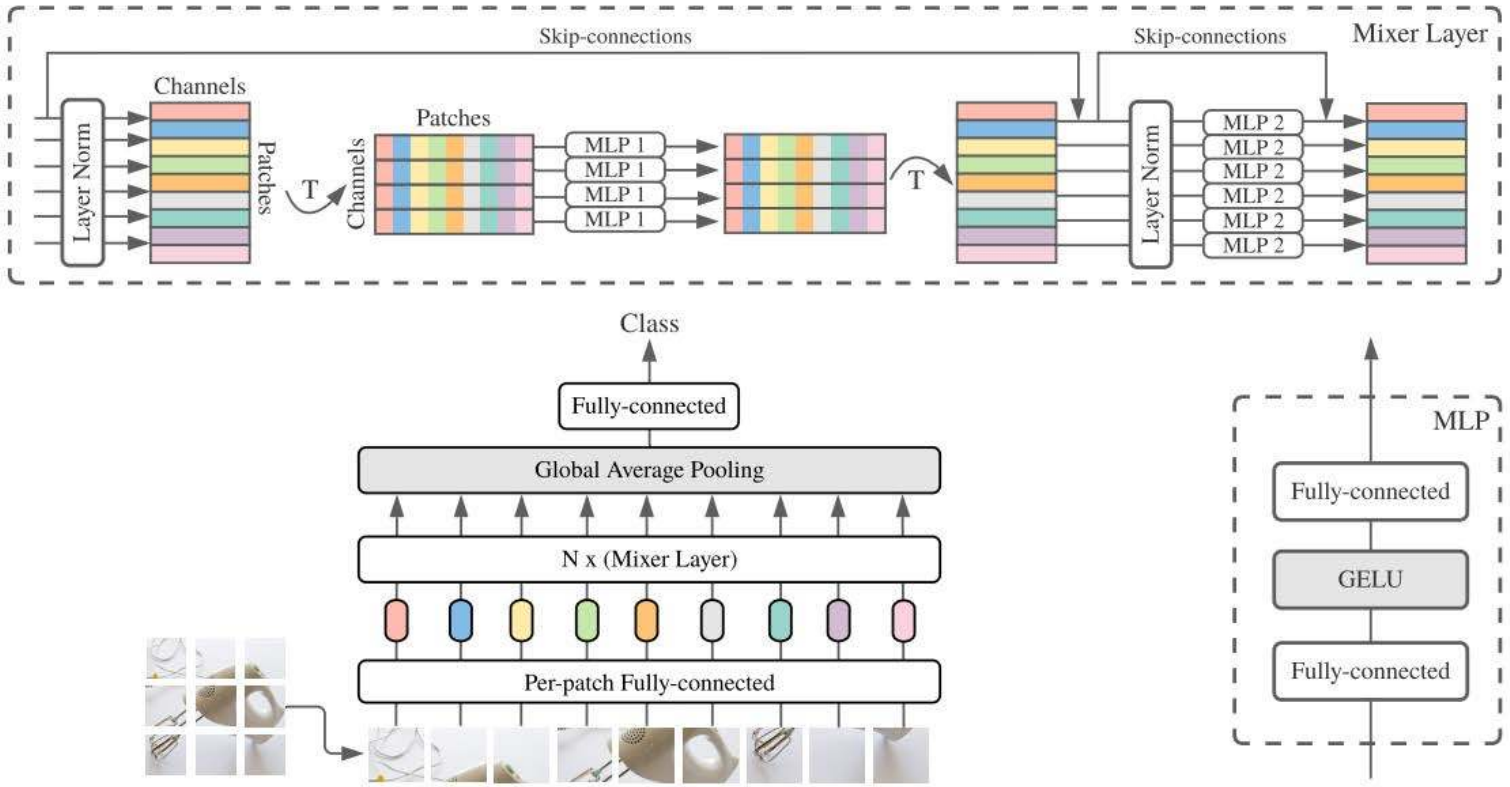
Ground truth

- Good start but not the end of the story.
- We will discuss this again during the delineation lecture.

Streets Of Toronto

— False negatives
— False positives

Transformers

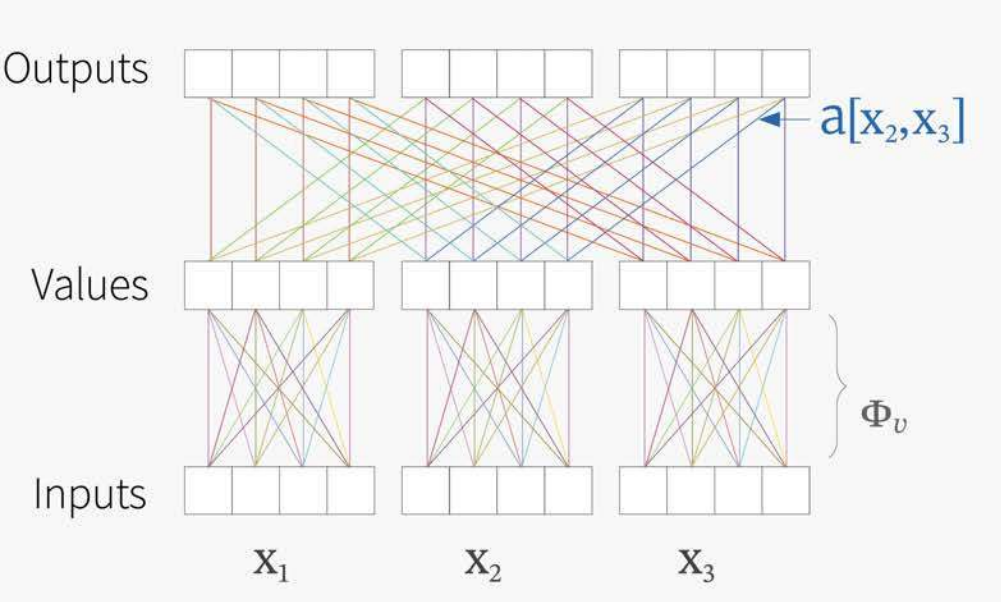


- Break up the images into square patches.
- Transform each path into a feature vector.
- Feed to a transformer architecture.

Tolstikhin et al. , ArXiv'21

Self Attention

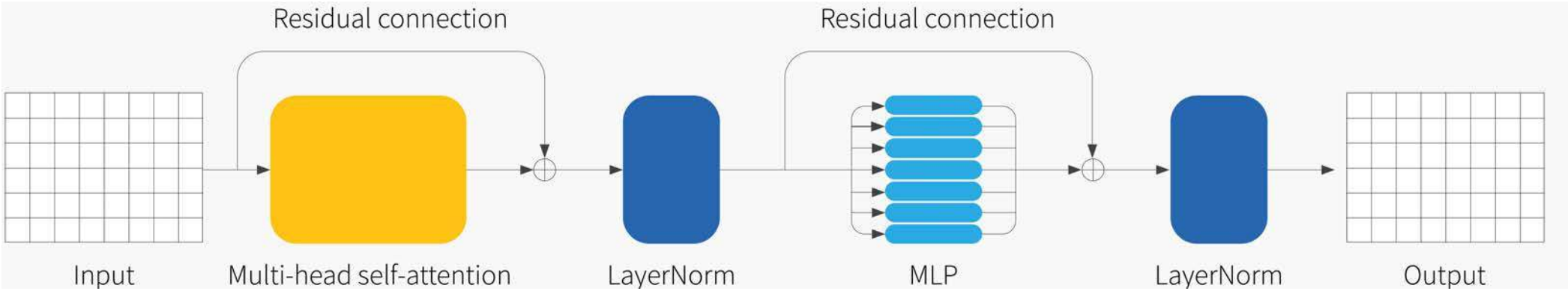
Given $\mathbf{X} = [\mathbf{x_1}, \ldots, \mathbf{x_I}]$:



- $a[\mathbf{x_i}, \mathbf{x_j}]$ is the attention that $\mathbf{x_i}$ gives to $\mathbf{x_j}$. It measures the influence of one on the other.
- It can be computed for all I and j using far fewer weights that in a fully connected layer.

—> Provides context.

Transformer Layer



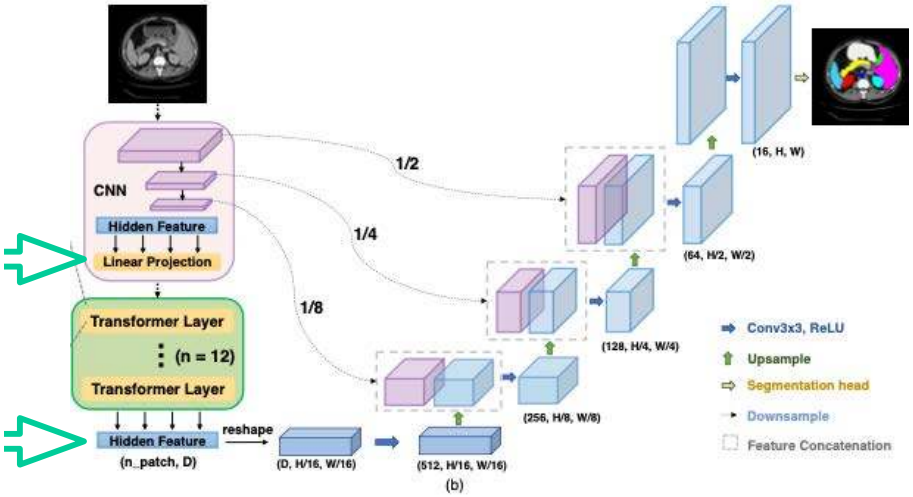
$$\mathbf{X} \leftarrow \mathbf{X} + Sa(\mathbf{X})$$

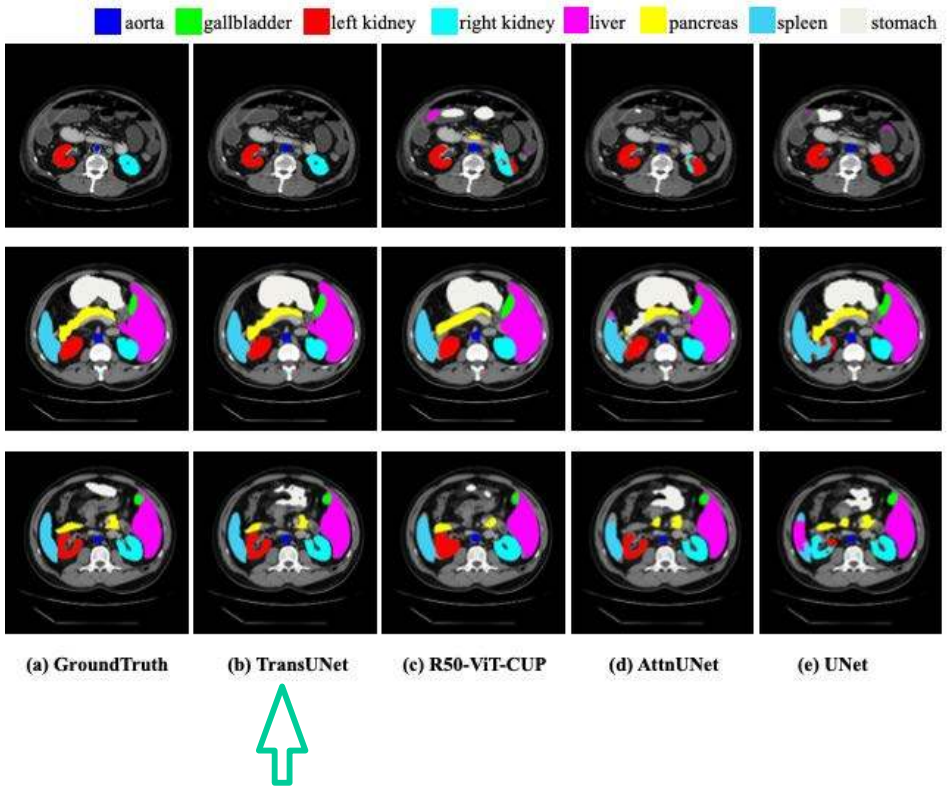
$$\mathbf{X} \leftarrow LayerNorm(\mathbf{X})$$

$$\mathbf{x}_i \leftarrow \mathbf{x}_i + mlp[\mathbf{x}_i] \quad \forall i$$

$$\mathbf{X} \leftarrow LayerNorm(\mathbf{X})$$

U-NET + Transformers

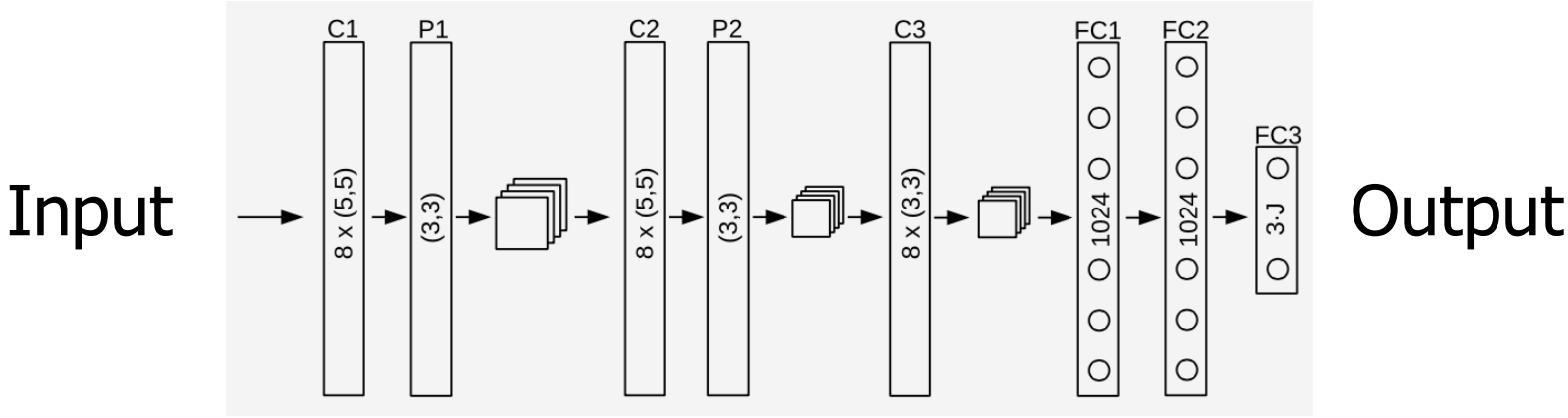




- A CNN operates at low-resolution and produces a feature vector.
- A transform operates on that feature vector.
- The upsampling is similar to that of U-Net

—> Best of both worlds?

Regression

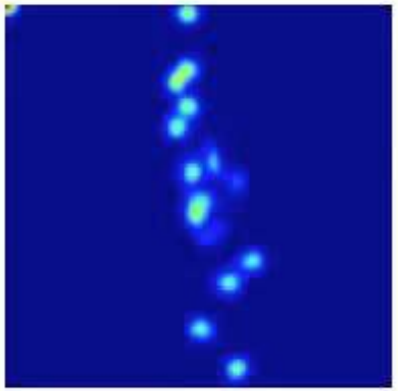


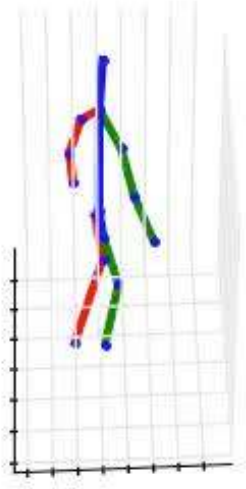
$$\min_{\mathbf{W}_l, \mathbf{B}_l} \sum_i ||\mathbf{F}(\mathbf{x}_i, \mathbf{W}_1, \ldots, \mathbf{W}_L, \mathbf{b}_1, \ldots, \mathbf{b}_L) - \mathbf{y}_i||^2$$

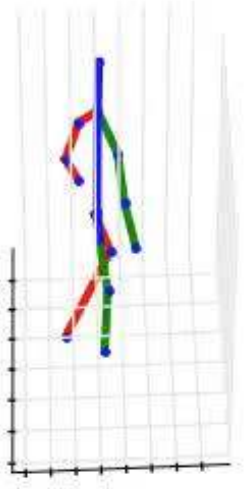
using

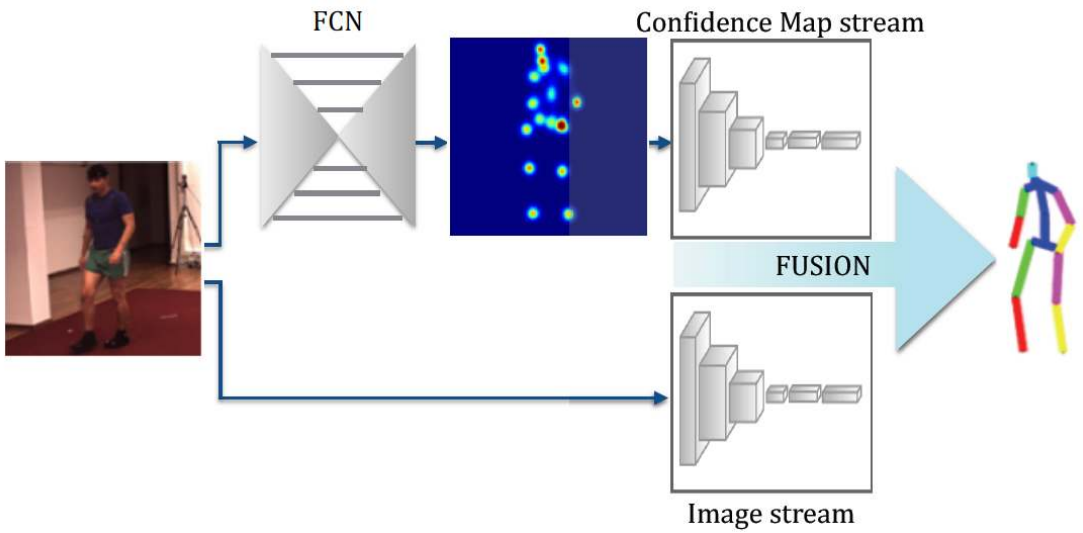
- stochastic gradient descent on mini-batches,
- dropout,
- hard example mining,
- ……

Monocular Pose Estimation



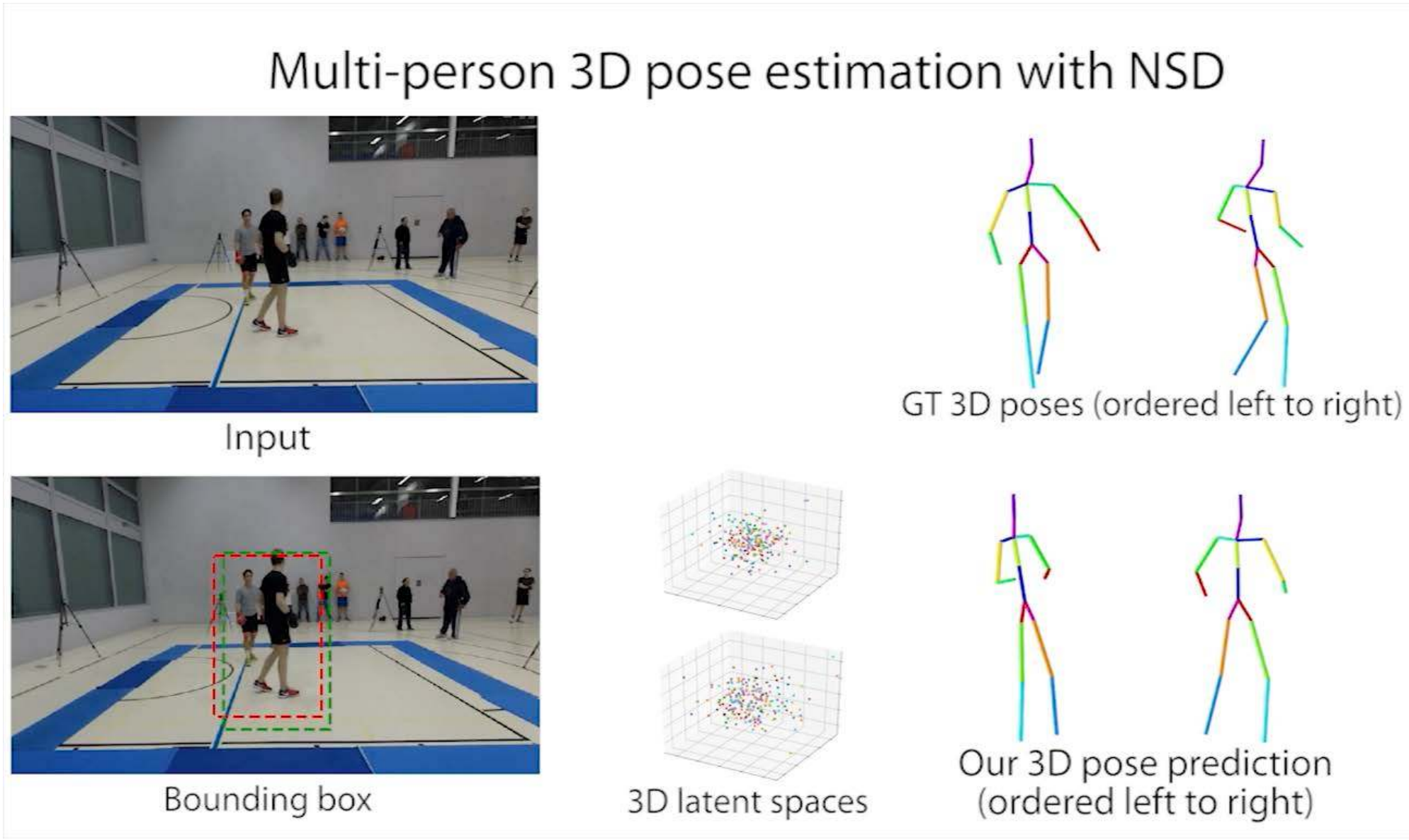






Tekin et al. , ICCV'17

Multiple People



People and their Clothes

Input Ours *(raw)* Ours *(post ref.)*

- Regress the body pose.
- Drape the garments on the body.
- Enforce consistency.

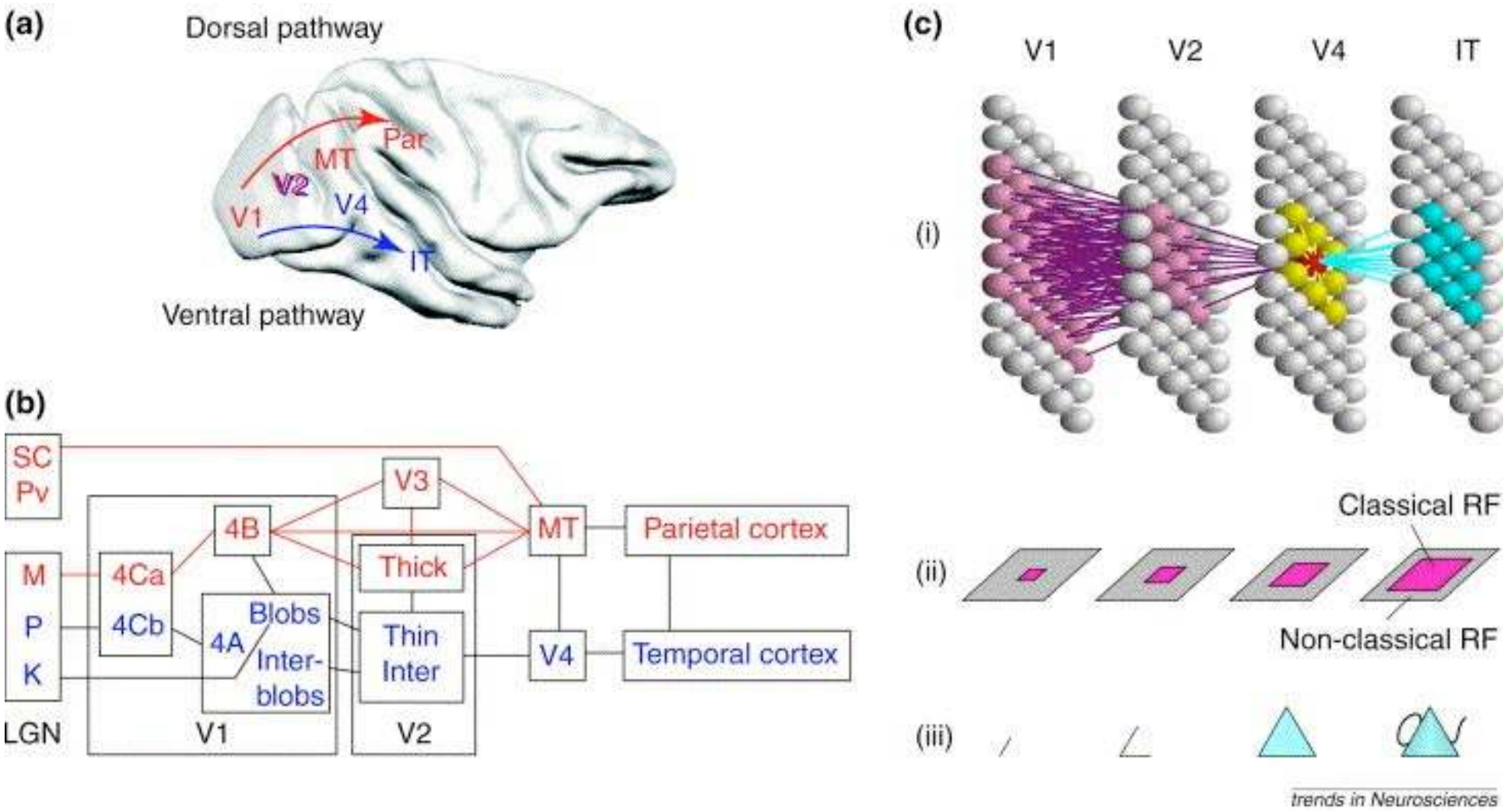
Crowd Counting

EPFL at lunchtime: The colors denote crowd density.

Brains vs Neural Networks

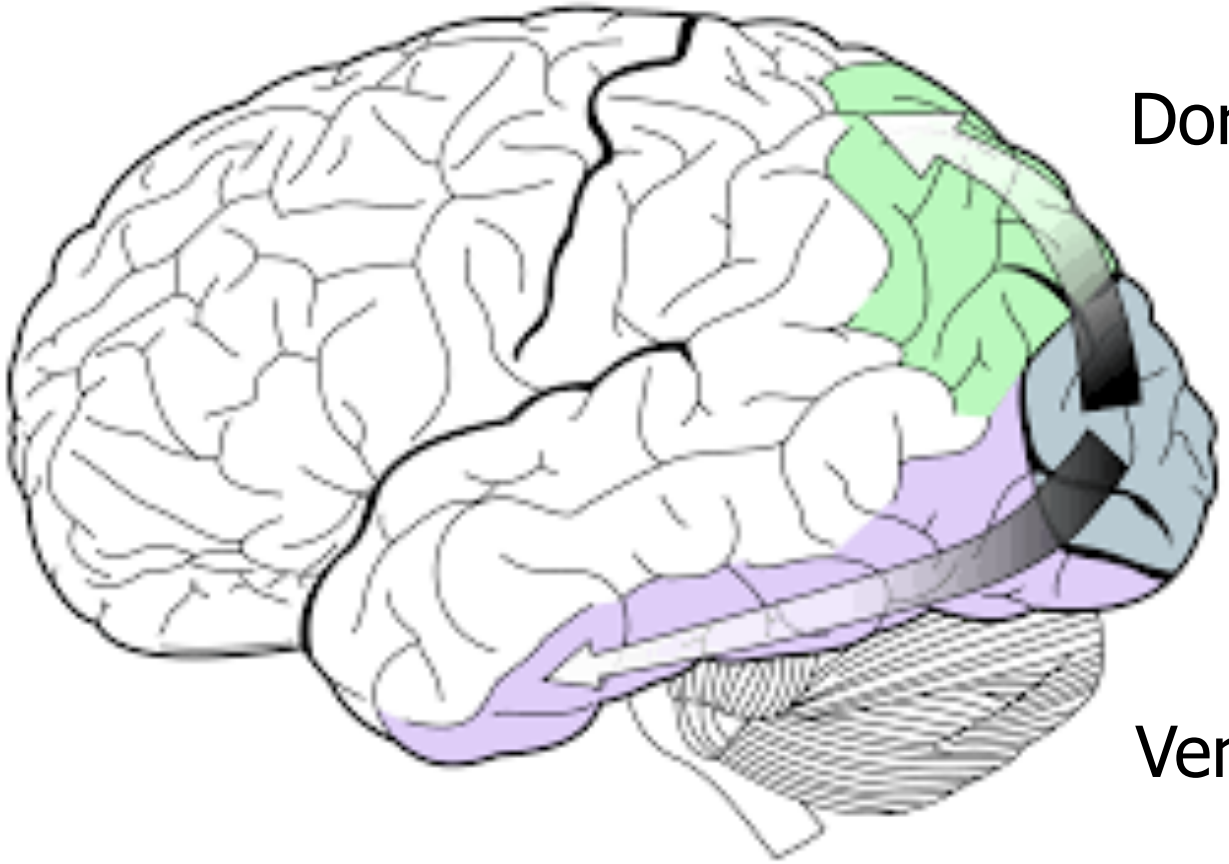
- Neural networks are said to “bio-inspired”.
- An excellent marketing argument but how true is it?

Monkey Cortex



Pink: Feed forward.
Cyan: Feed back.
Yellow: Horizontal

Human Visual Cortex



Dorsal pathway

Ventral pathway

Recognize And Classify: Animal /No Animal

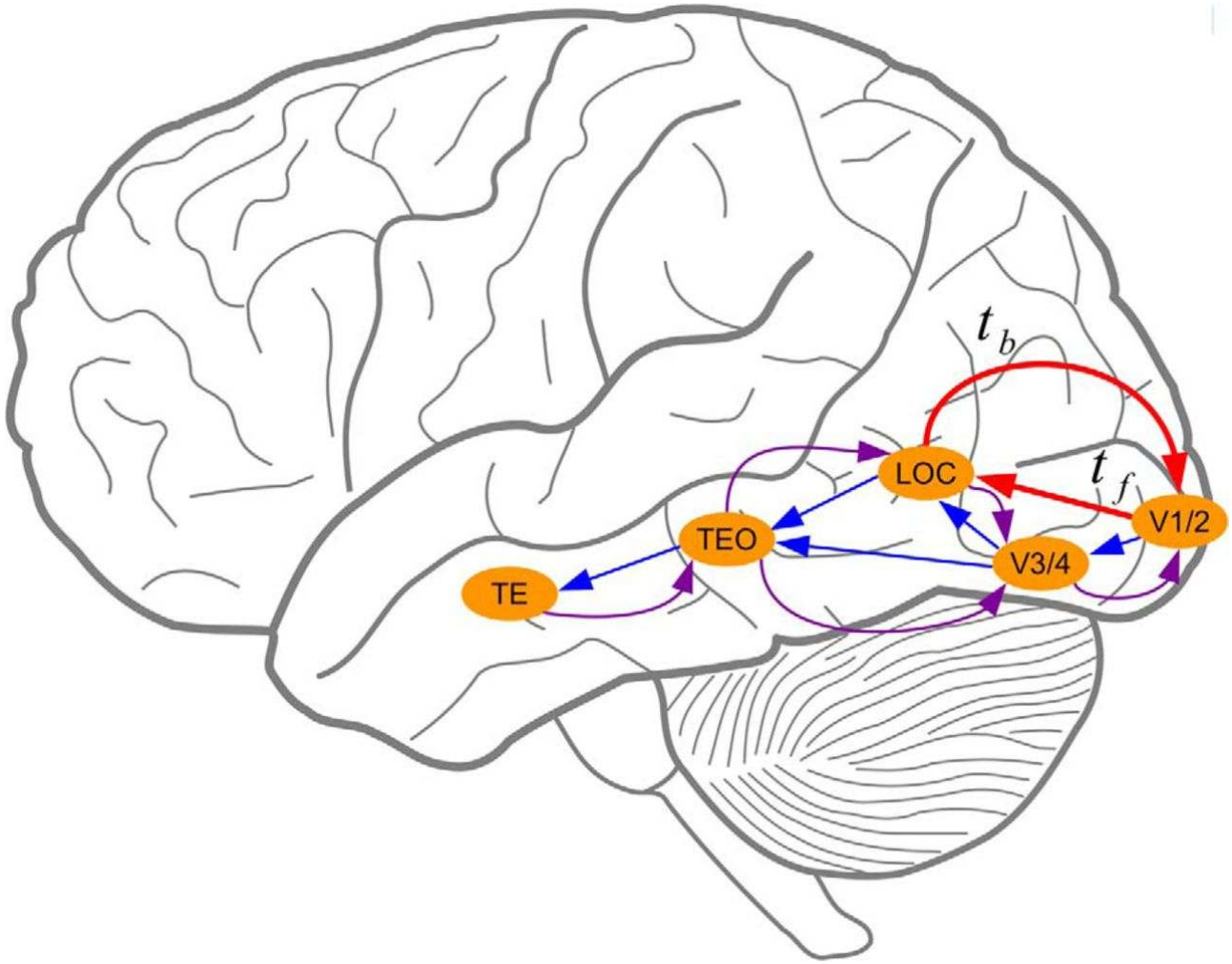
Subjects must raise their hand if they see an animal:

- 60 images
- 1 image per second

→ Measure their reaction time.

Simon Thorpe, *Nature*, 1996

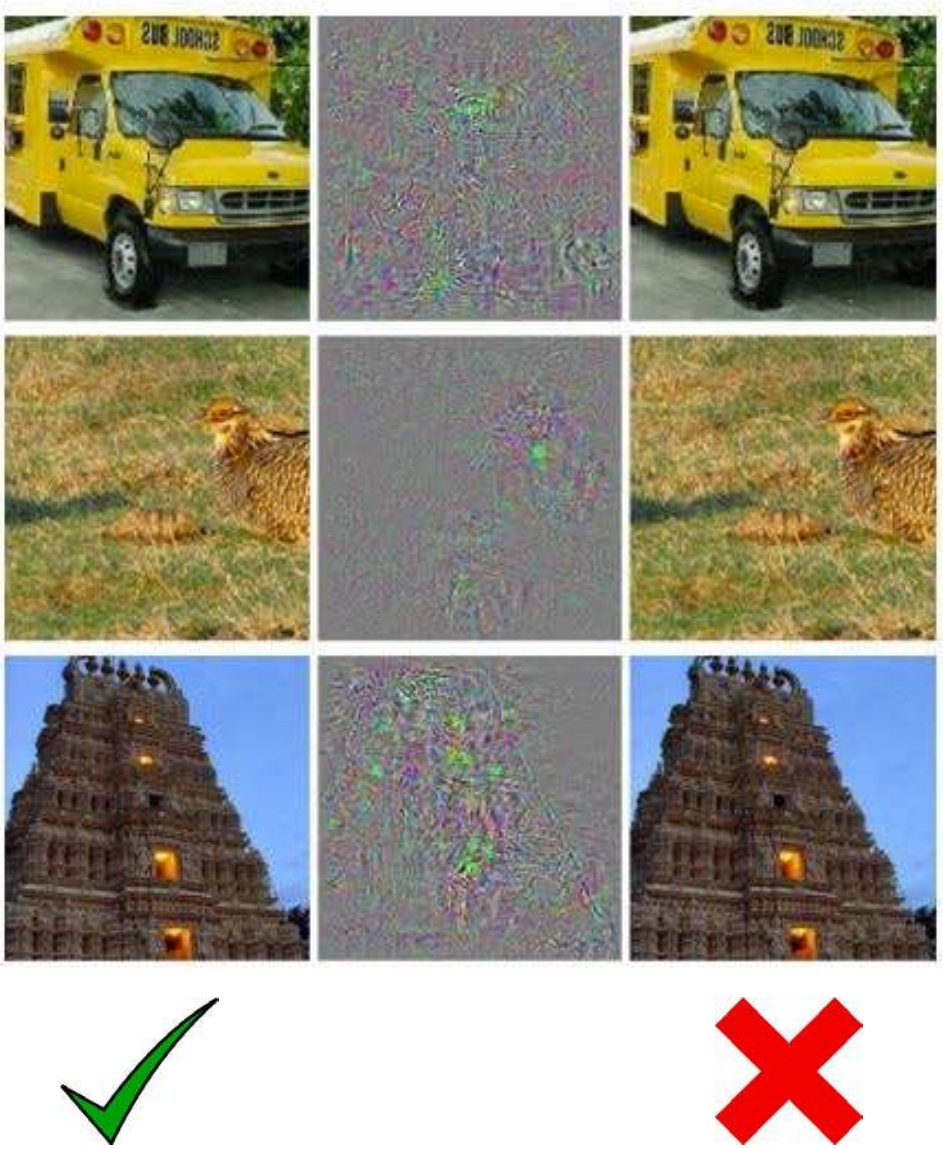
Reminder: Recurrent Pathways



"Shape stimuli are optimally reinforcing each other when separated in time by ~60 ms, suggesting an underlying recurrent circuit with a time constant (feedforward + feedback) of 60 ms."

Drewes et al. , *Journal of Neuroscience*, 2016

Adversarial Images



Szegedy et al. 2013

Brains vs Neural Networks

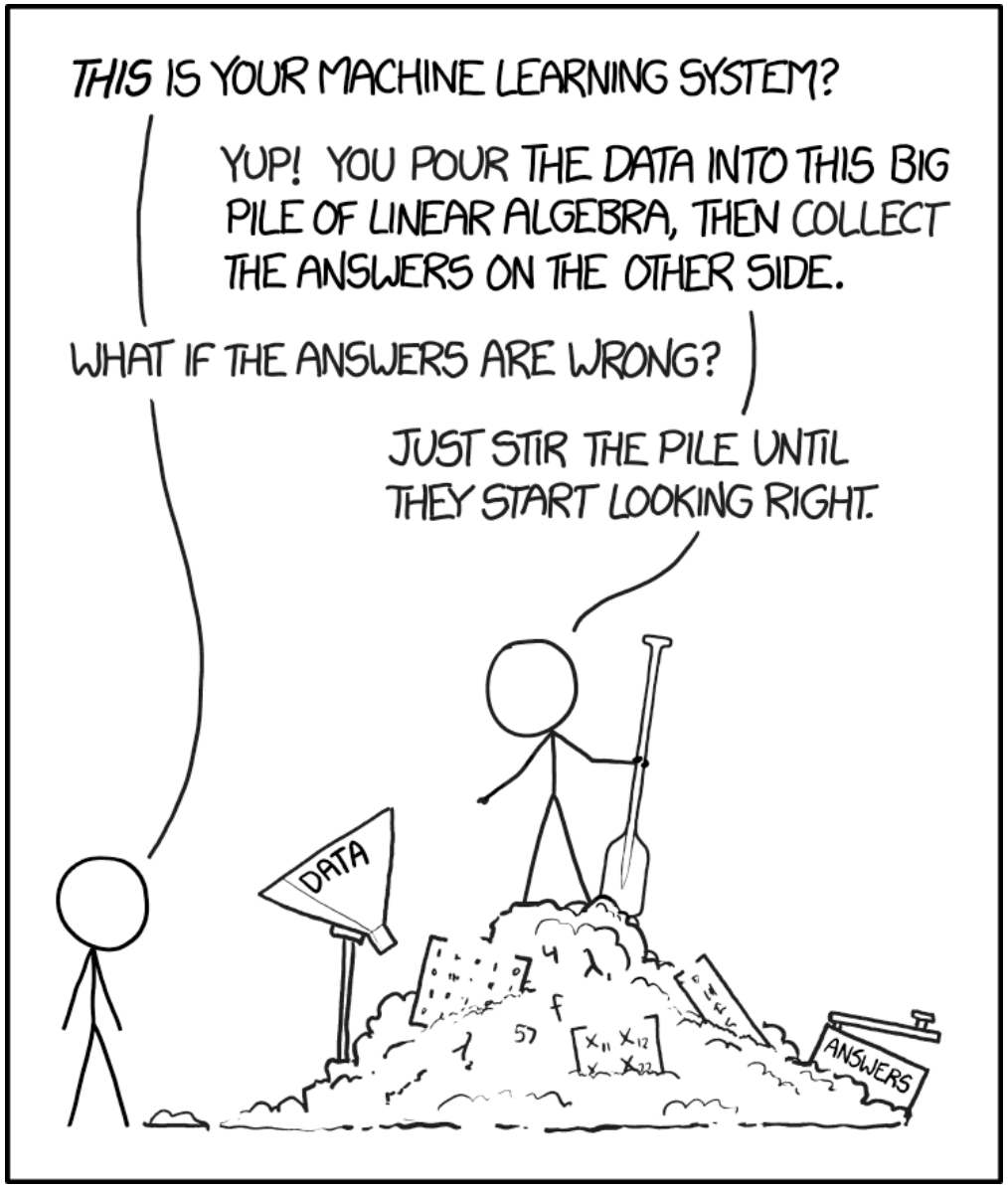
- Neural networks are said to "bio-inspired".
- An excellent marketing argument but how true is it?

Not that good:

- Much feedback is involved in biological systems.
- We don't need large databases to learn.
- We are not as susceptible to adversarial examples.

Neural nets are powerful but not the final answer!

XKCD's View On The Matter



https://xkcd.com/

Deep Nets in Short

- Deep Neural Networks can handle huge training databases.
- When the objective function can be minimized, the results are outstanding.
- There are failure cases and performance is hard to predict.

—> Many questions are still open and there is much theoretical work left to do.

Alpha Go

- Uses Deep Nets to find the most promising locations to focus on. (where to play)
- Performs Tree based search when possible. (what to play)
- Relies on reinforcement learning and other ML techniques to train.

—> Beat the world champion in 2017.

Combination of DL methods and exhaustion techniques