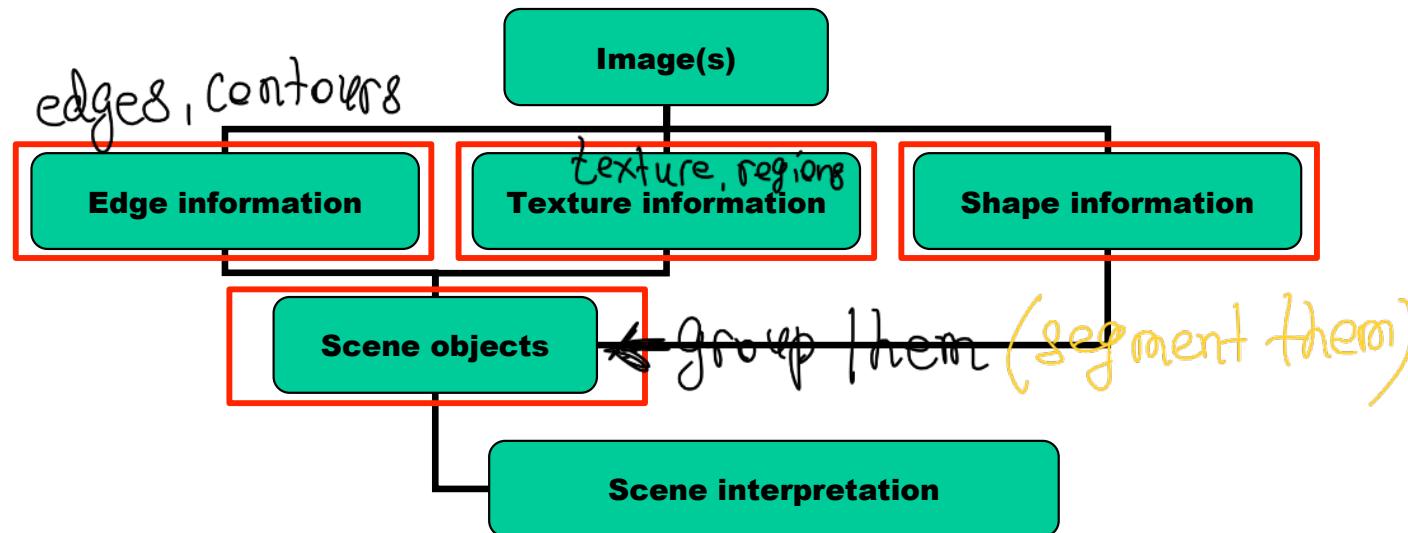


Reminder: A Teachable Scheme



Decomposition of the vision process into smaller manageable and implementable steps.

- > Paradigm followed in this course
- > May not be the one humans use

Shape From X

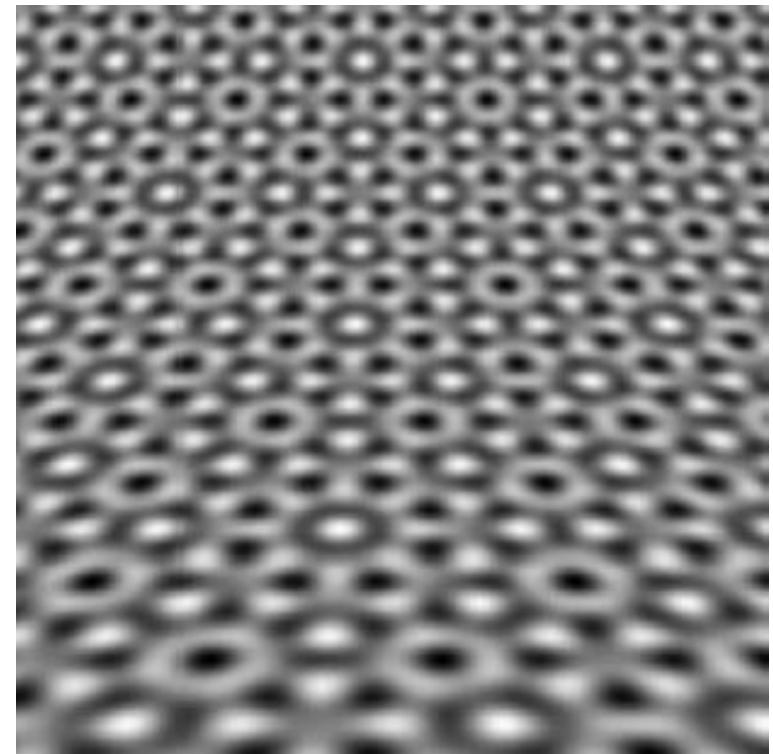
↳ can be many things

- One image:
 - **Shading**
 - Texture
- Two images or more:
 - Stereo
 - Contours
 - Motion



Shape From X

- One image:
 - Shading
 - **Texture**
- Two images or more:
 - Stereo
 - Contours
 - Motion



tells us sth about
shape

Shape From X

- One image:
 - Shading
 - Texture
- Two images or more:
 - **Stereo**
 - Contours
 - Motion



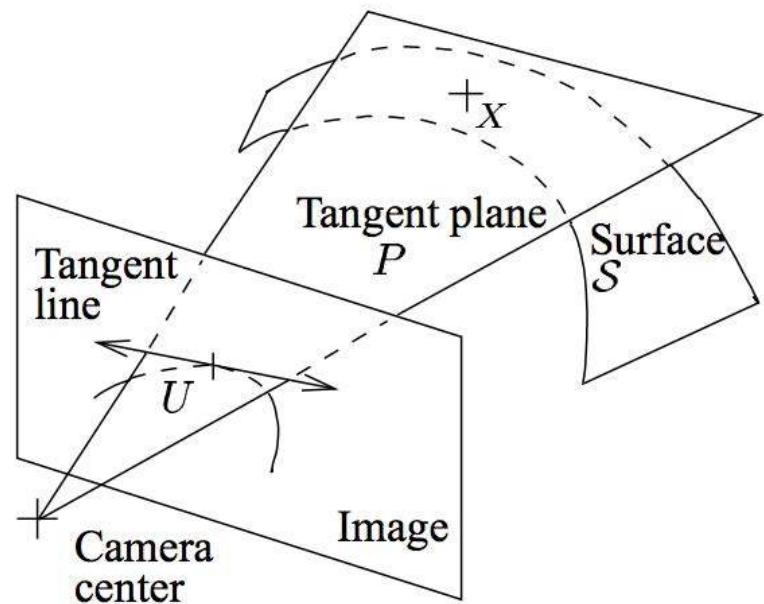
recover depth
(e.g. ramer img)

video sequences



Shape From X

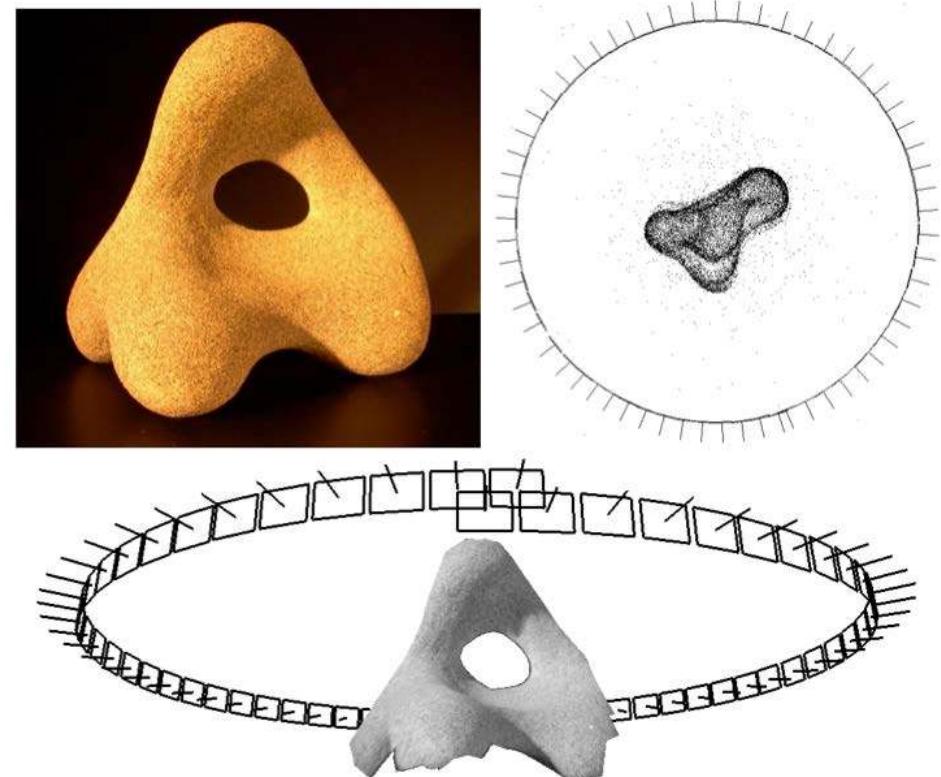
- One image:
 - Shading
 - Texture
- Two images or more:
 - Stereo
 - **Contours**
 - Motion



Shape From X

- One image:
 - Shading
 - Texture
- Two images or more:
 - Stereo
 - Contours
 - **Motion**

↙ whole video
of sequence



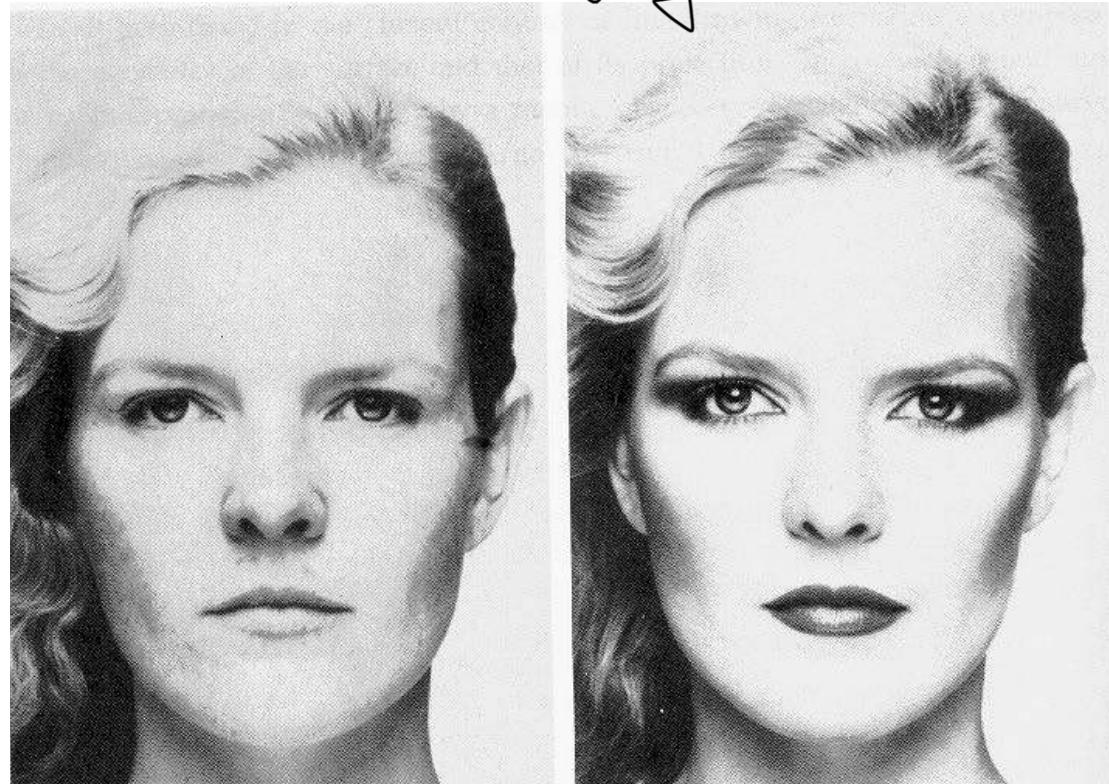
Shading

illumination

- Shading models
- Shape from shading
 - Variational Methods
 - Photometric Stereo

Shape-from-shading

No surgery

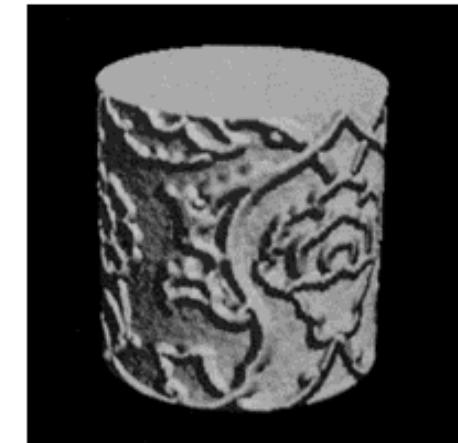
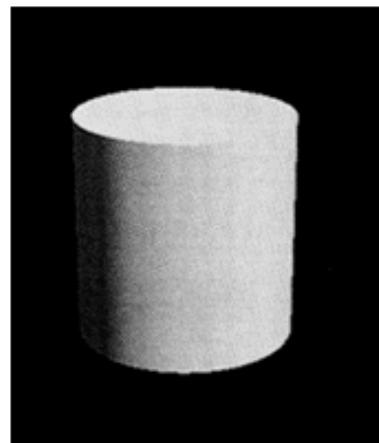
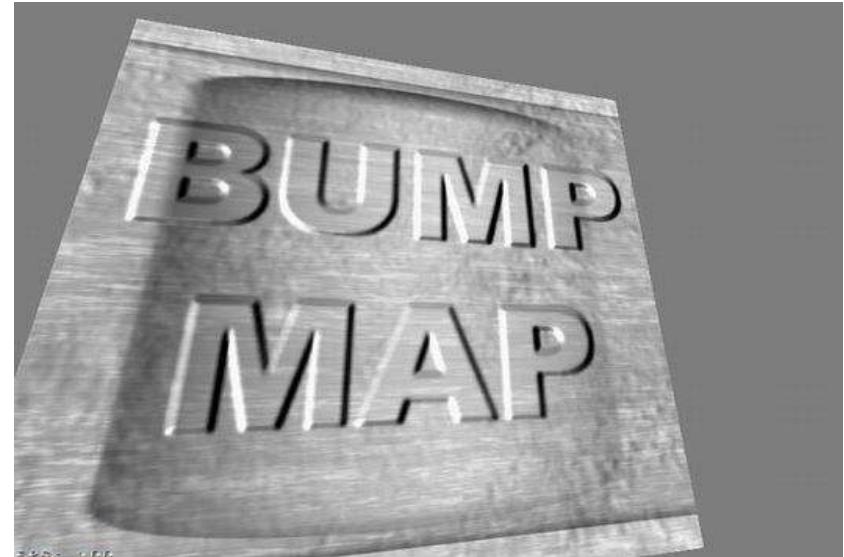
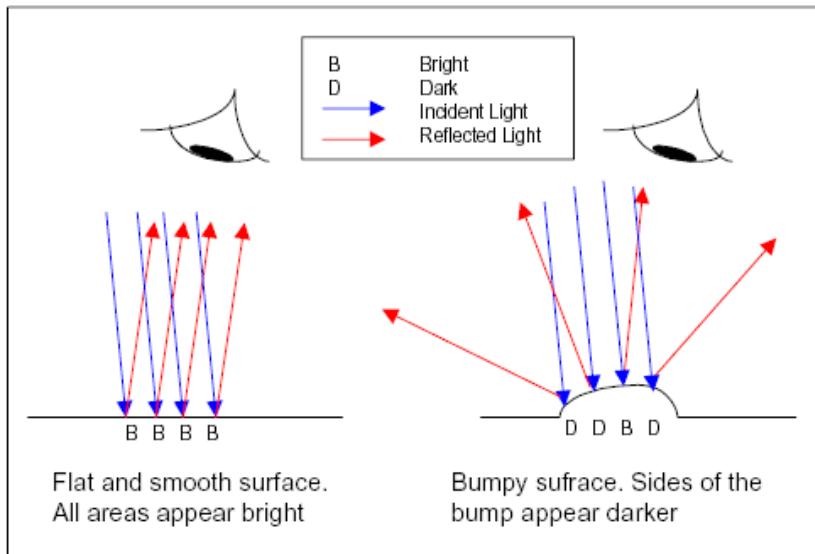


no makeup

makeup ✓

↳ change gray levels
of the image

Bump Mapping

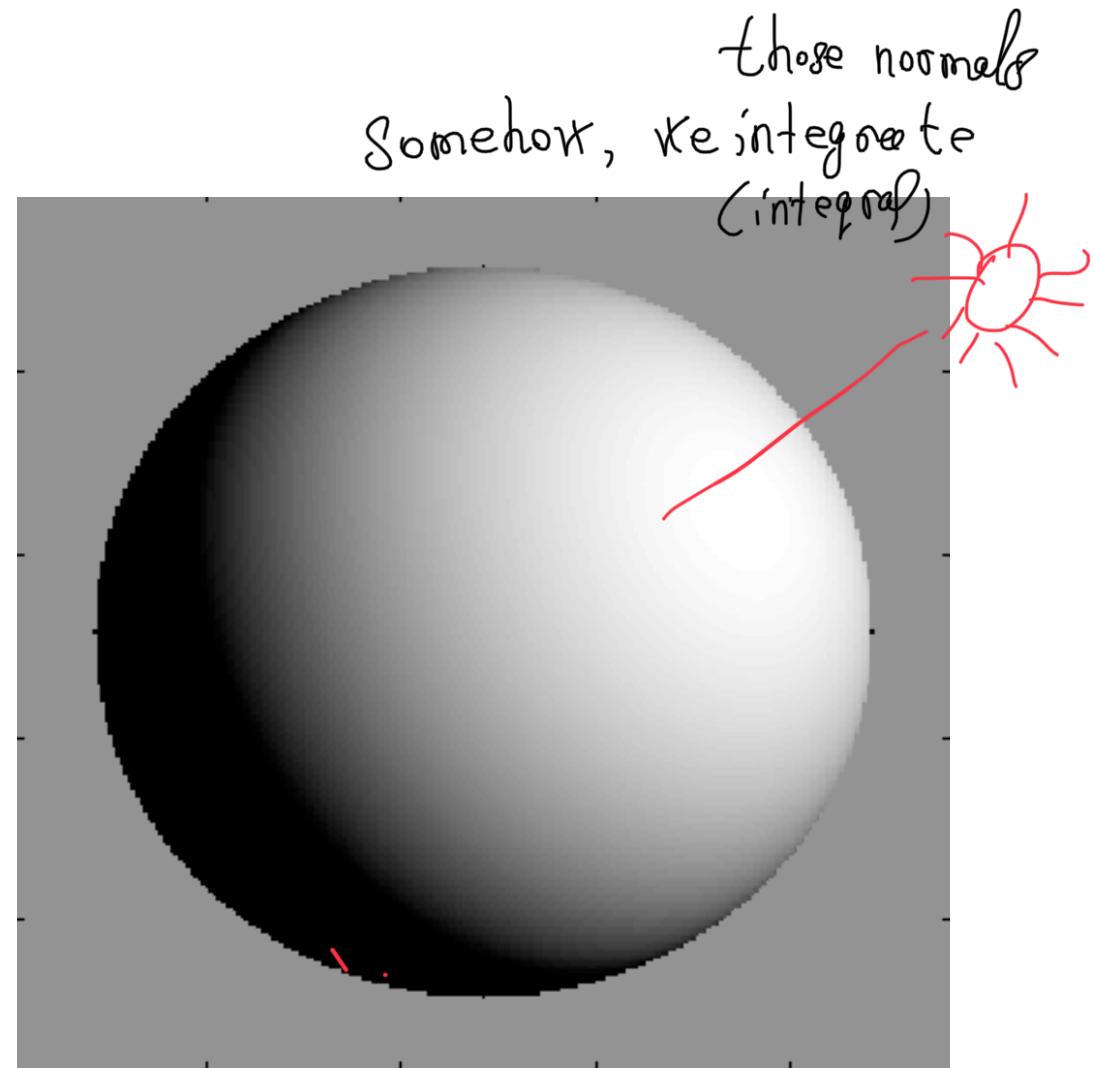
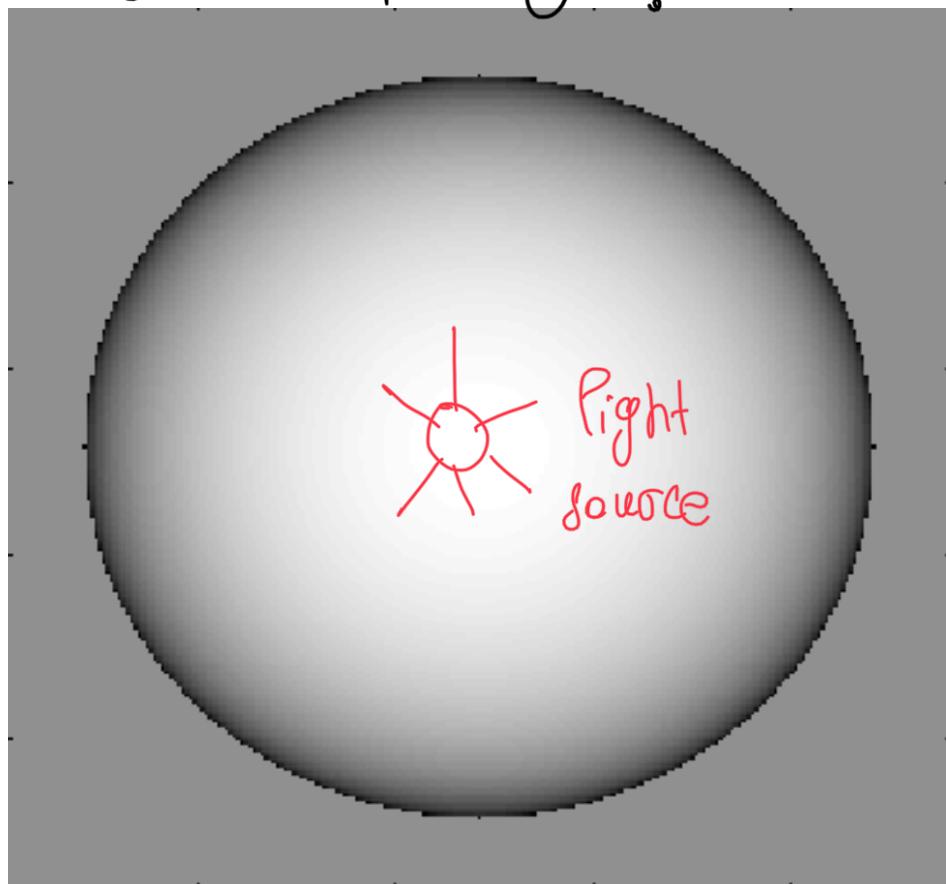


visually
looks 3D

Simple mesh + 2D bump map = Complex looking object

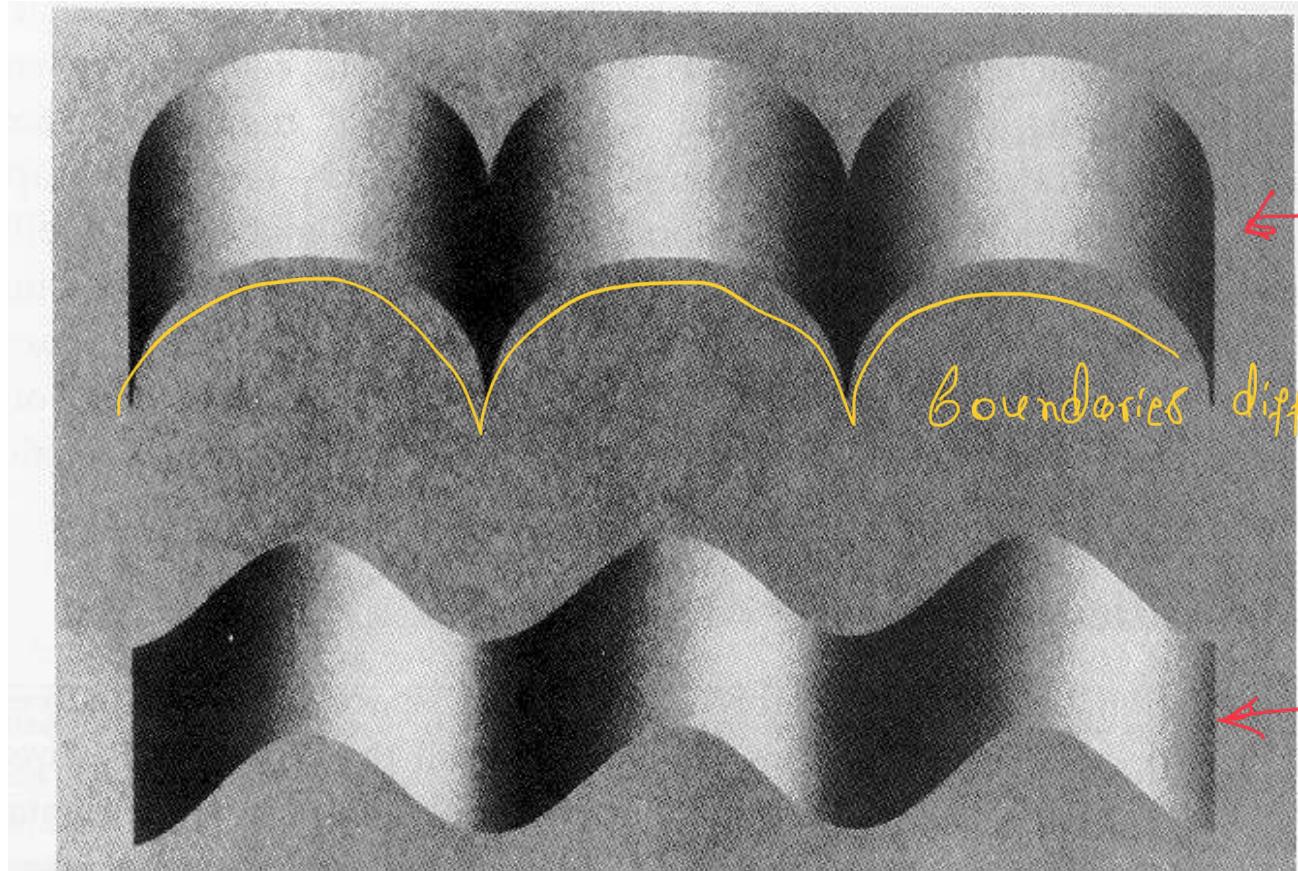
paint on the cup
bump map ↳ appropriate patterns of light and gray

Lambertian Half-Sphere



Gray level changes are interpreted as changes in the direction of the surface normal.

Solving an Inverse Problem



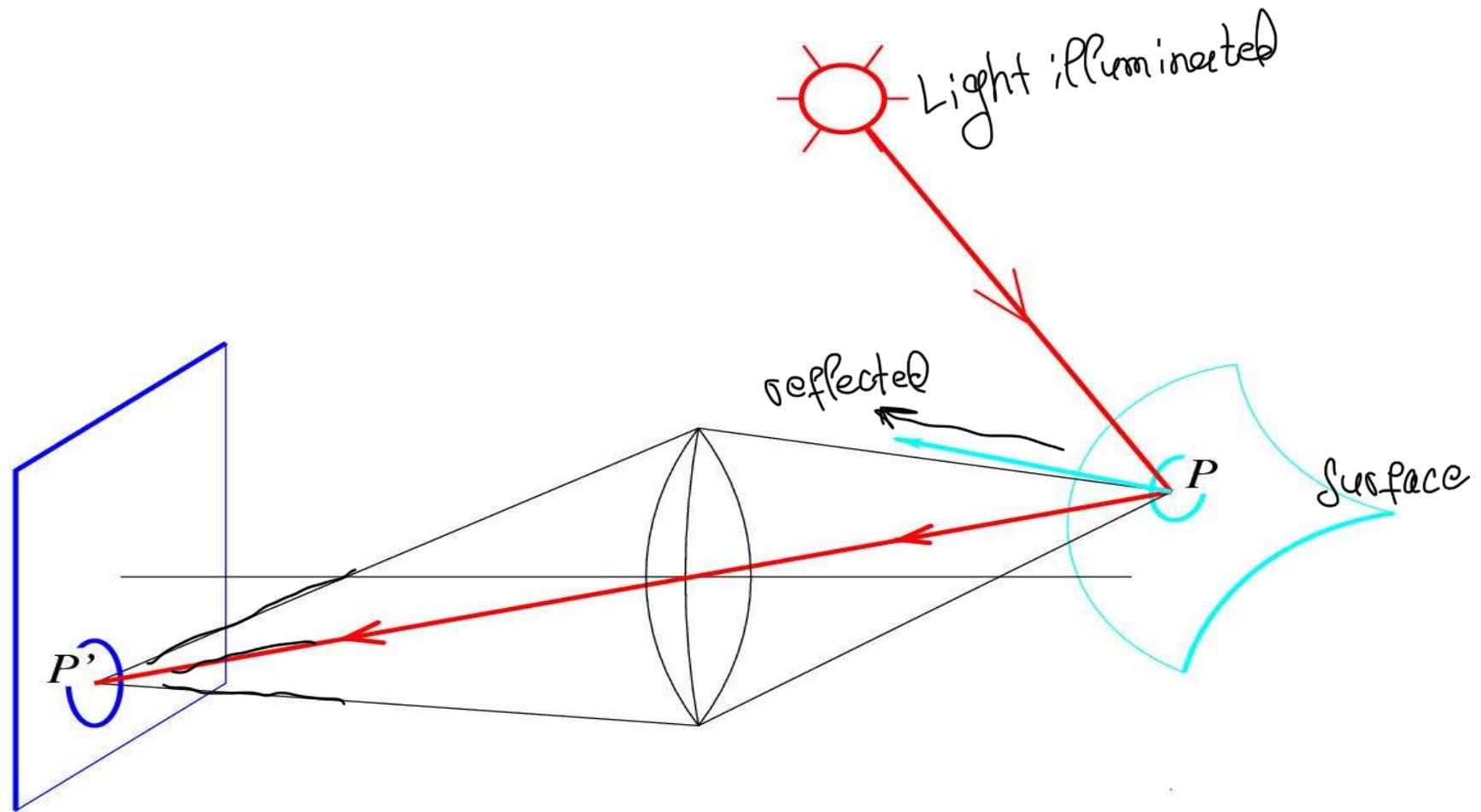
- Shading gives information about surface normals.
- Recovering the 3D surfaces amounts to solving a differential equation.
 - We use them to integrate those "ribbon" images
 - Boundary conditions are required to do so.

Boundary Conditions



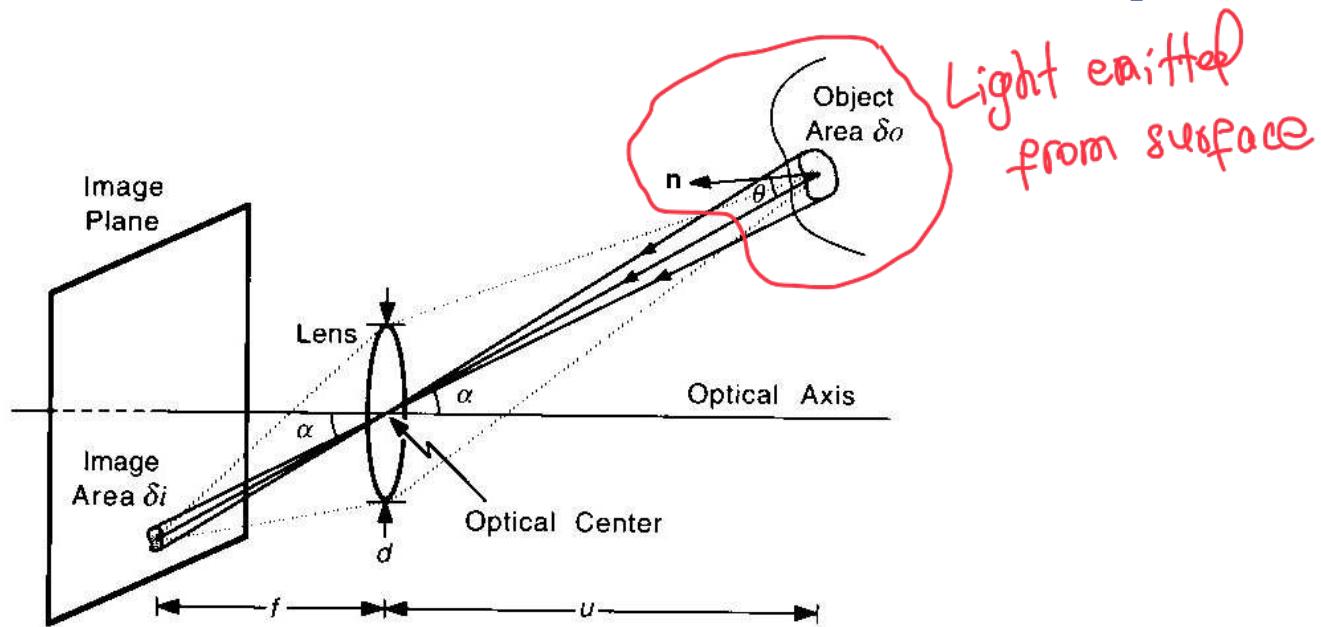
—> The carefully designed contour gives us an erroneous perception!

Reminder: Image Formation



- The light source illuminates a 3D surface.
- The 3D surface reemits some the light.
- It goes through a lens and forms an image on the image plane.

Reminder: Fundamental Radiometric Equation



Scene Radiance (Rad): Amount of light radiation emitted from a surface point (Watt / m² / Steradian)

Image Irradiance (Irr): Amount of light incident at the image of the surface point. (Watt / m²)

$$\text{Irr} = \frac{\pi}{4} \left(\frac{d}{f}\right)^2 \cos^4(\alpha) \text{Rad},$$

Radiometric eq'n

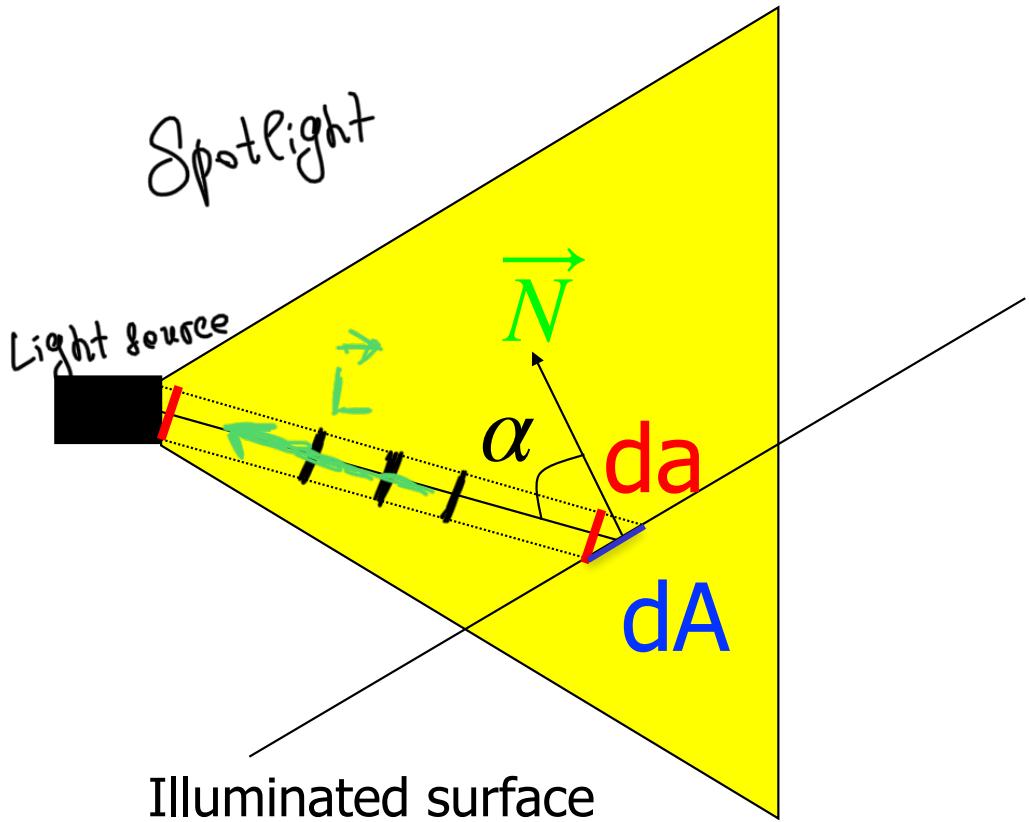
$$\Rightarrow I \propto \text{Rad},$$

Image intensity

when the camera is photometrically calibrated.

*can be eliminated by having a lens that's not strictly transparent
(darker in the middle) to compensate for that*

Lambertian Shading Model



Foreshortening:

$$da = \cos(\alpha) dA$$

$$\cos(\alpha) = \vec{L} \cdot \vec{N}$$

$d\alpha < dA$

\vec{L} direction of Light Source

Radiance:

$$\text{Rad} \propto \text{albedo } P/dA$$

$$P \propto da$$

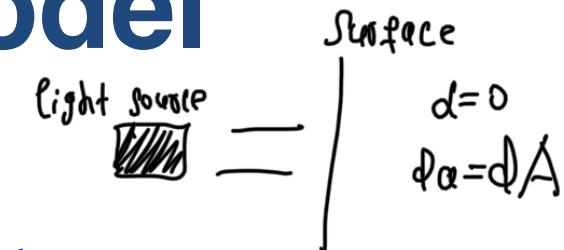
$$\Rightarrow \text{Rad} \propto \text{albedo } \underbrace{\vec{L} \cdot \vec{N}}_{\cos \alpha \text{ on } dA}$$

- The amount of light radiation P in the cylinder of section da in direction \vec{L} is spread over the surface area dA .

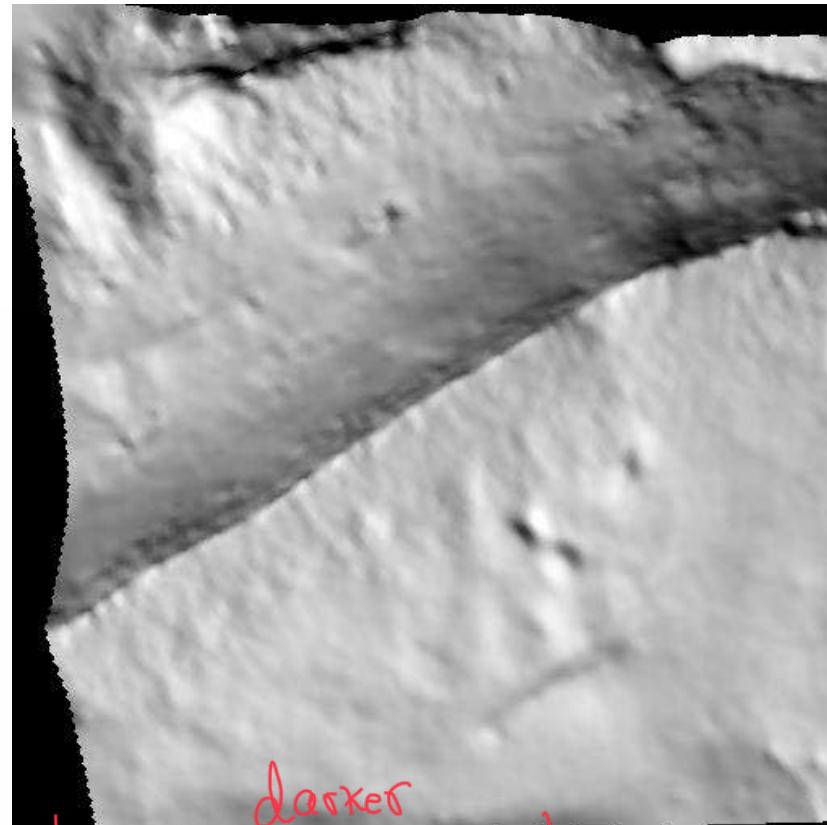
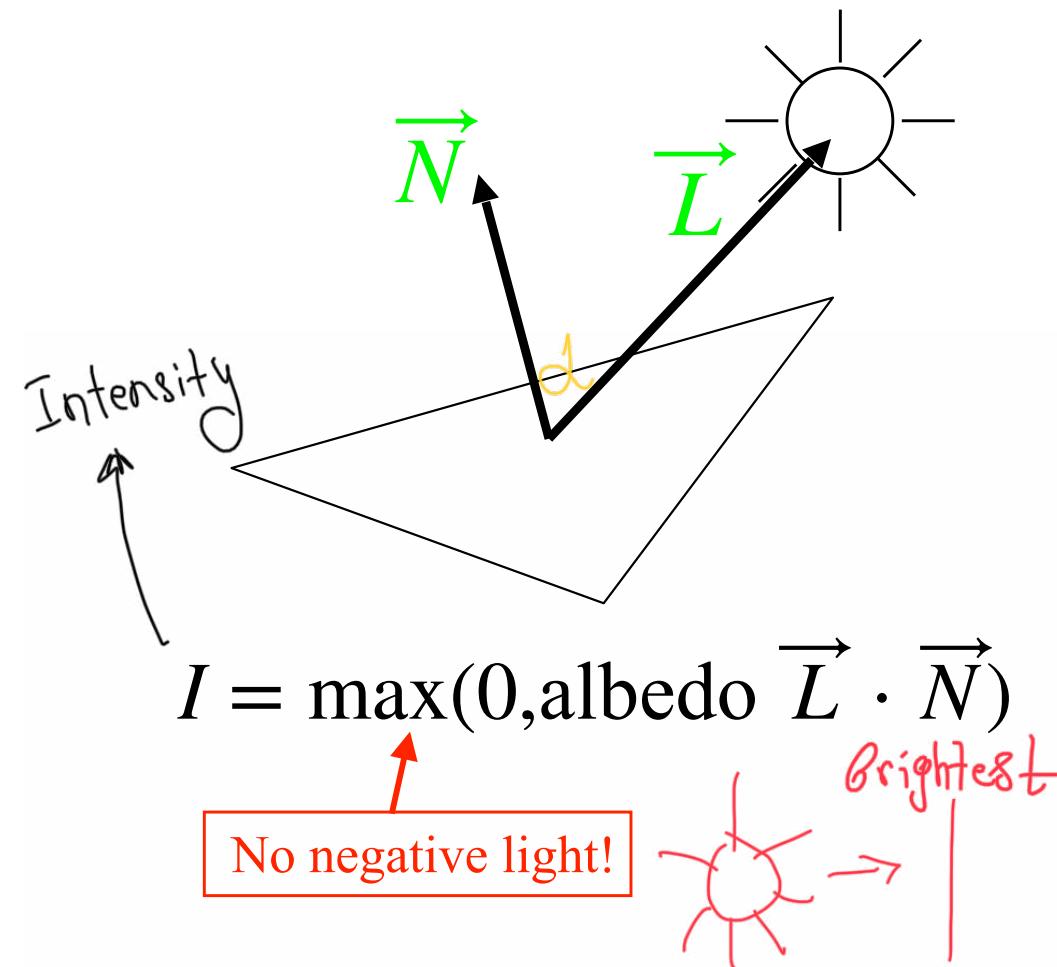
When you shine light

- Some light is absorbed ($0 < \text{albedo} < 1$).

EPFL on surface, gets reflected



Ideal Lambertian Surface

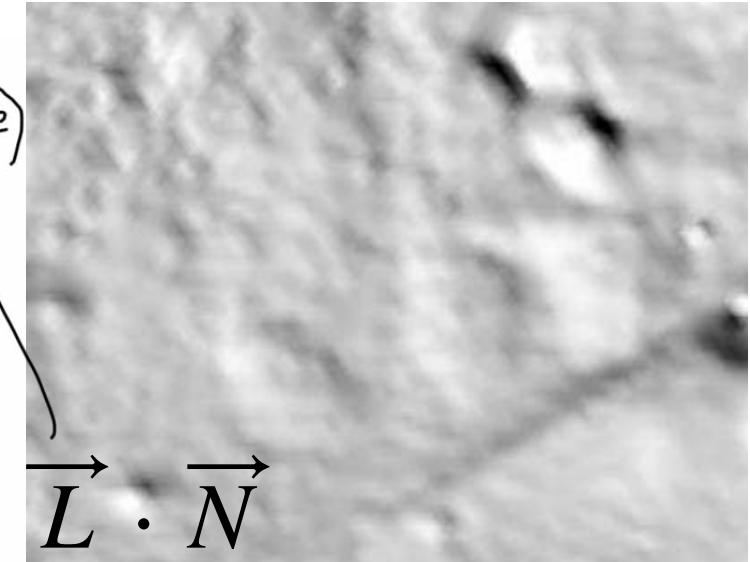


Perfectly matte surface: The radiance depends only on angle of incidence and not on viewing direction. This is known as **diffuse** reflection.

Estimated Albedo



sun is
(we know where)
Right
source



=



If ideal
Lambertian

Original image.

$$\underbrace{I}_{\text{Original image}} / \left(\overrightarrow{L} \cdot \overrightarrow{N} \right) \xrightarrow{\substack{\text{of terrain} \\ \text{+ shape but} \\ \text{physical properties}}}$$

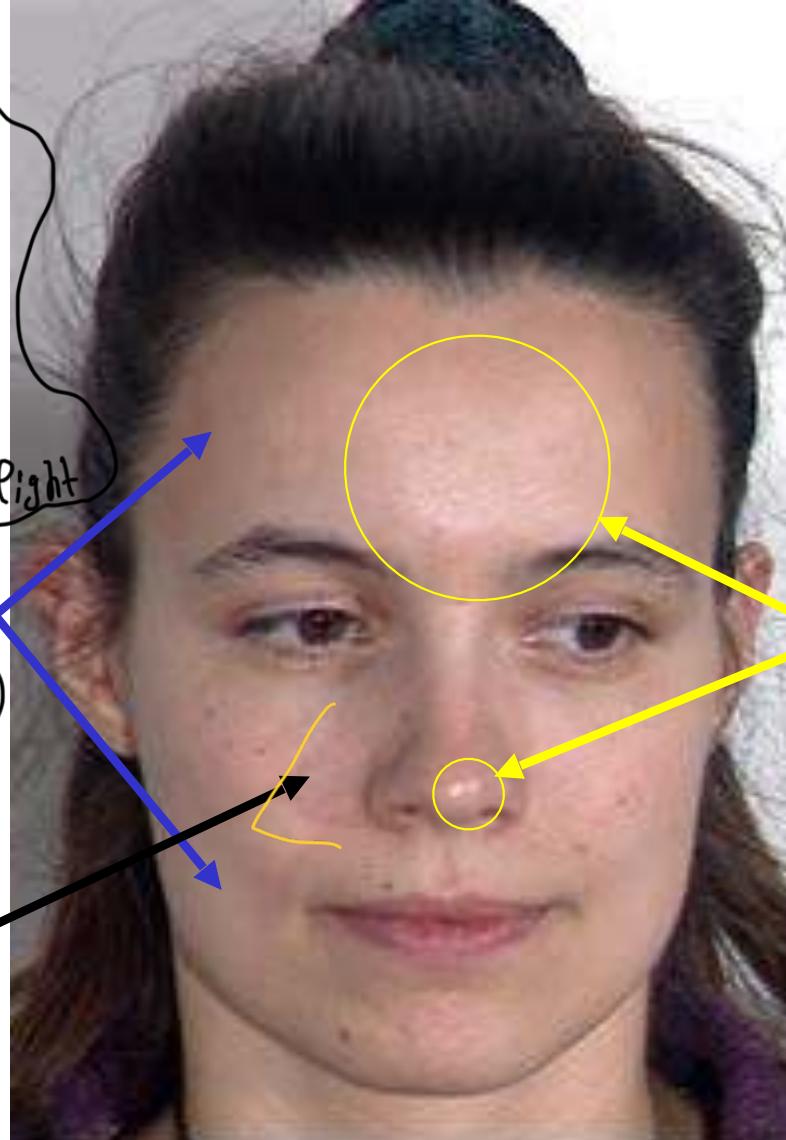
→ The “albedo” image looks much flatter than the original one.

Diffuse vs Specular

albedo {
physical properties
no geometry
~~shape of terrain~~
~~what surface is made of and how it interacts with light~~

Diffuse reflection
(Lambertian behavior)

Shadow
of nose



(Behaves like mirror)
Specularities
in direction you are looking at from

Real-world surfaces are not strictly Lambertian!

(oversimplification)

Radiance under Indirect Lighting



- The light source is not visible. Yet there still is light.
- The light enters through the windows and bounces off the walls.

Visualizing Secondary Illumination

Indirect illumination

Reflections produce indirect lighting.



Unique light source assumption does not allow correct albedo recovery.



Lambertian shading

Ideal



pretty constant

light albedo

Lambertian albedoes

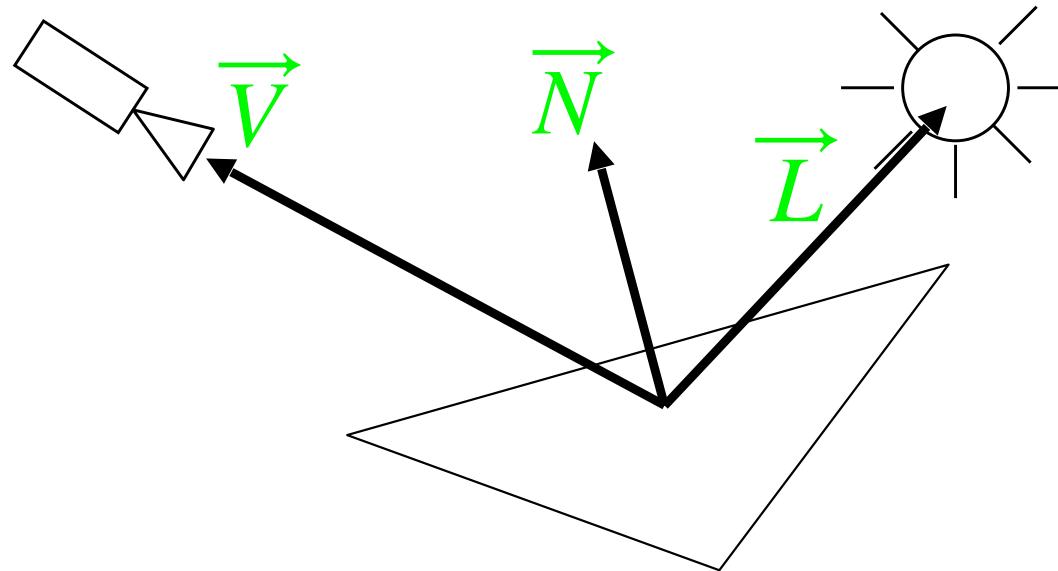
$I \cdot E \cdot N$

Because table is white,

Because of indirect lighting
Light bounces off the table and lights up glossy why?

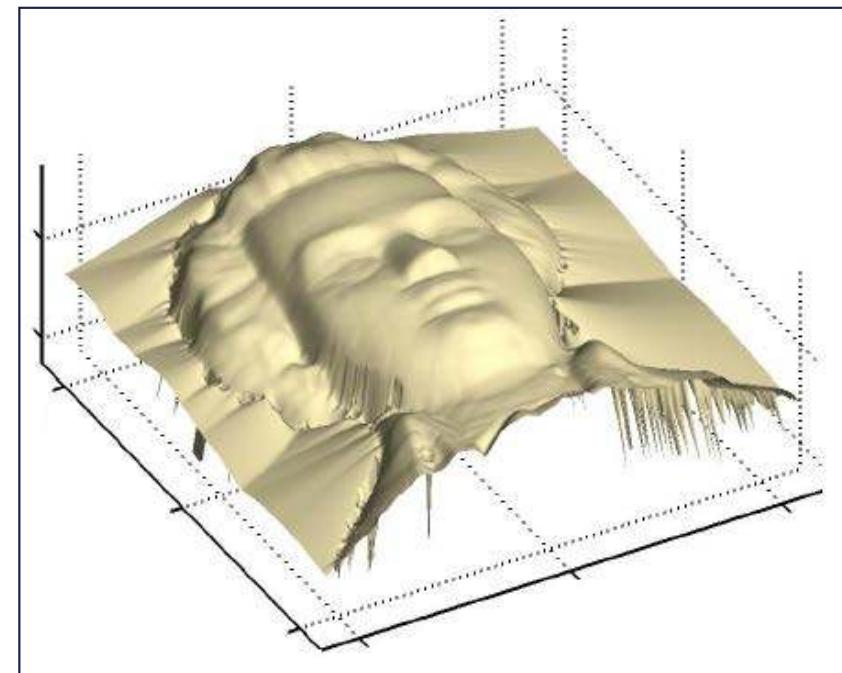
This is not right!

Simplifying Assumptions



- Accounting for secondary illumination in the computer vision context remains an open research problem.
- We will mostly ignore it in this class and make the following assumptions.
 - The illumination sources are distant from the imaged surfaces.
 - Secondary illumination is not significant.
 - There are no cast shadows.

Ideal Synthetic Case



Goal:

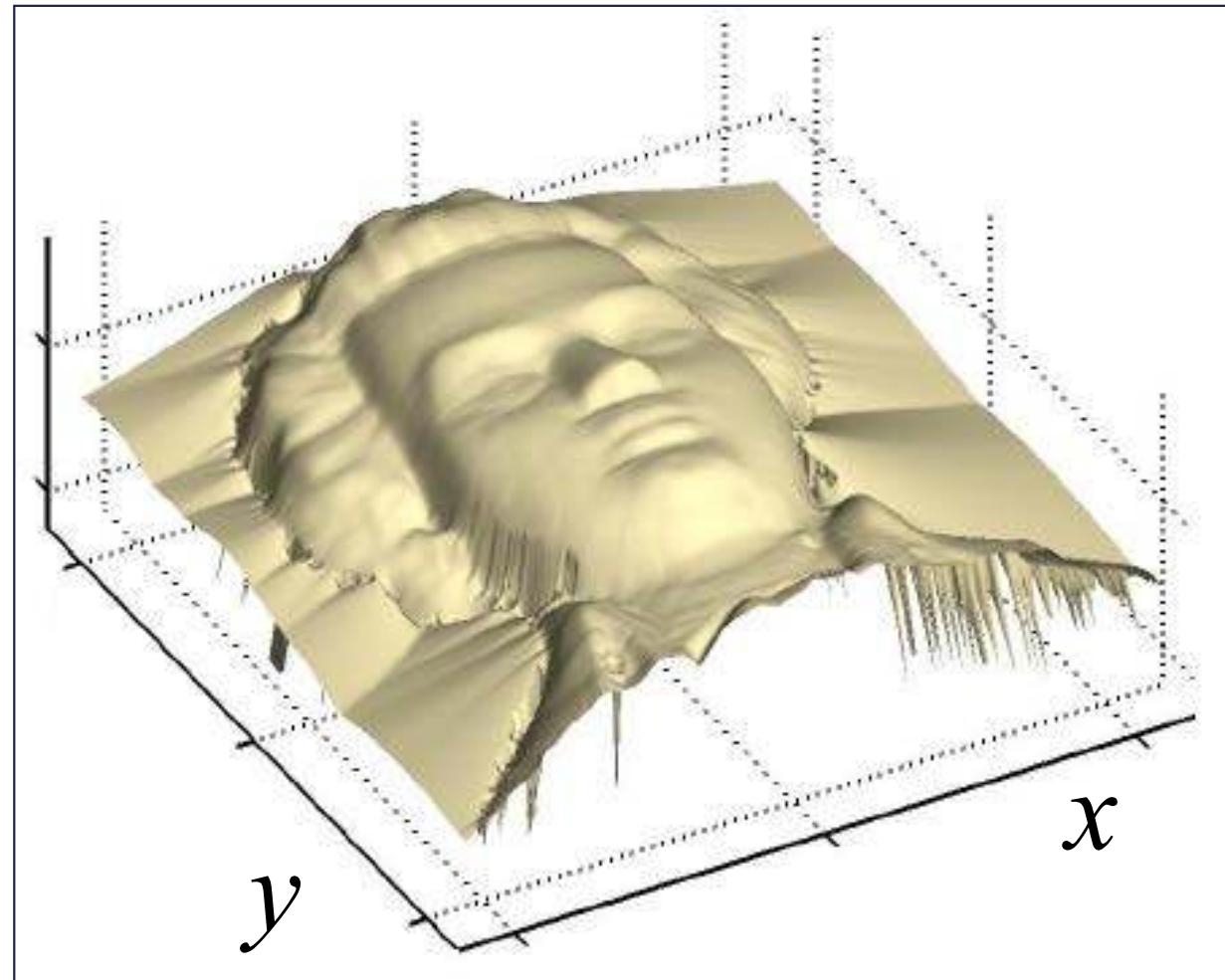
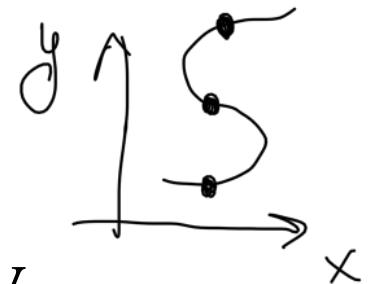
- Recover the 3D shape of the head from the 2D image.

Questions:

1. Given the surface normals, can we recover the surface? *Yes*
2. Can we recover the normals?

Monge Surface

- A Monge surface is defined by $z = f(x,y)$.
- Not all surfaces can be represented this way.

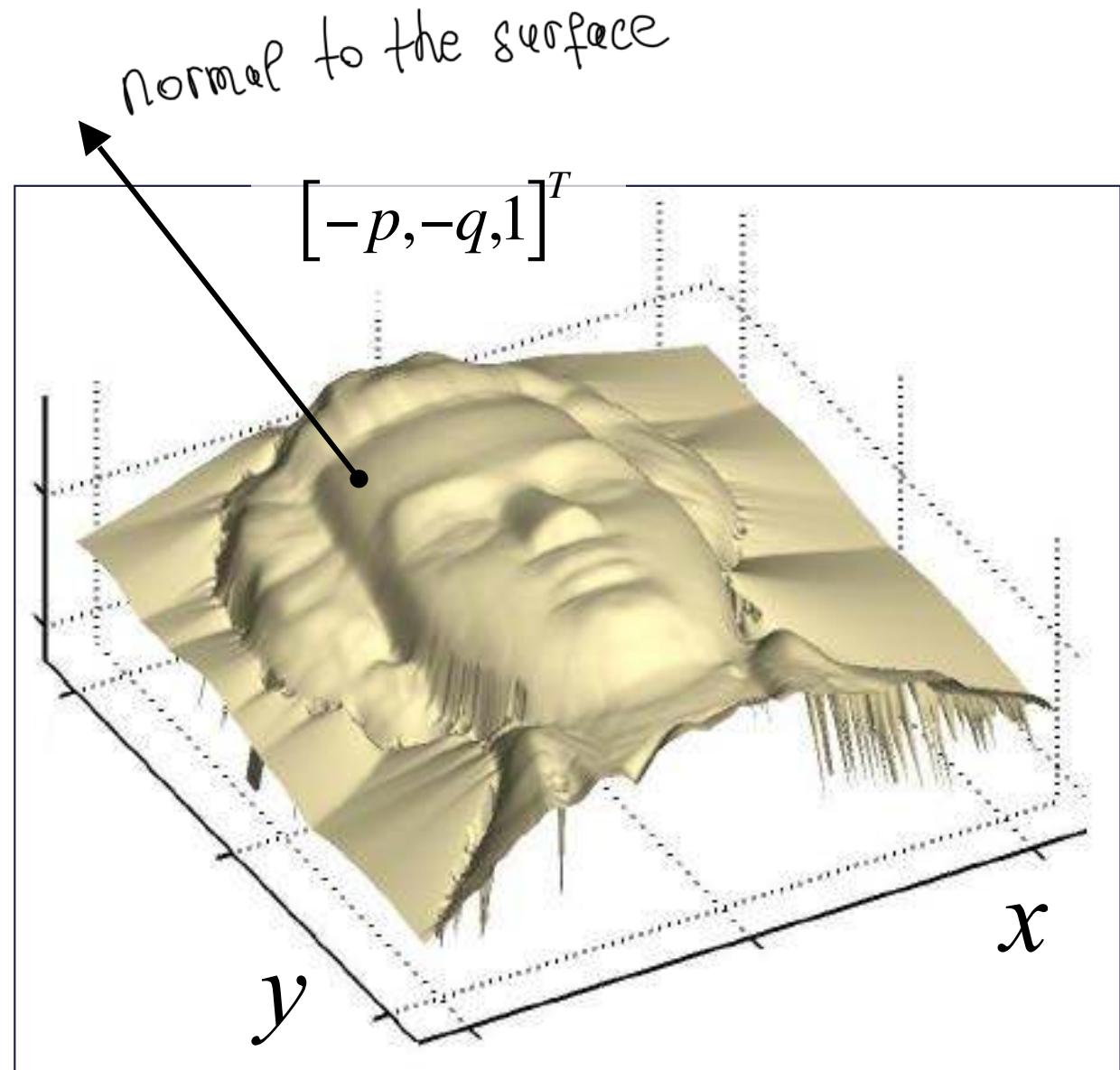


Surface Normals

$$z = f(x, y)$$

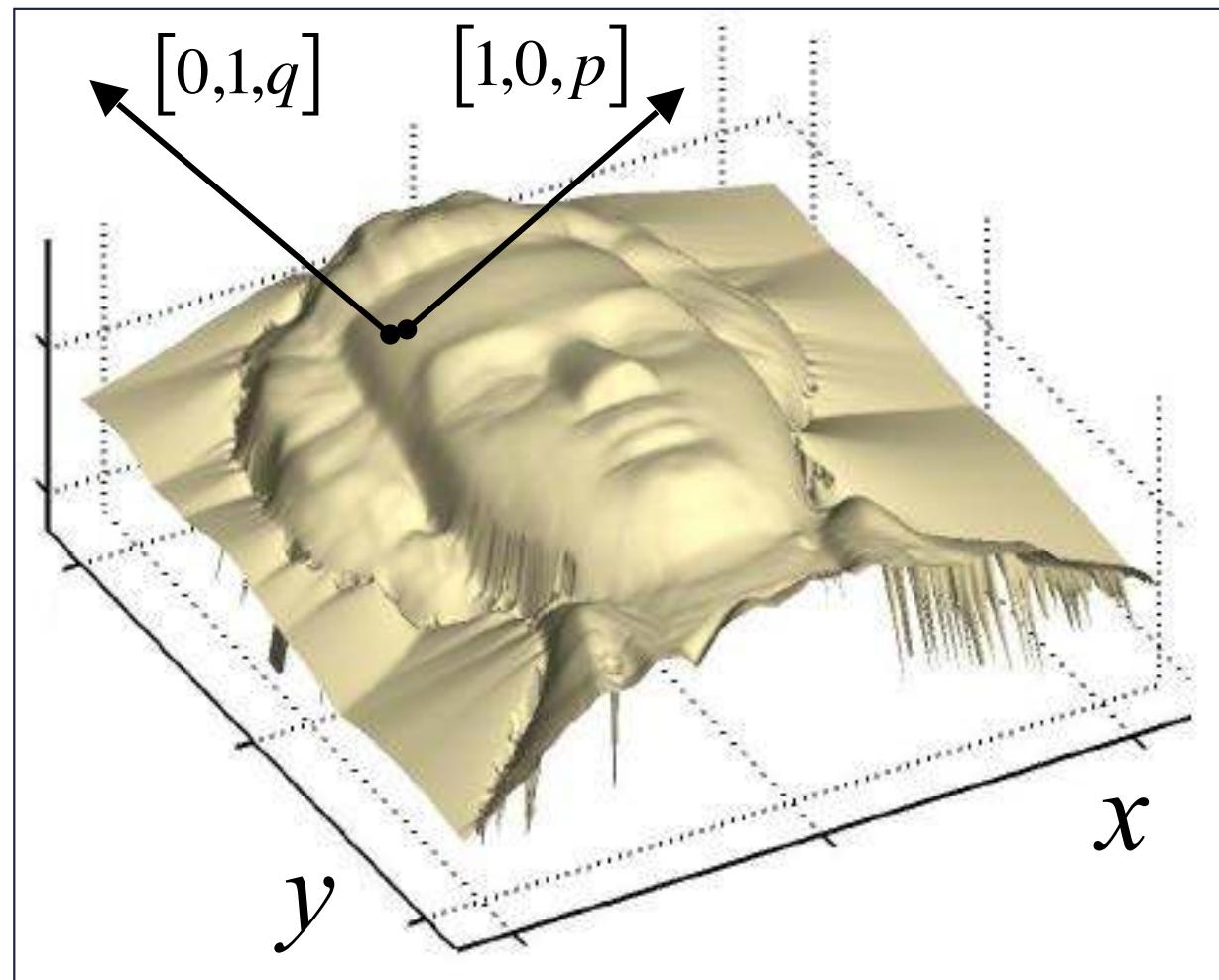
$$p = \frac{\delta z}{\delta x}$$

$$q = \frac{\delta z}{\delta y}$$

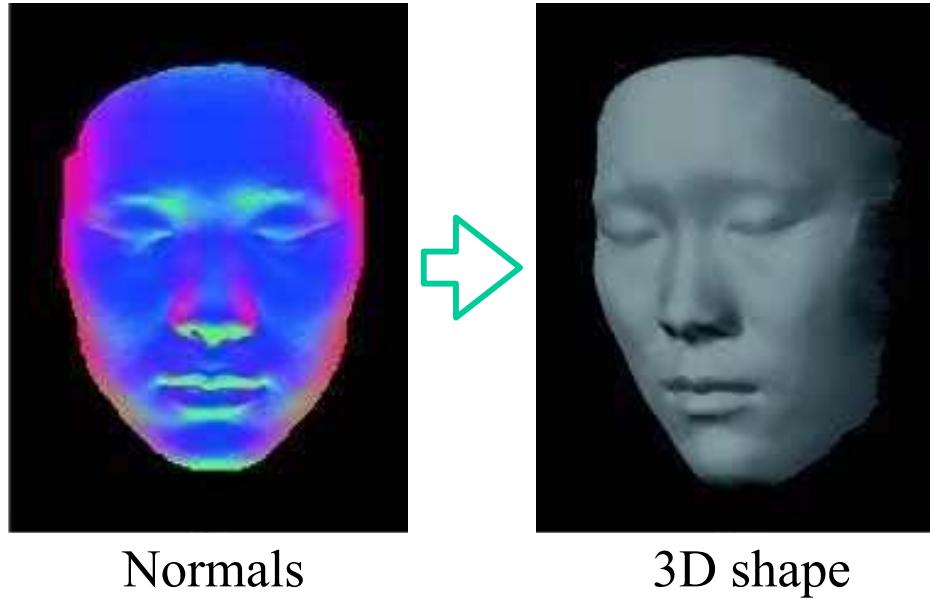


Tangent Vectors

$$\begin{aligned}z &= f(x, y) \\p &= \frac{\delta z}{\delta x} \\q &= \frac{\delta z}{\delta y}\end{aligned}$$



Shape from Normals



Elevation and normal:

$$z = f(x, y)$$

$$\mathbf{N} = \frac{1}{\sqrt{1 + p^2 + q^2}} \begin{bmatrix} -p \\ -q \\ 1 \end{bmatrix} \xrightarrow{\text{rotation}} \begin{bmatrix} -f_x \\ -f_y \\ 1 \end{bmatrix}$$

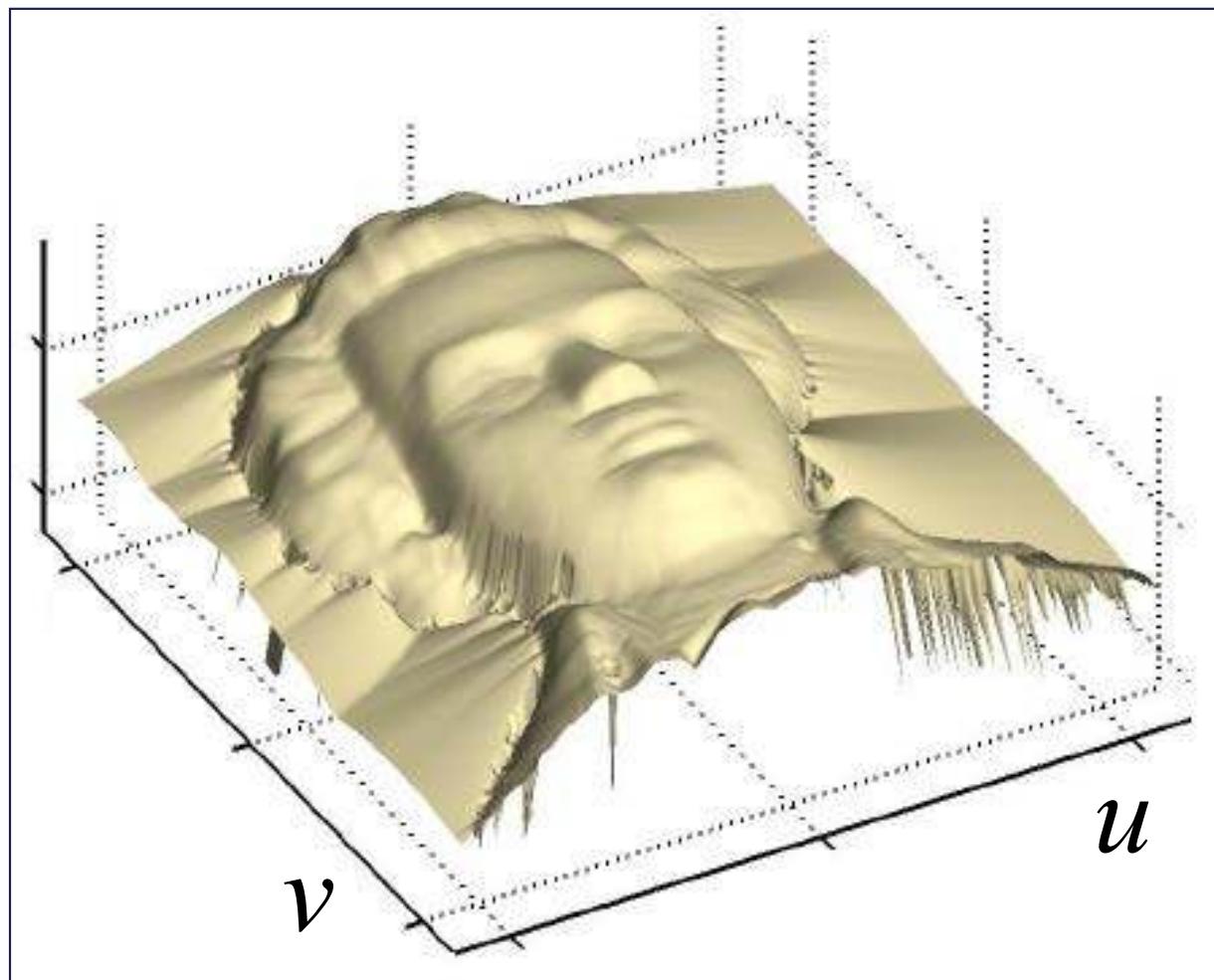
Orthographic projection:

$$\begin{aligned} u &= sx \\ v &= sy \end{aligned}$$

Camera with long focal length
so that 2D-coordinates (u, v) are
simply scaled version of (x, y)

Re-Parametrization

Perform a change of variables $z = f(u, v)$ where u and v are image coordinates.



Shape From Normals (1)

Since $u = sx$ and $v = sy$, the normal vector \mathbf{N} is

$$\begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \frac{1}{\sqrt{1 + \frac{\delta z}{\delta x}^2 + \frac{\delta z}{\delta y}^2}} \begin{bmatrix} -\frac{\delta z}{\delta x} \\ -\frac{\delta z}{\delta y} \\ 1 \end{bmatrix},$$

$$\Rightarrow \frac{\delta z}{\delta x} = -\frac{n_x}{n_z} \text{ and } \frac{\delta z}{\delta y} = -\frac{n_y}{n_z},$$

$$\Rightarrow \frac{\delta z}{\delta u} = -\frac{1}{s} \frac{n_x}{n_z} \text{ and } \frac{\delta z}{\delta v} = -\frac{1}{s} \frac{n_y}{n_z},$$

$$\Rightarrow \frac{\delta \bar{z}}{\delta u} = -\frac{n_x}{n_z} = n_1 \text{ and } \frac{\delta \bar{z}}{\delta v} = -\frac{n_y}{n_z} = n_2,$$

where $\bar{z} = \underline{sz}$ is the scaled distance.
↳ depends on camera

these are known

Shape From Normals (2)

- Let us assume we are given the normal at each pixel (u, v) .
- Let $n_1(u, v) = -n_x(u, v)/n_z(u, v)$ and $n_2(u, v) = -n_y(u, v)/n_z(u, v)$.
- From the previous slide, we have

$$\forall u, v \begin{cases} n_1(u, v) = \frac{\delta \bar{z}}{\delta u} \approx \bar{z}(u+1, v) - \bar{z}(u, v) \\ n_2(u, v) = \frac{\delta \bar{z}}{\delta v} \approx \bar{z}(u, v+1) - \bar{z}(u, v) \end{cases}$$

Finite difference between
neighbours

We are trying to compute \bar{z}

- We therefore have roughly twice as many equations as we have unknowns, the scaled distances $\bar{z}(u, v)$.
equations >> unknown
- This can be solved in the least squares sense.
solve with
over-constrained system

→ Given the normals at every pixel we can recover the distances up to a scale factor.

$$\bar{z} = \underline{s}$$

↳ we can compute the 3D-shape
 (\bar{z}, s)

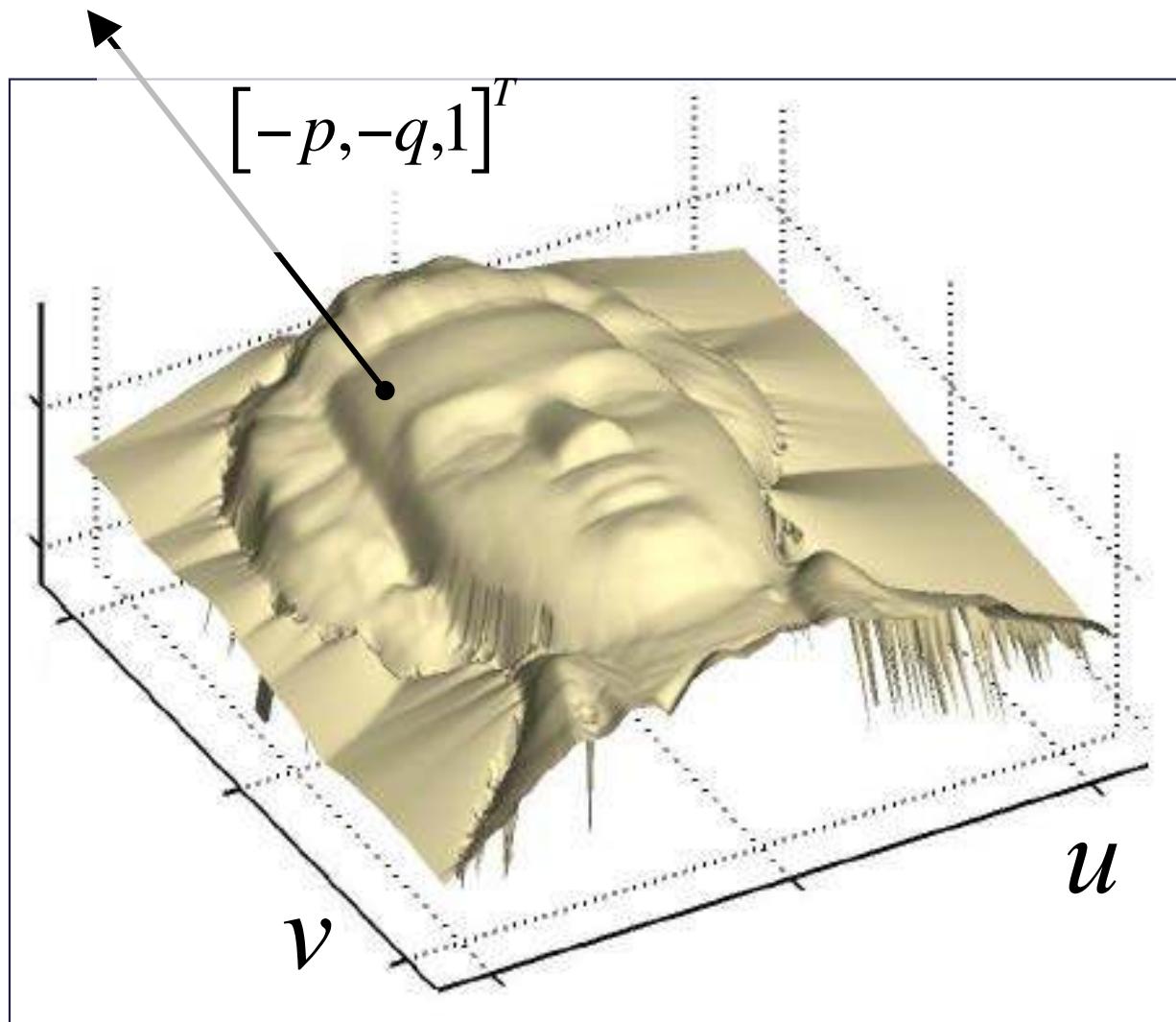
But how do we get the normals?

Back to Estimating the Normals

$$z = f(u, v)$$

$$p = \frac{\delta z}{\delta u}$$

$$q = \frac{\delta z}{\delta v}$$



What does the image tell us about them?

Reflectance Map

In the Lambertian case and for a constant albedo:

Intensity
 $I(u, v) \propto \mathbf{L} \cdot \vec{\mathbf{N}}$

$$\propto \mathbf{L} \cdot [-p(u, v), -q(u, v), 1]^T \quad (\text{For Lambertian case})$$

$$\propto \text{Ref}(p(u, v), q(u, v))$$

↙ Reflectance function

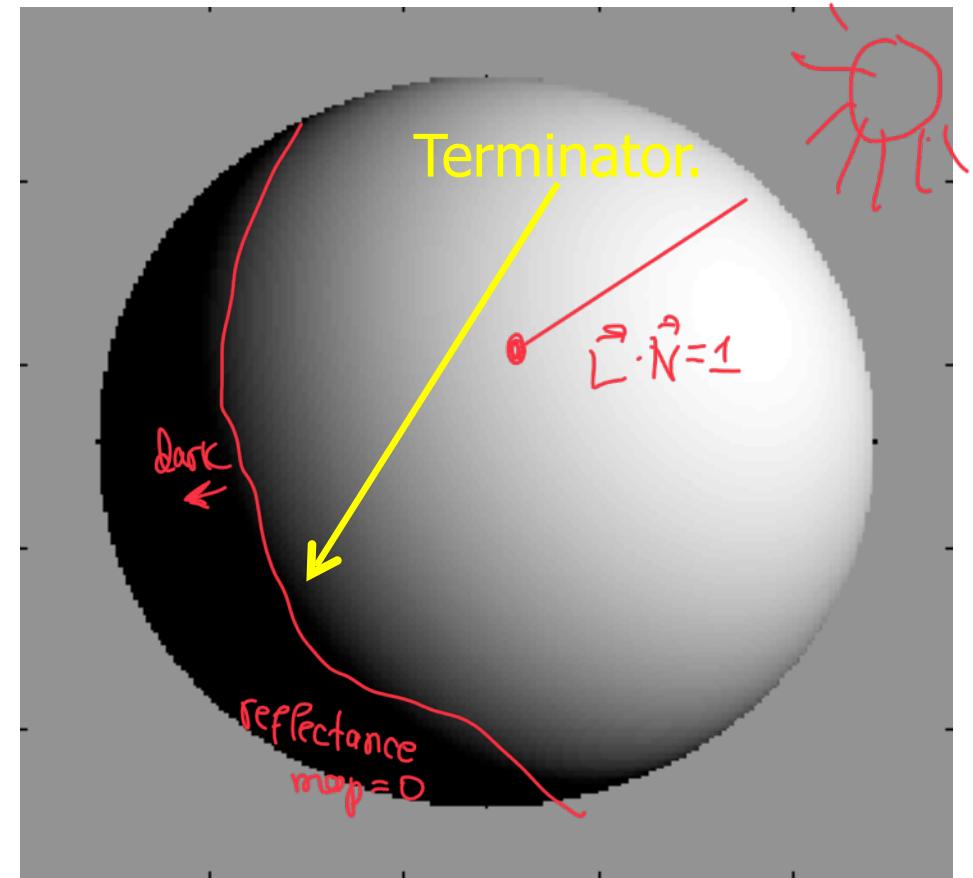
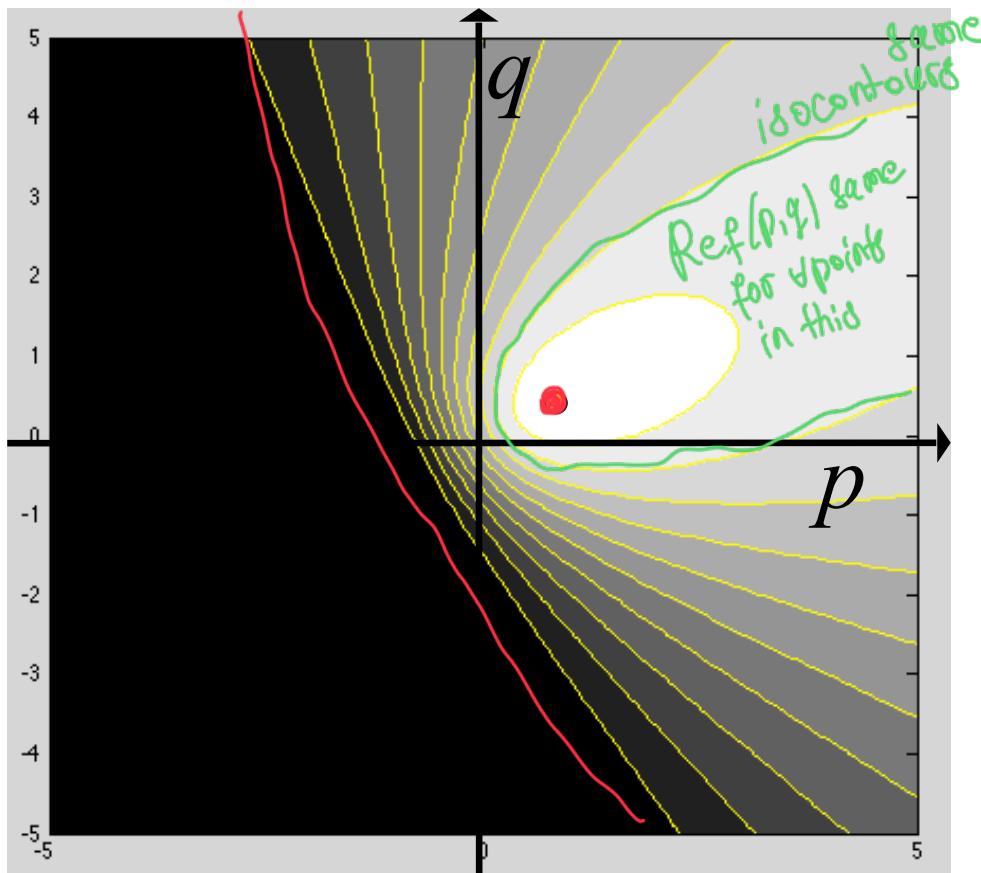
↑ it associates (p, q) with intensity observed
normal

- The function Ref is known as the reflectance map.
- For non-Lambertian surfaces it can be more complex.

↙ shapes you are looking at, but at the materials
(even for complex ones)

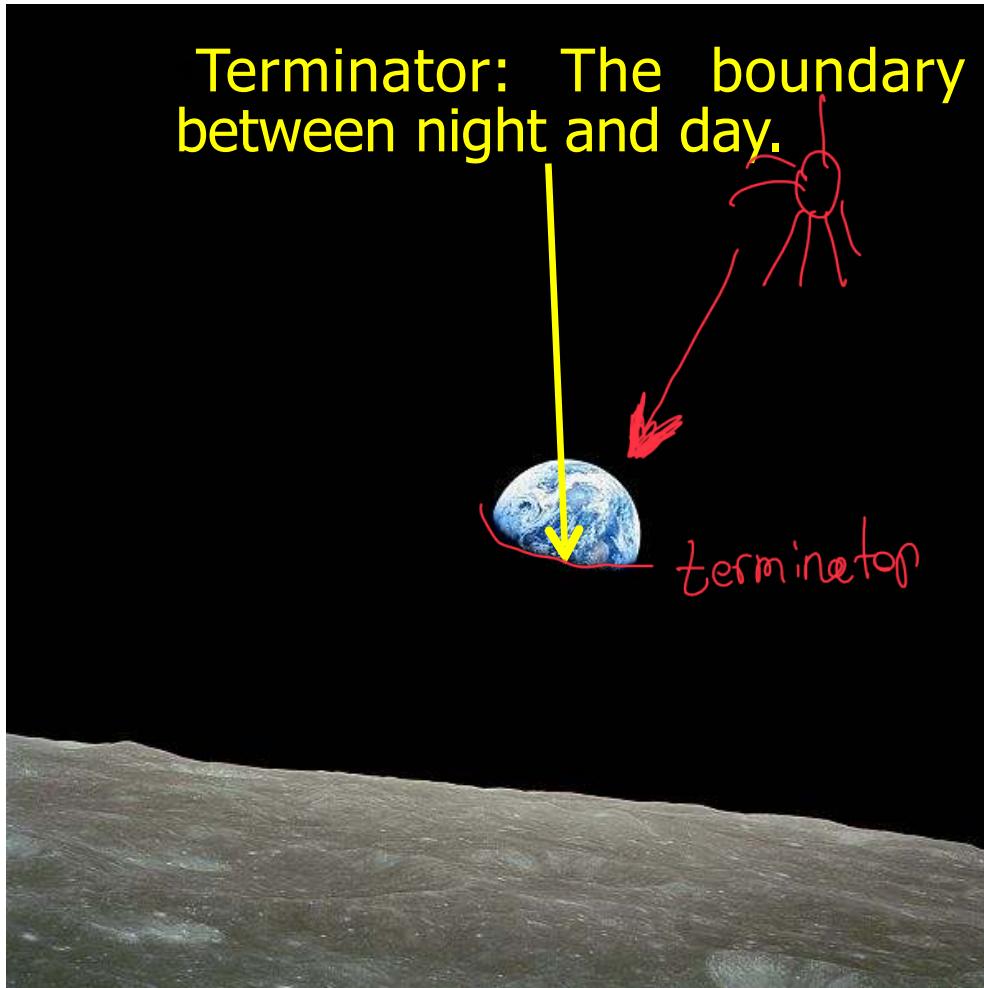
Lambertian Reflectance Map

$$\vec{L} \cdot \vec{N} = 0 \text{ (terminator)}$$



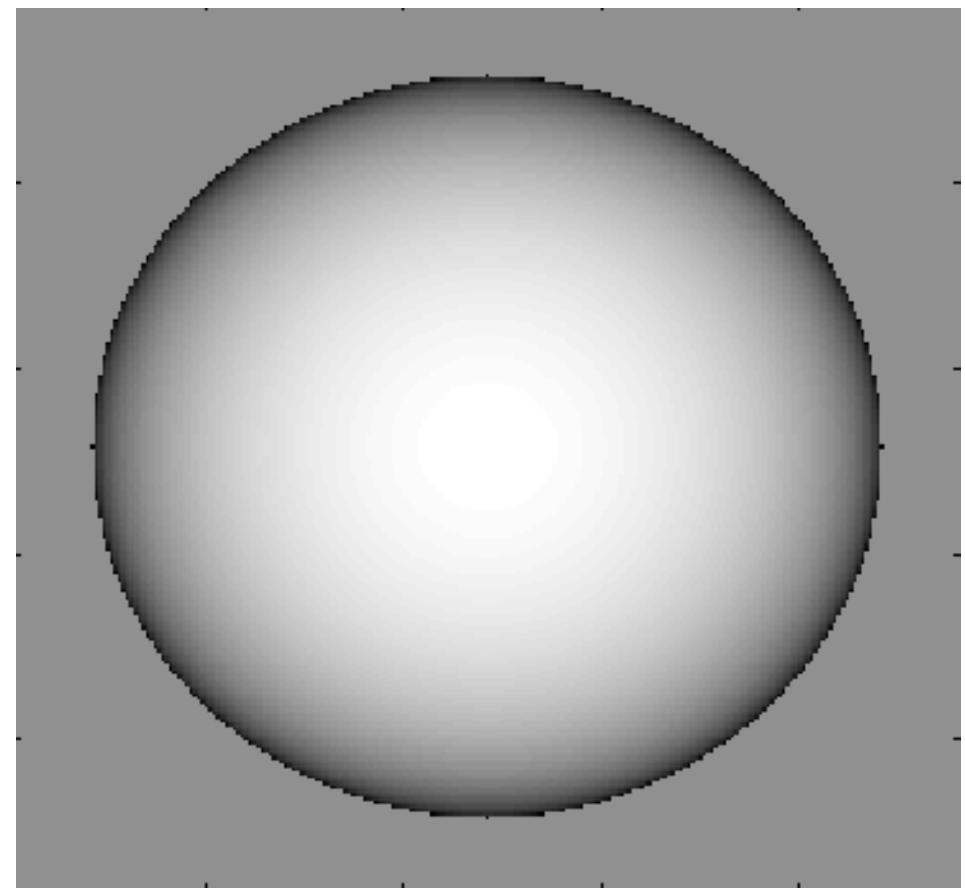
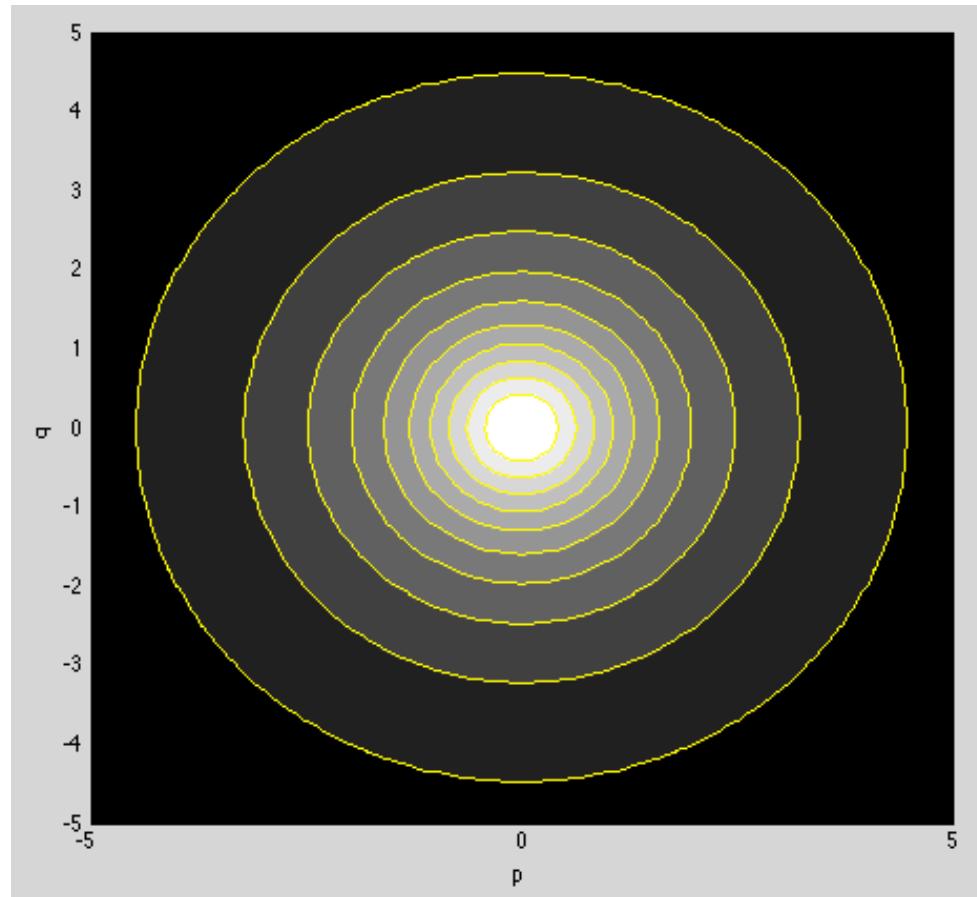
Reflectance map and shaded surface for Lambertian surface illuminated in the direction $[-1 -0.5 -1]$.

Earth Seen from the Moon



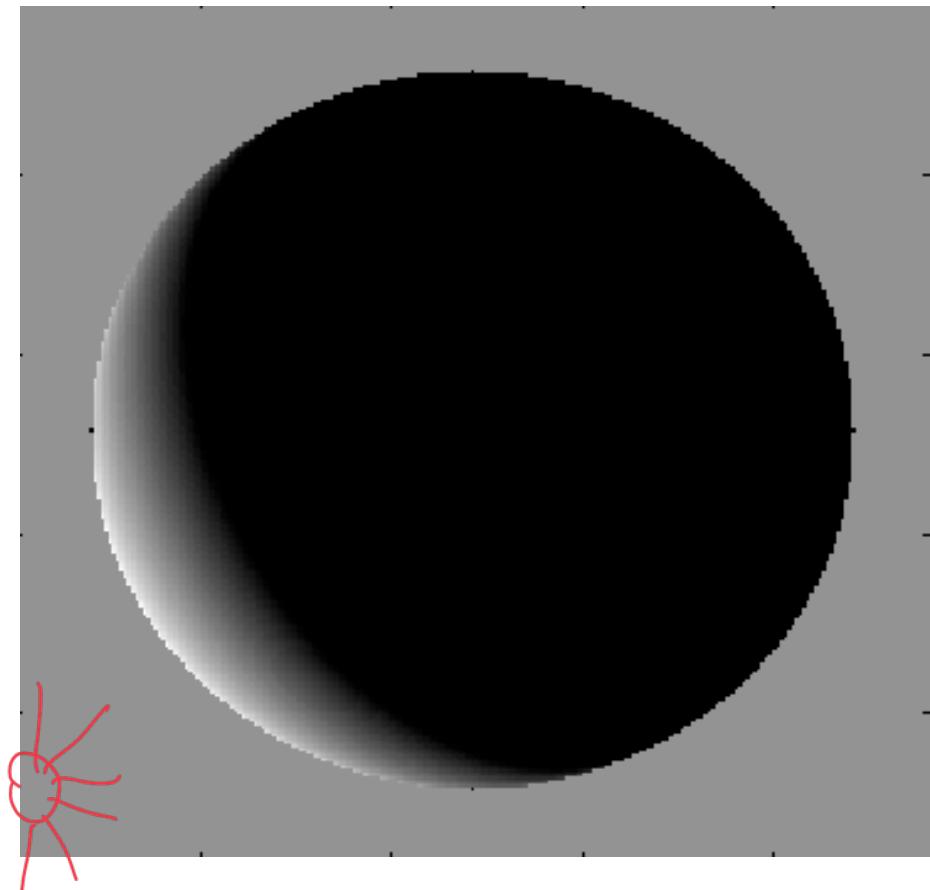
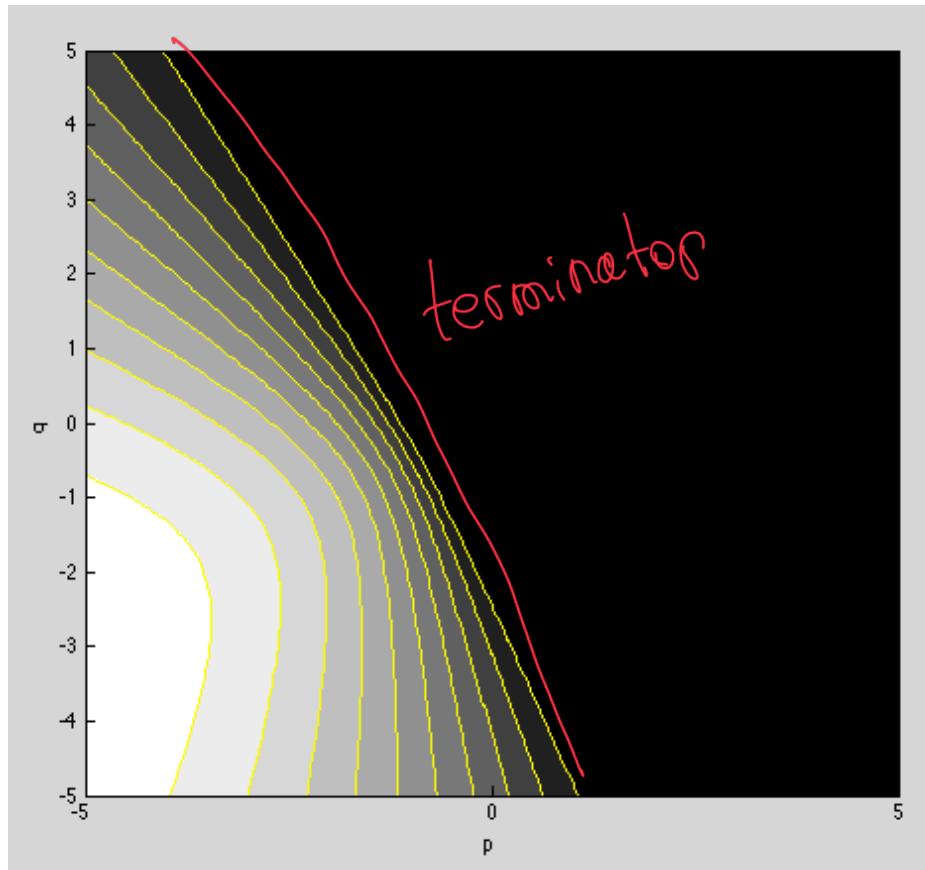
Close to Lambertian model

Lambertian Reflectance Map



Reflectance map and shaded surface for Lambertian surface illuminated in the direction [0 0 -1].

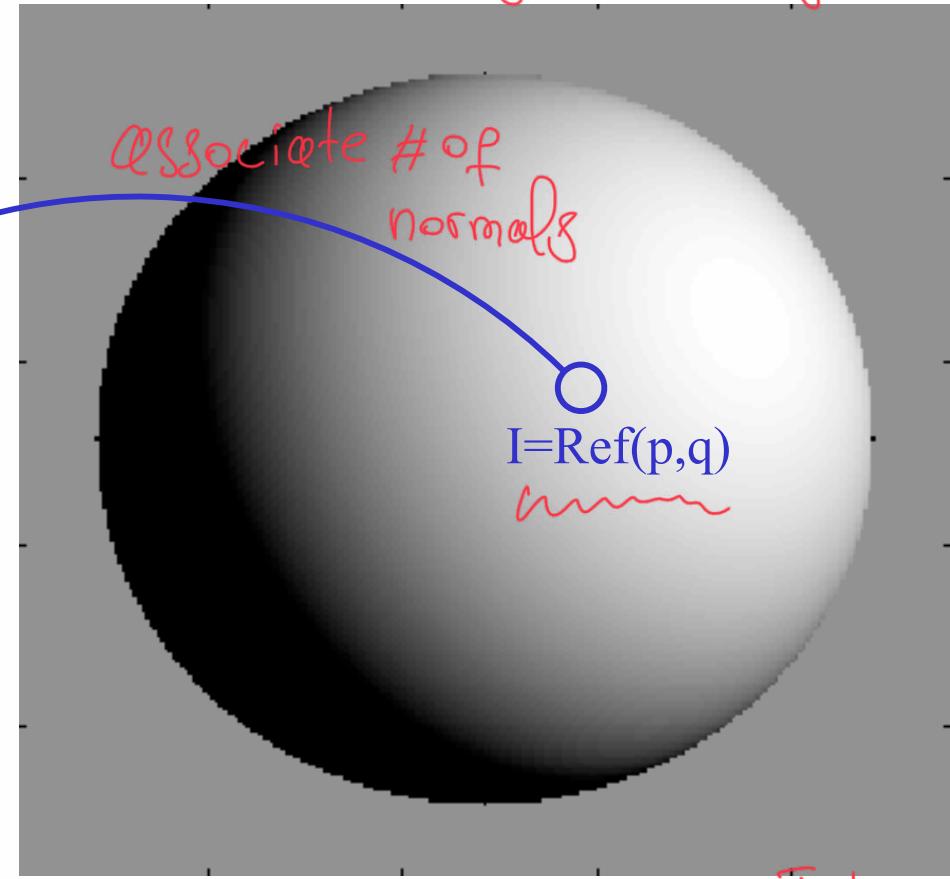
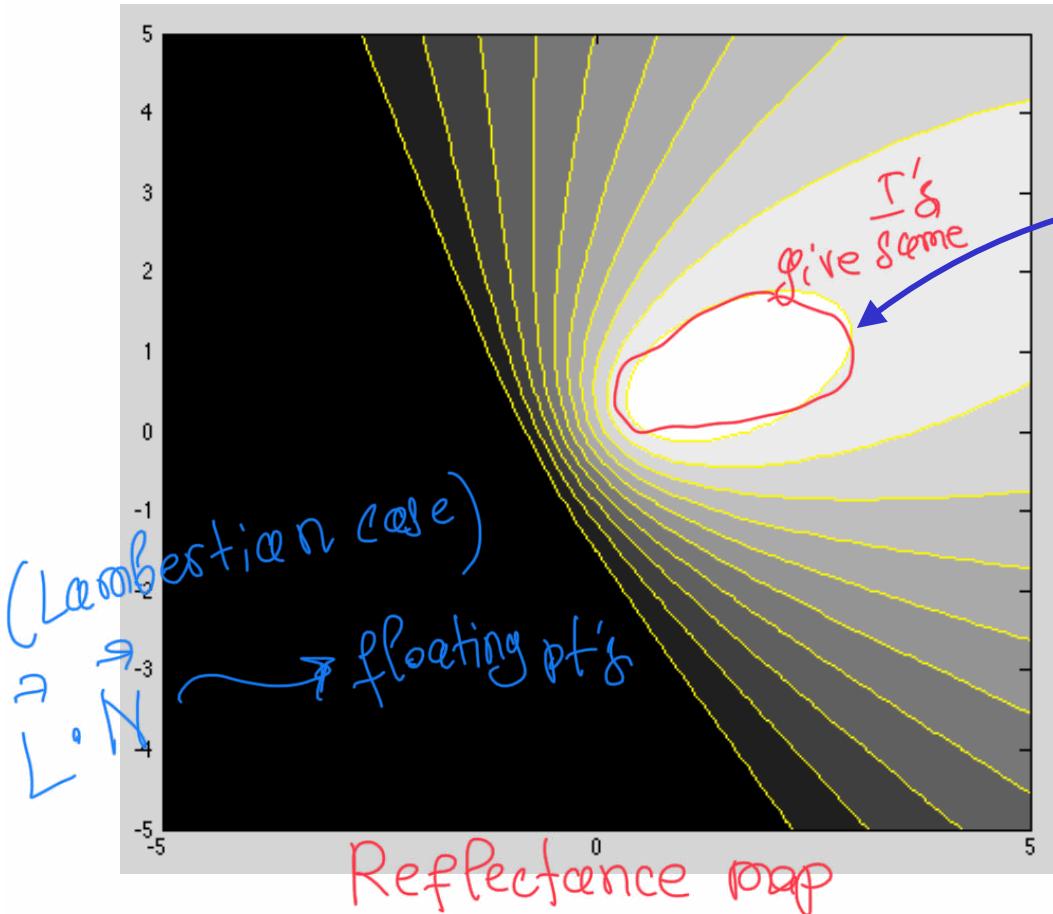
Lambertian Reflectance Map



Reflectance map and shaded surface for Lambertian surface illuminated in the direction $[1 \ 0.5 \ -1]$.

Inverse Problem (Shape from Shading)

Can we determine (p, q) uniquely for each image point independently?

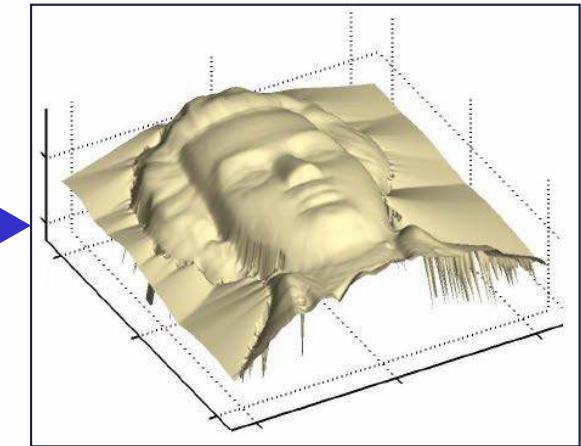


No because many p and q yield the same $\text{Ref}(p, q)$.

→ Global optimization required.

Variational Method (1)

for applying our points



Minimize:

$$\int \int \left([I(u, v) - Ref(p, q)]^2 + \lambda \left[\left(\frac{\delta p}{\delta u} \right)^2 + \left(\frac{\delta p}{\delta v} \right)^2 + \left(\frac{\delta q}{\delta u} \right)^2 + \left(\frac{\delta q}{\delta v} \right)^2 \right] + \mu \left(\frac{\delta p}{\delta v} - \frac{\delta q}{\delta u} \right)^2 \right) dudv$$

Data term

Smoothness term
(regularization)

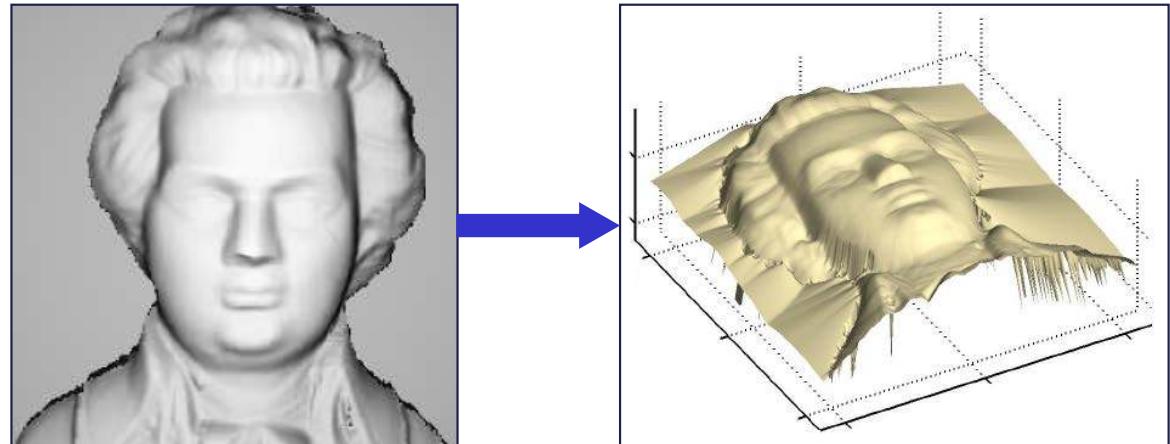
Integrability term

$$\frac{\delta z}{\delta u} = \frac{\delta z}{\delta v} = \frac{\delta^2 z}{\delta u \delta v}$$

1. Recover the normals and make them integrable.
2. Integrate to recover the 3D surface. RH8

Variational Method (2)

Minimize:



$$\int \int \left(\left[I(u, v) - Ref\left(\frac{\delta z}{\delta u}, \frac{\delta z}{\delta v}\right) \right]^2 + \lambda \left[\left(\frac{\delta^2 z}{\delta u^2} \right)^2 + \left(\frac{\delta^2 z}{\delta u \delta v} \right)^2 + \left(\frac{\delta^2 z}{\delta v^2} \right)^2 \right] \right) dudv$$

$\uparrow \quad \overbrace{P}^{\text{Data term}} \quad \overbrace{Q}^{\text{Smoothness term}}$
 $\uparrow \quad \text{Second derivatives of } z$

Smoothness term

Introduce z

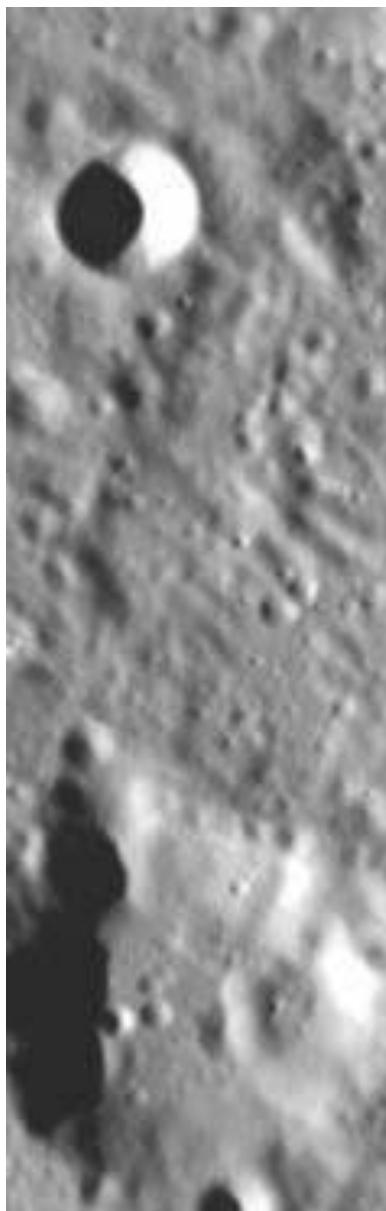
- Recover the surface directly, which means solving a second order differential equation instead of a first order one.
- Both approaches are valid and require boundary conditions.

for PDEs

Moonscape

($\alpha P_{\text{bedo}} \sim \text{constant}$)

same physical
properties over the
extensive areas



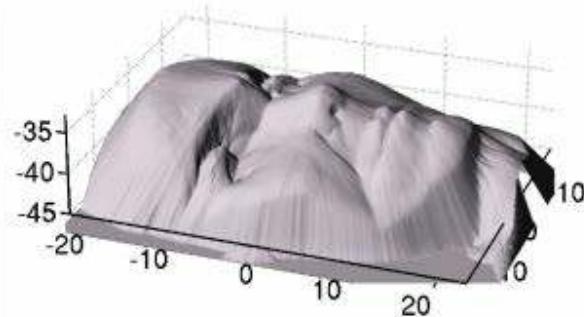
Moon image



Depth map

(what has been coded
is the altitude)

Faces from Shading



- Generic Monge surface
- Low resolution images

Prados and Faugeras, CVPR'05.

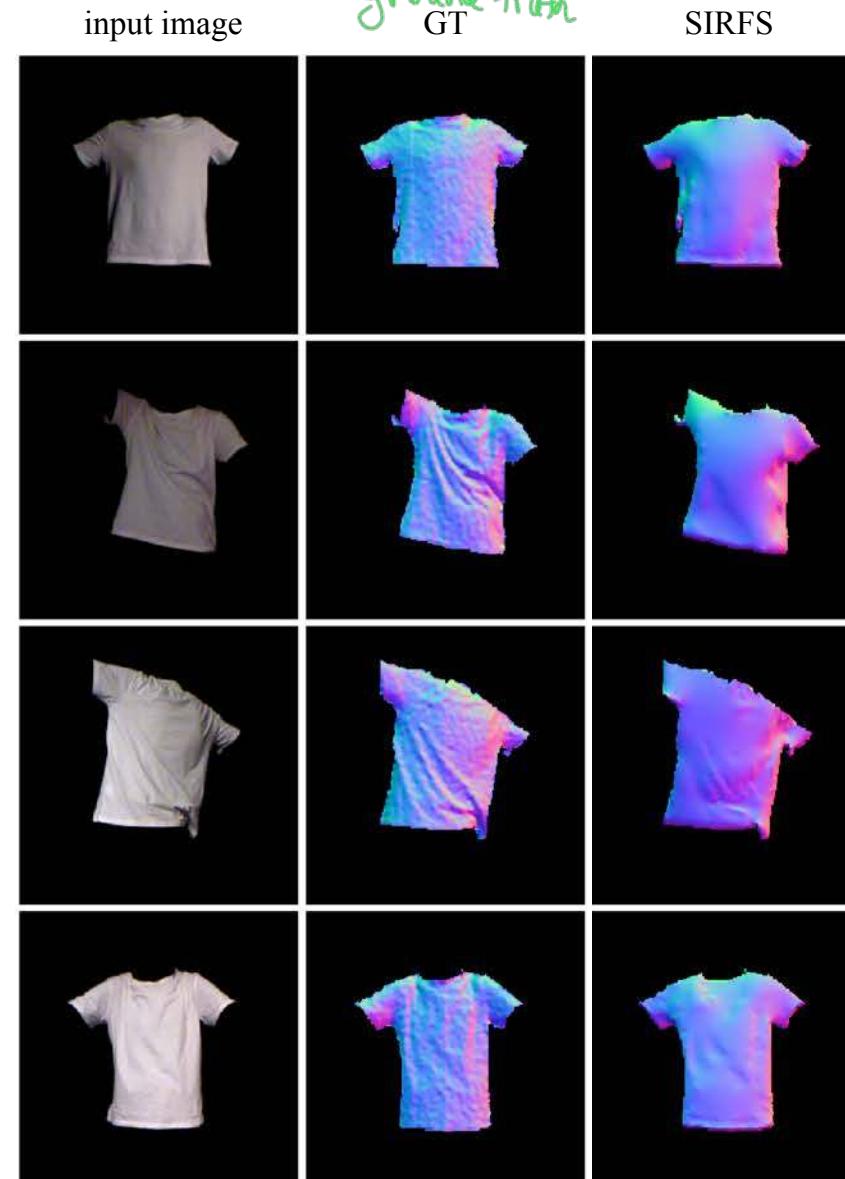
Combination of shape-from-shading
and powerful face models
(VAEs)



- Deep face model
- High resolution images

Bagautdinov et al., CVPR'18.

T-Shirt



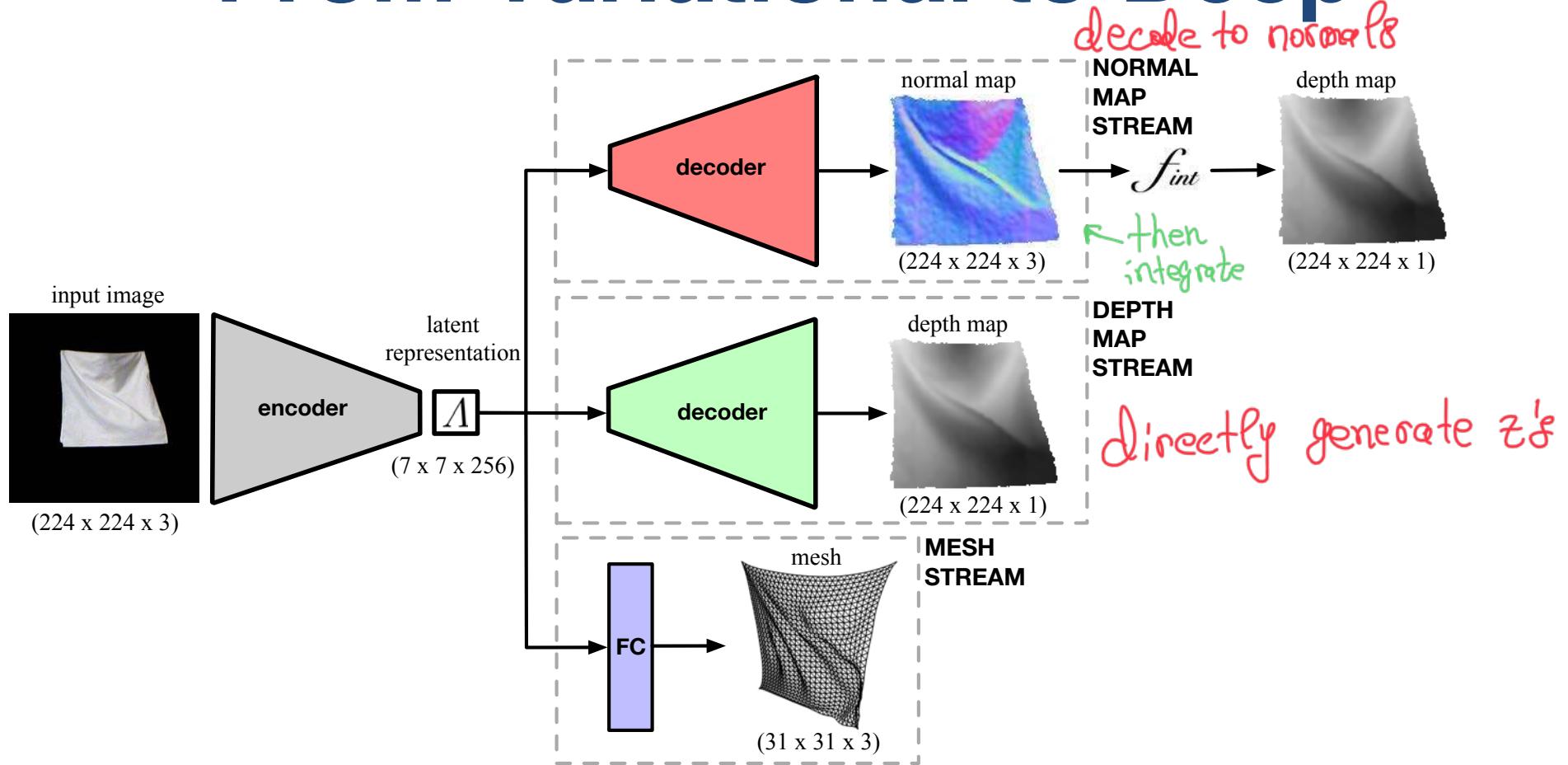
OPD approach

smoothness too much

T-shirt which has patterns \Rightarrow bad

Works because the albedo is constant!

From Variational to Deep

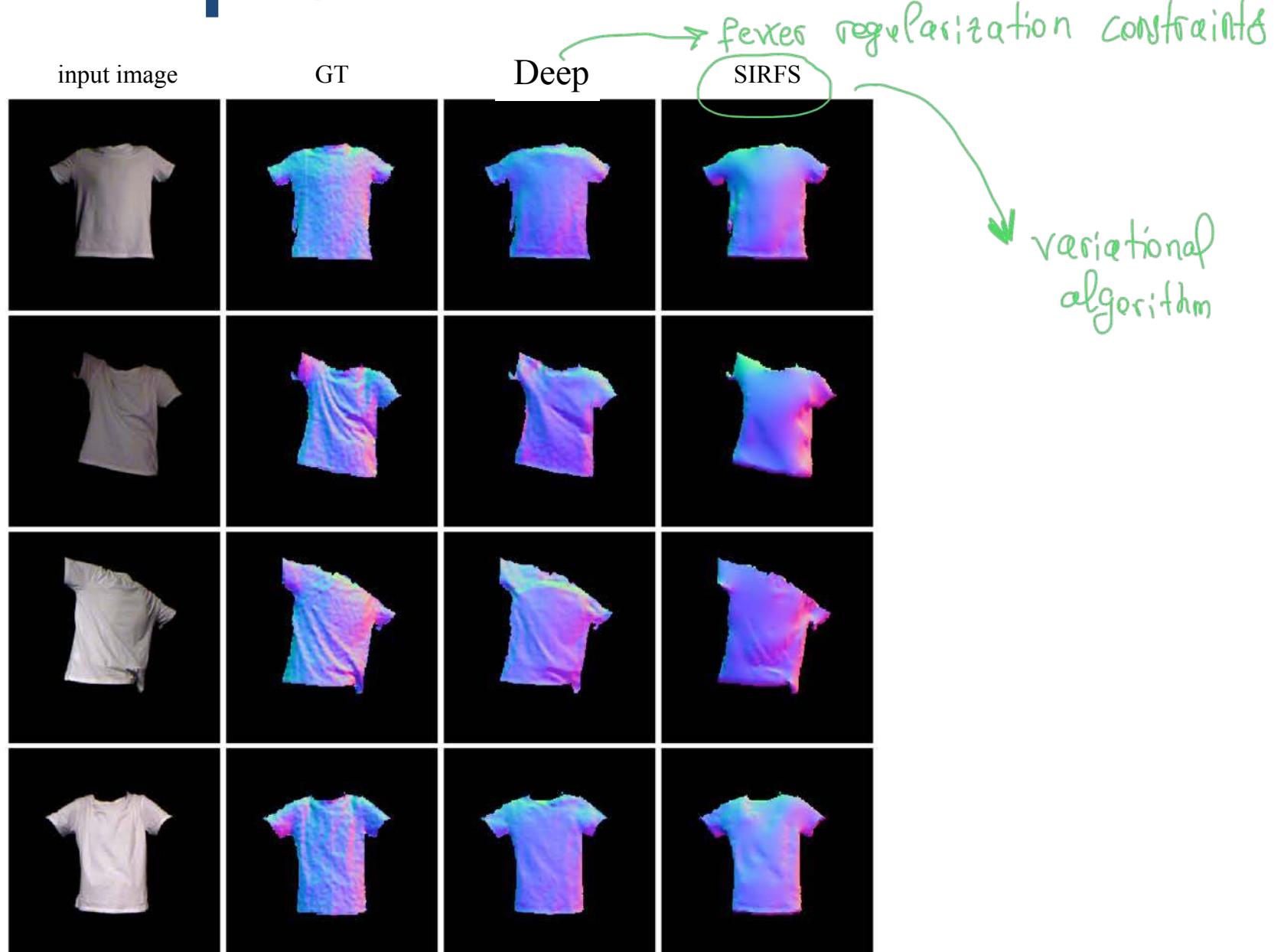


The deep net is trained to:

- produce both a depth map and a map of normals,
- ensure they are consistent with each other.

→ Can be understood as another way to solve the variational problem.

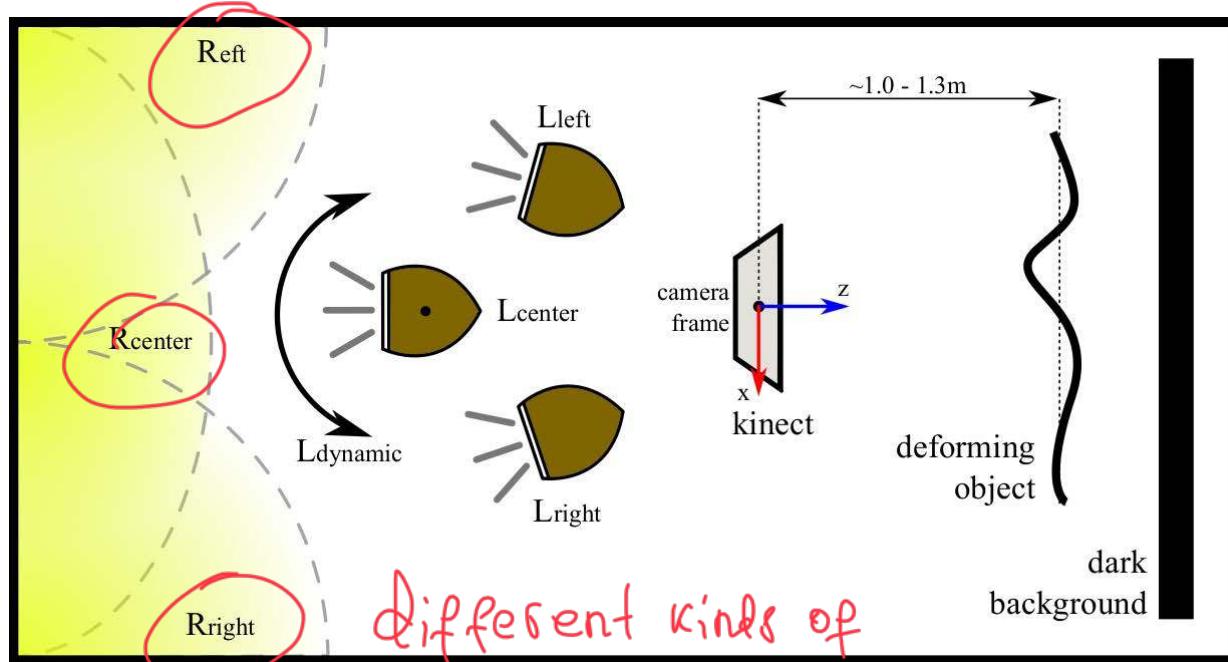
Improved Results



The deep net recovers more details

Training Data

... but required serious training.

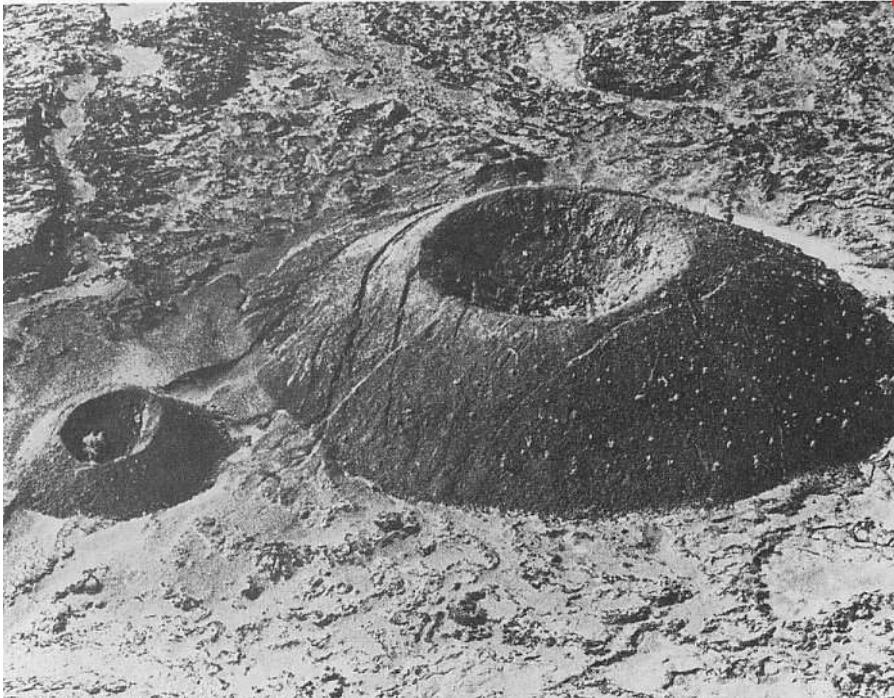


- 3 fixed light sources.
 - 1 mobile one.
- Not yet a generally applicable solution.

But getting closer. We'll talk about it again next week

Reminder: Ambiguities

We flipped the images
but light source → still same
Position of



- Back where we started.
- Let us look at them more closely.

Bas-Relief Ambiguity



Looks like a normal human head ...



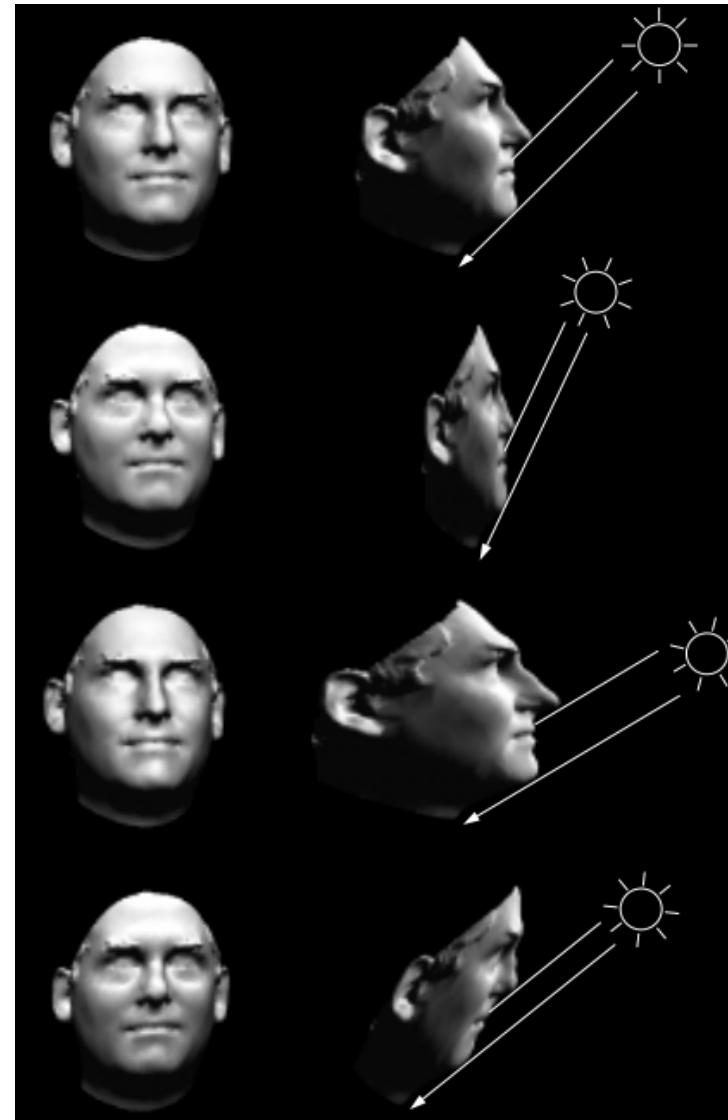
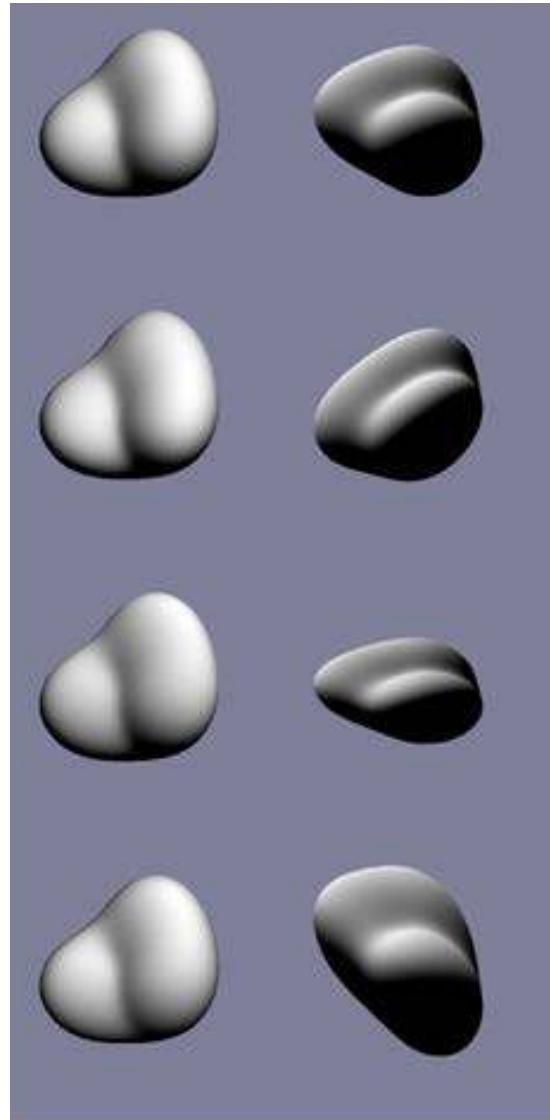
... but not when seen from the side.

Head is
squashed

← → narrow in this direction

Why is that?

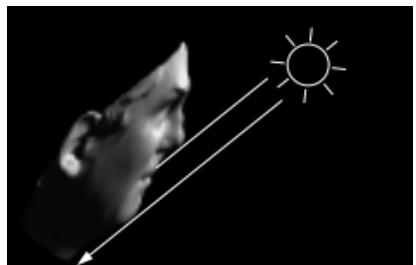
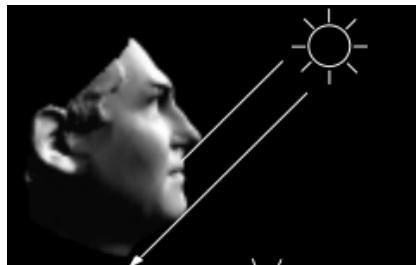
Bas-Relief Ambiguity



rendered as a
Lambertian
surface with const
albedo (equal grey)
values for off
surface pts

By moving the light source, it is possible to produce the same image for different 3D shapes.

Bas-Relief Ambiguity Explained (1)



$$Ref = \mathbf{N} \cdot \mathbf{L}$$

↳ Reflectance function

3×1 vector

For any invertible 3×3 linear transformation A:

similar intensities

$$\begin{aligned} (\mathbf{A}\mathbf{N}) \cdot (\mathbf{A}^{-T}\mathbf{L}) &= (\mathbf{A}\mathbf{N})^T(\mathbf{A}^{-T}\mathbf{L}) \\ &\quad \text{(} (\mathbf{A}^{-1})^T \text{)} \\ &= \mathbf{N}^T \mathbf{A}^T \mathbf{A}^{-T} \mathbf{L} \\ &= \mathbf{N}^T \mathbf{L} \\ &= \mathbf{N} \cdot \mathbf{L} \end{aligned}$$

move the light source
and get ~same result

- In theory, applying \mathbf{A} to the normals and \mathbf{A}^{-1} to the light source would not change the image.
- However, the normals must remain integrable, which means that not all transformations of the normals are valid.
- In particular, for a Monge surface $z = f(u,v)$, we must have

$$\frac{\frac{\delta z}{\delta u}}{\delta v} = \frac{\frac{\delta z}{\delta v}}{\delta u} = \frac{\delta^2 z}{\delta u \delta v}$$

Bas-Relief Ambiguity Explained (2)

Let us write the integrability constraint in our specific case:

$$\left. \begin{array}{lcl} \frac{\delta z}{\delta u} & = & -\frac{n_x^*}{n_z^*} \\ \frac{\delta z}{\delta v} & = & -\frac{n_y^*}{n_z^*} \end{array} \right\} \Rightarrow \frac{\delta \frac{n_x^*}{n_z^*}}{\delta v} = \frac{\delta \frac{n_y^*}{n_z^*}}{\delta u} \text{ with } \begin{bmatrix} n_x^* \\ n_y^* \\ n_z^* \end{bmatrix} = \mathbf{A}^{-T} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

$$\Rightarrow \mathbf{A} \text{ restricted to } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mu & \nu & \lambda \end{bmatrix}$$

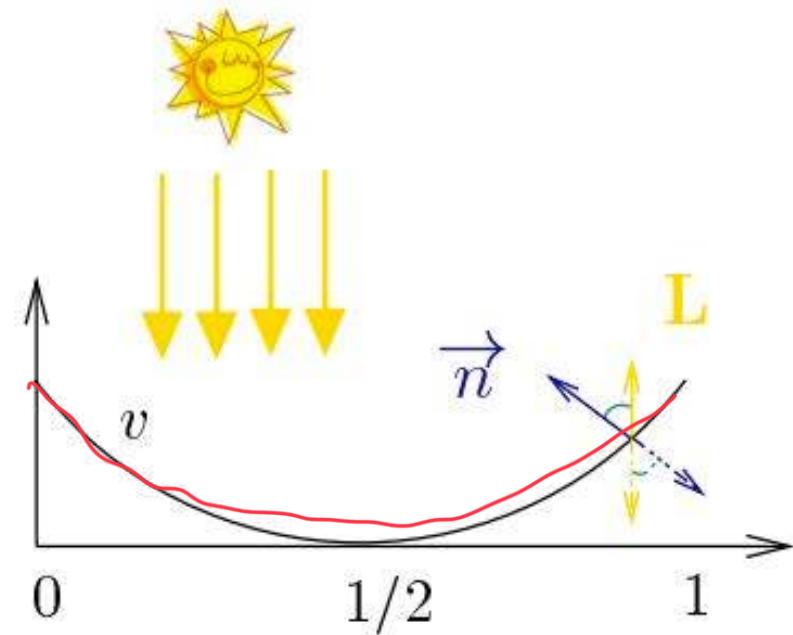
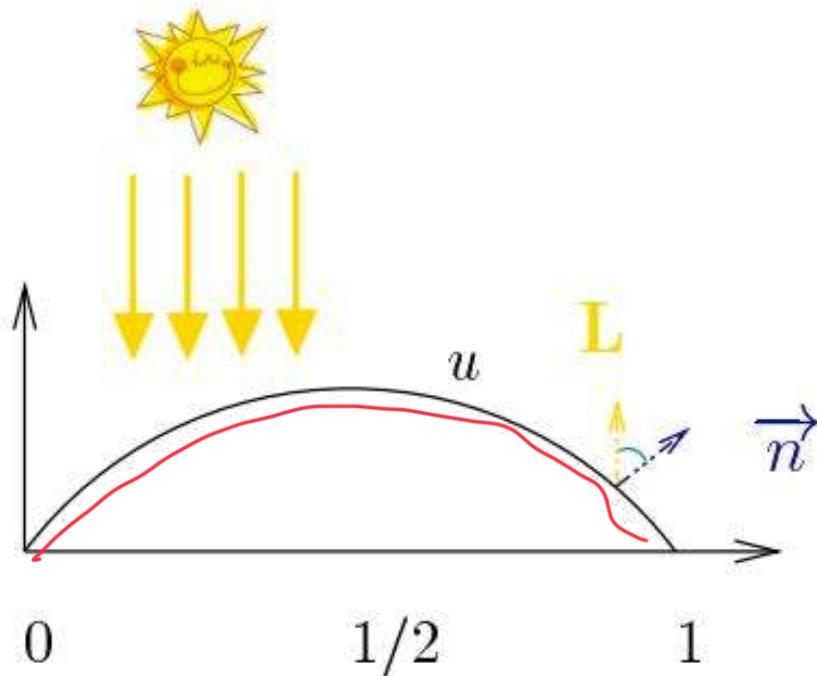
$\leq \neq$
 ER

→ The surface $f(u, v)$ can be changed into $\lambda f(u, v) + \mu u + \nu v$ and still produce the same image.

By simply moving the light source
in a correct way

Convex/Concave Ambiguity

in shape from shading

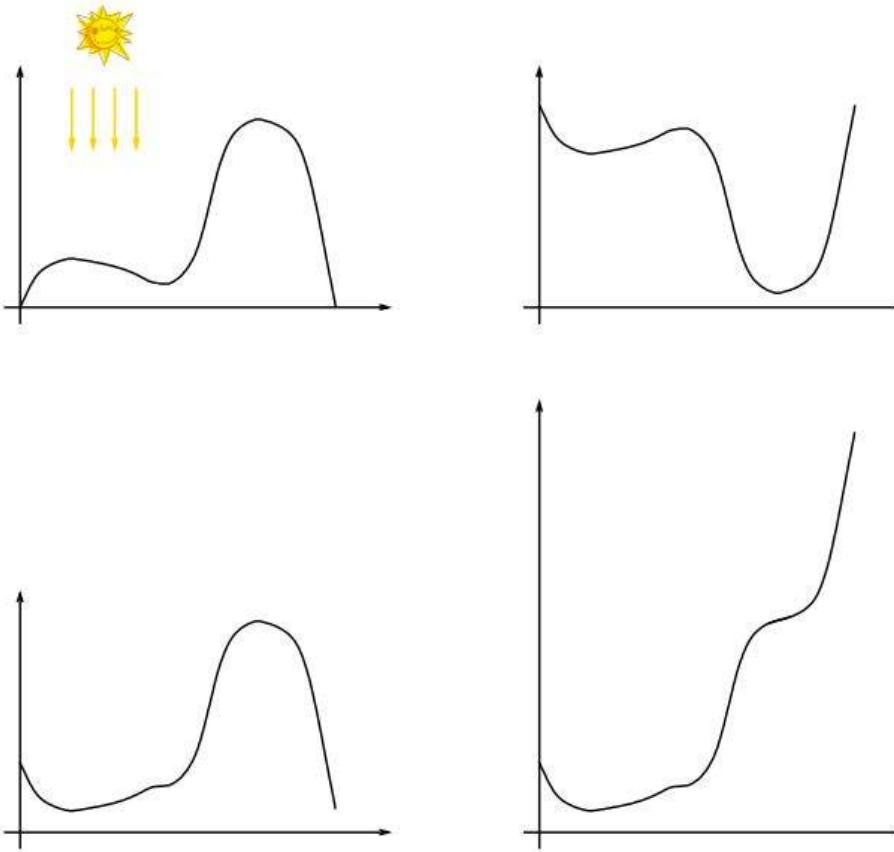


It can happen even when the light source is known!

You can't differentiate them just on the basis of their shading

- Convex/concave surface
- Light from is same even light source is far

Convex/Concave Ambiguity



- All four profiles will produce the same image under the illumination shown here.
- The SfS problem under orthographic projection with a distant light-source is ill-posed. *(Variational)* ↗ No unique sol'n
(one of them ⊥ starting)

Making the SfS Problem Well-Posed

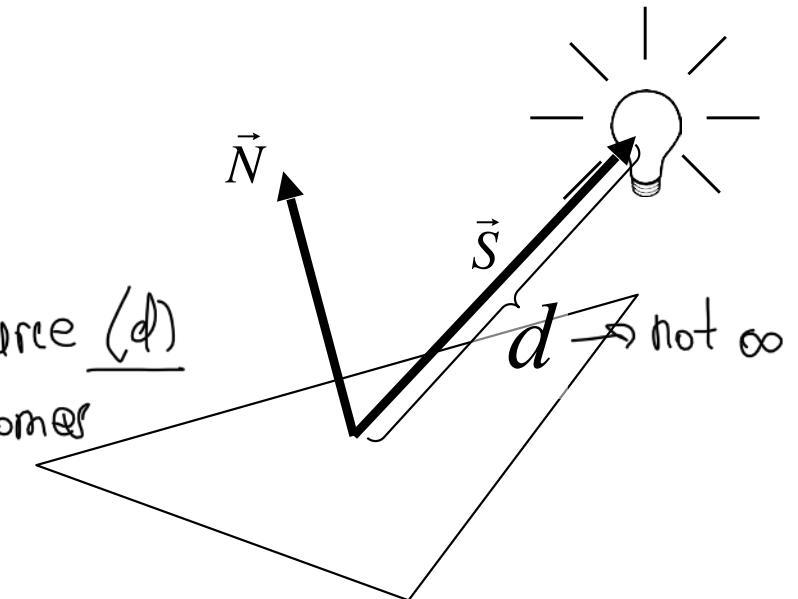
- Use perspective projection model;
- Radiance depends on distance to light source:

$$I = \frac{\text{Albedo} \cdot (\mathbf{N} \cdot \mathbf{S})}{d^2}$$

instead of

further away from light source (d)
 \rightarrow the dimmer image becomes

$$I = \text{Albedo} \cdot (\mathbf{N} \cdot \mathbf{S})$$



- Light source located at the optical center.

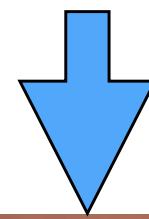
camera and light source at same place

-> Unique solution but more complex computations.

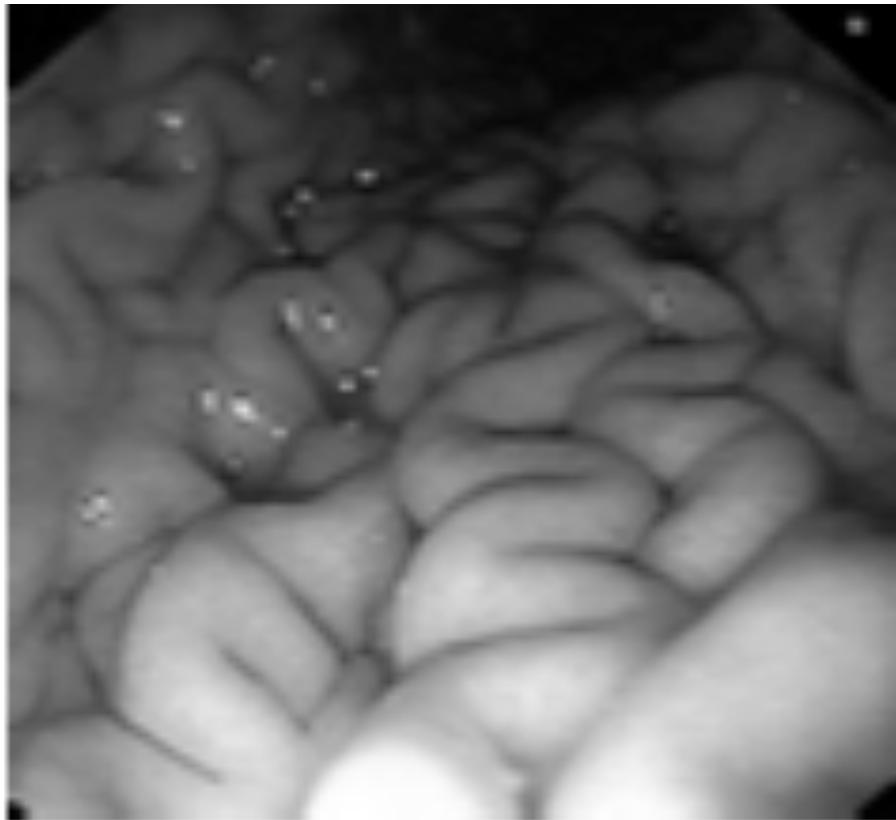
Endoscopy



+



Endoscopy 2005

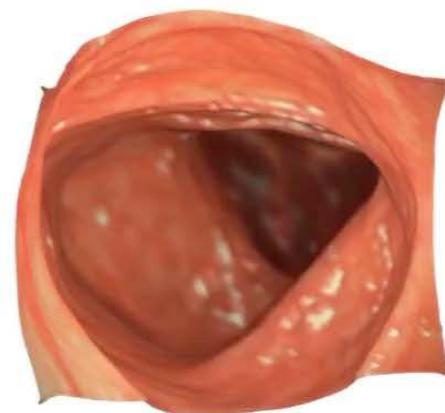
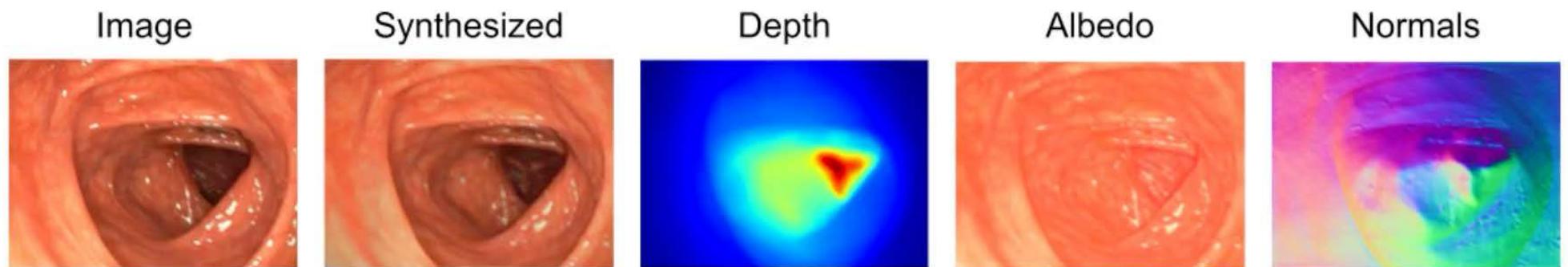


Light source and camera
→ at the same place (almost)

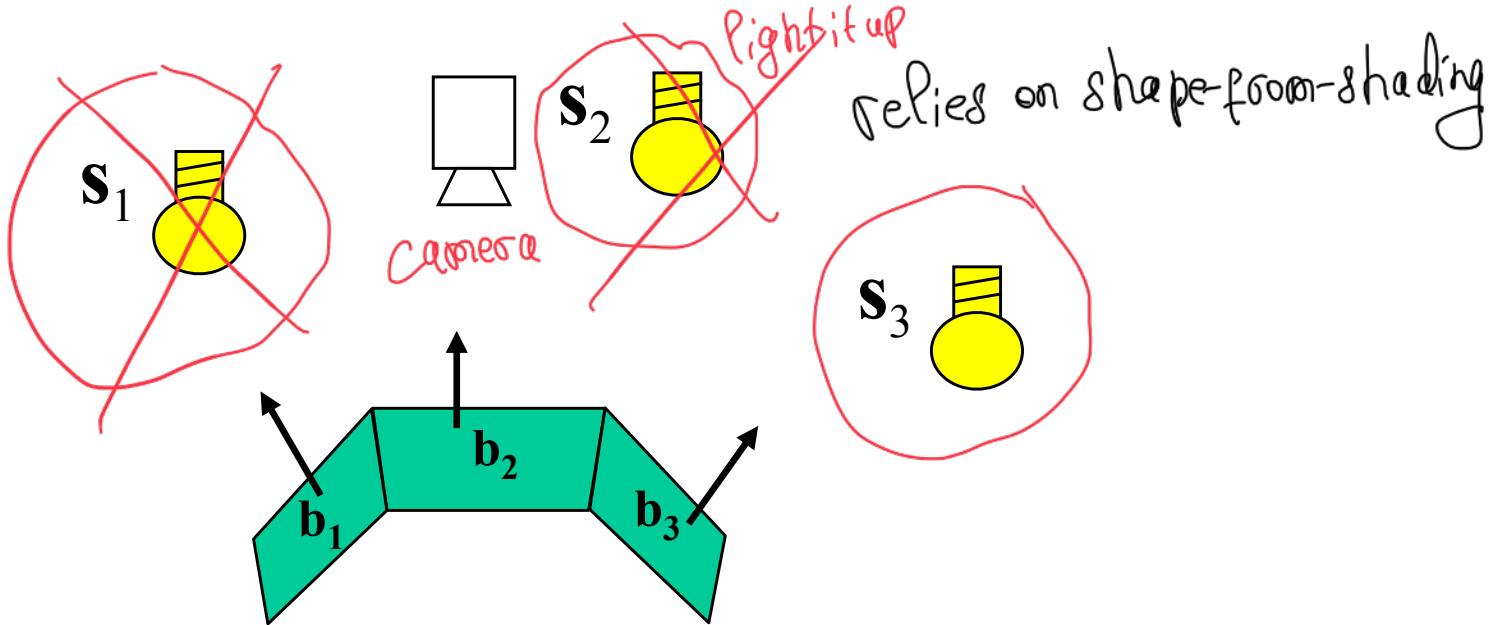


- The problem becomes well-posed but the variational approach assumes constant albedo, which is limiting.
- Can we take advantage of deep learning to overcome this problem?

Endoscopy 2023



Photometric Stereo

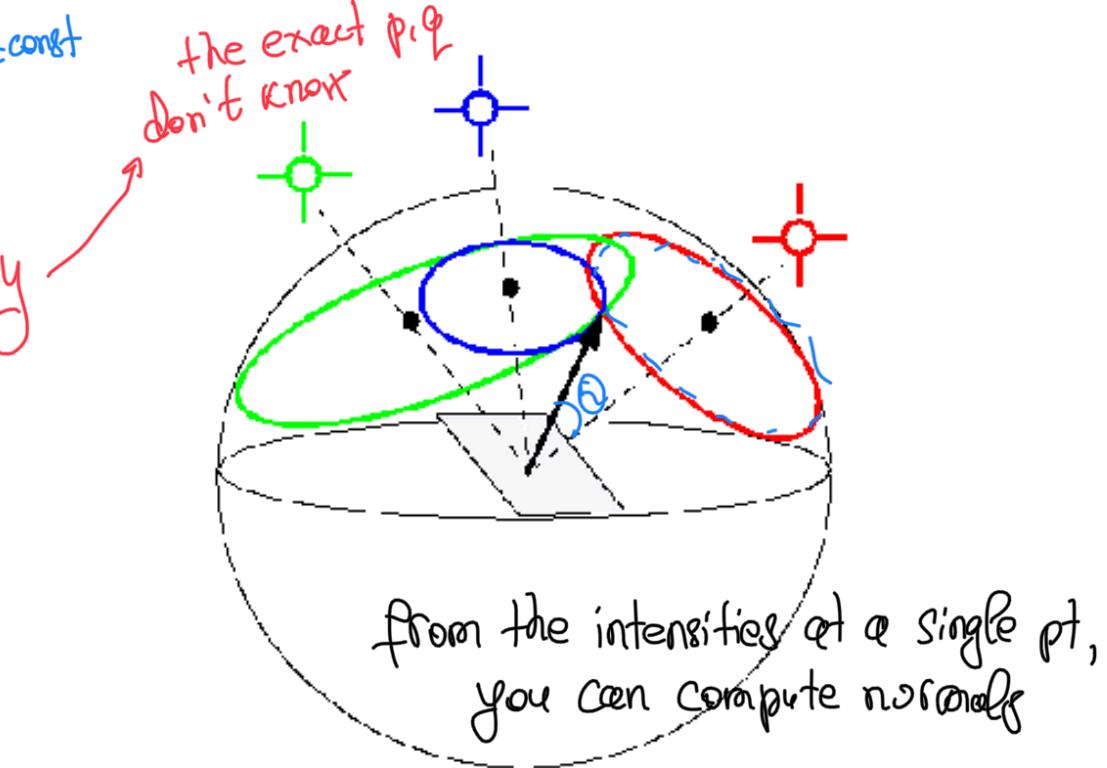
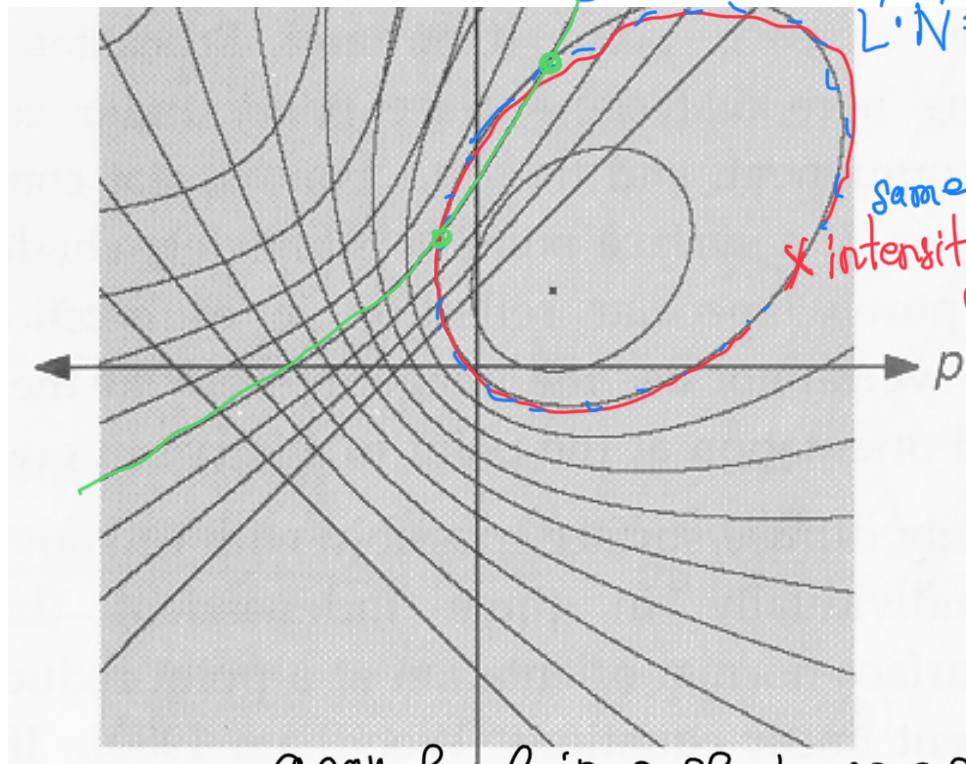


Given multiple images of the same surface under different known lighting conditions, can we recover the surface shape?

- Yes! (Woodham, 1978).
- This gave rise to an early and still practical application of computer vision.

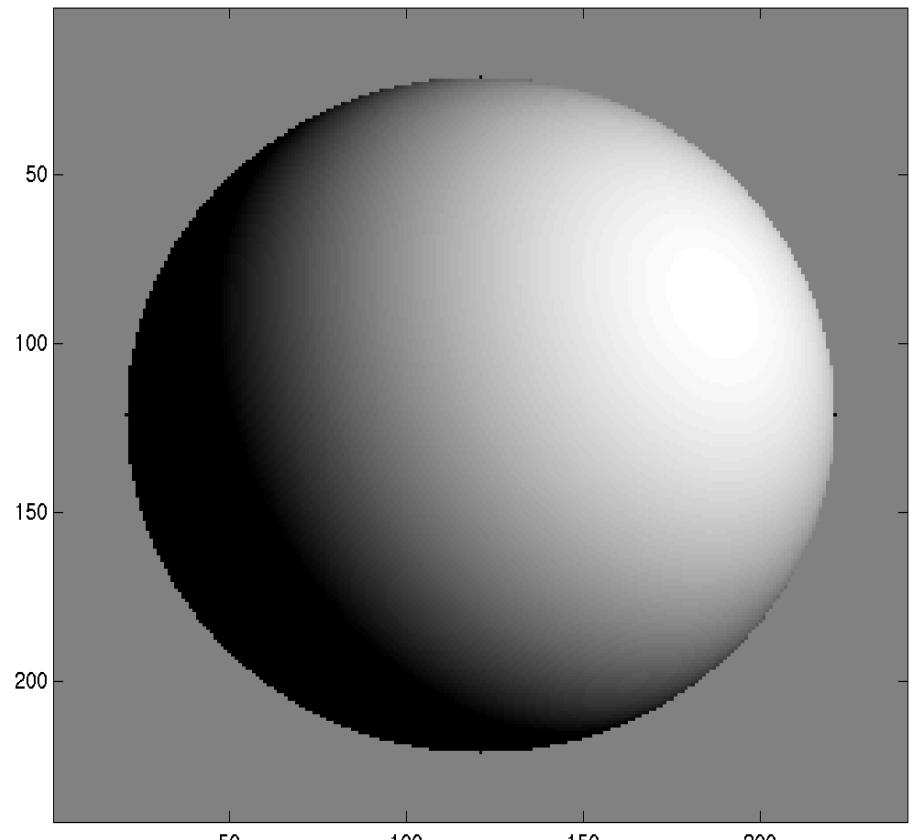
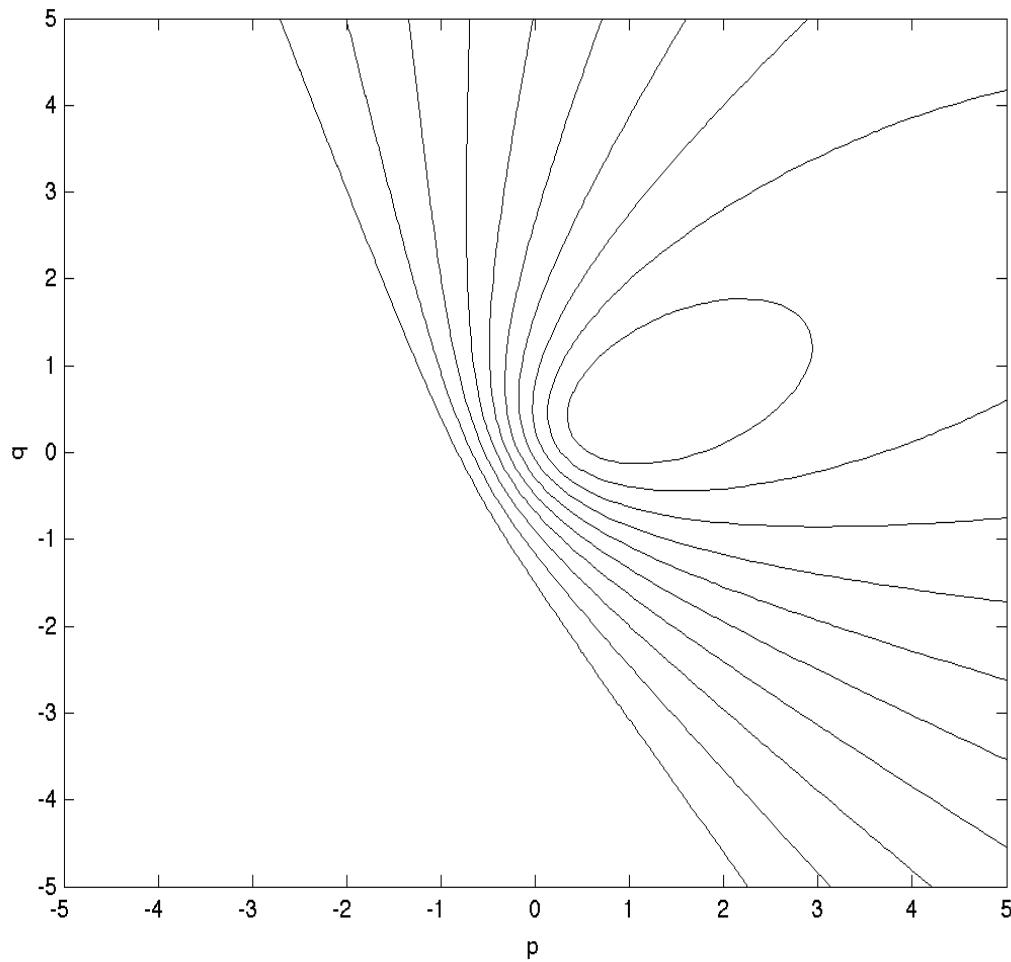
have the same angle θ with the direction of light source
go through this circle
all normals that

Basic Idea



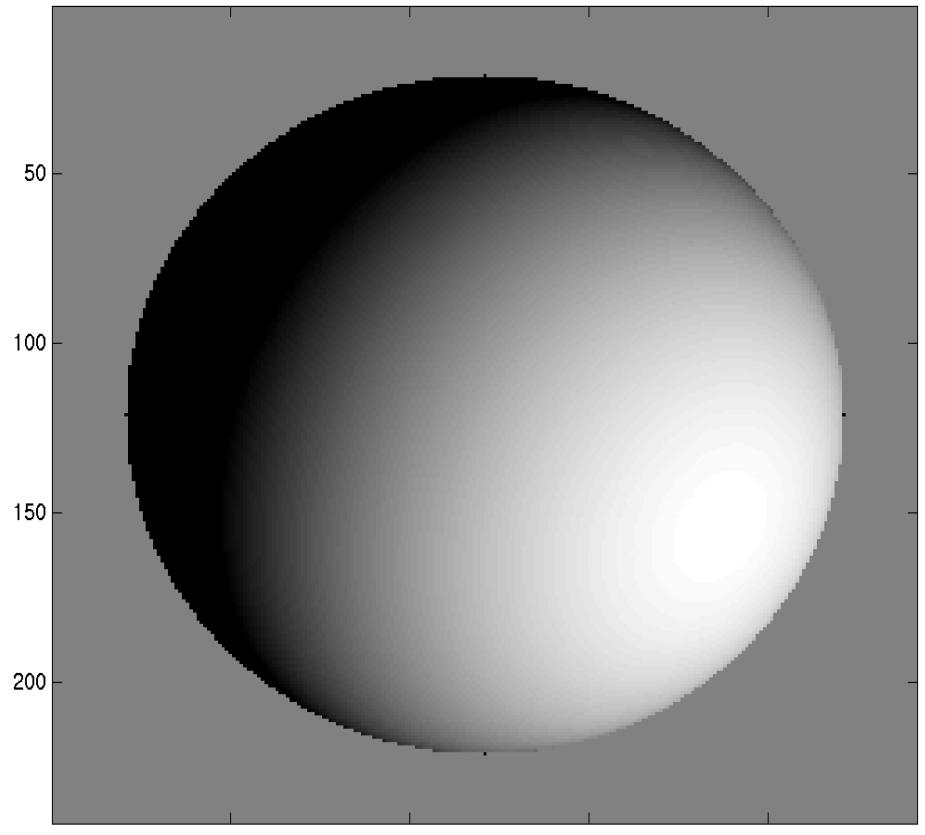
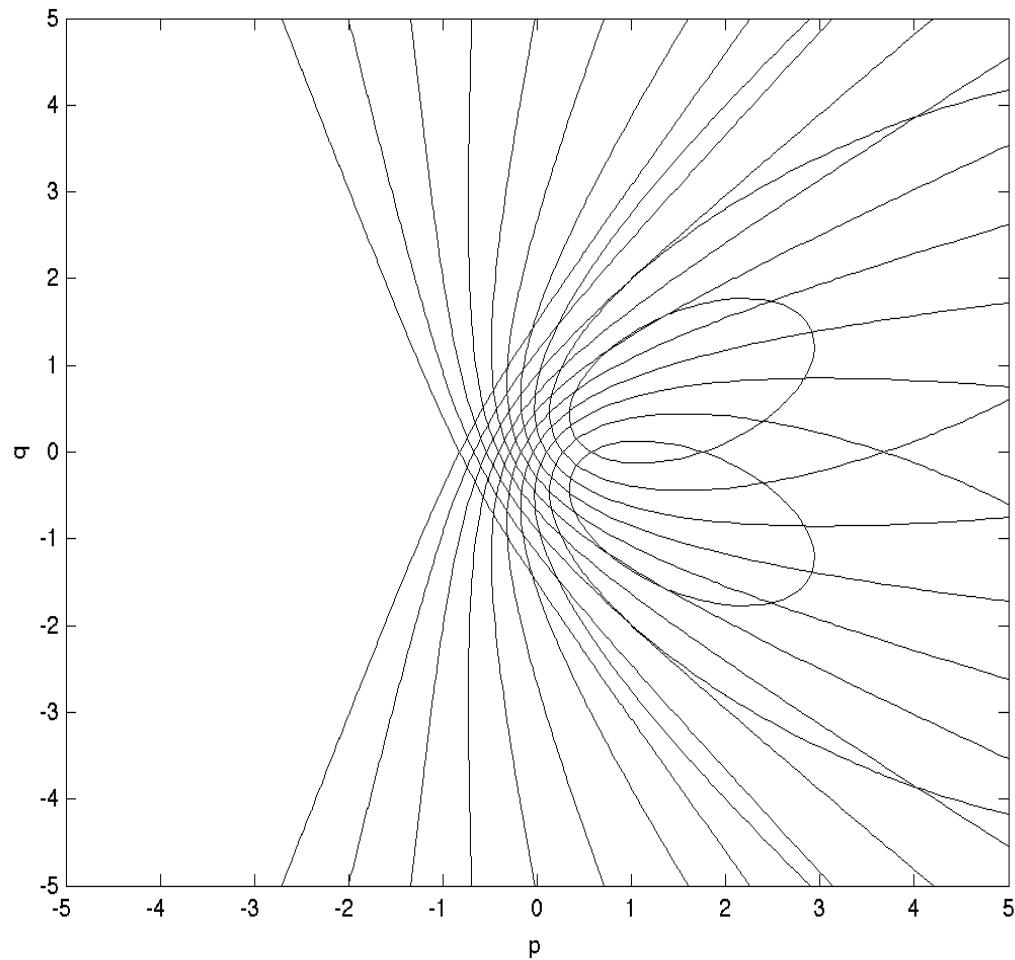
- Take several images under different lighting conditions.
- Infer the normals from the changes in illumination.
- Given at least three different lights, there are no more ambiguities.

One Single Light Source



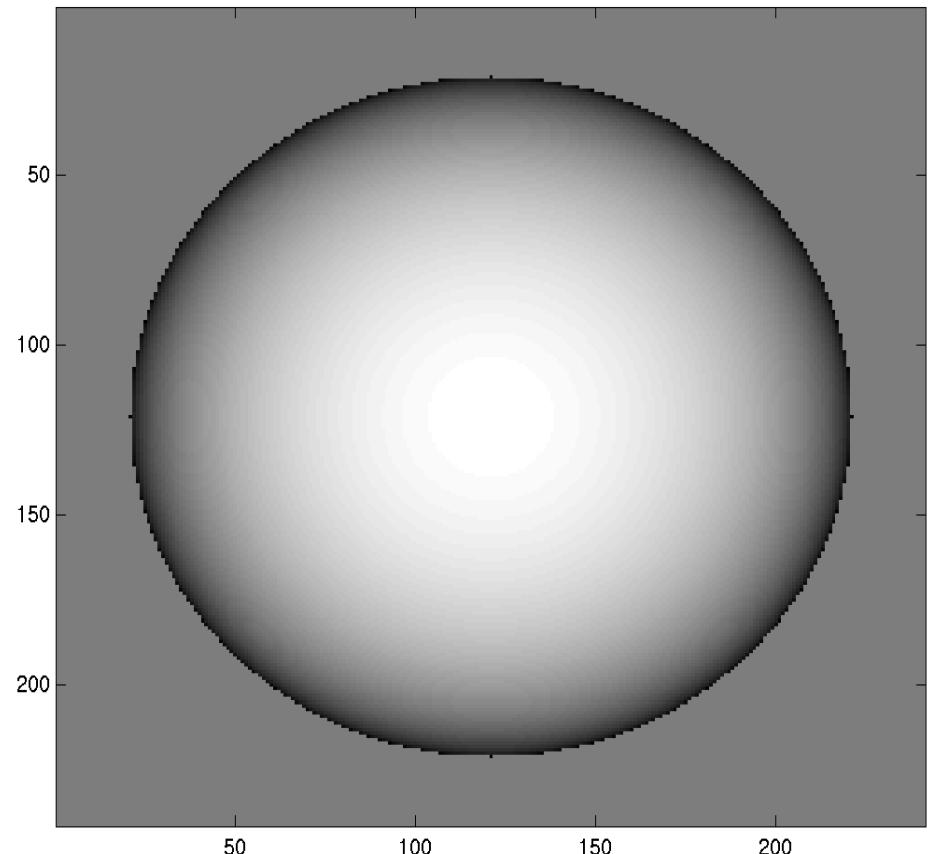
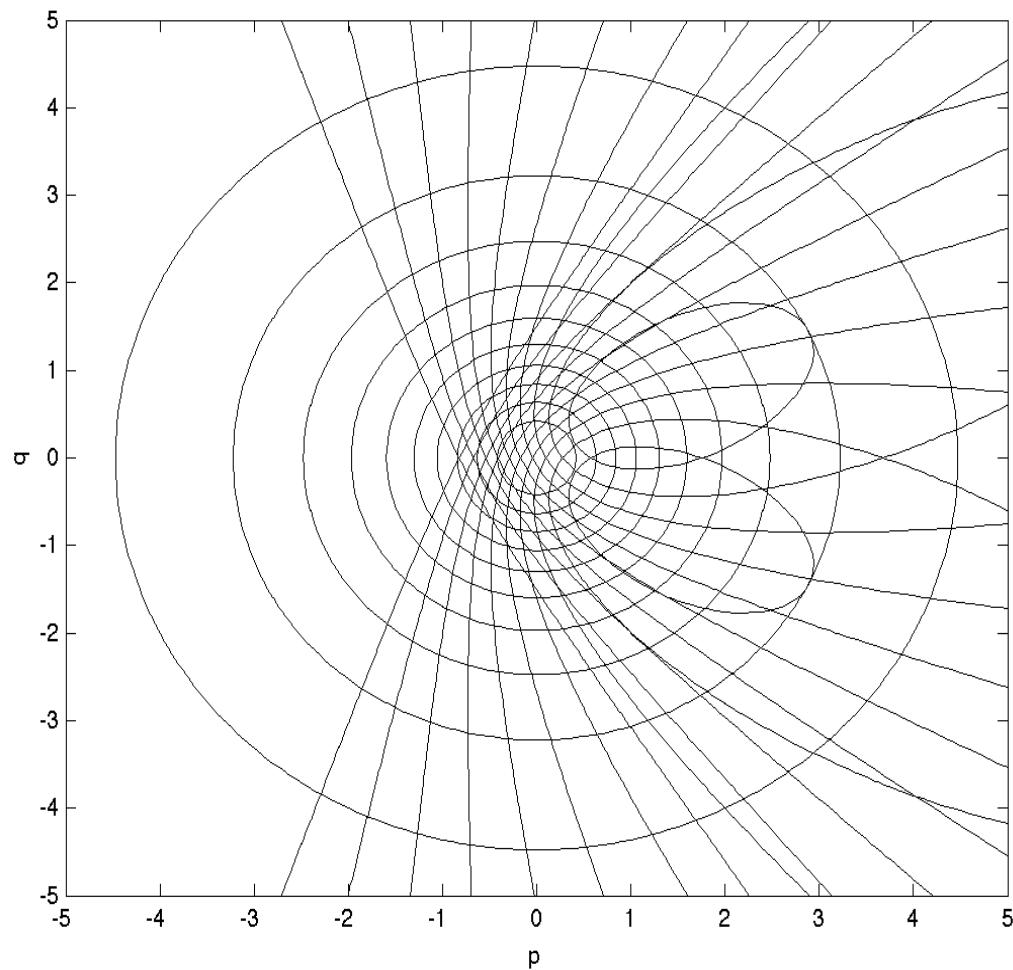
Many potential normals for each image point.

Two Light Sources



Still some ambiguities.

Three Light Sources



No more ambiguities even if the albedo is unknown.

Algebraic Formulation

Recover shape: estimate α, \mathbf{N} with known \mathbf{L}

Lambertian model:

$$I = \alpha (\mathbf{L} \cdot \mathbf{N})$$

Annotations:

- Intensity → Intensity
- α → alpha; unknown
- ($\mathbf{L} \cdot \mathbf{N}$) → estimate
- Light source: ✓ direction → unknown → normal vector

Unknown 3 vector that can be estimated by solving a 3x3 linear system.

Three light sources:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ \mathbf{L}_3 \end{bmatrix} \quad \mathbf{M}$$

Annotations:

- Vector of intensities: ✓
- Normals → $\mathbf{N} = \frac{\mathbf{M}}{\|\mathbf{M}\|}$
- Light source: ✓

\mathbf{N} and α can then be inferred from \mathbf{M} .

Using More Lights (≥ 3)

Photometric stereo

One can use as many lights as one wants:

$$\mathbf{I} = \mathbf{LM}, \text{ with } \mathbf{I} = \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} \text{ and } \mathbf{L} = \begin{bmatrix} \mathbf{L}_1 \\ \vdots \\ \mathbf{L}_n \end{bmatrix}$$

$$\Rightarrow \mathbf{L}^t \mathbf{LM} = \mathbf{L}^t \mathbf{I} \text{ (Least - squares solution)}$$

more equations $n \gg$ unknowns ($= 3$)

→ This is known as an over-constraint problem and, the more camera, the more robust to noise the solution is.

→ finding solution for M using ≥ 3 images

Discounting Shadows

In Reflectance maps, we didn't consider some pixels might shadow, thus appearing darker

be in

- Shadowed pixels for a given light source position do not conform to the model.
- Premultiplying by the intensities reduces their contributions because their intensities tend to be lower.

minimize $\sum_j (I_j - L_j M)^2$

Intensities on the diagonal

$$I = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_n \end{bmatrix} \quad LM = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_n \end{bmatrix}$$

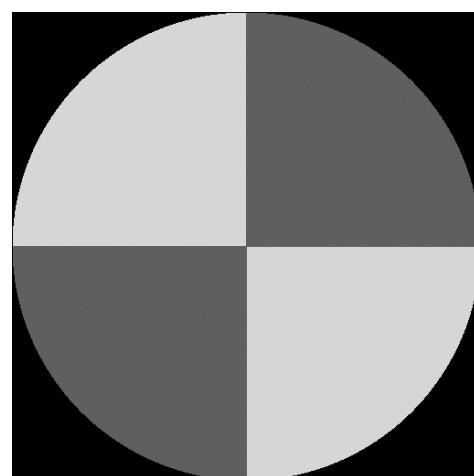
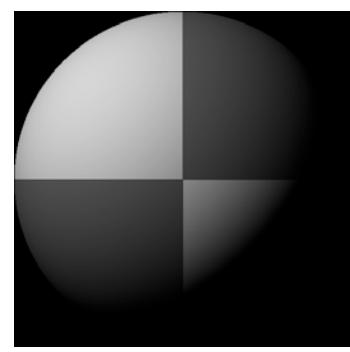
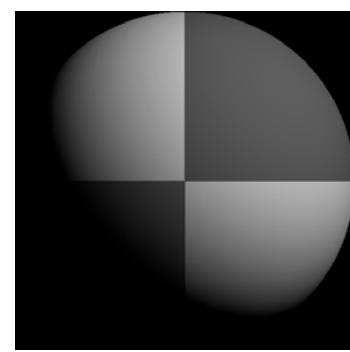
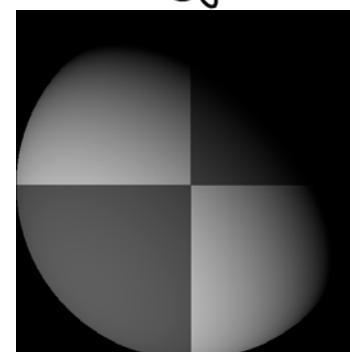
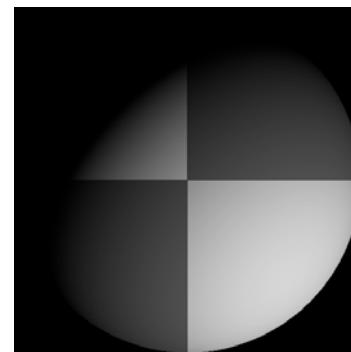
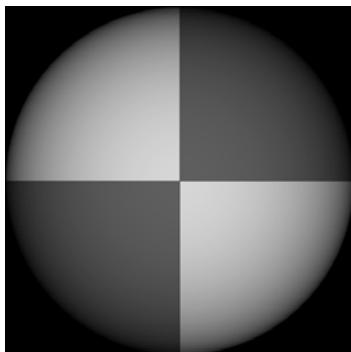
→ weights \propto intensities

→ pixels having low intensities have less

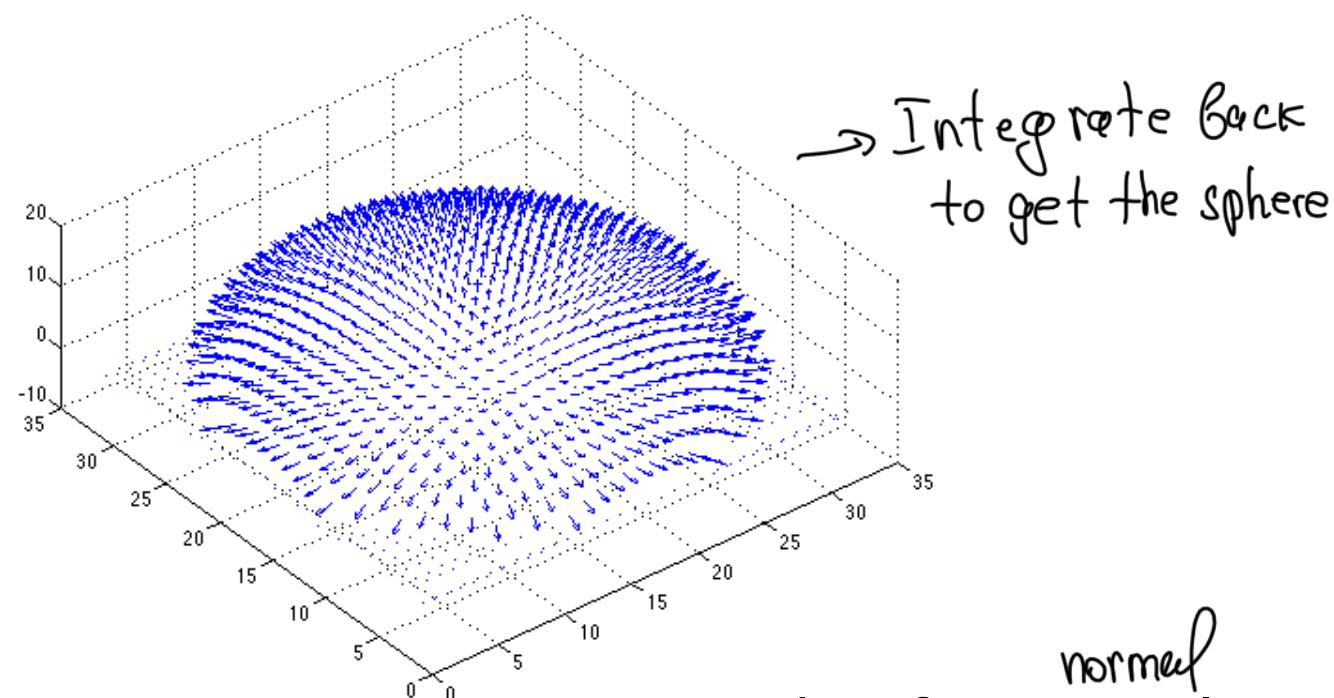
influence on others (those on shadows = \rightarrow)

Synthetic Sphere Images

Five different light sources:



Recovered albedo



Recovered surface normals

normal
map

Scanning Michel Angelo's Pieta



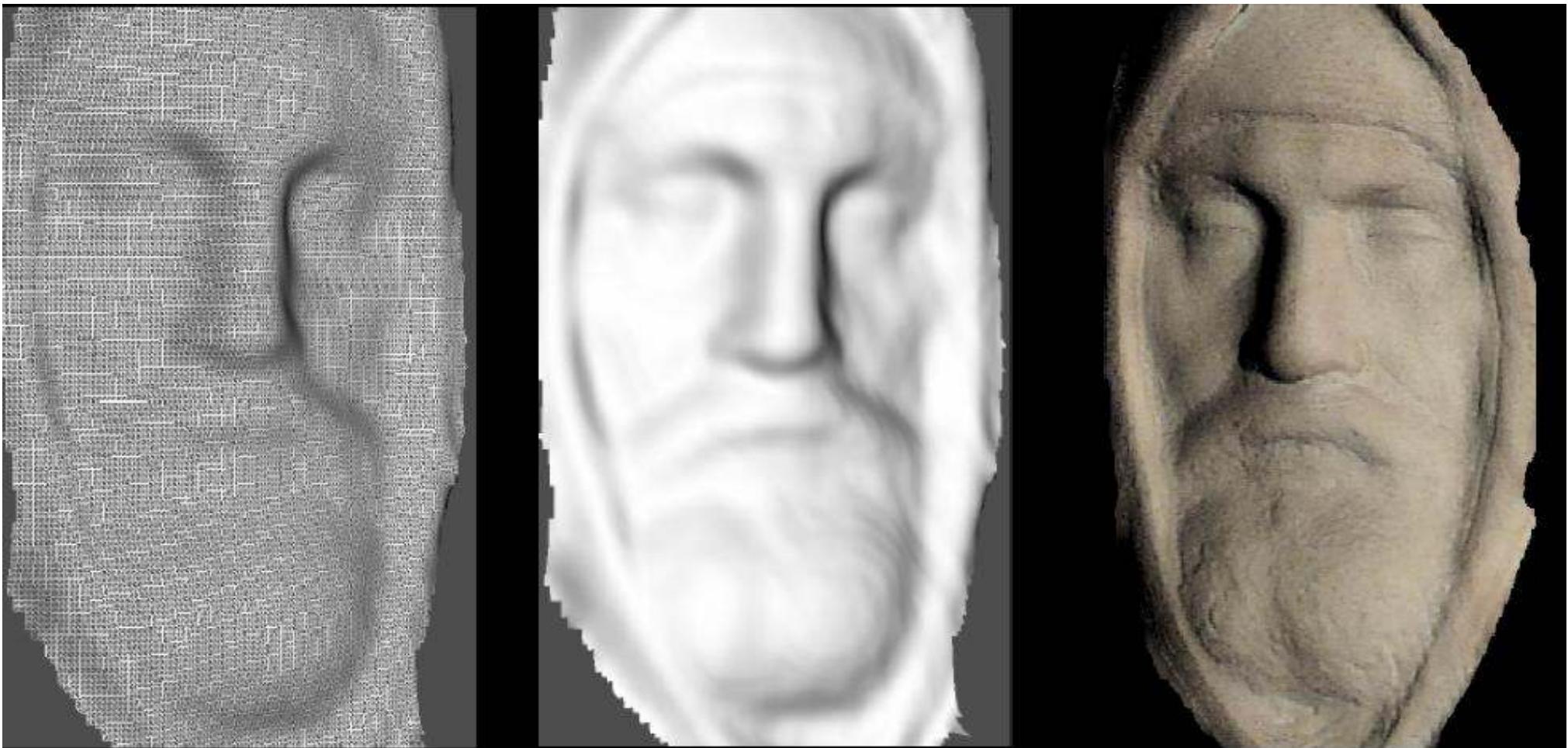
- One camera and five light sources. They know L_1, \dots, L_5
- The positions of the light sources w.r.t. the camera are exactly known.

Full 3D Model

Normals

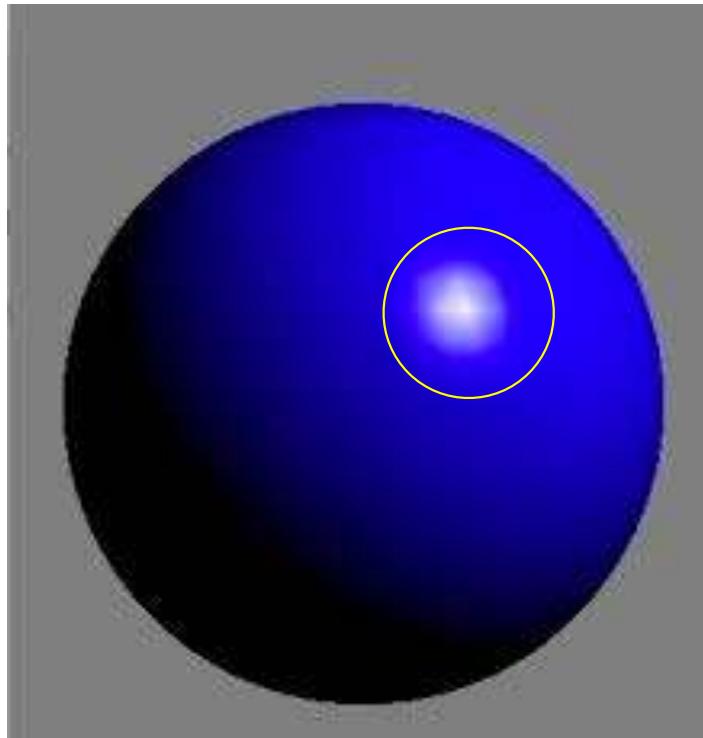
integrate
Shaded surface

using computer algebra
Full rendering



- This was done for the whole statue.
- All the fragments were then “glued” together.
—> A full 3D model that can be visualized from any viewpoint and under any illumination conditions. (even animated)

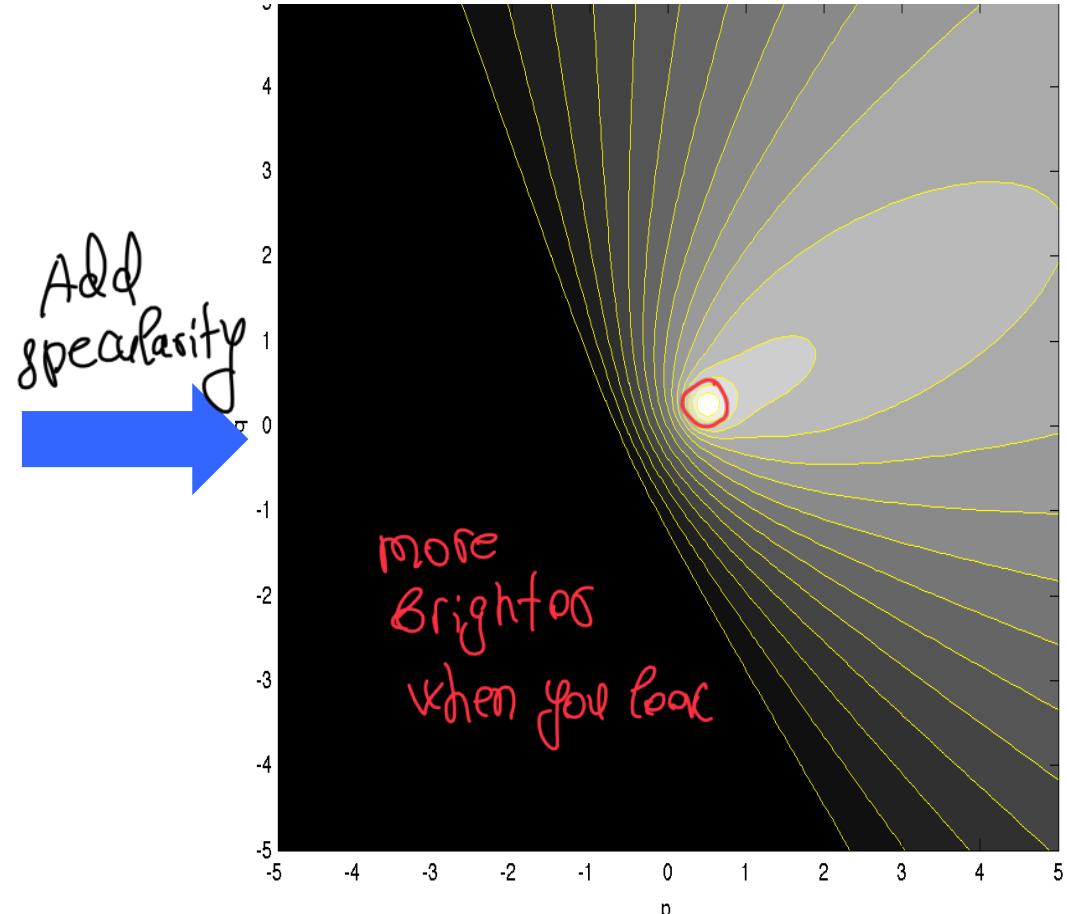
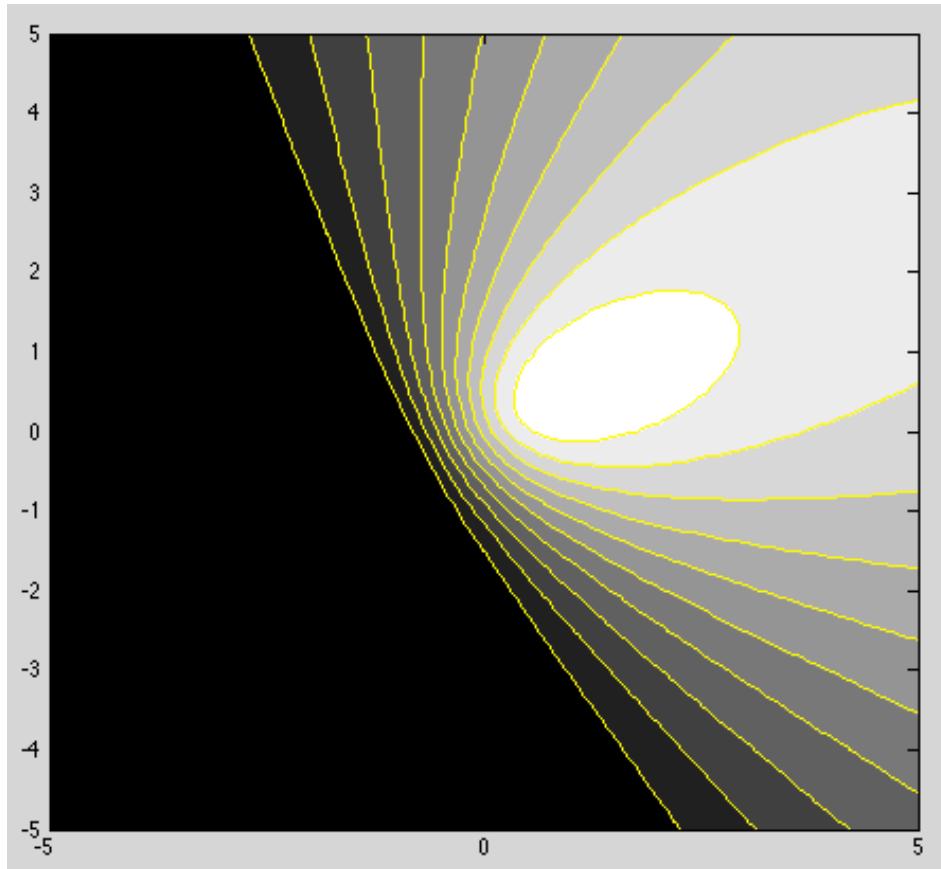
Specularities



- At specular points Lambertian assumptions are violated.
- The surface behaves like a mirror. (brighter than it should be)
- This is known as specular reflection.
- Most surfaces are a combination of diffuse and specular reflectors.

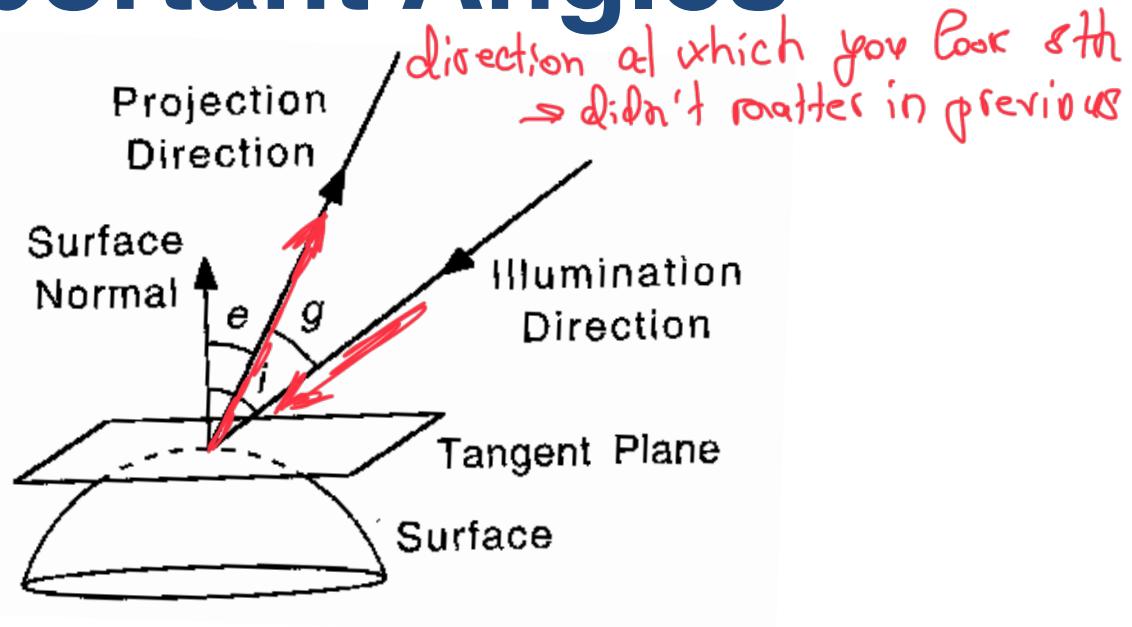
Lambertian + Specular Reflectance Map

The notion of reflectance map still applies but the function Ref becomes more complex than in the diffuse case:



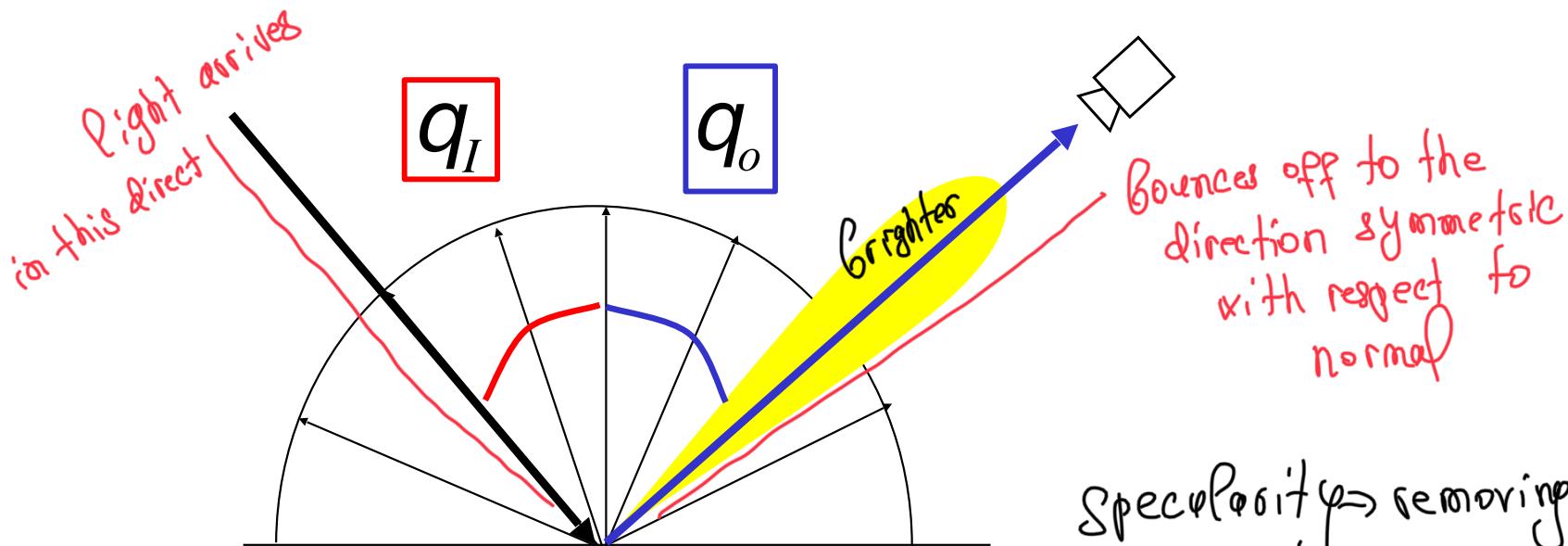
Reflectance map for glossy white paint: It is the weighted sum of a diffuse and a specular component.

Important Angles



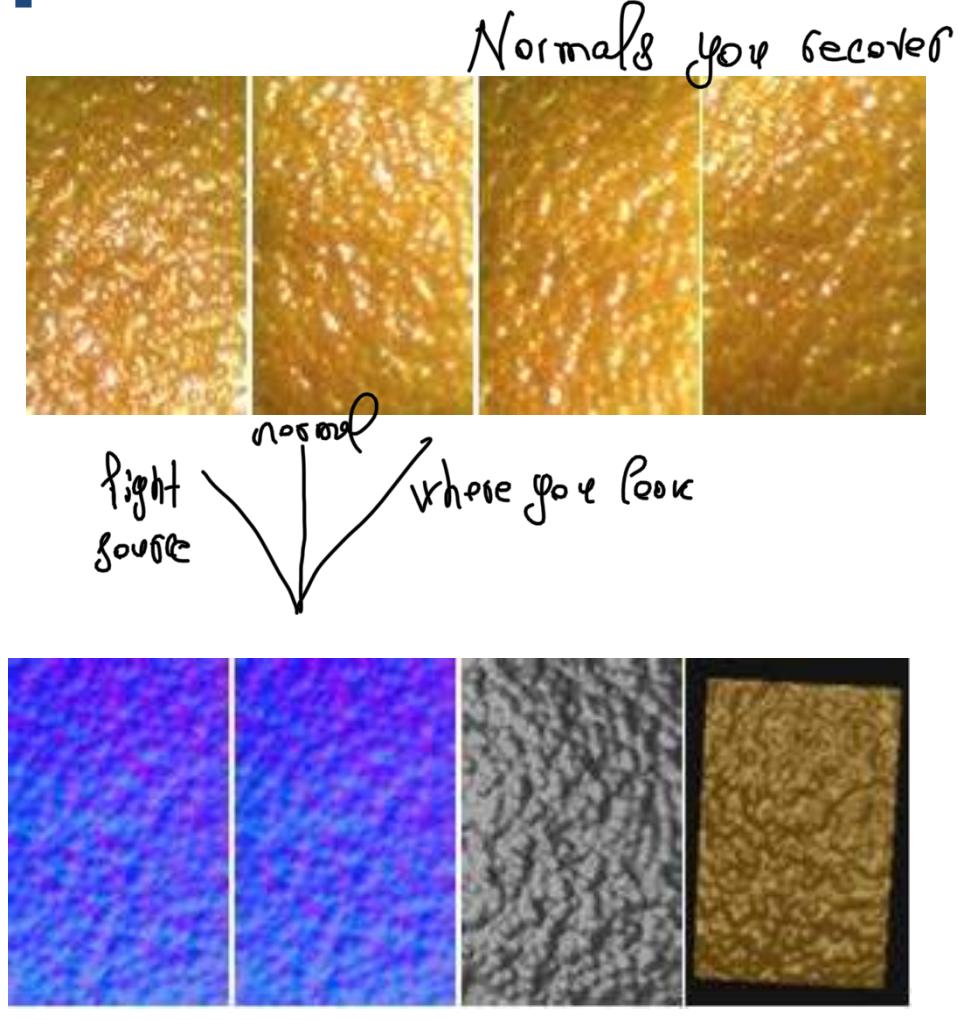
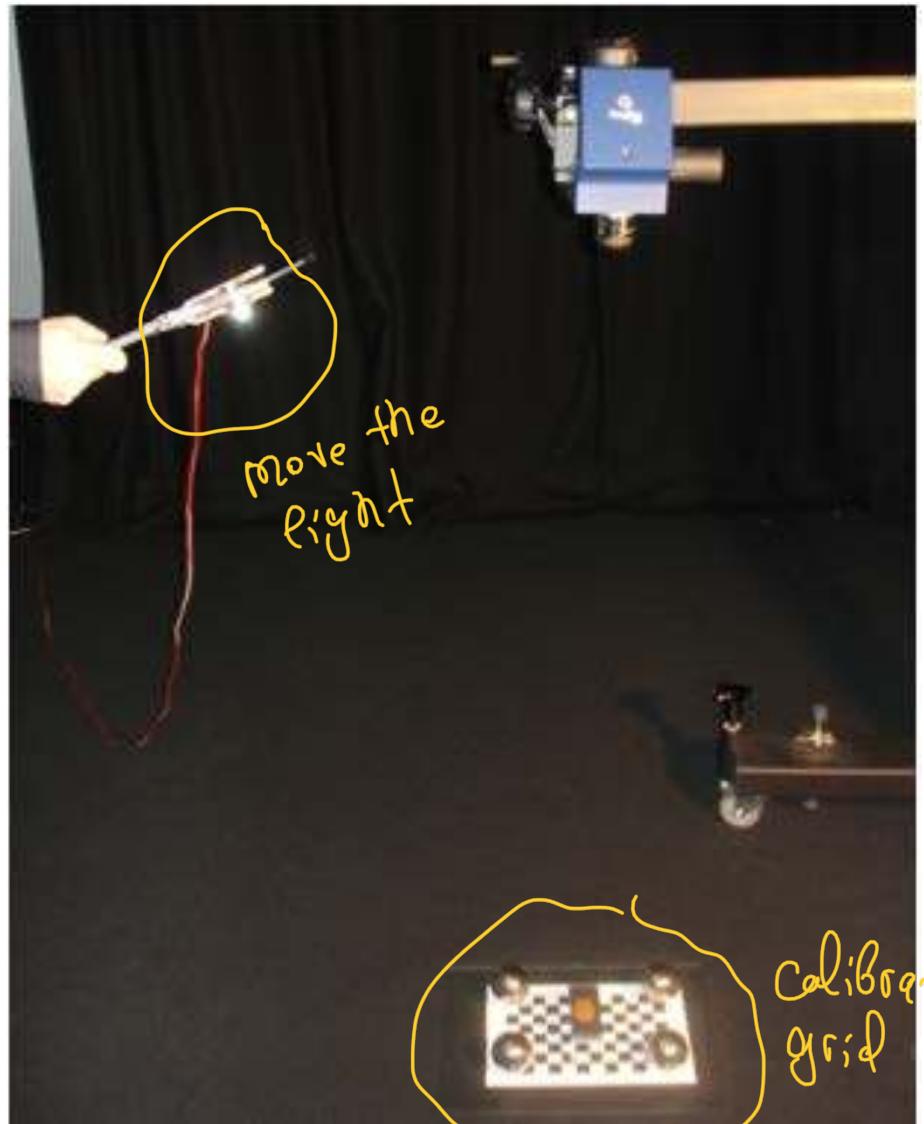
- **Angle of incidence (i):** Angle between the surface normal and the direction of the incident light ray. $\vec{L} \cdot \vec{N}$
- **Angle of emittance (e):** Angle between emitted light ray and surface normal.
- **Phase angle (g):** Angle between incident and emitted light ray.

Mirror-Like Behavior



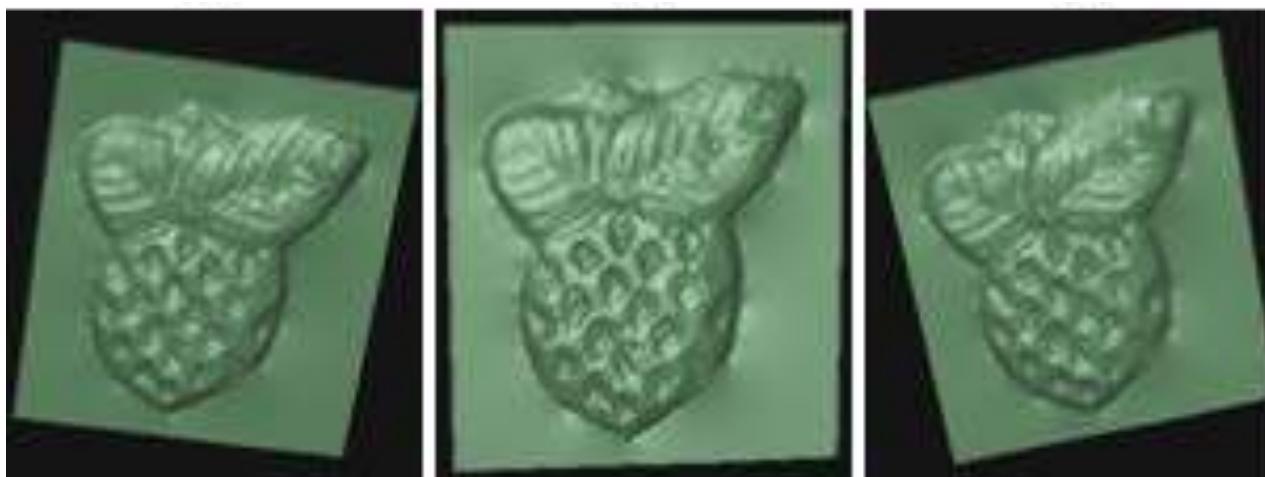
- Specularities occur when $i=e$ and $i+e=g$.
(do not obey Lambertian laws)
- In other words, when the two directions are symmetric with respect to the normal.
Solving LS in robust way
→ They can also be used to infer the normals.

Shape from Specularities



- Move the light around and compute its position each time.
- Find the bright spots and the image and assume they are specularities.
- Infer the normals at those points.

Shape from Specularities



Shiny images

- Excellent precision can be achieved because the specularities are very sensitive to the exact normal direction.
- However, this only works well for shiny, that is, highly specular, objects.

Specularities in Short



- Most materials are both diffuse and specular.
 - Specularities are very sensitive to motion. → they can provide additional info
Besides shape-from-shading
 - Specularities do not constrain all the degrees of freedom.
- They can either be discarded as noise or used to get additional information.

Shape-from-Shading in Short

Traditional Shape-from-Shading requires strong assumptions:

- Constant or piece-wise constant albedo
- No interreflections
- No shadows
- No specularities
ignore ↑

photometric stereo → useful ✓

→ In a single image context, it is most useful in conjunction with other information sources.