SEQUENCES AND TREES 4

COMPUTER SCIENCE 61A

February 19, 2015

1 List Comprehension

A **list comprehension** is a compact way to create a list whose elements are the results of applying a fixed expression to elements in another sequence.

```
[<map exp> for <name> in <iter exp> if <filter exp>]
```

Let's break down an example:

```
[x * x - 3 for x in [1, 2, 3, 4, 5] if x % 2 == 1]
```

In this list comprehension, we are creating a new list after performing a series of operations to our initial sequence `[1, 2, 3, 4, 5]`. We only keep the elements that satisfy the filter expression `x % 2 == 1` (`1`, `3`, and `5`). For each retained element, we apply the map expression `x*x - 3` before adding it to the new list that we are creating, resulting in the output `[-2, 6, 22]`.

Note: The `if` clause in a list comprehension is optional.

1.1 Questions

1. What would Python print?

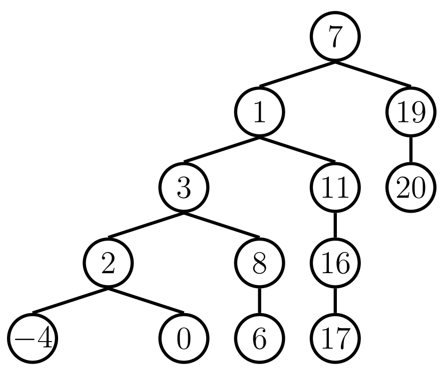
```
>>> [i + 1 for i in [1, 2, 3, 4, 5] if i % 2 == 0]

>>> [i * i for i in [5, -1, 3, -1, 3] if i > 2]

>>> [[y * 2 for y in [x, x + 1]] for x in [1, 2, 3, 4]]
```

2. Define a function `foo` that takes in a list `lst` and returns a new list that keeps only the even-indexed elements of `lst` and multiples each of those elements by the corresponding index.

```
def foo(lst):
    """
    >>> x = [1, 2, 3, 4, 5, 6]
    >>> foo(x)
    [0, 6, 20]
    """

    return [______________________________________________]
```

2 Trees

In computer science, **trees** are recursive data structures that are widely used in various settings. This is a diagram of a simple tree.



Notice that the tree branches downward – in computer science, the **root** of a tree starts at the top, and the **leaves** are at the bottom.

Some terminology regarding trees:

- **Parent node**: A node that has children. Parent nodes can have multiple children.
- **Child node**: A node that has a parent. A child node can only belong to one parent.
- **Root**: The top node of the tree. In our example, the node that contains 7 is the root.
- **Leaf**: A node that has no children. In our example, the nodes that contain -4, 0, 6, 17, and 20 are leaves.
- **Subtree**: Notice that each child of a parent is itself the root of a smaller tree. In our example, the node containing 1 is the root of another tree. This is why trees are *recursive* data structures: trees are made up of subtrees, which are trees themselves.

- **Depth**: How far away a node is from the root. In other words, the length of the path from the root to the node. In the diagram, the node containing 19 has depth 1; the node containing 3 has depth 2. We define the depth of the root of a tree is 0.
- **Height**: The depth of the lowest leaf. In the diagram, the nodes containing -4, 0, 6, and 17 are all the "lowest leaves," and they have depth 4. Thus, the entire tree has height 4.

In computer science, there are many different types of trees – some vary in the number of children each node has, and others vary in the structure of the tree.

2.1 Implementation

A tree has both a root value and a sequence of branches. In our implementation, we represent the branches as lists of subtrees.

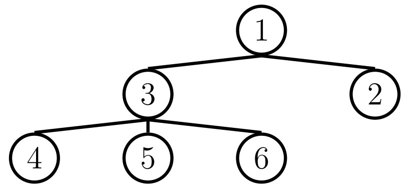
- The arguments to the constructor, `tree`, as a value for the root and a list of branches.
- The selectors are `root` and `branches`.

```
# Constructor
def tree(value, branches=[]):
    for branch in branches:
        assert is_tree(branch), 'branches must be trees'
    return [value] + list(branches)

# Selectors
def root(tree):
    return tree[0]

def branches(tree):
    return tree[1:]
```

We have also provided two convenience functions, `is_leaf` and `is_tree`:

```
def is_leaf(tree):
    return not branches(tree)

def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

It's simple to construct a tree. Let's try to create the following tree:




```
t = tree(1,
         [tree(3,
               [tree(4),
                tree(5),
                tree(6)]),
          tree(2)])
```

The use of whitespace can help with legibility, but it is not required.

2.2 Questions

1. Define a function `square_tree(t)` that squares every item in the tree `t`. It should return a new tree. You can assume that every item is a number.

```
def square_tree(t):
    """Return a tree with the square of every element in t"""
```

2. Define a function `height(t)` that returns the height of a tree. Recall that the height of a tree is the length of the longest path from the root to a leaf.

```
def height(t):
    """Return the height of a tree"""
```

3. Define a function `tree_size(t)` that returns the number of nodes in a tree.

```
def tree_size(t):
    """Return the size of a tree."""
```

4. Define a function `tree_max(t)` that returns the largest number in a tree.

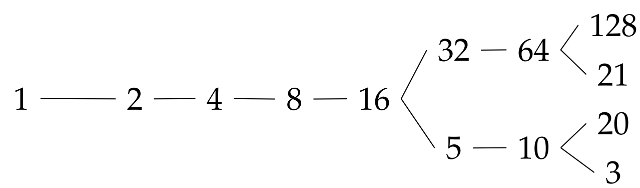
```
def tree_max(t):
    """Return the max of a tree."""
```

2.3 Extra Questions

1. An **expression tree** is a tree that contains a function for each non-leaf root, which can be either `add` or `mul`. All leaves are numbers. Implement `eval_tree`, which evaluates an expression tree to its value. You may find the functions `reduce` and `apply_to_all`, introduced during lecture, useful.

```
def reduce(fn, s, init):
    reduced = init
    for x in s:
        reduced = fn(reduced, x)
    return reduced

def apply_to_all(fn, s):
    return [fn(x) for x in s]

from operator import add, mul
def eval_tree(tree):
    """Evaluates an expression tree with functions as root
    >>> eval_tree(tree(1))
    1
    >>> expr = tree(mul, [tree(2), tree(3)])
    >>> eval_tree(expr)
    6
    >>> eval_tree(tree(add, [expr, tree(4)]))
    10
    """
```

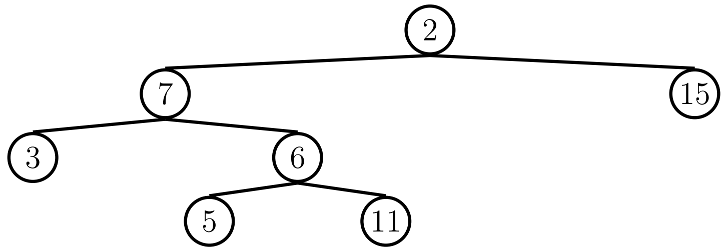
2. We can represent the hailstone sequence as a tree in the figure below, showing the route different numbers take to reach 1. Remember that a hailstone sequence starts with a number n, continuing to $n/2$ if n is even or $3n + 1$ if n is odd, ending with 1. Write a function `hailstone_tree(n, h)` which generates a tree of height `h`, containing hailstone numbers that will reach `n`.




```
def hailstone_tree(n, h):
    """Generates a tree of hailstone numbers that will reach N
       , with height H.
    >>> hailstone_tree(1, 0)
    [1]
    >>> hailstone_tree(1, 4)
    [1, [2, [4, [8, [16]]]]]
    >>> hailstone_tree(8, 3)
    [8, [16, [32, [64]], [5, [10]]]]
    """
```

3. Define the procedure `find_path(tree, x)` that, given a rooted tree `tree` and a value `x`, returns a list containing the nodes along the path required to get from the root of `tree` to a node `x`. If `x` is not present in `tree`, return `False`. Assume that the elements in `tree` are unique.

   For the following tree, `find_path(t, 5)` should return `[2, 7, 6, 5]`




```
def find_path(tree, x):
    """ Returns a path in a tree to a leaf with value X,
    False if such a leaf is not present.
    >>> t = tree(2, [tree(7, [tree(3), tree(6, [tree(5), tree
        (11)])]), tree(15)])
    >>> find_path(t, 5)
    [2, 7, 6, 5]
    >>> find_path(t, 6)
    [2, 7, 6]
    >>> find_path(t, 10)
    False
    """
```