A Synopsis of Special Relativity

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Relativity has been studied for the past one-hundred years. This is generic derivation of Special Relativity.

Special Relativity was developed by three prominent Physicsists and Mathematicians.

1. Einstien
2. Lorentz
3. Poincaré

Later, after originally published, Special Relativity was studied by Minkowski.

This paper is a synopsis of the Mathematical Derivation of Special Relativity. The derivation is taking directly from Maxwell's equations of Light, using the Gallilean Transform (Gallilean Relativity).

1 The Gallilean Transform

The Gallilean Transform consists of the following equations:

$$
\begin{aligned}
x' &= x - vt \\
y' &= y \\
z' &= z \\
t' &= t
\end{aligned}
$$

And

$$
\begin{aligned}
x &= x' + vt \\
y &= y' \\
z &= z' \\
t &= t'
\end{aligned}
$$

Where the "prime" frame is the frame of motion and the non-"prime" frame is the "stationary" frame.

As postulated by Einstein, there is no "stationary" frame. Hence, this idea is merely a basic understanding of the difference between the observer in motion vs. the stationary observer. For the purposes of this paper, the stationary observer is the observer that "sees" the other observer in motion.

2 Maxwell's Equations

Simplistic version of Maxwell's equation is as follows:

$$\nabla^2\varphi = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}$$

3 Gallilean Relativity into Maxwell's Equations

Putting Gallilean Transforms into Maxwell's Equations, the equations are the following:

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\varphi}{\partial x'}\cancelto{1}{\frac{\partial x'}{\partial x}} + \cancel{\frac{\partial\varphi}{\partial y'}}\cancelto{0}{\frac{\partial y'}{\partial x}} + \cancel{\frac{\partial\varphi}{\partial z'}}\cancelto{0}{\frac{\partial z'}{\partial x}} + \cancel{\frac{\partial\varphi}{\partial t'}}\cancelto{0}{\frac{\partial t'}{\partial x}}$$
$$\therefore \frac{\partial\varphi}{\partial x} = \frac{\partial\varphi}{\partial x'}$$

And hence:

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{\partial^2\varphi}{\partial x'^2}$$

$$\frac{\partial\varphi}{\partial y} = \cancel{\frac{\partial\varphi}{\partial x'}}\cancelto{0}{\frac{\partial x'}{\partial y}} + \frac{\partial\varphi}{\partial y'}\cancelto{1}{\frac{\partial y'}{\partial y}} + \cancel{\frac{\partial\varphi}{\partial z'}}\cancelto{0}{\frac{\partial z'}{\partial y}} + \cancel{\frac{\partial\varphi}{\partial t'}}\cancelto{0}{\frac{\partial t'}{\partial y}}$$
$$\therefore \frac{\partial\varphi}{\partial y} = \frac{\partial\varphi}{\partial y'}$$

And hence:

$$\frac{\partial^2\varphi}{\partial y^2} = \frac{\partial^2\varphi}{\partial y'^2}$$

$$\frac{\partial\varphi}{\partial z} = \cancel{\frac{\partial\varphi}{\partial x'}}\cancelto{0}{\frac{\partial x'}{\partial z}} + \cancel{\frac{\partial\varphi}{\partial y'}}\cancelto{0}{\frac{\partial y'}{\partial z}} + \frac{\partial\varphi}{\partial z'}\cancelto{1}{\frac{\partial z'}{\partial z}} + \cancel{\frac{\partial\varphi}{\partial t'}}\cancelto{0}{\frac{\partial t'}{\partial z}}$$
$$\therefore \frac{\partial\varphi}{\partial z} = \frac{\partial\varphi}{\partial z'}$$

And hence:

$$\frac{\partial^2\varphi}{\partial z^2} = \frac{\partial^2\varphi}{\partial z'^2}$$

$$\frac{\partial\varphi}{\partial t} = \frac{\partial\varphi}{\partial x'}\cancelto{-v}{\frac{\partial x'}{\partial t}} + \frac{\partial\varphi}{\cancel{\partial y'}}\cancelto{0}{\frac{\partial y'}{\partial t}} + \frac{\partial\varphi}{\cancel{\partial z'}}\cancelto{0}{\frac{\partial z'}{\partial t}} + \frac{\partial\varphi}{\partial t'}\cancelto{1}{\frac{\partial t'}{\partial t}}$$

$$\therefore \left.\frac{\partial\varphi}{\partial t} = \frac{\partial\varphi}{\partial t'} - v\frac{\partial\varphi}{\partial x'}\right\}\alpha$$

Thus:

$$\frac{\partial\varphi^2}{\partial t^2} = \frac{\partial[\alpha]}{\partial t} = \frac{\partial[\alpha]}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial[\alpha]}{\partial t'}\frac{\partial t'}{\partial t}$$

$$\frac{\partial\varphi^2}{\partial t^2} = \frac{\partial[\alpha]}{\partial x'}\cancelto{-v}{\frac{\partial x'}{\partial t}} + \frac{\partial[\alpha]}{\partial t'}\cancelto{1}{\frac{\partial t'}{\partial t}}$$

$$\frac{\partial\varphi^2}{\partial t^2} = \frac{\partial[\alpha]}{\partial t'} - v\frac{\partial[\alpha]}{\partial x'}$$

$$\frac{\partial\varphi^2}{\partial t^2} = \frac{\partial}{\partial t'}[\frac{\partial\varphi}{\partial t'} - v\frac{\partial\varphi}{\partial x'}] - v\frac{\partial}{\partial x'}[\frac{\partial\varphi}{\partial t'} - v\frac{\partial\varphi}{\partial x'}]$$

$$\frac{\partial^2\varphi}{\partial t^2} = \frac{\partial^2\varphi}{\partial t'^2} - 2v\frac{\partial\varphi^2}{\partial x\partial t'} + v^2\frac{\partial^2\varphi}{\partial x'^2}$$

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t'^2} - \frac{2v}{c^2}\frac{\partial\varphi^2}{\partial x\partial t'} + \frac{v^2}{c^2}\frac{\partial^2\varphi}{\partial x'^2}$$

Which follows:

$$\frac{\partial^2\varphi}{\partial x'^2} + \frac{\partial^2\varphi}{\partial y'^2} + \frac{\partial^2\varphi}{\partial z'^2} = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t'^2} - \frac{2v}{c^2}\frac{\partial\varphi^2}{\partial x\partial t'} + \frac{v^2}{c^2}\frac{\partial^2\varphi}{\partial x'^2}$$

$$\therefore (1 - \frac{v^2}{c^2})\frac{\partial^2\varphi}{\partial x'^2} + \frac{\partial^2\varphi}{\partial y'^2} + \frac{\partial^2\varphi}{\partial z'^2} + \frac{2v}{c^2}\frac{\partial\varphi^2}{\partial x\partial t'} = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t'^2}$$

$$\boxed{(1 - \frac{v^2}{c^2})\frac{\partial^2\varphi}{\partial x'^2} + \frac{\partial^2\varphi}{\partial y'^2} + \frac{\partial^2\varphi}{\partial z'^2} + \frac{2v}{c^2}\frac{\partial\varphi^2}{\partial x\partial t'} = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t'^2}}$$

This shows that plugging in Galilean Relativity into Maxwell's Equations **is not** invariant. This shows a problem with the equations as related to Galilean Relativity.

Under the given situation of non-invariance, the question arises what would the relativistic equations be to make Maxwell's equations invariant?

From a mathematical point of view, the answer lies in the expression: $\boxed{\frac{2v}{c^2}\frac{\partial\varphi^2}{\partial x\partial t'}}$

Which would imply a direct **relation to time and space**.

4 A Correction to Galilean Relatity

Based on the section above, a new set of Relativity equations need to be developed in such a way as to ensure invariance between reference frames.

$$\begin{aligned} x' &= Ax + Bvt \\ y' &= y \\ z' &= z \\ t' &= Dt + Ex \end{aligned}$$

Letting $x' = 0$, $x = vt$, and $-Avt = Bvt$
$\therefore B = -A$ and

$$x' = A(x - vt)$$

with:

$$t' = Dt + Ex$$

These equations can be substituted into Maxwell's equations in the similar way as previous.

$$\begin{aligned} \frac{\partial \varphi}{\partial x} &= \frac{\partial \varphi}{\partial x'} \cancelto{A}{\frac{\partial x'}{\partial x}} + \cancel{\frac{\partial \varphi}{\partial y'}} \cancelto{0}{\frac{\partial y'}{\partial x}} + \cancel{\frac{\partial \varphi}{\partial z'}} \cancelto{0}{\frac{\partial z'}{\partial x}} + \frac{\partial \varphi}{\partial t'} \cancelto{E}{\frac{\partial t'}{\partial x}} \\ \therefore \frac{\partial \varphi}{\partial x} &= \left. A\frac{\partial \varphi}{\partial x'} + E\frac{\partial \varphi}{\partial t'} \right\} \alpha \end{aligned}$$

Since $y' = y$ and $z' = z$, there is no change to these original equations.

$$\begin{aligned} \frac{\partial \varphi}{\partial t} &= \frac{\partial \varphi}{\partial x'} \cancelto{-Av}{\frac{\partial x'}{\partial t}} + \cancel{\frac{\partial \varphi}{\partial y'}} \cancelto{0}{\frac{\partial y'}{\partial t}} + \cancel{\frac{\partial \varphi}{\partial z'}} \cancelto{0}{\frac{\partial z'}{\partial t}} + \frac{\partial \varphi}{\partial t'} \cancelto{E}{\frac{\partial t'}{\partial t}} \\ \therefore \frac{\partial \varphi}{\partial t} &= \left. E\frac{\partial \varphi}{\partial t'} - Av\frac{\partial \varphi}{\partial x'} \right\} \beta \end{aligned}$$

Which leads to: