$$
\begin{aligned}
LG(x, y) &= \nabla^2(G_\sigma * E(x, y)) = (\nabla^2 G_\sigma) * E(x, y) \\
&= \left(\frac{\partial^2}{\partial x^2}G_\sigma + \frac{\partial^2}{\partial y^2}G_\sigma\right) * E(x, y) \\
&= \frac{1}{2\pi\sigma^2}\left[\frac{x^2 + y^2 - 2\sigma^2}{\sigma^4}\right]e^{-\frac{x^2+y^2}{2\sigma^2}} * E(x, y)
\end{aligned}
$$