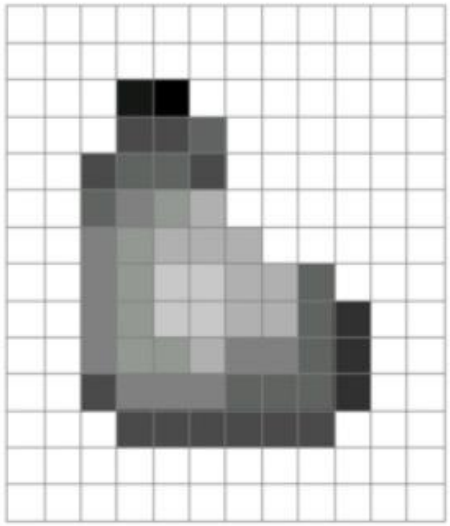
Vision and Image Processing 2018-18
Filtering and edges

Basic image processing



What is an image?

- A grid (matrix) of intensity values



=

255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255
255	255	127	145	145	175	127	127	95	47	255	255
255	255	74	127	127	127	95	95	95	47	255	255
255	255	255	74	74	74	74	74	74	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255

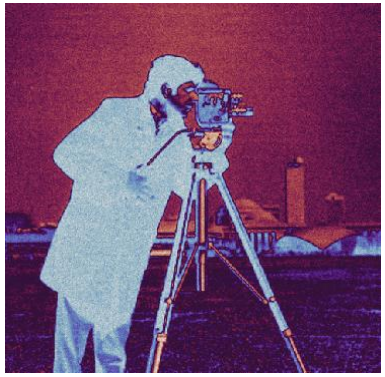
(common to use one byte per value: 0 = black, 255 = white)

Display of images in your report

- Please display single-valued functions (gray scale images) in gray scale and not using some arbitrary color map.

Colormap "greys"

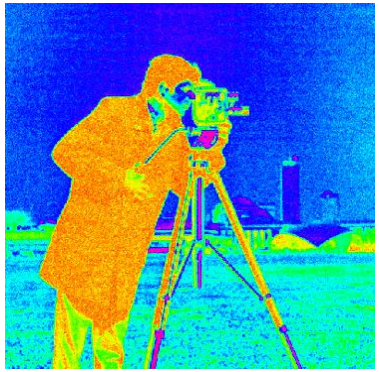
Colormap "cool"



Colormap "twilight"

Colormap "gist_stern"

Colormap "jet"



Colormap "hsv"

What is filtering good for ?

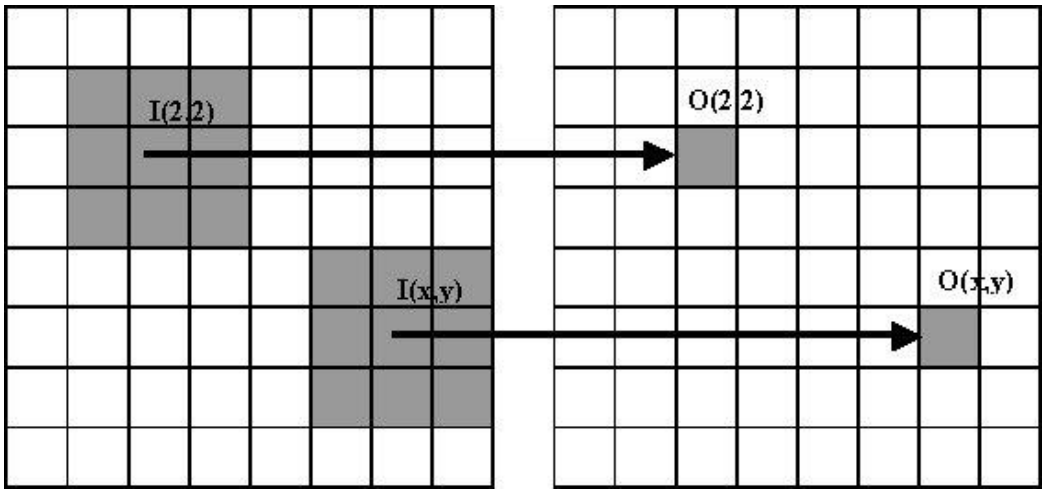
1. Noise reduction
2. Elimination of image quantization effects and conversion from integers to floats
3. Anti aliasing when up/down-scaling
4. Estimation of image derivatives
5. Enhancement of specific structures
6. Detection of image features such as edges and corners

Examples

Filtering

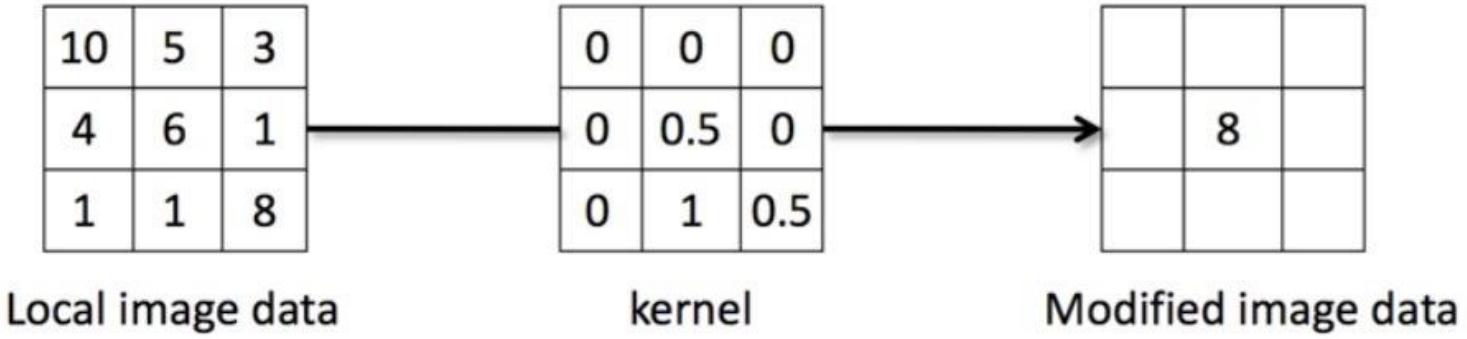
- Almost all filtering is local, i.e. the filter response in a pixel is computed based on an image patch centered at the pixel.

- Many filters are linear and position invariant, implying that the filtering may be described by a convolution.



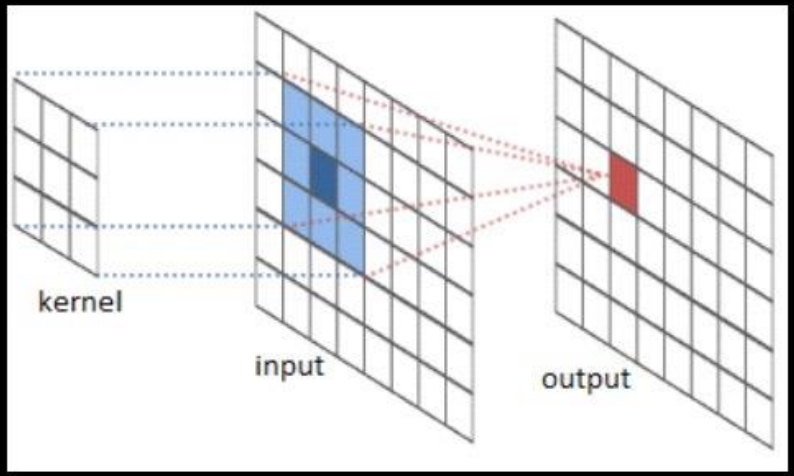
Linear filtering

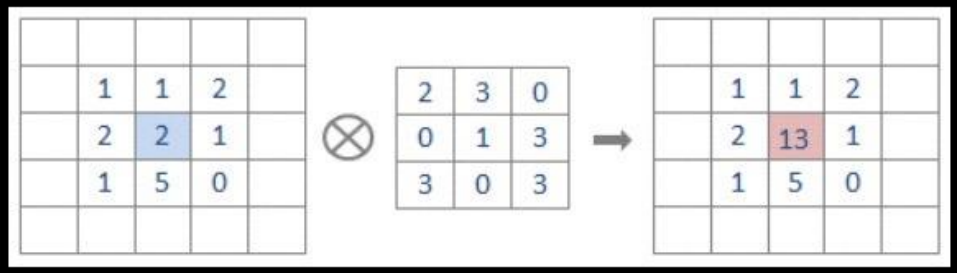
- One simple version: linear filtering (cross-correlation, convolution)
  - Replace each pixel by a linear combination of its neighbors
- The prescription for the linear combination is called the "kernel"



Source: L. Zhang

2D convolution





(1*2) + (1*3) + (2*0) +
(2*0) + (2*1) + (1*3) +
(1*3) + (5*0) + (0*3)
= 13.

illustrations from http://intellabs.github.io/RiverTrail/tutorial/

Convolutions

- Filtering by discrete convolution
  - A filter is defined by a filter kernel $h(x,y)$
  - Filtering the image $I(x,y)$ with the kernel $h(x,y)$ is defined as

$$R(x,y) = \sum_{u,v} I(u,v)h(x-u,y-v) \equiv I(x,y) * h(x,y)$$

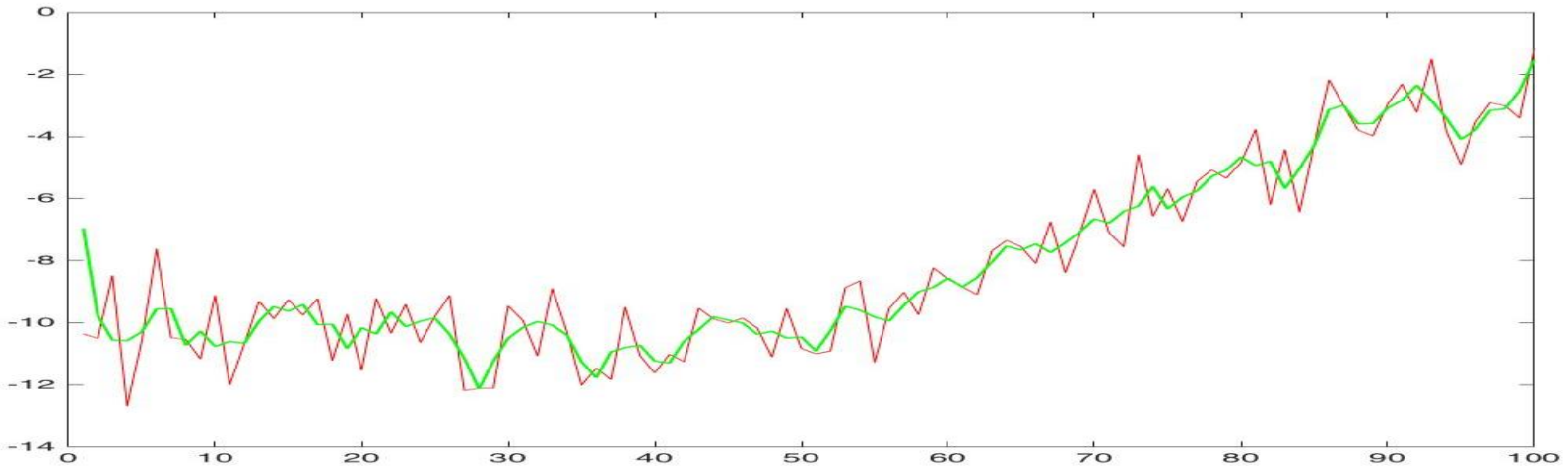
- $I*h = h*I$. Convolving the image with the kernel or the kernel with the image is the same.
- Consider filter kernels as images
  - Filtering slides the reverse filter kernel (or the reverse image) across the image and compute the product sum at each location
  - The result $R(x,y)$ is called the filter response

Example

The most simple filter has all values 1. In this case the filter response is just a summation of the neighboring values in the support area S.

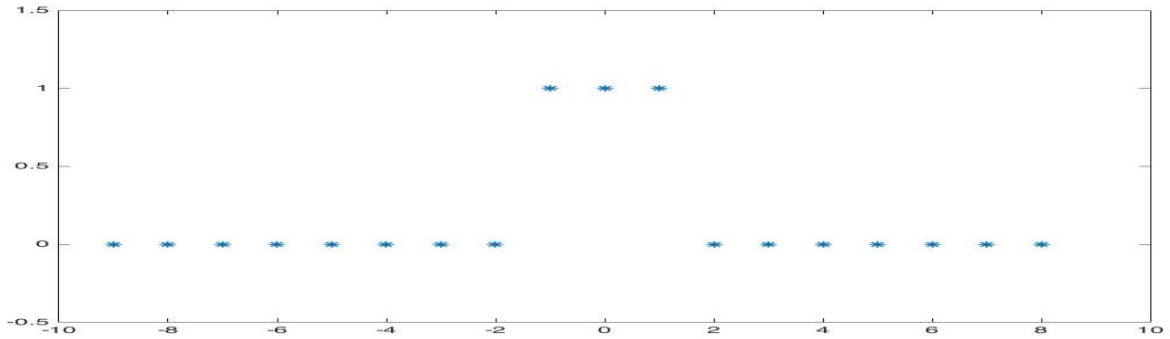
$$R(x,y) = \sum_{(u,v) \in S} I(u,v)$$

Assume the image is 1D and, support area is 3 pixels, and the filter/kernel values are all 1/3. Then the result will be a local running average.



Class exercise

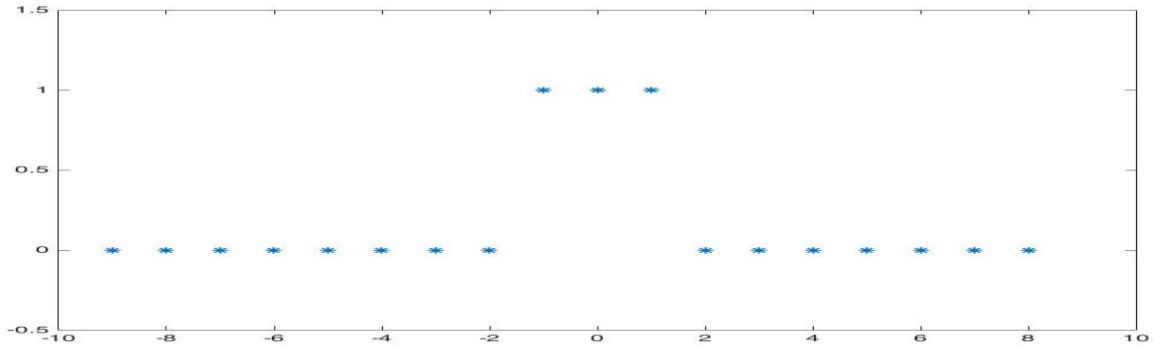
- Let f be:



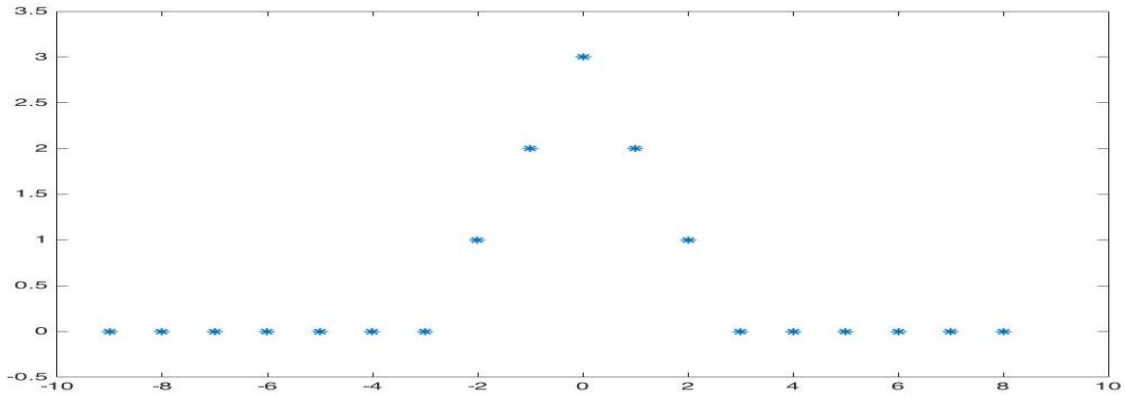
- Please use paper and pencil (yes) now (yes!!) and compute the f*f. What is the shape of the resulting signal ?

Class exercise

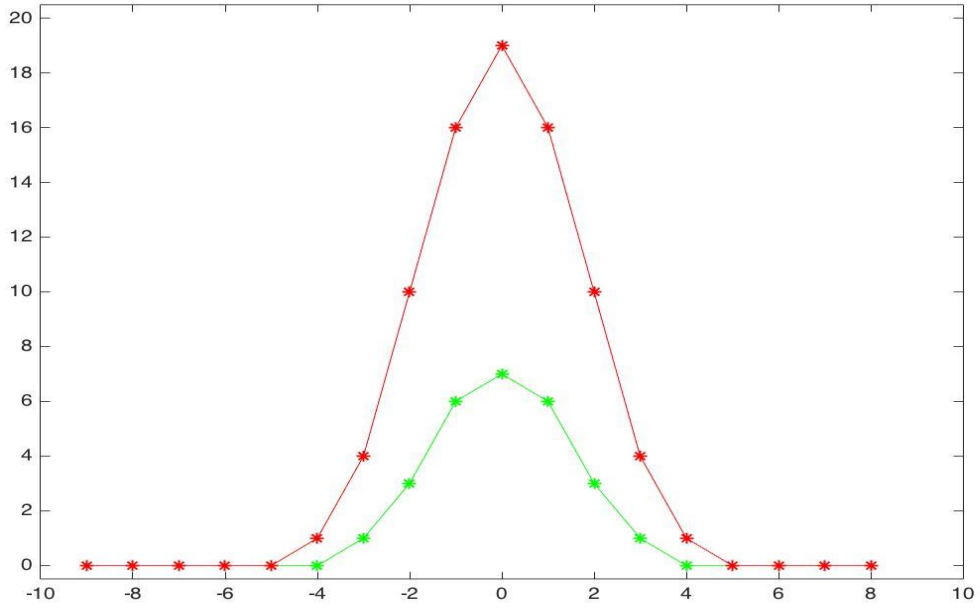
- Let f be:



- Please use paper and pencil (yes) now (yes!!) and compute $f*f$.
  What is the shape of the resulting signal ?



What happens if we compute f*f*f and f*f*f*f



Except from increasing its value the result becomes smoother and gradually more similar to a Gaussian (Normal distribution) (not normalized though)

However

- Remember, that in the general case, convolution is:

$$R(x,y) = \sum_{u,v} I(u,v)h(x-u,y-v) \equiv I(x,y) * h(x,y)$$

- i.e. we have to:

1. Mirror one of the images/functions
2. Slide it to the position (x,y) that we want to measure the convolution result at
3. Multiply the two images/functions
4. Sum the product over all pixels, e.g. assuming they are extended by zero values.

What about image borders

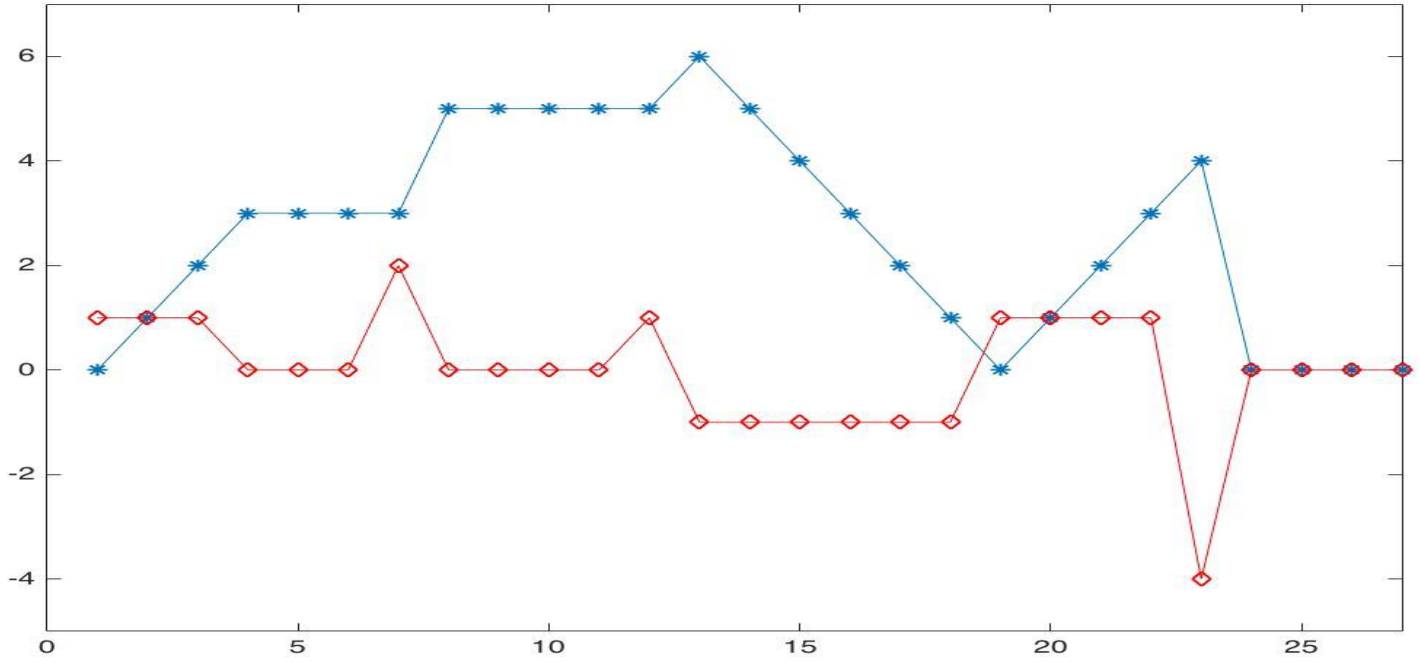
- For very small and large values of (x,y), x-u may become negative or y-v become larger than image size ?

$$R(x,y) = \sum_{u,v} I(u,v)h(x-u,y-v) \equiv I(x,y) * h(x,y)$$

- We don't know what is outside the image!
  - Assume zero values outside image border, or
  - Assume image is mirrored, or
  - Assume image is extended periodically, or
  - Don't! Leave border pixels unaltered

Another example

- Now lets try to convolve some data with a difference filter [1,-1]

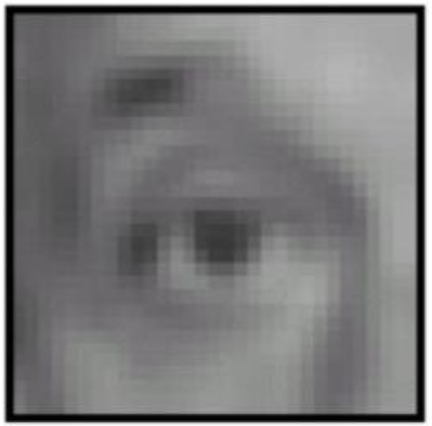


The difference filter computes an estimate of the rate of change, i.e. the derivative of the data.

Linear filters: examples

Original

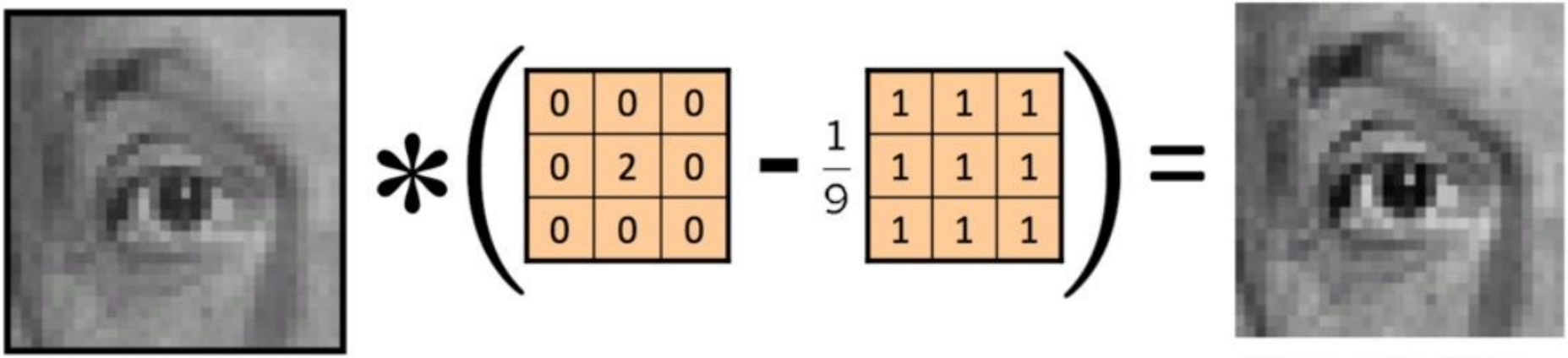
$$* \quad \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$



Blur (with a mean filter)

Source: D. Lowe

Linear filters: examples



Original

Sharpening filter
(accentuates edges)

Source: D. Lowe

Calculus for convolutions

$$f * g = g * f$$
$$f * (g + h) = f * g + f * h$$
$$f * (g * h) = (f * g) * h$$
$$D[f * g] = D[f] * g = f * D[g]$$

If *f* is discrete but *g* is continuous, the convolution result will be continuous.

If *f* is our image and *g* the noise smoothing filter, we may differentiate *f* by convolving with the derivative of *g*.

Correlations

- Correlation is very similar to convolution

$$R(x,y) = \sum_{u,v} I(x+u, y+v)h(u,v) \circ I(x,y) * h(x,y)$$

$$R(x,y) = \sum_{u,v} I(u,v)h(x+u, y+v) \equiv I(x,y) * h(x,y)$$

The "-" is replaced by a "+".
Note for the mathematicians: Really, h should be complex conjugated, but in this course, we assume the filter h real.

If $h(x,y)$ is real and symmetric, convolutions and correlations are identical. In any case: It's still just a local weighted average.

Correlations are often used to find image positions where the image is locally similar to a given image patch

Normalized cross-correlation

- The 1D normalized cross correlation (used in previous result) is given by:

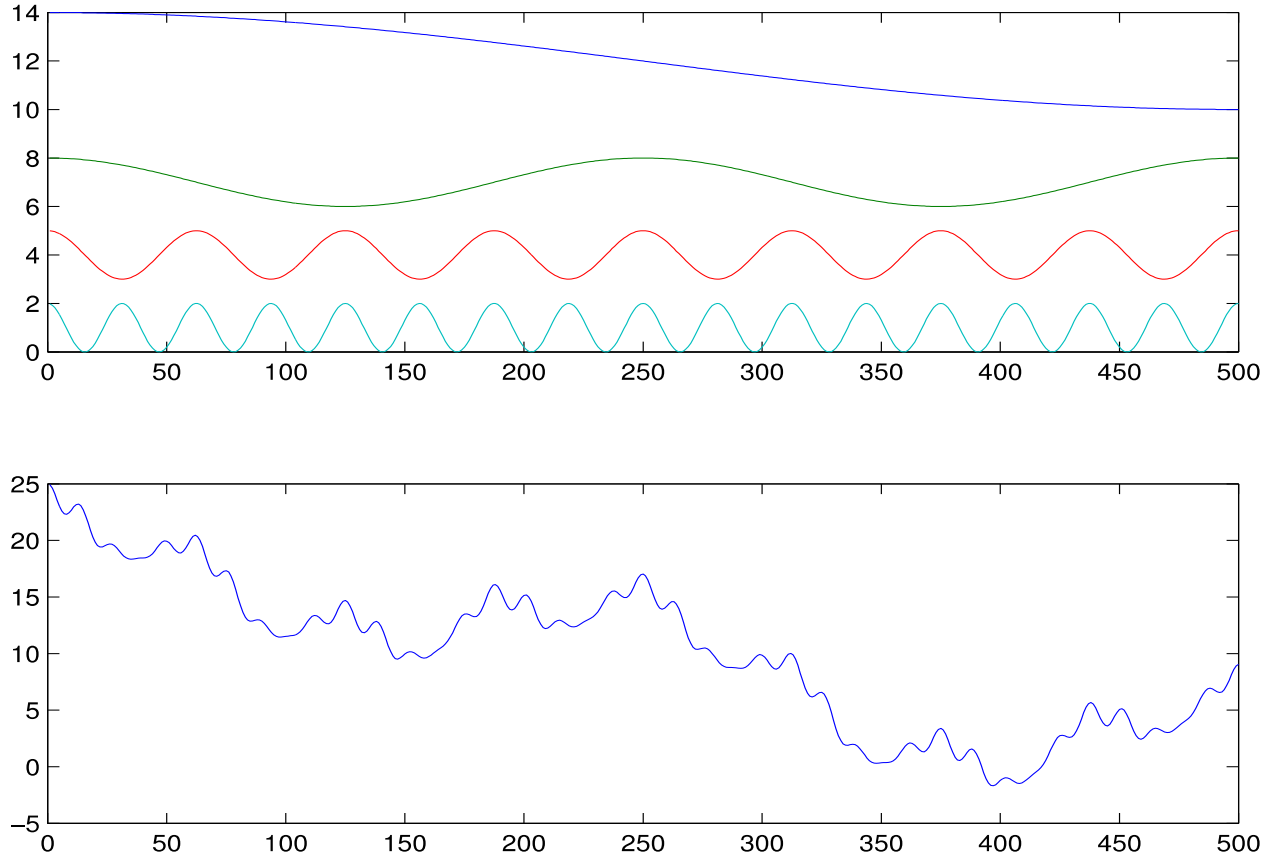
$$k(f,g) = \frac{\sum (f(x) - \bar{f})(g(x) - \bar{g})}{\sigma_f \, \sigma_g}$$

with $\bar{f}$ the mean value of f and σ the spread (or standard deviation).

The correlation coefficient is a real number between -1 and 1 with the latter indicating a strong linear relationship between the two functions. The extension to 2D is straightforward.

Frequency analysis

- Similar to sound/music all images may be decomposed into a sum of waves of varying frequencies and orientations.

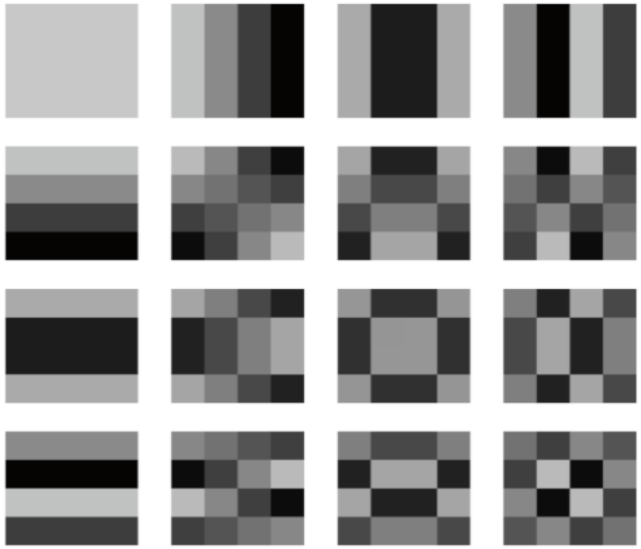


Cosine frequency decomposition

- There exists a number of frequency transformations including The Fourier transform (after the mathematician J. B. Joseph Fourier 1768-1830) and the Cosine transform (used in jpeg).

$$J(x,y) = \sum_{u=1}^{N} \sum_{v=1}^{N} C(u,v) K(x,y;u,v)$$

All images J may be constructed as a weighed sum of basis images $K(\mathrm{x},\mathrm{y})$



Frequency filtering

- Some filters are best formulated in the frequency domain.

- All linear and position invariant filters may be implemented in both domains, i.e. the spatial domain or the frequency domain.

- The Fourier transform $F\{f\}$ and its inverse $F^{-1}\{g\}$ provide a tool for going from one domain to the other.

- Filtering with large kernels (or filtering with many different kernels) may be implemented faster in the frequency domain than in the spatial domain.

The convolution theorem

- The Fourier transform of a convolution is the product of the Fourier transforms:

$$F\{(f * g)(x,y)\} = F\{f(x,y)\} \bullet F\{g(x,y)\} = \tilde{f}(u,v) \bullet \tilde{g}(u,v)$$

the tilde denotes functions in the frequency domain.

Similar equation holds for correlations.

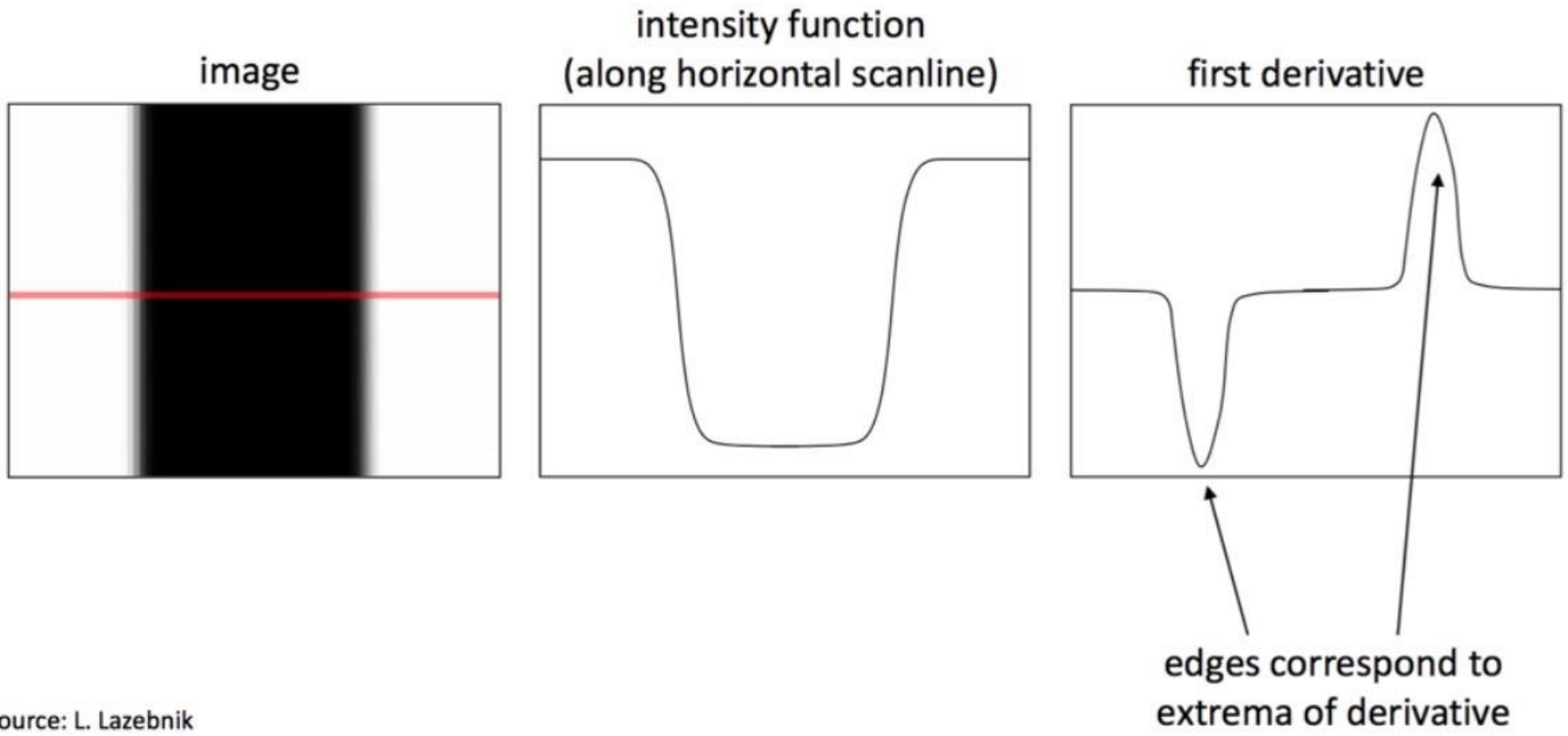
For differentiation, another important result is:

$$F\{\frac{\partial^p}{\partial x^p}\frac{\partial^q}{\partial y^q} f(x,y)\} = (-2\pi i)^{p+q} u^p v^q F\{f(x,y)\}$$

Where "*i*" is the imaginary complex unit. So, if we can Fourier transform and multiply with a polynomial, then it's easy to differentiate (in theory).

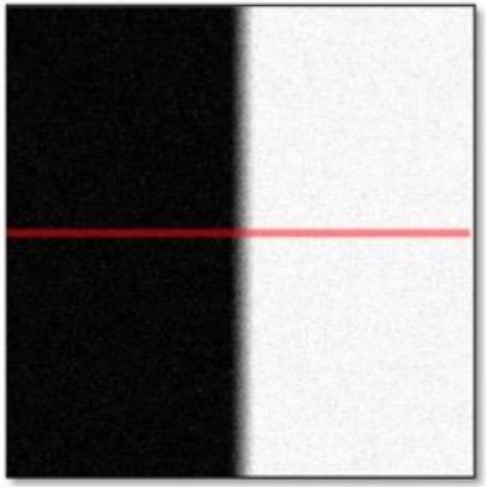
Characterizing edges

- An edge is a place of rapid change in the image intensity function



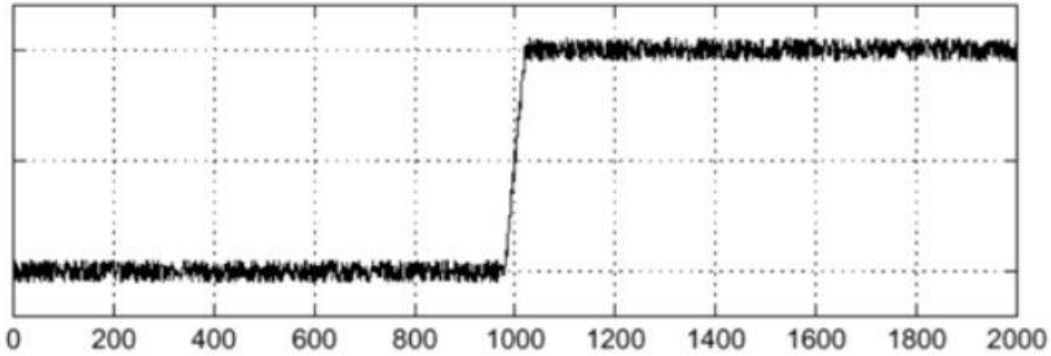
Source: L. Lazebnik

Effects of noise

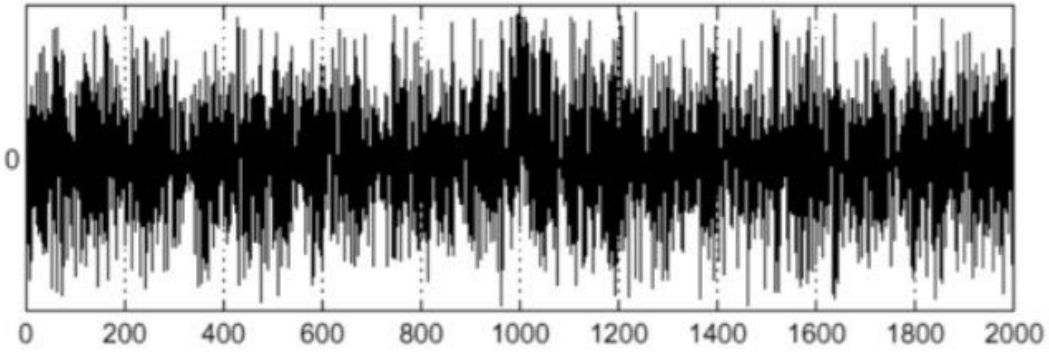


Noisy input image

$f(x)$



$\frac{d}{dx}f(x)$

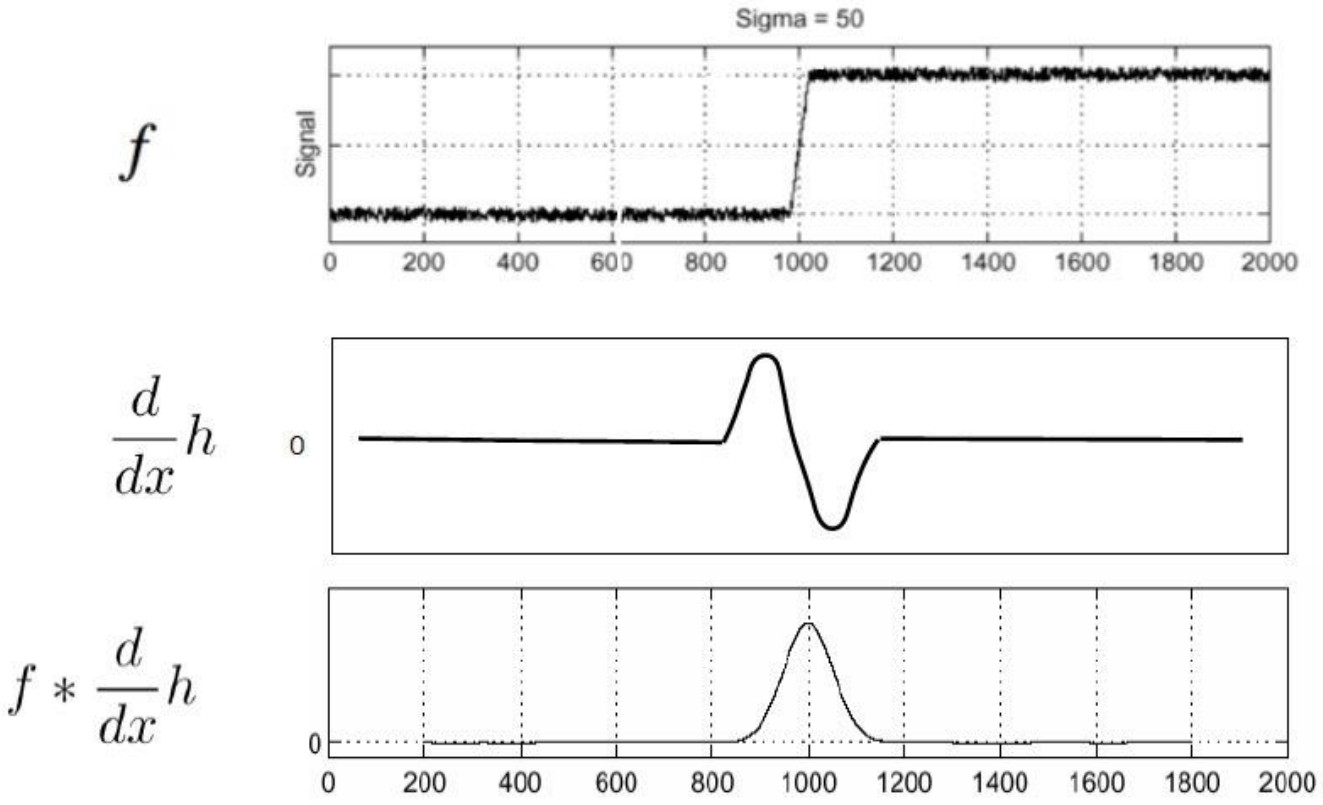


Where is the edge?

Source: S. Seitz

Associative property of convolution

- Differentiation is convolution, and convolution is associative: $\frac{d}{dx}(f * h) = f * \frac{d}{dx}h$

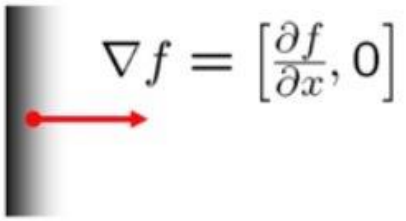


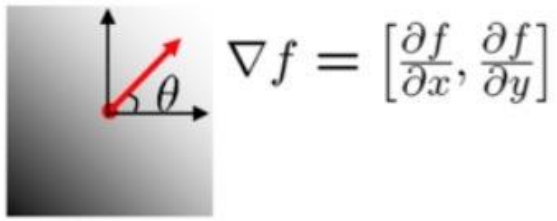
Source: S. Seitz

Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The gradient points in the direction of most rapid increase in intensity





The *edge strength* is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The gradient direction is given by:

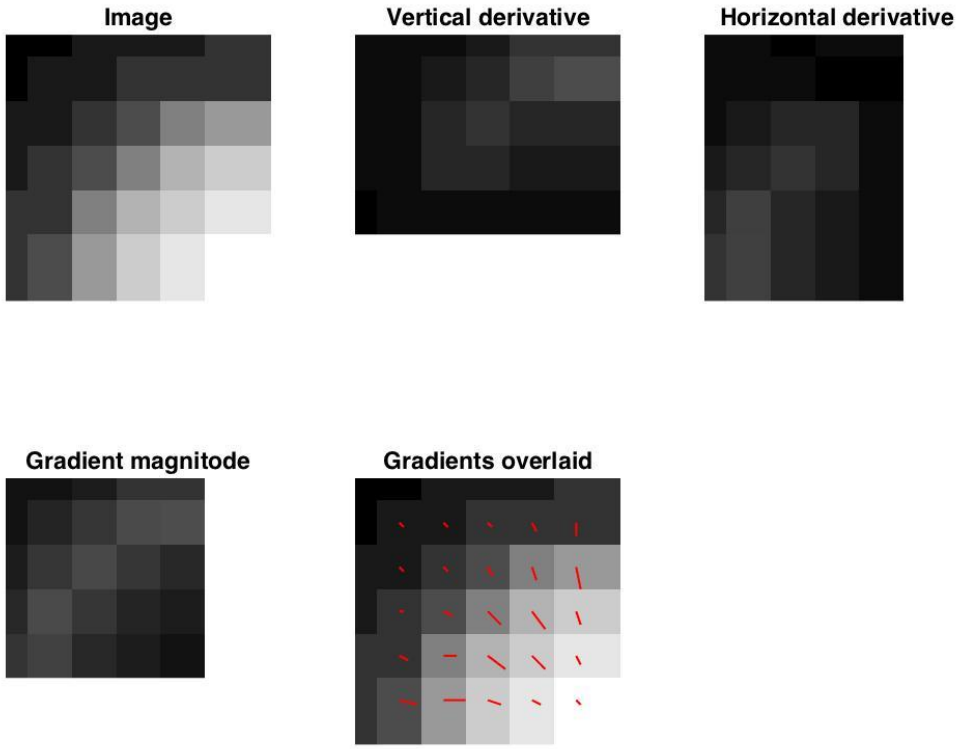
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

Source: Steve Seitz

Differentiation

- When we differentiate a function f(x), we compute the rate of change.
- In 2D there are partial derivatives in the x- and y-axis directions.
- The vector of first order partial derivatives is called the gradient. It points at the direction of the largest change and its length, the gradient magnitude, is the contrast.



Why compute image derivatives

- Image positions of locally maximal contrast form edges and capture a significant element of the semantics of the image.

- Statistical analysis of the local gradient image provides a tool for detecting corners and interest points

- Ridges and valleys in the image intensity landscape may be computed from the first and second order partial derivatives.

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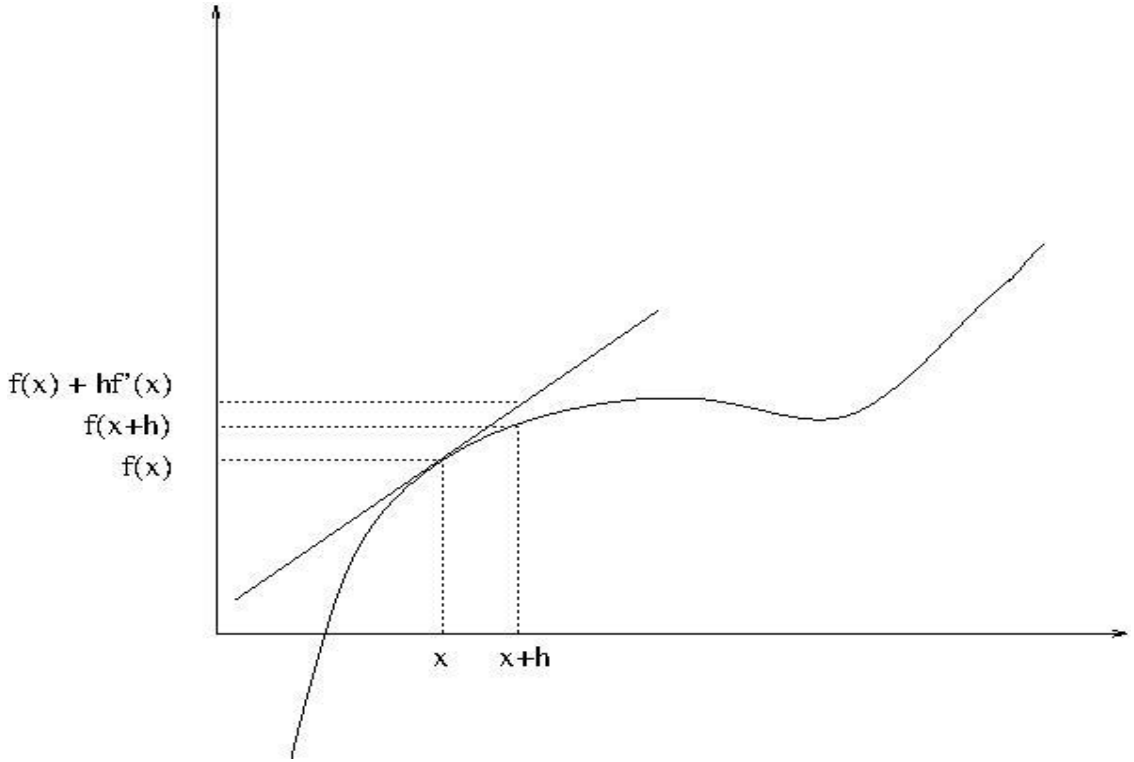
How differentiate a discrete image

- A Taylor series expansion of a continuous differentiable 1D function

$$f(x+h) = f(x) + hf'(x) + \frac{h^2 f''(x)}{2} + \frac{h^3 f'''(x)}{6} + \cdots + \frac{h^n f^{(n)}(x)}{n!} + \cdots$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2 f''(x)}{2} - \frac{h^3 f'''(x)}{6} + \cdots + (-1)^n \frac{h^n f^{(n)}(x)}{n!} + \cdots$$



Difference approximations

- Ignoring higher order terms we may obtain three discrete difference approximations to the derivative (via Taylor's Theorem):

$$
\begin{aligned}
f'(x) &= \frac{f(x+h)-f(x)}{h} - \frac{hf''(x)}{2} - \cdots \\
f'(x) &= \frac{f(x)-f(x-h)}{h} + \frac{hf''(x)}{2} - \cdots \\
f'(x) &= \frac{f(x+h)-f(x-h)}{2h} - \frac{h^2 f'''(x)}{6} - \cdots
\end{aligned}
$$

The last central difference approximation is symmetric which may improve interpretation.

What is the derivative of a color image

- A color image is a 3-valued function, so there is 6 first order partial derivatives:

$$J = \begin{bmatrix} \frac{\partial R}{\partial x} & \frac{\partial G}{\partial x} & \frac{\partial B}{\partial x} \\ \frac{\partial R}{\partial y} & \frac{\partial G}{\partial y} & \frac{\partial B}{\partial y} \end{bmatrix}$$

Gaussian filters

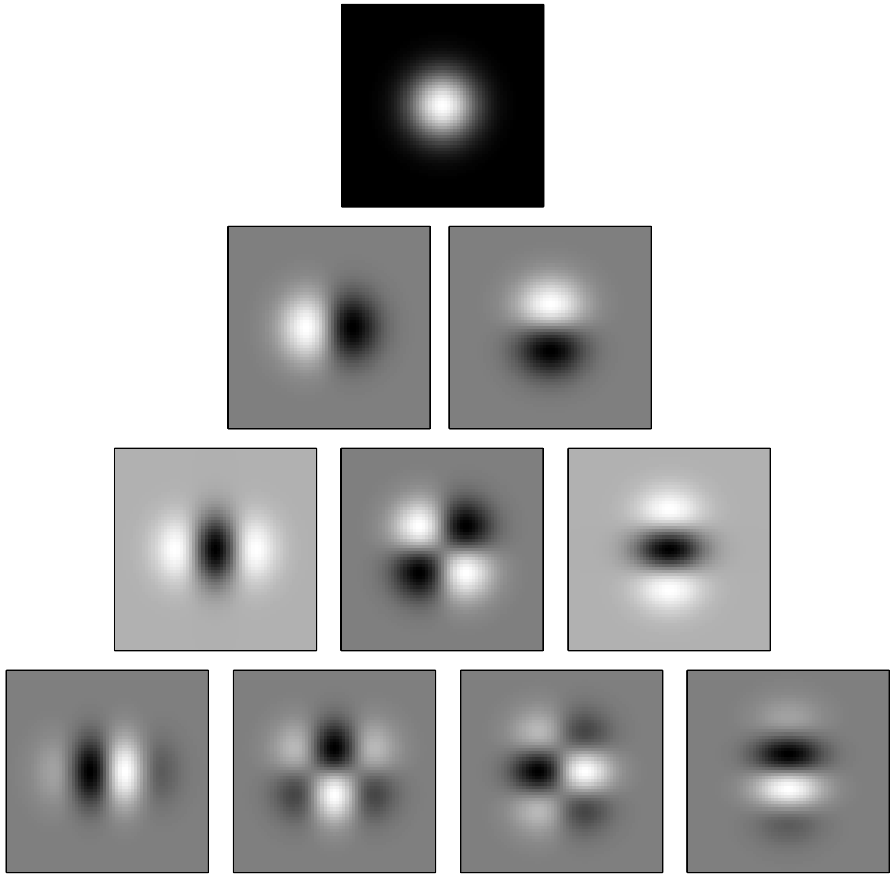
- The box-filter (with constant values) and the Gaussian filter are two of the most used filters.

- The box-filter is fast. The Gaussian has a lot of nice mathematical advantages.

- The Gaussian is a normal distribution with a mean value of zero. In 2D we get:

$$G(x,y,s) = \frac{1}{2ps^2} e^{-\frac{(x^2+y^2)}{2s^2}}$$

Gaussian derivative filter kernels



$$G(x,y;s) = \frac{1}{2ps^2}\exp\left(-\frac{x^2+y^2}{2s^2}\right)$$

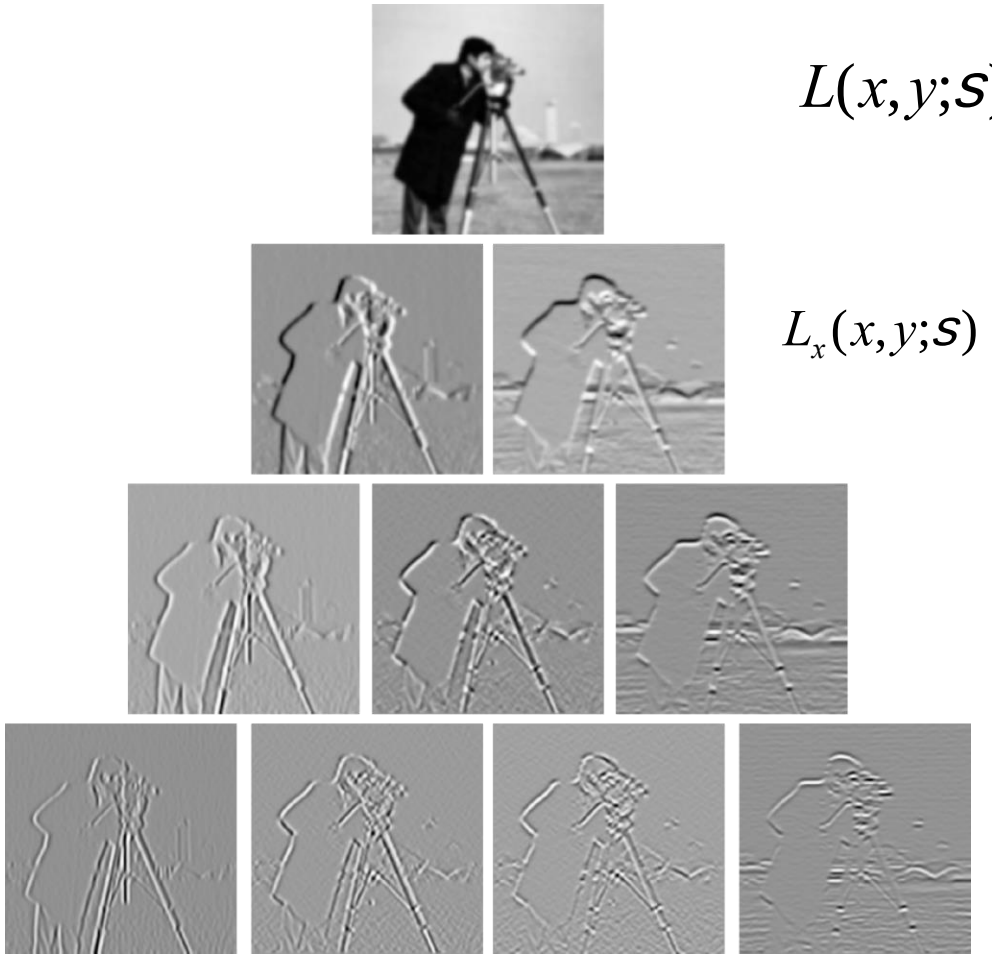
$$\frac{\partial}{\partial x}G\,,\quad \frac{\partial}{\partial y}G$$

$$\frac{\partial^2}{\partial x^2}G\,,\quad \frac{\partial^2}{\partial x\,\partial y}G\,,\quad \frac{\partial^2}{\partial y^2}G$$

$$\frac{\partial^3}{\partial x^3}G\,,\quad \frac{\partial^3}{\partial x^2\partial y}G\,,\quad \frac{\partial^3}{\partial x\,\partial y^2}G\,,\quad \frac{\partial^3}{\partial y^3}G$$

Aside: Neurophysiological experiments by Young (1987) show that the receptive field profiles in the human retina and visual cortex can be modeled by Gaussian derivatives.

Image derivatives with Gaussian filters



$$L(x,y;\sigma) = (I * G)(x,y;\sigma)$$

$$L_x(x,y;\sigma) = I * \frac{\partial}{\partial x} G \;,\; L_y(x,y;\sigma) = I * \frac{\partial}{\partial y} G$$

$$L_{x^2} = I * \frac{\partial^2}{\partial x^2} G \;,\; L_{xy} = I * \frac{\partial^2}{\partial x \partial y} G \;,$$

$$L_{y^2} = I * \frac{\partial^2}{\partial y^2} G$$

$$L_{x^3} = I * \frac{\partial^3}{\partial x^3} G \;,\; L_{x^2y} = I * \frac{\partial^3}{\partial x^2 \partial y} G \;,$$

$$L_{xy^2} = I * \frac{\partial^3}{\partial x \partial y^2} G \;,\; L_{y^3} = I * \frac{\partial^3}{\partial y^3} G$$

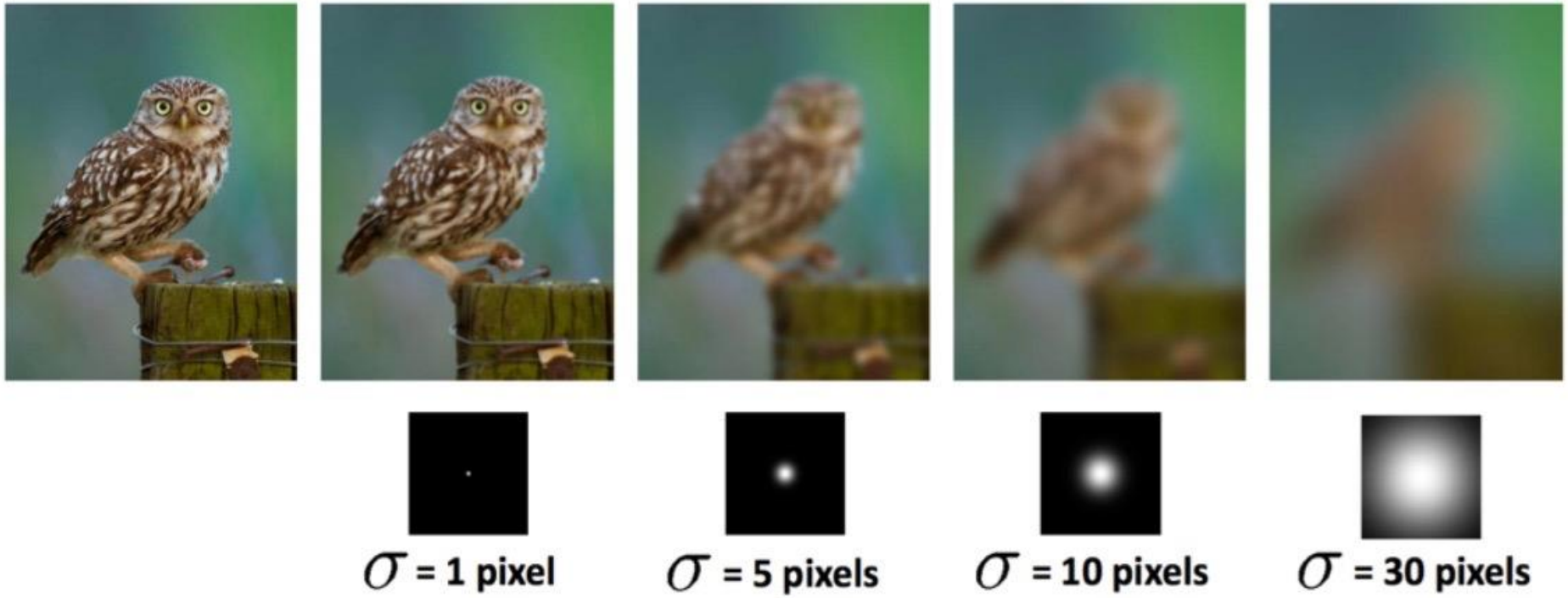
Gaussians

- Gaussians are the most common smoothing filter because of it simplicity, its unique mathematical properties, and because it is parameterized only by a single parameter, s, the scale parameter.

- A complete research field area: Scale space analysis has emerged with the focus of analyzing images at several resolution levels, using Gaussians as the basis filter.

- The basic problem is that a priori we don't know the size of what we a looking at. We need to use several filters adapted to the range of possible sizes.

Gaussian filters



Prewitt and Sobel Filters

- Prewitt filters

  x direction

-1	0	1
-1	0	1
-1	0	1

  y direction

1	1	1
0	0	0
-1	-1	-1

- Sobel Filters

  x direction

-1	0	1
-2	0	2
-1	0	1

  y direction

1	2	1
0	0	0
-1	-2	-1

Simple 3x3 filters/convolutions

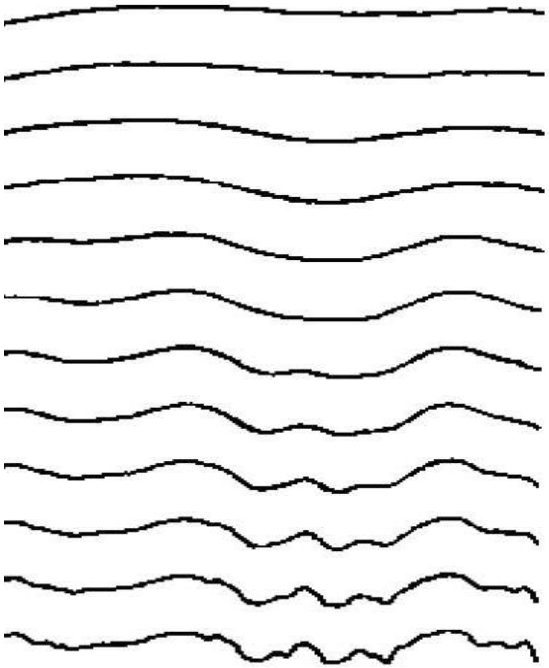
Are Prewitt or Sobel as good as Gaussians?

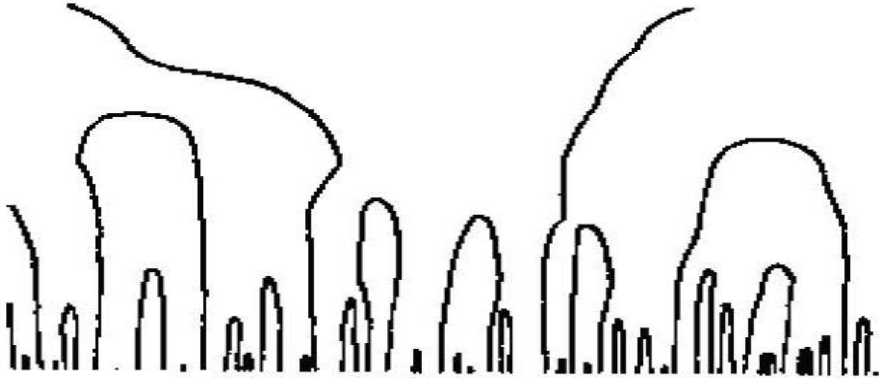
- No, they are fixed scale 3x3 masks/kernels, and their mathematical properties lack behind

- Can't I just do Gaussian filtering and then apply say Sobel ?
  Yes, you can, but it's not the same as a derivative of a Gaussian

- Would it make a big difference in practice?
  Probably no, but scale-space control might be less obvious.

Scale Space (Witkin 1983)





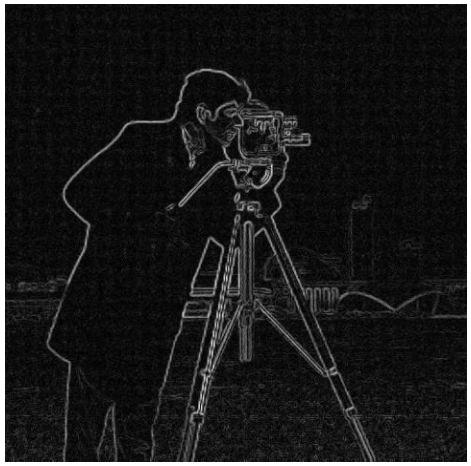
When a signal is smoothed with gradually increasing sigma, details disappear in a orderly fashion where local minima and maxima meet and annihilate. The Gaussian is the only filter with this property.

Edges at different scales

Depending on the scale parameter value and a gradient threshold value, edges often are broken, disappear, or are connected in unexpected ways.

Edge detection

- Edges mark high contrast image structures and capture a large part of the semantics of an image.
- For contrast measurement convolution with the derivative of Gaussians often are used (although this can be shown not to be optimal).

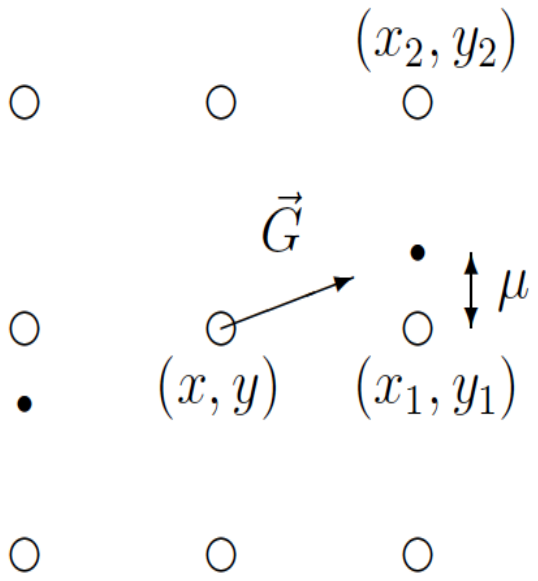


Thresholding the gradient magnitude leads to fat edges

Non-maximum suppression

For each pixel *(x,y)*, we interpolate the two gradient magnitude values at points on the grid in the positive and negative gradient direction G.

If the gradient magnitude at *(x,y)* is larger than both values we mark the point as an edge point.



The Laplacian of a Gaussian (LoG)

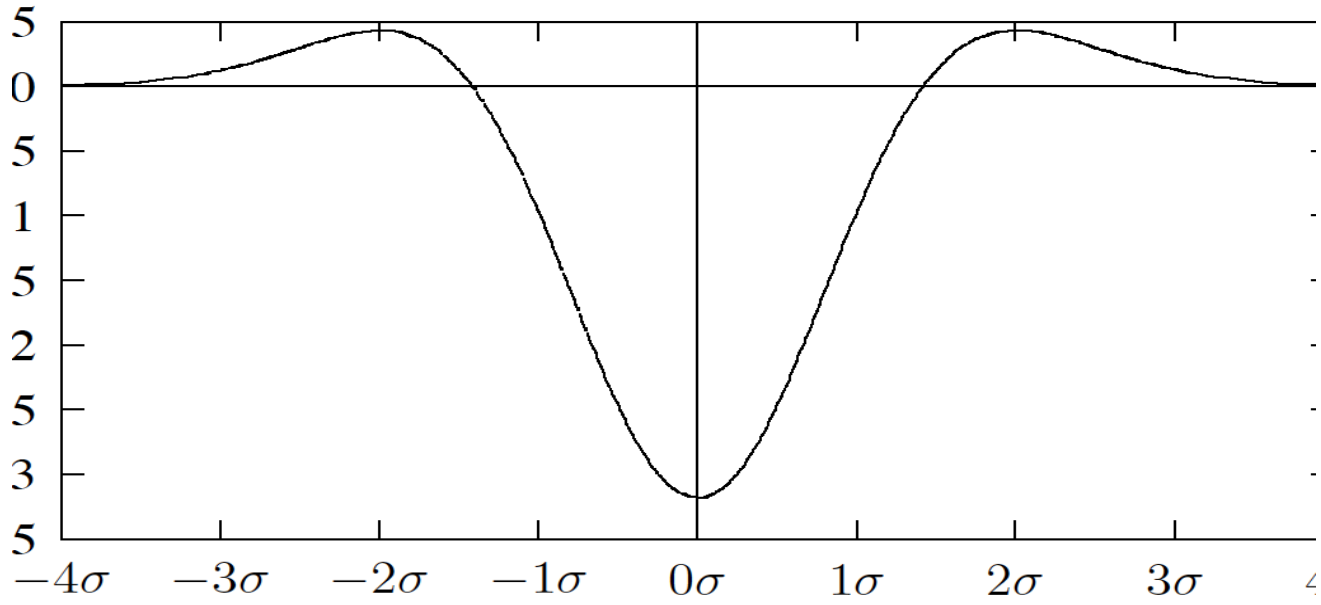
- The Laplacian is the sum of the two second order partial derivatives:

$$
\begin{aligned}
LG(x,y) &= \nabla^2(G_\sigma * E(x,y)) = (\nabla^2 G_\sigma) * E(x,y) \\
&= (\frac{\partial^2}{\partial x^2}G_\sigma + \frac{\partial^2}{\partial y^2}G_\sigma) * E(x,y) \\
&= \frac{1}{2\pi\sigma^2}\left[\frac{x^2+y^2-2\sigma^2}{\sigma^4}\right] e^{-\frac{x^2+y^2}{2\sigma^2}} * E(x,y)
\end{aligned}
$$

- ~~The filter some times are referred to as the Mexican hat filter.~~

- The filter is the king of blob-filters, may be used to detect edges, and has a center-surround shape similar to ganglion cells in the mammalian visual pathway.

Cross section of $\nabla^2 G_s$



The filter computes the deviation of the central values from the surroundings.

The filter responds maximally to a white/black spot on a black/white background.

Difference of Gaussians (DoG)

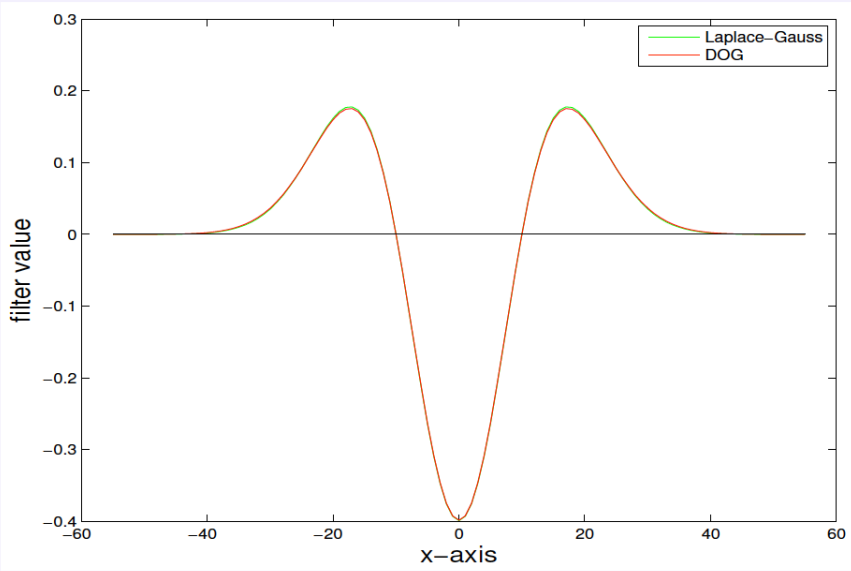
A fast approximation to $\nabla^2 G_\sigma$ is a scaled version of the difference of two Gaussians with different and carefuly selected σ's.

$$B(G_{\sigma_1} - G_{k\sigma_1})$$

where

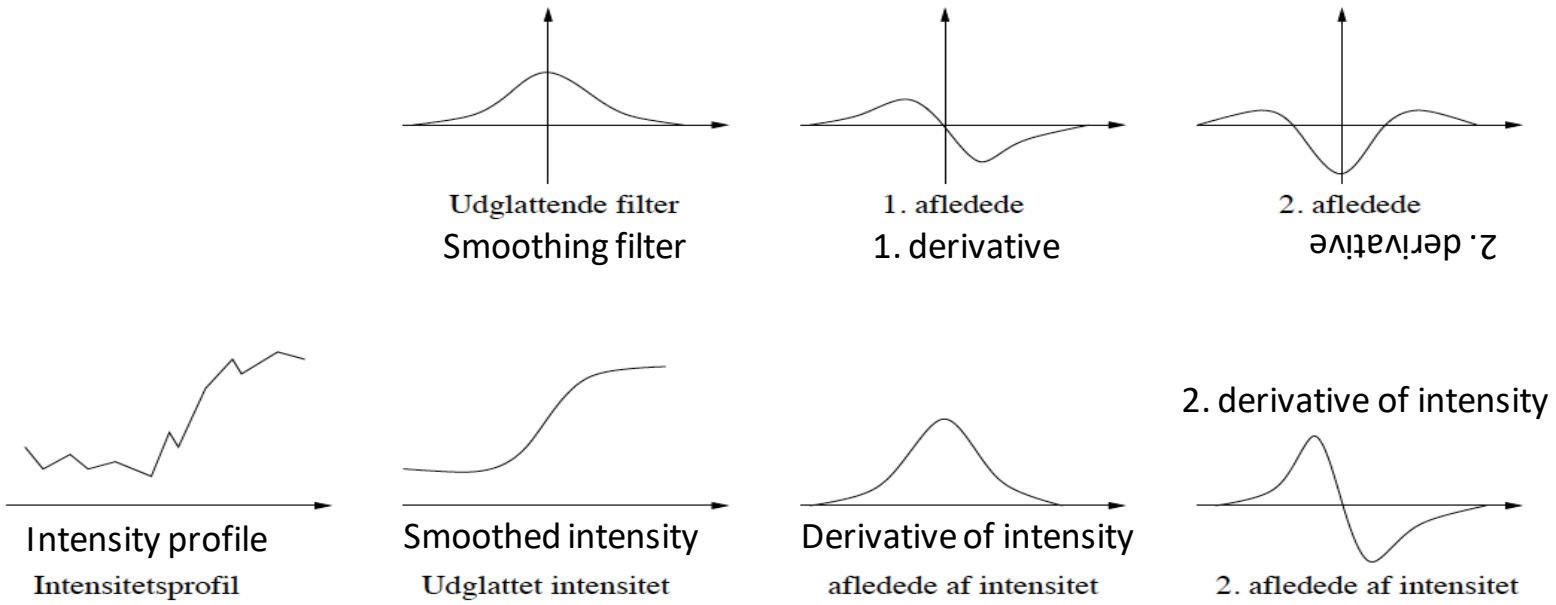
$$B = 2\frac{\sigma_1^2}{\sigma^4}\frac{k^2}{1-k^2} \quad \text{and} \quad \sigma_1^2 = \sigma^2 \frac{k^2-1}{k^2 \ln k^2}$$

and where $1 < k < 2$ gives acceptable approximations.



Edge detection using Laplacian Gaussian

- The second order derivative is zero where the first order derivative has a maximum.



Zero crossings of the Laplacian of a Gaussian form very good edges

Blob detection

- Blob detection using extremum detection in the convolution of an image with the Laplacian of a Gaussian ideally mark circular spots with a diameter of $2\sqrt{2}s$

Blobs are a main building stone in feature point based image description and will play a central role in the remaining part of the course.

Interest points

- When we want to match images or perform Content Based Image Retrieval (CBIR) we need to identify interest points in the images.

- Blobs and corners are the most used interest points

- Next time we will look at corner detectors and how we may attribute interest points with descriptors.

Summary

- Today we have been through:

1. Filtering: Convolutions and correlations
2. The convolution theorem
3. Frequency representation
4. Differentiation and discrete difference approximations
5. Gaussians and derivative of Gaussians
6. Scale Space
7. Edge detection
8. Blob detection

2D filter masks

- 2D filter kernels may be viewed as small images patches or masks. Examples are Prewitt, Box, smoothing, mixed 2nd order derivative.

-1	0	1
-1	0	1
-1	0	1

1	1	1
1	1	1
1	1	1

0.04	0.14	0.04
0.14	0.28	0.14
0.04	0.14	0.04

0.04	0.02	0.00	-0.02	-0.04
0.02	0.01	0.00	-0.01	-0.02
0.00	0.00	0.00	0.00	0.00
-0.02	-0.01	0.00	0.01	0.02
-0.04	-0,02	0.00	0.02	0.04