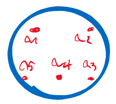
MINIMUM ENCLOSING BALL PROBLEM

GIVEN A FINITE SET OF VECTORS $A = \{a_1, \dots, a_m\} \subset \mathbb{R}^n$

COMPUTE THE MINIMUM ENCLOSING BALL OF A MEB(A)

✱ WE WANT TO FIND $B_{c,\varrho} = \{x \in \mathbb{R}^n : \|x - c\| \le \varrho\}$

s.t.



$A \subset B_{c,\varrho}$

ϱ AS SMALL AS POSSIBLE

MODEL USED IN CLUSTERING, DATA CLASSIFICATION, FACILITY LOCATION, COMPUTER GRAPHICS

FORMULATION

$$\min_{c,\varrho} \ \varrho^2$$
$$\|a_i - c\|^2 \le \varrho^2 \quad i = 1, \dots, m$$

RADIUS

SET A IS CONTAINED IN $B_{\varrho,c}$

$\varrho^* = \sqrt{\phi(u^*)}$

$C^* = \sum_{i=1}^{m} u_i^* a_i$

$\phi(u^*)$

REWRITE AS (WE SQUARE CONSTRAINTS & SET $\gamma = \varrho^2$)

$\varrho^2 = \gamma$

$$\min_{c,\gamma} \ \gamma$$
$$a_i^T a_i - 2 a_i^T c + c^T c \le \gamma \quad i = 1, \dots, m$$

$A = [a_1 \dots a_m]$

BUILD LAGRANGIAN DUAL

$$\max_u \ \phi(u) = \sum_{i=1}^{m} \|a_i\|^2 u_i - \left(\sum_{i=1}^{m} u_i a_i\right)^T \left(\sum_{i=1}^{m} u_i a_i\right)$$
$$\text{s.t.} \quad \sum_{i=1}^{m} u_i = 1, \quad u_i \ge 0 \quad i = 1, \dots, m$$

EQUIVALENTLY

$$\min \ u^T A^T A u - \sum_{i=1}^{m} \|a_i\|^2 u_i$$

(P1) $\quad e^T u = 1, \ u \ge 0$ UNIT SIMPLEX

EXERCISE

GENERATE A SET OF m POINTS $A = \{a_1, \dots, a_m\}$ WITH $a_i \in \mathbb{R}^n$ $i = 1, \dots, m$

SOLVE PROBLEM (P1) TO GET MEB(A)

USE

1. FRANK-WOLFE ALGORITHM
2. AWAY-STEP FRANK-WOLFE ALGORITHM
3. PROJECTED GRADIENT

PROJECTION OVER UNIT SIMPLEX

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proj_simplex_vector = @(y) max(y-max((cumsum(sort(y,1,'descend'),1)-1)./(1:size(y,1))'),0);
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