

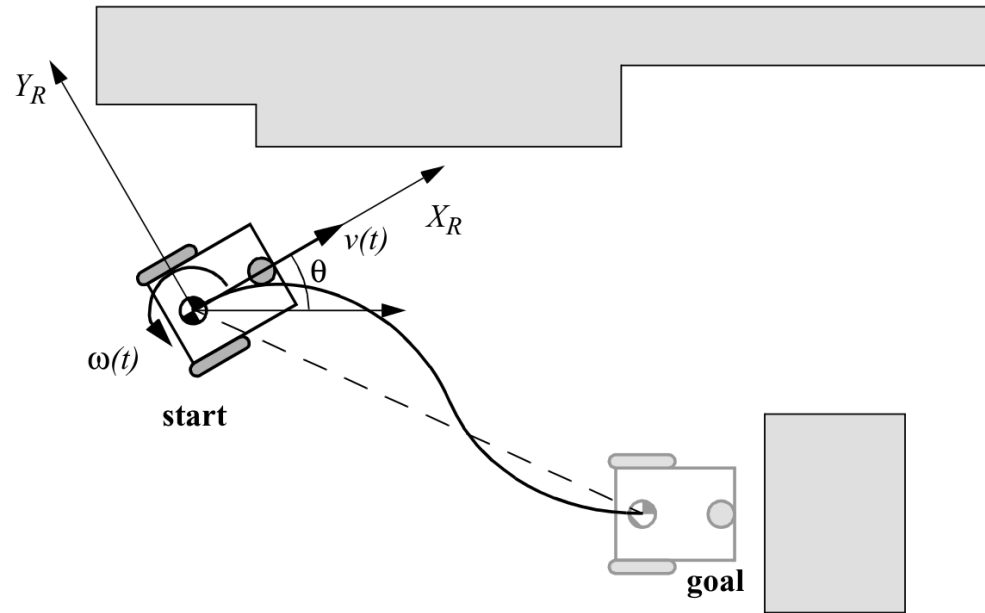
# Mobile Robot Kinematics

- Differential drive kinematic model

$$v = \frac{r\dot{\phi}_r}{2} + \frac{r\dot{\phi}_l}{2}$$
$$\omega = \frac{r\dot{\phi}_r}{2l} - \frac{r\dot{\phi}_l}{2l}$$

- Robot description: *wheel speeds +intrinsic*  $\leftrightarrow$  *robot speed*
- More complex description:
  - Include forces, dynamic constraints
  - What forces for particular kinematic behavior?

# Controllability

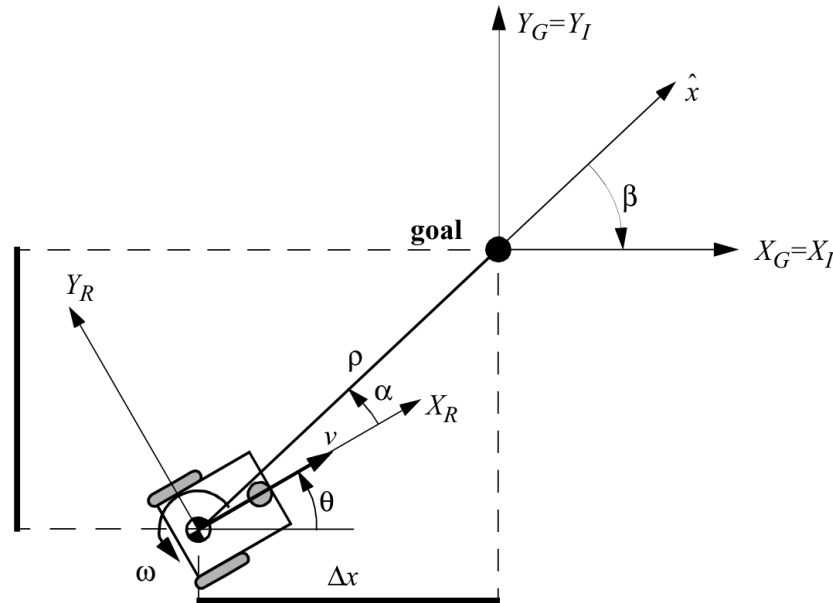


- How can the robot reach a goal position from its current state?

# Control

- Holonomic / Non-holonomic
- Open-loop / Feedback control
- Open-loop:
  - Trajectory synthesis → follow set-points
  - Can't adapt to dynamic changes in environments
  - Imperfect execution of trajectories (*boundaries, lines...*)
- Feedback:
  - Always knows current state  $\leftrightarrow$  execute next step

# Control



- **Kinematic model:** Wheel speeds needed for achieving particular robot motion
- **Control:** What robot motion (speeds) will be needed to get to the goal state

# Control

- Error :-  $e = {}^R[x, y, \theta]^T$

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K {}^R \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \Rightarrow K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix} \quad \text{with } k_{ij} = k(t, e)$$

- Proportional control: Robot not actuated when at the goal position
- Find K such that  $\lim_{t \rightarrow \infty} e(t) = 0$

# Control

- Motion of the robot in inertial frame
- Kinematic of differential-drive robot in Inertial frame:

$${}^I \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

- Cartesian  $\rightarrow$  Polar
  - Easier description for the given problem
- What does the controller need?
  - Position  $\rightarrow$  decide speed
  - Heading  $\rightarrow$  for alignment to goal location
  - Orientation  $\rightarrow$  for aligning with goal orientation

# Control

- Inertial  $\rightarrow$  Polar transformation

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \text{atan2}(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$

- Kinematics model:-

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos \alpha & 0 \\ \frac{\sin \alpha}{\rho} & -1 \\ -\frac{\sin \alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

**NOTE:**  
heading  $\leftrightarrow$  speed  
relation

# Control Law

- Simple linear control law:

$$v = k_{\rho}\rho$$

$$\omega = k_{\alpha}\alpha + k_{\beta}\beta$$

- Closed-loop system description:

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_{\rho}\rho \cos \alpha \\ k_{\rho} \sin \alpha - k_{\alpha}\alpha - k_{\beta}\beta \\ -k_{\rho} \sin \alpha \end{bmatrix}$$



# Local Stability

- Linearizing the system around equilibrium ( $\cos x = 1$ ;  $\sin x = x$ )

- $$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_{\rho} & 0 & 0 \\ 0 & -(k_{\alpha} - k_{\rho}) & -k_{\beta} \\ 0 & -k_{\rho} & 0 \end{bmatrix} \begin{bmatrix} \rho \\ \alpha \\ \beta \end{bmatrix}$$

- Locally exponentially stable **A** if the eigenvalues of the matrix  $A$  have -ve real part.

- Characteristic polynomial of  $A$  (  $\det(A - \lambda I)$  ):

- $$(\lambda + k_{\rho})(\lambda^2 + \lambda(k_{\alpha} - k_{\rho}) - k_{\rho}k_{\beta})$$

# Local Stability

- Negative real parts for all the roots of characteristic polynomial:

$$k_{\rho} > 0 \ ; \quad -k_{\beta} > 0 \ ; \quad k_{\alpha} - k_{\rho} > 0$$

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