VIET NAM GENERAL CONFEDERATION OF LABOR

TON DUC THANG UNIVERSITY

FACULTY OF INFORMATION TECHNOLOGY

NGUYỄN QUANG HUY - 523H0140

NGUYỄN THANH TÙNG - 523H0192

FINAL REPORT

APPLIED CALCULUS FOR IT

HO CHI MINH CITY, 2024

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Instructor

Master of Science Phạm Kim Thủy

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Ho Chi Minh city, January 4, 2024

Author

(Sign and write your full name)

THIS FINAL REPORT HAD BEEN COMPLETED AT TON DUC THANG UNIVERSITY

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Ho Chi Minh city, January 4, 2024

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LIST OF ABBREVIATIONS

MSc	Master of Science
M.A	Master of Art
Pcs	Pieces

CHAPTER 1: SOLUTIONS

1.1 Question 1

1.1.1 Topic

Tell whether the following functions are even, odd, or neither. Give reasons for your answer.

- $f(x) = x^2 + x$
- $f(x) = x^3 + x$
- $f(x) = \dfrac{4}{x^4 - 4}$
- $f(x) = \dfrac{x^3}{x^4 - 4}$

1.1.2. Solution

- To solve the problem of determining whether a function is even, odd or not we do the following:
- Find the domain D of the function.
- Based on the definition of even and odd functions as follows:
  - A function $f(x)$ is considered even if, for every x in its domain:

$$f(x) = -f(x)$$

  - A function $f(x)$ is considered odd if, for every x in its domain

$$f(x) = -f(-x) \text{ or } -f(x) = f(-x)$$

1.1.3. Final result

- $f(x) = x^2 + x$
  - The domain of the function $f(x): D = \mathbb{R}$
  - For every $x \in \mathbb{R} \to -x \in \mathbb{R}$, we have:

$$f(-x) = (-x)^2 + (-x) = x^2 - x \neq f(x)$$

- Because of $f(-x) \neq -f(x)$, we can conclude that the **function *f(x)* = x^2 + *x* does not have parity**.

- $f(x) = x^3 + x$
  - The domain of the function $f(x): D = \mathbb{R}$
  - For every $x \in \mathbb{R} \to -x \in \mathbb{R}$, we have:

$$f(-x) = (-x)^3 + (-x) = -x^3 - x = -(x^3 + x) = -f(x)$$

  - Therefore, we can conclude that the **function *f(x)* = x^3 + *x* is odd**.

- $f(x) = \dfrac{4}{x^4 - 4}$
  - The domain of the function $f(x)$:

$$x^4 - 4 \neq 0$$

$$\to \begin{cases} x \neq \sqrt{2} \\ x \neq -\sqrt{2} \end{cases} \quad \to D = \mathbb{R} \backslash \{-\sqrt{2}; \sqrt{2}\}$$

  - For every $x \in \mathbb{R} \backslash \{-\sqrt{2};\ \sqrt{2}\} \to -x \in \mathbb{R} \backslash \{-\sqrt{2};\ \sqrt{2}\}$, we have:

$$f(-x) = \frac{4}{(-x)^4 - 4} = \frac{4}{x^4 - 4} = f(x)$$

  - Therefore, we can conclude that **the function $\boldsymbol{f(x) = \dfrac{4}{x^4 - 4}}$ is even**.

- $f(x) = \dfrac{x^3}{x^4 - 4}$
  - The domain of the function $f(x)$:

$$x^4 - 4 \neq 0$$

$$\to \begin{cases} x \neq \sqrt{2} \\ x \neq -\sqrt{2} \end{cases} \quad \to D = \mathbb{R} \backslash \{-\sqrt{2}; \sqrt{2}\}$$

  - For every $x \in \mathbb{R} \backslash \{-\sqrt{2}; \sqrt{2}\} \to -x \in \mathbb{R} \backslash \{-\sqrt{2}; \sqrt{2}\}$, we have:

$$f(-x) = \frac{(-x)^3}{(-x)^4 - 4} = -\frac{x^3}{x^4 - 4} = -f(x)$$

  - Finally, we can conclude that **the function $\boldsymbol{f(x) = \dfrac{x^3}{x^4 - 4}}$ is odd**.

1.2 Question 2

1.2.1 Topic

Find the following limit $\lim \dfrac{555}{x^2-25}$ as:

- $x \to 5^+$
- $x \to 5^-$
- $x \to -5^+$
- $x \to -5^-$

1.2.2 Solution

- Simplify the given function.
- Because when substituting the limit value of x into the function, the function becomes undefined, so consider the numerator and denominator separately to determine the limit:
  - The numerator is a constant.
  - The denominator is a polynomial that contains x substitutions for each case to find the limit of the denominator.
- Conclusion, with the numerator being a constant:
  - The denominator has a limit approaching positive tense $\rightarrow$ given expression has lim $= +\infty$
  - The denominator has a limit approaching negative tense → given expression has lim $= -\infty$

1.2.3 Final result

- $\lim\limits_{x \to 5^+} \dfrac{555}{x^2-25}$

  Consider:
  - The numerator: 555 is a constant.
  - The denominator: $\lim(x+5)(x-5) = 0^+$ at x → 5^+.

$$\Rightarrow \lim_{x\to 5^+} \frac{555}{(x-5)(x+5)} = +\infty$$

- $\lim_{x\to 5^-} \frac{555}{x^2-25}$

Consider:

- The numerator: 555 is a constant.
- The denominator: $\lim(x+5)(x-5) = 0^-$ *at* $x \to 5^-$.

$$\Rightarrow \lim_{x\to 5^-} \frac{555}{(x-5)(x+5)} = -\infty$$

- $\lim_{x\to -5^+} \frac{555}{x^2-25}$

Consider:

- The numerator: 555 is a constant.
- The denominator: $\lim(x+5)(x-5) = 0^+$ *at* $x \to -5^+$.

$$\Rightarrow \lim_{x\to -5^+} \frac{555}{(x-5)(x+5)} = +\infty$$

- $\lim_{x\to -5^-} \frac{555}{x^2-25}$

Consider:

- The numerator: 555 is a constant.
- The denominator: $\lim(x+5)(x-5) = 0^-$ at $x \to -5^-$

$$\Rightarrow \lim_{x\to -5^-} \frac{555}{(x-5)(x+5)} = -\infty$$

1.3 Question 3

1.3.1 Topic

Find the derivatives $\frac{dy}{dx}$ **of the following functions:**

- $y = \frac{\sqrt{x} - 4}{\sqrt{x} + 4}$

- $y = \left(\frac{\sqrt{x}}{10} - 1 \right)^{-10}$

1.3.2 Final result.

- $y = \frac{\sqrt{x} - 4}{\sqrt{x} + 4}$

$$\frac{dy}{dx} = \left(\frac{\sqrt{x} - 4}{\sqrt{x} + 4} \right)'$$

$$= \frac{\left(\sqrt{x} - 4\right)'\left(\sqrt{x} + 4\right) - \left(\sqrt{x} - 4\right)\left(\sqrt{x} + 4\right)'}{\left(\sqrt{x} + 4\right)^2}$$

$$= \frac{\frac{\sqrt{x} + 4}{2\sqrt{x}} - \frac{\sqrt{x} - 4}{2\sqrt{x}}}{\left(\sqrt{x} + 4\right)^2}$$

$$= \frac{\frac{8}{2\sqrt{x}}}{\left(\sqrt{x} + 4\right)^2}$$

$$= \frac{4}{\sqrt{x}\left(\sqrt{x} + 4\right)^2}$$

- $y = \left(\frac{\sqrt{x}}{10} - 1 \right)^{-10}$

$$\frac{dy}{dx} = \left[\left(\frac{\sqrt{x}}{10} - 1 \right)^{-10} \right]' = \left[\left(\frac{\sqrt{x} - 10}{10} \right)^{-10} \right]'$$

$$= -10 \cdot \left(\frac{\sqrt{x} - 10}{10} \right)^{-11} \cdot \left(\frac{\sqrt{x} - 10}{10} \right)'$$

$$= -10 \cdot \left(\frac{\sqrt{x} - 10}{10} \right)^{-11} \cdot \left(\frac{\frac{10}{2\sqrt{x}}}{10^2} \right)$$

$$= -10 \cdot \left(\frac{\sqrt{x} - 10}{10} \right)^{-11} \cdot \left(\frac{1}{20\sqrt{x}} \right)$$

$$= \left(\frac{-1}{2\sqrt{x}} \right) \cdot \left(\frac{\sqrt{x} - 10}{10} \right)^{-11}$$

1.4 Question 4

1.4.1 Topic

Find an equation of the tangent line to the graph of $y = 1 + 2e^x$ at the point where $x = 0$.

1.4.2 Solution

- Based on the tangent equation: $y - y_0 = f'(x_0)(x - x_0)$ to do exercise 4.
- Find the coordinates of the point with $x = 0$ by substituting x into the function to find y.
- Apply the derivative formula to calculate the derivative y'.
- Calculate the value of y' at $x = 0$.
- Set to $y - y_0 = f'(x_0)(x - x_0)$ the formula to find the tangent equation.

1.4.3 Final result

- Because the tangent line to the graph of $y = 1 + 2e^x$ goes through the graph at the point $A(0, y_0)$, *so the point A belongs to the graph.*

$$y(0) = 1 + 2e^0 = 1 + 2 = 3$$

- So the coordinates of point A: $A(0,3)$.
- We have: $y'(x) = (1 + 2e^x)' = 2e^x$

$$\rightarrow y'(x_0) = y'(0) = 2e^0 = 2.1 = 2$$

- Tangent equation of the graph of a function $y = 1 + 2e^x$:

$$y - y_0 = y'(x_0)(x - x_0)$$
$$\rightarrow y - 3 = 2(x - 0)$$
$$\rightarrow y - 3 = 2x$$
$$\rightarrow y = 2x + 3$$

- So we can conclude that the equation of the tangent to the function $y = 1 + 2e^x$ at the point $x = 0$ is: $y = 2x + 3$.

1.5 Question 5

1.5.1 Topic

Given the derivative $f'(x) = (\sin x + \cos x)(\sin x - \cos x)$, $0 \le x \le 2\pi$.

- What are the critical numbers of f ?
- On what open intervals is f increasing or decreasing?
- At what points, if any, does f assume local maximum and minimum values?

1.5.2 Solution

- Give the function $f'(x) = 0$ to find the critical number.
- Draw a table of variations to determine whether the function increases or decreases at the intervals found when given: $f'(x) = 0$
- Identify Local Maxima and Minima Points:

- Local Maximum: Points where the function transitions from decreasing to increasing. This occurs when the derivative changes sign from positive to negative.
- Local Minimum: Points where the function transitions from increasing to decreasing. This occurs when the derivative changes sign from negative to positive.

1.5.3 Final result

- What are the critical numbers of f ?

$$\rightarrow (sin\,x + cos\,x)(sin\,x - cos\,x) = 0$$

$$\rightarrow \left[\begin{array}{l} sin\,x + cos\,x = 0 \\ sin\,x - cos\,x = 0 \end{array}\right.$$

$$\rightarrow \left[\begin{array}{l} sin\,x = -cos\,x \\ sin\,x = cos\,x \end{array}\right.$$

$$\rightarrow \left[\begin{array}{l} tan\,x = -1 \\ tan\,x = 1 \end{array}\right.$$

$$\rightarrow \left[\begin{array}{ll} \begin{cases} x = \dfrac{3\pi}{4} \\ x = \dfrac{7\pi}{4} \end{cases} & (0 \le x \le 2\pi) \\ \begin{cases} x = \dfrac{\pi}{4} \\ x = \dfrac{5\pi}{4} \end{cases} & (0 \le x \le 2\pi) \end{array}\right.$$

- So the critical number of the function *f(x)* is $\left(\frac{3\pi}{4}; \frac{7\pi}{4}; \frac{\pi}{4}; \frac{5\pi}{4}\right)$ suitable for the given condition $0 \le x \le 2\pi$.
- And we have the following variation table:

x	$+\infty$	0		$\frac{\pi}{4}$		$\frac{3\pi}{4}$		$\frac{5\pi}{4}$		$\frac{7\pi}{4}$		2π	$+\infty$
f'(x)			-	0	+	0	-	0	+	0	-		
f(x)			$\searrow$		$\nearrow$		$\searrow$		$\nearrow$		$\searrow$		

- On what open intervals is f increasing or decreasing?
  - According to the variation table above, we can see that:
    - The function decreases at $\left(0,\frac{\pi}{4}\right)$, $\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$, $\left(\frac{7\pi}{4},2\pi\right)$
    - The function increases at $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$ and $\left(\frac{5\pi}{4},\frac{7\pi}{4}\right)$
- At what points, if any, does *f* assume local maximum and minimum values?
  - At $x=\frac{\pi}{4}$, $x=\frac{5\pi}{4}$, We see the value of $f(x)$ from increasing to decreasing or the derivative value changing sign from negative to positive.

    → The value of *f(x)* reaches its local minimum value at $x=\frac{\pi}{4}$, $x=\frac{5\pi}{4}$
  - At $x=\frac{3\pi}{4}$, $x=\frac{7\pi}{4}$, We see the value $f(x)$ from decreasing to increasing or the derivative value $f'(x)$ changing sign from positive to negative.

    → The value of *f(x)* reaches its local maximum value at $x=\frac{3\pi}{4}$, $x=\frac{7\pi}{4}$

1.6 Question 6

1.6.1 Topic

Find all curves through a point where *x = 1* whose arc length is the following L value:

$$L=\int_{1}^{5}\sqrt{1+\frac{1}{x^2}}\;dx$$

1.6.2 Solution

- According to the bibliography Applied Calculus for IT, wrote by M.A. Phạm Kim Thủy, Division of Computer Science, TON DUC THANG University, we have an equation to measure the arc length of the curve *y = f(x)* from *x = a* to *x = b* is

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$$

- So that we have

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\,dx = \int_1^5 \sqrt{1 + \frac{1}{x^2}}\,dx$$

$$\rightarrow \sqrt{1 + (f'(x))^2} = \sqrt{1 + \frac{1}{x^2}} \quad \left((f'(x))^2 \geq 0,\ \frac{1}{x^2} \geq 0 \right)$$ *Square both sides*

$$\rightarrow 1 + (f'(x))^2 = 1 + \frac{1}{x^2}$$ *Subtract both side 1*

$$\rightarrow (f'(x))^2 = \frac{1}{x^2}$$

$$\rightarrow \left[\begin{aligned} f'(x) &= \frac{1}{x} \\ f'(x) &= \frac{-1}{x} \end{aligned} \right.$$

$$\rightarrow \left[\begin{aligned} \int f'(x)dx &= \int \frac{1}{x}dx \\ \int f'(x)dx &= -\int \frac{1}{x}dx \end{aligned} \right.$$ *Implication both sides*

$$\rightarrow \left[\begin{aligned} f(x) &= \ln|x| + C_1 \\ f(x) &= -\ln|x| + C_2 \end{aligned} \right.$$

- Given that the curves pass through the point where *x = 1*, we can find the constants C_1 and C_2 by substituting *x = 1* and $y = y_0$:

$$y_0 = \ln|1| + C_1 \to C_1 = y_0$$
$$y_0 = -\ln|1| + C_2 \to C_2 = y_0$$

1.6.3 Final result:

So all the curves that pass through the point (1, y_0) and satisfy the given arc length:

$$y = \ln|x| + y_0 \text{ and } y = -\ln|x| + y_0$$

1.7 Question 7

1.7.1 Topic

Given that $a_1, a_2, a_3, \ldots, a_n, \ldots$ are real numbers fulfilling the following conditions:

- $a_n > 0, n \in \mathbb{Z}^+$
- $a_1 \geq a_2 \geq a_3 \geq \ldots \geq a_n \geq \ldots$
- The series $a_2 + a_4 + a_8 + a_{16} + \ldots + a_{2^n} + \ldots$ diverges.

Determine the convergence or divergence of the following series. Explain.

$$a_1 + \frac{a_2}{2} + \frac{a_3}{3} + \frac{a_4}{4} \ldots + \frac{a_n}{n} + \ldots$$

1.7.2 Final result

- We have the divergence series S:

$$\sum_{n=1}^{\infty} a_{2^n} = a_2 + a_4 + a_8 + \ldots + a_{2^n} = S$$

- Suppose that we have the T series following:

$$a_1 + \frac{a_2}{2} + \frac{a_3}{3} + \frac{a_4}{4} \ldots + \frac{a_n}{n} + \ldots$$

  - Apply the comparison test:

$$0 < a_{2^n} \leq a_n$$

- Also we known that $\sum_{n=1}^{\infty} a_{2^n}$ is a divergence series.
- So $\sum_{n=1}^{\infty} a_n$ is a divergence series either.
- At the same time, with $a_n > 0$ so that $\boldsymbol{\sum_{n=1}^{\infty} \frac{a_n}{n}}$ **is also a divergence series**.

1.8 Question 8

1.8.1 Topic

Find all values of *x* such that the following series is absolutely convergent:

$$\sum_{n=1}^{\infty} \frac{nx^n}{(n+1)(2x+1)^n}$$

1.8.2 Solution

- *Apply the ratio test:*

$$\lim_{x\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{x\to\infty}\left|\frac{(n+1)x^{n+1}}{(n+2)(2x+1)}\cdot\frac{(n+1)(2x+1)}{nx^n}\right| = \lim_{x\to\infty}\left|\frac{(n+1)^2 x}{(n+2)n}\right|$$

$$= \lim_{x\to\infty}\left|\frac{n^2+2nx+x}{n^2+2n}\right| \overset{L'H}{=} \lim_{x\to\infty}\left|\frac{2n+2x}{2n+2}\right| \overset{L'H}{=} \lim_{x\to\infty}\left|\frac{2x}{2}\right| = \lim_{x\to\infty}|x| = L$$

- So the above series is absolutely convergent, if:

$$0 \le L < 1 \quad \to \quad 0 \le |x| < 1 \quad \to \quad -1 < x < 1$$

1.8.3 Final result

So with **every x ∈ (-1,1),** $\sum_{n=1}^{\infty} \frac{nx^n}{(n+1)(2x+1)^n}$ **will be absolutely convergent.**

1.9 Question 9

1.9.1 Topic

One thousand earphones sell for \$55 each, resulting in a revenue of (1000) (\$55) = \$55,000. For each \$5 increase in the price, 20 fewer earphones are sold. For ex., if the price of each earphone is \$60, there will be 980 (1000 – 20) earphones sold; if the price of each earphone is \$65, there will be 960 (1000 – 20 – 20) earphones sold; so on. Find the revenue in case the price of each earphone is \$255

1.9.2 Solution

- We call T is the revenue (units \$).
- We call S is the total earphones sold (units pcs).
- We call G is the original cost of each earphones (units \$).
- We call N is the topic requires cost of each earphones (units \$).
- From the topic, we know that: If we increase \$5 in price for each earphone, 20 fewer earphones are sold, so we have the following equation:

$$S = 1000 - \frac{N - G}{5} \times 20 \quad (pcs)$$

- So we can find the total earphones sold by the above equation:

$$S = 1000 - \frac{225 - 55}{5} \times 20 = 200 \quad (pcs)$$

- Finally, find the revenue if we sell \$225 each earphones:

$$T = S \times N = 200 \times 255 = \$51000$$

1.9.3 Final result

If we sold 200 piece earphones by \$225 for each, we can find the revenue is \$51000.

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