Math 1172 — Midterm 1 Cheatsheet (Front & Back)

Built to be self-sufficient: set-ups, identities, missing (anti)derivatives, mini-examples, and checklists.
Basic antiderivatives already on the official sheet are intentionally omitted.

0) Rapid Rules You'll Actually Use

Product $(uv)' = u'v + uv'$. **Quotient** $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$. **Chain** $(f\circ g)' = f'(g)\,g'$.

$$\frac{d}{dx}a^x = a^x \ln a, \quad \frac{d}{dx}\log_a x = \frac{1}{x\ln a}, \quad \frac{d}{dx}\ln|g| = \frac{g'}{g}.$$

Inverse trig: $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$.

Missing antiderivatives (not on basic list):

$$\int \tan x\,dx = -\ln|\cos x| + C, \quad \int \cot x\,dx = \ln|\sin x| + C,$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C, \quad \int \csc x\,dx = -\ln|\csc x + \cot x| + C,$$

$$\int \sin(kx)\,dx = -\frac{1}{k}\cos(kx) + C, \int \cos(kx)\,dx = \frac{1}{k}\sin(kx) + C,$$

$$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C, \int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C.$$

Chain-pattern spotting: $\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$ (e.g. $\int \frac{2x}{1+x^2}\,dx = \ln(1+x^2)+C$).

1) Trig Identities (minimal but mighty)

Pythagorean: $\sin^2 x + \cos^2 x = 1$, $1+\tan^2 x = \sec^2 x$, $\sec^2 x - \tan^2 x = 1$.

$\sin(2x) = 2\sin x\cos x$, $\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$.

Power-reduction: $\sin^2 x = \frac{1}{2}(1-\cos 2x)$, $\cos^2 x = \frac{1}{2}(1+\cos 2x)$.

$\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$, $\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$.

Exact trig values:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef

2) Area Between Curves

W.r.t. x: $A = \int_a^b (y_{\text{top}} - y_{\text{bot}})\,dx$; **w.r.t.** y: $A = \int_c^d (x_{\text{right}} - x_{\text{left}})\,dy$.

Split at intersection(s) when top/bottom (or right/left) switch.

Mini-Example (swap top/bottom): Between $y = x$ and $y = x^2$ on $[0,1]$: $A = \int_0^1 (x - x^2)dx = \frac{1}{6}$.

Intersection tricks: set $f(x) = g(x)$; if messy, consider factoring, completing square, or switching to y-slices.

3) Accumulated Cross-Sections

Perpendicular slices; area depends on shape:

$$V = \int \text{Area(slice)}\,ds, \quad s = x \text{ or } y.$$

Shapes: square $A = w^2$; equilateral tri. $A = \frac{\sqrt{3}}{4}w^2$; semicircle $A = \frac{\pi}{8}w^2$; isos. right tri. $A = \frac{1}{2}w^2$. (w = *distance between bounding curves along the slicing direction.*)

Mini-Example (semicircles, vertical slices): Region $y = \sqrt{x}$ to $y = 0$ on $[0,4]$; $w = \sqrt{x}$; $V = \frac{\pi}{8}\int_0^4 x\,dx = \pi$.

4) Solids of Revolution

Washers (slices ⊥ axis): $V = \pi\int (R_{\text{out}}^2 - R_{\text{in}}^2)\,ds$. **Shells (slices ∥ axis):** $V = 2\pi\int(\text{radius})(\text{height})\,ds$.

Radii templates: About $x = c$: with x-slices (shells) radius $= |x - c|$, height $= \text{top}(x) - \text{bot}(x)$. About $y = k$: with x-slices (washers) $R = |\,\text{curve} - k\,|$; if using y-slices, swap roles.

Mini-Example (washers about $y = 3$): Region $y = \sqrt{x}$ to 0 on $[0,4]$: $R_{\text{out}} = 3$, $R_{\text{in}} = |3 - \sqrt{x}|$, $V = \pi\int_0^4 (9 - (3-\sqrt{x})^2)\,dx = 24\pi$.

Mini-Example (shells about $x = 4$): Region between $y = x$ and $y = 0$ on $x \in [0,3]$: radius $= |x - 4| = 4 - x$, height $= x - 0$, $V = 2\pi\int_0^3 (4-x)\,x\,dx = 2\pi\Big[2x^2 - \frac{1}{3}x^3\Big]_0^3 = \frac{30}{1}\pi$.

5) Arc Length

Graph $y = f(x)$: $L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$. **Inverse** $x = g(y)$: $L = \int_c^d \sqrt{1+[g'(y)]^2}\,dy$. **Parametric:** $L = \int_\alpha^\beta \sqrt{(x')^2 + (y')^2}\,dt$.

Perfect-square trick: if $1+[f'(x)]^2$ simplifies to $(ax+b)^2$, length becomes $\int |ax+b|\,dx$.

Mini-Example: $y = \frac{1}{3}x^{3/2}$ on $[0,4]$: $f'(x) = \frac{1}{2}x^{1/2}$, $L = \int_0^4 \sqrt{1+\frac{1}{4}x}\,dx = \frac{8}{3}(2^{3/2} - 1)$.

6) Physical Apps (Mass/Work)

Mass (1D wire): $m = \int_a^b \rho(x)\,dx$.

Work to pump: $W = \int \rho g\,A(y)\,D(y)\,dy$. *Units:* ρg (weight density, e.g. lb/ft^3), $A(y)$ (ft^2), $D(y)$ (ft) $\Rightarrow$ ft-lb.

Mini-Example (cylinder, pump over top): $A(y) = \pi R^2$, $D(y) = H - y$: $W = \rho g\pi R^2\int_0^H (H-y)\,dy = \frac{1}{2}\rho g\pi R^2 H^2$.

Mini-Example (sphere radius R; bottom at $y = 0$): Slice at height y: $r(y) = \sqrt{2Ry - y^2}$, $A(y) = \pi r(y)^2 = \pi(2Ry - y^2)$, outlet at $y = H$: $W = \rho g\int_0^{2R} \pi(2Ry - y^2)\,(H-y)\,dy$.

7) Integration by Parts (IBP)

$\int u\,dv = uv - \int v\,du$, choose u by **LIATE**.

Tabular IBP (poly × trig/exp): For $\int x^2 e^x dx$:

u	dv
x^2	$e^x dx$
$2x$	$e^x dx$
2	$e^x dx$
0	$e^x dx$

$\Rightarrow x^2e^x - 2xe^x + 2e^x + C$.

Mini-Examples:

$$\int xe^{2x}dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C, \qquad \int x\sin(2x)dx = -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C.$$

8) Trig Integrals — Picking the Method

Odd sin**:** save one sin, convert rest, $u = \cos x$. **Odd** cos**:** save one cos, convert rest, $u = \sin x$. **Both even:** power-reduction. sec / tan**:** if tan odd $\Rightarrow$ save $\sec^2 x\,dx$ ($u = \tan x$); if sec even $\Rightarrow$ save $\sec^2 x\,dx$.

Mini-Examples:

$$\int \sin^3 x\,dx = \int \sin x(1-\cos^2 x)\,dx, \ u = \cos x \Rightarrow \int (1-u^2)(-du).$$

$$\int \sec^4 x\,dx = \int \sec^2 x\cdot\sec^2 x\,dx, \ u = \tan x \Rightarrow \int (1+u^2)\,du.$$

9) Trig Substitution — Map & Back-Sub

$$\begin{array}{c|l} \sqrt{a^2-x^2} & x=a\sin\theta,\ \sqrt{a^2-x^2}=a\cos\theta \\ \sqrt{a^2+x^2} & x=a\tan\theta,\ \sqrt{a^2+x^2}=a\sec\theta \\ \sqrt{x^2-a^2} & x=a\sec\theta,\ \sqrt{x^2-a^2}=a\tan\theta \end{array}$$

Completing square first if needed: $x^2+bx+c=(x+\frac{b}{2})^2+\left(c-(\frac{b}{2})^2\right)$.

Mini-Examples:

$$\int \frac{dx}{\sqrt{9-x^2}},\ x=3\sin\theta \Rightarrow \arcsin(x/3)+C.$$

$$\int \frac{x^2}{\sqrt{4x^2-25}}\,dx,\ x=\tfrac{5}{2}\sec\theta,\ \sqrt{4x^2-25}=\tfrac{5}{2}\tan\theta.$$

10) Partial Fractions (PFD)

Proper? If $\deg N \ge \deg D$, long divide first. **Factor** D over $\mathbb{R}$: linears $(x-a)$, irreducible quadratics (x^2+bx+c).

$$\frac{P(x)}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b},\qquad \frac{P}{(x-a)^k}=\sum_{j=1}^{k}\frac{A_j}{(x-a)^j},$$

$$\frac{P(x)}{(x^2+bx+c)^m}=\sum_{j=1}^{m}\frac{A_jx+B_j}{(x^2+bx+c)^j}.$$

Heaviside (distinct linears): A for $\frac{A}{x-a}$ is $N(a)/D'(a)$ when $D=(x-a)\tilde{D}$.

Mini-Examples (form only):

$$\frac{3x+1}{x(x-2)(x^2+1)}=\frac{A}{x}+\frac{B}{x-2}+\frac{Cx+D}{x^2+1}.$$

$$\frac{2x^2+1}{(x-1)^2(x+3)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+3}.$$

11) Improper Integrals & FTC sanity

Improper if ∞ limit(s) or vertical asymptote in $[a,b]$.

$$\int_a^\infty f=\lim_{t\to\infty}\int_a^t f,\quad \int_a^b f=\lim_{t\to c^-}\int_a^t f+\lim_{t\to c^+}\int_t^b f.$$

p-tests: $\int_1^\infty \frac{dx}{x^p}$ converges iff $p>1$; $\int_0^1 \frac{dx}{x^p}$ converges iff $p<1$.

Comparison near ∞: compare to $\frac{1}{x^p}$. Near 0^+: compare to $\frac{1}{x^p}$. *Monotone positive* integrands simplify comparison.

Mini-Examples:

$$\int_1^\infty \frac{dx}{x(\ln x)^2} \text{ converges (stronger than } 1/x). \qquad \int_0^1 \frac{dx}{\sqrt{x}}=2 \text{ (converges)}.$$

12) Fast Set-Up Templates (Fill & Go)

Area (x-slices): $A=\int_a^b \left(f_{\text{top}}(x)-g_{\text{bot}}(x)\right)dx.$

Cross-sections (vertical): $V=\int_a^b \kappa\,[\text{top}(x)-\text{bot}(x)]^2\,dx,\quad \kappa\in\{1,\frac{\sqrt{3}}{4},\frac{\pi}{8},\frac{1}{2}\}.$

Washers about $x=c$ (vertical axis, x-slices):

$$V=\pi\int_a^b\left((x-c)_{\text{out}}^2-(x-c)_{\text{in}}^2\right)dx.$$

Shells about $x=c$ (vertical axis, x-slices):

$$V=2\pi\int_a^b |x-c|\cdot(\text{top}-\text{bot})\,dx.$$

Work (pump to height H): $W=\rho g\int A(y)\,[H-y]\,dy.$

Arc length (graph): $L=\int_a^b\sqrt{1+[f'(x)]^2}\,dx.$

IBP: choose u by LIATE; dv is the rest.

Trig-int choice: odd sin / cos $\Rightarrow$ save one; both even $\Rightarrow$ reduce.

Trig-sub map: $\sqrt{a^2-x^2}\to x=a\sin\theta$; $\sqrt{a^2+x^2}\to x=a\tan\theta$; $\sqrt{x^2-a^2}\to x=a\sec\theta$.

PFD: long divide if needed; linear factors $\to$ constants, irreducible quadratics $\to Ax+B$.

13) Algebra/Calc Micro-Toolkit (saves minutes)

Completing square: $x^2+bx+c=(x+\frac{b}{2})^2+\left(c-(\frac{b}{2})^2\right)$. **Factoring patterns:** $a^2-b^2=(a-b)(a+b)$; $x^3-y^3=(x-y)(x^2+xy+y^2)$. **Linearization:** for small h, $\ln(1+h)\approx h$ (for quick reasonableness checks).

Solving intersections quickly: Set equal, move all to one side, try: common factoring, quadratic formula, or switching variables.

Domain reminders: square roots $\Rightarrow$ inside ≥ 0; denominators $\ne 0$; logs $\Rightarrow$ inside >0.

Inverse-function derivative: If $y=f(x)$ invertible, then $\frac{dy}{dx}=\frac{1}{f'(f^{-1}(y))}$. (*Useful in arc length/implicit spots.*)

14) Worked Micro-Checks (answer included)

A: $\int\frac{2x}{1+x^2}\,dx=\ln(1+x^2)+C$. **B:** $\int\tan x\,dx=-\ln|\cos x|+C$. **C:** $\int_0^{\pi/2}\sin^2 x\,dx=\frac{\pi}{4}$ (power-reduction).

D (PFD form): $\frac{x^2+3}{(x-1)^2(x^2+4)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{Cx+D}{x^2+4}.$

E (shells about $x=1$): Region between $y=x^2$ and $y=x$ on $[0,1]$: radius $=|x-1|=1-x$, height $=x-x^2$, $V=2\pi\int_0^1(1-x)(x-x^2)\,dx=2\pi\left[\frac{1}{12}\right]=\frac{\pi}{6}$.

F (arc length set-up only): $y=\ln(\cosh x)$ on $[0,a]$: $y'=\tanh x$, so $L=\int_0^a\sqrt{1+\tanh^2 x}\,dx=\int_0^a \operatorname{sech}^{-1}x$ (recognize simplification if known).

15) Exam Flow Hints (decision cheats)

Area $\to$ top-bottom or right-left; **split** if ordering changes.
Volume: axis vertical $\Rightarrow$ x-slices shells / y-slices washers; axis horizontal $\Rightarrow$ x-slices washers / y-slices shells. Pick the slicing that avoids solving hard inverses.
Work: pick vertical coordinate, compute $A(y)$ from geometry, $D(y)$ to outlet.
Arc length: check if $1+(f')^2$ simplifies; else substitution.
Trig integrals: odd/even rules; reduce before expanding.
Trig sub: match radicand to the map; complete square first.
PFD: divide $\Rightarrow$ factor $\Rightarrow$ write form $\Rightarrow$ solve constants (cover-up where allowed).
IBP: if it gets worse, change u (LIATE) or try substitution first.