

Part 1:

Time complexity

let from street and avenue, one of them is maximum so let it be n.

So, reoccurrence relation is when n is zero its takes constant time complexity.

otherwise, its $T(n-1) + T(n-1) + 1$

$$T(n) = 2T(n-1) + 1$$

$$T(n) = 2(2T(n-2) + 1) + 1 = 2^2T(n-2) + 2 + 1$$

$$T(n) = 2(2(T(n-3) + 1) + 1) + 1 = 2^3T(n-3) + 2^2 + 2 + 1$$

.....

$$T(n) = 2^{(n-1)}T(1) + 2^{(n-2)} + \dots + 2^2 + 2 + 1$$

$$T(n) = 2^nT(0) + 2^{(n-1)} + \dots + 2^2 + 2 + 1$$

$$T(0) = 1$$

So,

$$T(n) = 2^n(1) + 2^{(n-1)} + \dots + 2^2 + 2 + 1$$

$$T(n) = 2^n + (2^{(n-1)} - 1)/2$$

so, we can write it as $T(n) = 2^n$

So, the time complexity for this function is 2^n

Part 2:

Time complexity

Here, the time complexity for this function depends on the length of T most of the time.

So, when T or S is empty, complexity will be constant and otherwise

let n is the total length of T, so from $T(n)$ the reoccurrence relation should be

$$T(n) = T(n-1) + 1 \quad (\text{because complexity depends on size of T})$$

where n is size of array and the n goes from size to first element of array, so

$$T(n) = T(n-2) + 1 + 1$$

$$T(n) = T(n-3) + 1 + 1 + 1$$

....

$$T(n) = T(1) + 1 + 1 + \dots + 1$$

$$T(n) = T(0) + 1 + 1 + 1 + \dots + 1$$

$$\text{so, } T(n) = n(1)$$

The time complexity of this function is $O(n)$.