

Question 2:

a)

Time complexity of 3 way merge sort

Dividing the arrays gives constant time complexity of 3 way merge sort $O(1)$

The size of 3 sub-arrays are $N/3$, so the relation should be $T(N/3)$ and it recursively sort three sub-arrays so it is $3T(N/3)$.

While merging the sub-arrays, the time complexity is linear as it place the elements in correct order using comparison fo 3 sub-arrays so its $O(N)$

So,

$$T(N) = 3T(N/3) + N$$

$$T(N) = 3(3T(N/9) + N/3) + N$$

$$T(N) = 3(3(3T(N/27) + N/9) + N/3) + N$$

$$T(N) = 3^k(T(N/3^k)) + N/3^{(k-1)} + N/3^{(k-2)} + \dots + N$$

Here, $N/3^k = 1$ for base case

so, $k = \log(N)$ of base 3.

$$\text{So, } T(N) = 3^{\log(N)} * T(1) + N/3^{(\log N - 1)} + \dots + N$$

$$\text{Now, } 3^{\log N} = N$$

and

$$N/3^{(\log N - 1)} = N/(N/3) = 3$$

So, $N/3^{(\log N - 1)} + \dots + N$ gives us complexity $O(N)$

$$T(N) = NT(1) + O(N)$$

$$T(N) = O(N)$$

So, the complexity of 3 way merge sort is $O(N)$

b)

2 way merge sort has better time complexity than 3 way merge sort because 2 way merge sort divide arrays more quickly than 3 way merge sort mostly if the size is large. So, 2 way merge sort has better time complexity and performance than 3 way merge sort.

c)

In place merge sort algorithm using space complexity $O(n)$

merge(array[], start, mid, end)

int a1[start to mid]

int a2[mid+1 to end]

for(i from 0 to size of a1)

a1[i] = array[start+i]

for(i from 0 to size of a2)

a2[i] = array[mid+i]

// Merging array a1 and a2 to original array

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i and j = 0 and k = start
while(i < size of a1 and j < size of a2)
    if(a1[i] <= a2[j])
        array[k++] = a1[i]
        i++
    else
        array[k++] = a2[j]
        j++

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//copying remaining elements to original array
while(i is less than size of a1) {
    array[k++] = a1[i++]
}
while(j is less than size of a2) {
    array[k++] = a2[j++]
}

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mergeSort(array[], start, end)
    if(start >= end)
        return
    else
        mid = half of the size of array

        mergeSort(array, start, mid)
        mergeSort(array, mid+1, end)

        merge(array, start, mid, end)

```

Here, merge function merge the two arrays to original array using two arrays a1 and a2
 mergeSort function divides the array into two halves and then sort them