University of Pittsburgh
Department of Electrical and Computer Engineering ECE 0301 – Spring 2021

In-Class Assignment #4

Programming concepts:
- Looping instructions: `for`, `while`, `do/while`
- Reading and writing text files

ECE concepts:
- Analysis of resistive circuits: KVL, KCL, Ohm's Law
- The voltage divider
- The Wheatstone bridge

Background: Voltage divider and Wheatstone bridge circuits

The voltage divider, shown in Figure 1, should be familiar to you from the study of resistive circuits. To analyze this or any other circuit, we must determine all of the unknown voltages and currents (in this case, I, V_1 and V_2) from the given information (in this case, the source voltage V_s and the resistances R_1 and R_2.)

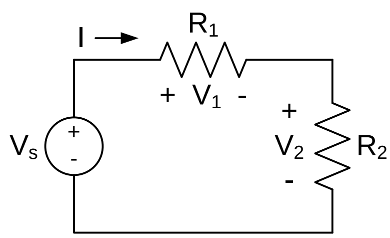

Figure 1: Voltage divider.

The analysis is very simple, and begins with KVL and Ohm's Law,

$$V_s = V_1 + V_2 = IR_1 + IR_2$$

$$I = \frac{V_s}{R_1 + R_2} \qquad V_1 = V_s \frac{R_1}{R_1 + R_2} \qquad V_2 = V_s \frac{R_2}{R_1 + R_2}.$$

By now, it should be easy for you to visualize how to write a `C++` program to implement these equations, and compute $\{I, V_1, V_2\}$ for any values of $\{V\text{s}, R_1, R_2\}$.

The Wheatstone bridge circuit shown in Figure 2 is considerably more complex. Nevertheless, we can perform the analysis armed only with KVL, KCL, Ohm's Law, and elementary algebra. We can also take advantage of some notational simplifications that are easy to implement as computer instructions.

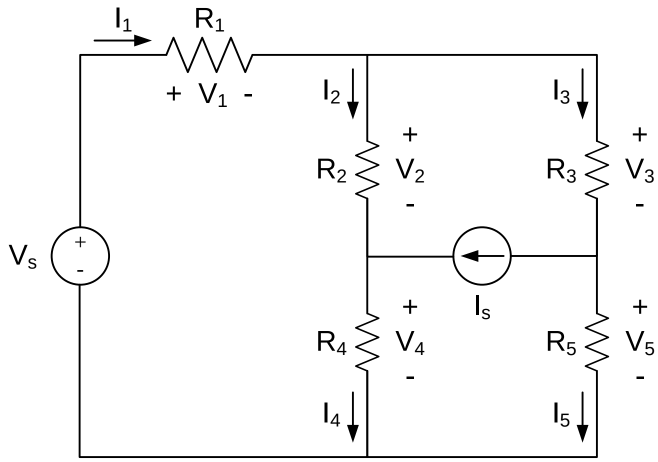

Figure 2: Wheatsone bridge circuit[1]

Begin by noting that all resistor voltages and currents are related through Ohm's Law,

$$V_1 = I_1 R_1 \qquad V_2 = I_2 R_2 \qquad V_3 = I_3 R_3 \qquad V_4 = I_4 R_4 \qquad V_5 = I_5 R_5 .$$

Therefore, we can solve for the voltages and find the currents directly. By KVL, we can also express two of the voltages in terms of the others,

$$V_4 = V_s - V_1 - V_2 \tag{1}$$

$$V_5 = V_s - V_1 - V_3. \tag{2}$$

At this point, we need only solve for $\{V_1, V_2, V_3\}$ to determine the remaining voltages and all of the currents. Next, if we apply KCL at the nodes at either end of the current source,

$$I_2 + I_s = I_4 \qquad \Rightarrow \quad \frac{V_2}{R_2} + I_s = \frac{V_4}{R_4}$$

$$I_3 = I_s + I_5 \quad \Rightarrow \qquad \frac{V_3}{R_3} = I_s + \frac{V_5}{R_5},$$

then by substitution, we have

$$\frac{V_2}{R_2} + I_s = \frac{V_s - V_1 - V_2}{R_4} \qquad \Rightarrow \qquad V_1 + V_2\left(1 + \frac{R_4}{R_2}\right) = V_s - I_s R_4$$

$$\frac{V_3}{R_3} = I_s + \frac{V_s - V_1 - V_3}{R_5} \qquad \Rightarrow \qquad V_1 + V_3\left(1 + \frac{R_5}{R_3}\right) = V_s + I_s R_5.$$

We now have two linearly independent equations relating the unknown voltages $\{V_1, V_2, V_3\}$. We can obtain a third equation by applying KCL at the node at the top of R_2 and R_3,

$$I_1 = I_2 + I_3 \quad \Rightarrow \quad \frac{V_1}{R_1} = \frac{V_2}{R_2} + \frac{V_3}{R_3} \quad \Rightarrow \quad V_1 = V_2\left(\frac{R_1}{R_2}\right) + V_3\left(\frac{R_1}{R_3}\right).$$

We have analyzed the circuit to obtain three linearly independent equations in the three unknown voltages, of the general form

[1] This is not a typical form of the Wheatstone bridge. In the original application for this circuit, a voltmeter is used in place of the current source, and R_5 is an unknown resistance one wishes to measure, usually by varying R_4 until the voltmeter reads zero.

$$V_1 + aV_2 = b$$
$$V_1 + cV_3 = d$$
$$V_1 = eV_2 + fV_3. \tag{3}$$

To solve any system of equations like this, we may substitute the expression for V_1 from the last equation into the other two, yielding

$$\begin{aligned} eV_2 + fV_3 + aV_2 &= b \\ eV_2 + fV_3 + cV_3 &= d \end{aligned} \quad \Rightarrow \quad \begin{aligned} \left(\frac{a+e}{b}\right)V_2 + \left(\frac{f}{b}\right)V_3 &= 1 \\ \left(\frac{e}{d}\right)V_2 + \left(\frac{c+f}{d}\right)V_3 &= 1 \end{aligned}$$

Now we have a pair of equations in two unknowns of the general form

$$\begin{aligned} wV_2 + xV_3 &= 1 \\ yV_2 + zV_3 &= 1 \end{aligned} \quad \Rightarrow \quad \begin{aligned} wyV_2 + xyV_3 &= y \\ wyV_2 + wzV_3 &= w \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_3 &= \frac{y-w}{xy-wz} && (4) \\ V_2 &= \frac{1-xV_3}{w} && (5) \end{aligned}$$

These results provide us with an algorithm for solving the Wheatstone bridge circuit:

- Use the given values for $\{V_s, I_s, R_1, R_2, R_3, R_4, R_5\}$ to compute $\{a, b, c, d, e, f\}$.
- Use $\{a, b, c, d, e, f\}$ to compute $\{w, x, y, z\}$.
- Use equations (4) and (5) to compute V_3 and V_2.
- Use equation (3) to compute V_1.
- Use equations (1) and (2) to compute V_4 and V_5.

Instructions: Develop software according to the following specifications and submit whatever portion you have completed to the class repository before midnight tonight. **For all in-class assignments, you must include line-by-line comments to explain what your does and why! You must also choose meaningful names for all variables so that it will be easy for a reader to understand your code.**

1. Write a computer program that will display the following introductory message to standard output:
   ```
   ECE 0301: Circuit Solver for Voltage Divider
   and Wheatstone bridge example circuits.
   ```

   Don't forget to include comments! Type them in your code as you complete each item.

2. Create a text file with the word `Divider` on the first line, and save it to a file named "`divider_wheatstone_circuits.txt`" in the directory where your program is stored. For the remainder of this assignment, all input will be read from this file.

   Declare an `ifstream` object, use it to open the input file, and read the first line with the `>>` operator. If the first line is anything other than `Divider` or `Wheatstone`, the program should exit with the error message (printed to standard output)
   ```
   ERROR! Invalid header.
   ```
   and a return value of -1.

Test your program by changing the first line of the text file, and make sure that the program exits under the proper conditions.

3. If the first line of the file is `Divider`, the program should read three more lines from the file, which hold values for the voltage source V_s and the resistors R_1 and R_2. Store V_s in a `double` variable, and store R_1 and R_2 in `int` variables.

   Define an `ofstream` object, and use it to open a text file for output named "`divider_wheatstone_solutions.txt`". Write messages to the output file to report the values that were read from the file.

   Type the following into the input file and use it to test your program.

```
Divider
5
1000
1500
```

4. Modify your program so that, if the first line of the file is `Divider`, it will solve the voltage divider circuit by computing the loop current I and the resistor voltages V_1 and V_2, and write messages to the output file reporting the computed values.

   Test your program using the input file from step 3, and make sure the computed values are correct. The required format for the output file is shown below for this case.

```
ECE 0301: Circuit Solver for Voltage Divider
and Wheatstone bridge example circuits.
-----
Circuit #1 (Voltage Divider)
---
Source voltage: Vs = 5 Volts.
Resistor: R1 = 1000 Ohms.
Resistor: R2 = 1500 Ohms.

Loop current: I = 0.002 Amperes.
Resistor voltage: V1 = 2 Volts.
Resistor voltage: V2 = 3 Volts.
```

   Modify the text file by changing the component values, and test again. The loop current and resistor voltage will be reported with more digits if the answers are not integers or terminating decimals. Can your program solve any voltage divider problem?

   When you are certain your program is correct, save it in a file named:

```
ece0301_ICA04_step04.cpp
```

   Submit this file to the class repository.

5. If the first line of the file is `Wheatstone`, the program should read seven more lines from the file, which hold values for the voltage source V_s, the current source I_s, and the resistors R_1 through R_5. Store V_s and I_s in `double` variables, and store the resistances in `int` variables. Write messages to the output file to report the values that were read from the file.

   Test your program with this file:

```
Wheatstone
12.0
0.1
150
150
150
150
150
```

6. Modify your program so that, if the first line of the file is `Wheatstone`, it will solve the Wheatstone bridge circuit by computing the resistor voltages V_1 through V_5, and resistor currents I_1 through I_5, and write messages to the output file reporting the computed values.

   Test your program using the input file from step 5, and verify that the computed values are correct. The required output file format is shown below for this case.

```
ECE 0301: Circuit Solver for Voltage Divider
and Wheatstone bridge example circuits.
-----
Circuit #1 (Wheatstone Bridge)
---
Source voltage: Vs = 12 Volts.
Source current: Is = 0.1 Amperes.
Resistor: R1 = 150 Ohms.
Resistor: R2 = 150 Ohms.
Resistor: R3 = 150 Ohms.
Resistor: R4 = 150 Ohms.
Resistor: R5 = 150 Ohms.

Resistor voltage: V1 = 6 Volts.
Resistor current: I1 = 0.04 Amperes.
Resistor voltage: V2 = -4.5 Volts.
Resistor current: I2 = -0.03 Amperes.
Resistor voltage: V3 = 10.5 Volts.
Resistor current: I3 = 0.07 Amperes.
Resistor voltage: V4 = 10.5 Volts.
Resistor current: I4 = 0.07 Amperes.
Resistor voltage: V5 = -4.5 Volts.
Resistor current: I5 = -0.03 Amperes.
```

7. The method we derived for solving the Wheatstone bridge circuit will not work in all cases. Your program should produce a divide-by-zero error if any of the following conditions are true (go back and look at the equations to see why):

$$R_2 = 0 \qquad V_s - I_s R_4 = 0$$
$$R_3 = 0 \qquad V_s + I_s R_5 = 0\,.$$

Furthermore, if either of the quantities in the right column are very small but not zero, then round-off error in representing them could occur when they are used as the denominator of a division operation. Round-off error for double-precision floating-point calculations is on the order of 10^{-15}, and we will require that all denominators be at least 100 times larger (in magnitude) than this value.

Modify your program so that, if it is directed to solve the Wheatstone bridge circuit, and any one of the following conditions are true

$$R_2 = 0 \qquad -10^{-13} < V_s - I_s R_4 < 10^{-13}$$
$$R_3 = 0 \qquad -10^{-13} < V_s + I_s R_5 < 10^{-13}\,.$$

then the program will print the error message

```
ERROR! Unstable floating-point division.
```

to standard output and exit with a return value of -1.

Modify your text file to try each of the four cases that produce errors, and confirm that your program generates errors as expected.

When you are certain your program is correct, save it in a file named:

```
ece0301_ICA04_step07.cpp
```

Submit this file to the class repository.

8. Modify your program so that it will read an additional line from the input file containing an integer *n*. This will be line 5 of the file for a voltage divider circuit and line 9 of the file for a Wheatstone bridge circuit. Add nested loops to your program so that it solves the circuit repeatedly for each resistor value taking the values {*R*, 2*R*, …, n*R*}. For example, if the text file contains:

```
Divider
5
1000
1500
3
```

then the program should solve the voltage divider circuit with the source set to V_s = 5V, and 3^2 = 9 different combinations of resistor values:

$$R_1 = 1000,\ R_2 = 1500 \qquad R_1 = 1000,\ R_2 = 3000 \qquad R_1 = 1000,\ R_2 = 4500$$
$$R_1 = 2000,\ R_2 = 1500 \qquad R_1 = 2000,\ R_2 = 3000 \qquad R_1 = 2000,\ R_2 = 4500$$
$$R_1 = 3000,\ R_2 = 1500 \qquad R_1 = 3000,\ R_2 = 3000 \qquad R_1 = 3000,\ R_2 = 4500\,.$$

For this case, the required output file format is summarized below,

```
ECE 0301: Circuit Solver for Voltage Divider
and Wheatstone bridge example circuits.
-----
Circuit #1 (Voltage Divider)
---
Source voltage: Vs = 5 Volts.
Resistor: R1 = 1000 Ohms.
Resistor: R2 = 1500 Ohms.

Loop current: I = 0.002 Amperes.
Resistor voltage: V1 = 2 Volts.
Resistor voltage: V2 = 3 Volts.
---
Source voltage: Vs = 5 Volts.
Resistor: R1 = 1000 Ohms.
Resistor: R2 = 3000 Ohms.

Loop current: I = 0.00125 Amperes.
Resistor voltage: V1 = 1.25 Volts.
Resistor voltage: V2 = 3.75 Volts.
---
(etc.)
```

Note that you must place a line of three dashes between each solution.

On the other hand, if the input file contains:

```
Wheatstone
12.0
0.1
150
150
150
150
150
4
```

then the program should solve the Wheatstone bridge problem for V_s = 12V, I_s = 0.1A, and $4^5 = 1,024$ different combinations of resistor values:

$$R_1 = 150,\ R_2 = 150,\ R_3 = 150,\ R_4 = 150,\ R_5 = 150$$
$$R_1 = 150,\ R_2 = 150,\ R_3 = 150,\ R_4 = 150,\ R_5 = 300$$
$$R_1 = 150,\ R_2 = 150,\ R_3 = 150,\ R_4 = 150,\ R_5 = 450$$
$$R_1 = 150,\ R_2 = 150,\ R_3 = 150,\ R_4 = 150,\ R_5 = 600$$
$$R_1 = 150,\ R_2 = 150,\ R_3 = 150,\ R_4 = 300,\ R_5 = 150$$
$$R_1 = 150,\ R_2 = 150,\ R_3 = 150,\ R_4 = 300,\ R_5 = 300$$
$$\vdots$$
$$R_1 = 600,\ R_2 = 600,\ R_3 = 600,\ R_4 = 600,\ R_5 = 300$$

$$R_1 = 600,\ R_2 = 600,\ R_3 = 600,\ R_4 = 600,\ R_5 = 450$$
$$R_1 = 600,\ R_2 = 600,\ R_3 = 600,\ R_4 = 600,\ R_5 = 600.$$

When you are certain your program is correct, save it in a file named:

`ece0301_ICA04_step08.cpp`

Submit this file to the class repository.

9. Modify your program so that it continues to read lines from the file until the end of the file is reached. After it has read all of the lines in the file for a given circuit, and solved the circuit and written the corresponding messages to the output file, the program should read the next line of the file. This line should contain either `Divider` or `Wheatstone`, just as on line 1, and your program should proceed to take the appropriate action, either reading more lines and solving the circuit, or exiting with return value -1.

   Test your program with this file:

```
Divider
5
1000
1500
3
Wheatstone
12.0
0.1
150
150
150
150
150
4
```

   For this case, the required output file format is summarized below. **Note that you must increment the number in the circuit title for each new circuit**.

```
ECE 0301 – Circuit Solver for Voltage Divider
and Wheatstone bridge example circuits.
-----
Circuit #1 (Voltage Divider)
---
Source voltage: Vs = 5 Volts.
Resistor: R1 = 1000 Ohms.
Resistor: R2 = 1500 Ohms.

Loop current: I = 0.002 Amperes.
Resistor voltage: V1 = 2 Volts.
Resistor voltage: V2 = 3 Volts.
---
Source voltage: Vs = 5 Volts.
Resistor: R1 = 1000 Ohms.
```

```
Resistor: R2 = 3000 Ohms.

Loop current: I = 0.00125 Amperes.
Resistor voltage: V1 = 1.25 Volts.
Resistor voltage: V2 = 3.75 Volts.
---
...(seven more solutions for Circuit #1)...
-----
Circuit #2 (Wheatstone Bridge)
---
Source voltage: Vs = 12 Volts.
Source current: Is = 0.1 Amperes.
Resistor: R1 = 150 Ohms.
Resistor: R2 = 150 Ohms.
Resistor: R3 = 150 Ohms.
Resistor: R4 = 150 Ohms.
Resistor: R5 = 150 Ohms.

Resistor voltage: V1 = 6 Volts.
Resistor current: I1 = 0.04 Amperes.
Resistor voltage: V2 = -4.5 Volts.
Resistor current: I2 = -0.03 Amperes.
Resistor voltage: V3 = 10.5 Volts.
Resistor current: I3 = 0.07 Amperes.
Resistor voltage: V4 = 10.5 Volts.
Resistor current: I4 = 0.07 Amperes.
Resistor voltage: V5 = -4.5 Volts.
Resistor current: I5 = -0.03 Amperes.
---
Source voltage: Vs = 12 Volts.
Source current: Is = 0.1 Amperes.
Resistor: R1 = 150 Ohms.
Resistor: R2 = 150 Ohms.
Resistor: R3 = 150 Ohms.
Resistor: R4 = 150 Ohms.
Resistor: R5 = 300 Ohms.

Resistor voltage: V1 = 6.81818 Volts.
Resistor current: I1 = 0.0454545 Amperes.
Resistor voltage: V2 = -4.90909 Volts.
Resistor current: I2 = -0.0327273 Amperes.
Resistor voltage: V3 = 11.7273 Volts.
Resistor current: I3 = 0.0781818 Amperes.
Resistor voltage: V4 = 10.0909 Volts.
Resistor current: I4 = 0.0672727 Amperes.
Resistor voltage: V5 = -6.54545 Volts.
Resistor current: I5 = -0.0436364 Amperes.
---
```

```
...(1022 more solutions for Circuit #2)
```

Notes:

The check for unstable floating-point division should occur after

```
Circuit #[num] (Wheatstone Bridge)
```

is printed to the output file and before any

```
---
```

is printed to the output file. This way there won't be any circuit # mismatches when the autograder tests the case where the program exits for unstable floating-point division.

When you are certain your program is correct, save it in a file named:
`ece0301_ICA04_step09.cpp`
Submit this file to the class repository.

Don't forget to include comments! You will lose credit if you leave them out, even if your code functions as directed!

Templates Key Map:

If any of the following test file are supposed to have corresponding output file, this file will have the same file name as the test file with the word "_out" added at the end of the file name.

Step number	File name	Explanation
Step 4	step04_test_invalid_header	input file with invalid header
Step 4	step04_test_divider	divider circuit
Step 7	step07_test_wheatstone	Wheatstone circuit test
Step 7	step07_test_divide_by_zero_1 step07_test_divide_by_zero_2 step07_test_divide_by_zero_3 step07_test_divide_by_zero_4	Check if properly handles unstable floating-point division
Step 8	step08_test_divider_n	Divider circuit (n > 1)
Step 8	step08_test_wheatstone_n	Wheatstone circuit (n > 1)
Step 9	step09_test_many	Sample Divider and Wheatstone circuits with many circuits
Step 9	step09_test_many_divide_by_zero	Sample Divider and Wheatstone circuits with many circuits and an unstable floating-point division case
Step 9	step09_test_many_invalid_header	Sample Divider and Wheatstone circuits with many circuits and an invalid header