Spectrum Sensing and SNR Walls When Primary User Has Multiple Power Levels

Chen Qian, Han Qian, and Feifei Gao

***Abstract*—In this paper, we study the spectrum sensing problem when the primary user (PU) transmits over more than one power levels and when there exists noise uncertainty at the secondary user (SU) side. Energy detection is, again, proved to be the optimal for detecting the "on-off" status of PU and could also be used for recognizing the power levels of PU. We derive the closed-form decision region and compute the corresponding closed-form decision probability. Meanwhile, it is shown that when the signal-to-noise ratio (SNR) is lower than certain thresholds, both the detection and recognition of some power levels could suffer from SNR wall effects, i.e., detection or recognition fails no matter how long the sensing time is. Finally, simulation results are provided to cohobate the proposed studies.**

I. Introduction

Cognitive radio is deemed as the solution to the spectrum scarcity by allowing the unlicensed users to access the white space in licensed spectrum. Spectrum sensing is then the key technology due to its capability of locating the spectrum holes. Popular spectrum sensing techniques include matched filter detection [1], energy detection [2] and cyclostationary detection [3], among which energy detection is the most widely used one because it requires the least information about the primary user's (PU) signal and has lower complexity.

In traditional CR, PU is assumed to have only two status, i.e., either being absent or operating under a fixed power. The task of spectrum sensing is then to detect the binary status of PU. However, it is easily seen from many current standards, e.g., IEEE 802.11 series, GSM, and the future standards, e.g., LTE, LTE-A that the licensed users could and should be working under different transmit power levels in order to cope with different situations. The spectrum sensing problem when PU has multiple transmit power levels (MPTP) was recently studied in [4], where both the PU's on-off status as well as the PU's power levels could be recognized. It was also demonstrated that sensing in MPTP has many differences from conventional binary sensing.

One the other hand, many conventional binary sensing works assume perfect knowledge of noise variance when designing the spectrum sensing algorithms. However, in a practical system, the noise variance cannot be perfectly measured, due to initial calibration error, temperature variation, changes in low noise amplifier gain by thermal variation, and interference, etc [5]. Noise uncertainty has negative effects on the precision of spectrum sensing and sometimes even fails the sensing. Such influence has been studied in several works, including [6], [7]. Especially, Tandra and Sahai [5] found that when the signal-to-noise ratio (SNR) is lower than a certain value, the performance of PU detection cannot be optimized by increasing the sensing time. This phenomena is well known as SNR wall and has attracted much attention [8], [9].

Motivated by the above discussions, we here study the spectrum sensing in MPTP scenario considering the noise uncertainty at the secondary user (SU) side. We first prove that the energy detection is the optimal for both PU detection and power level recognition. We then derive the closed-form solutions for the decision region and compute the closed-form decision probabilities. Similar as in the binary sensing [5], we find that SNR walls also exist for MPTP scenario but interestingly, there exist multiple SNR walls, two for each power level. The exact position of those SNR walls are also computed. Finally, simulating results are provided to corroborate the proposed studies.

II. Problem Formulation

A. Signal Model

Consider a simple CR system with a pair of PU and SU, where PU could possibly operate under any power level $P_k, k \in \{0, 1, \cdots, M\}$ with $P_0 = 0$ representing the absence of PU. The signal received at SU under time index n can be expressed as

$$\mathcal{H}_k : x_n = \sqrt{P_k}\sqrt{\gamma}e^{j\phi}s_n + \omega_n, \quad k = 0, \cdots, M, \quad (1)$$

where $\mathcal{H}_k$ stands for the hypothesis that the PU is operating under power P_k; s_n denotes the n-th symbol transmitted from PU which is assumed to follow circularly symmetric complex Gaussian (CSCG) distribution with zero mean and unit variance; $\sqrt{\gamma}$ is the channel gain and ϕ is the channel phase; ω_n represents the additive noise which is assumed to be CSCG with zero mean and variance σ^2. Hence, x_n also follows CSCG distribution

$$\mathcal{H}_k : \quad x_n \sim \mathcal{CN}(0, P_k\gamma + \sigma^2). \quad (2)$$

Note that SU could have the knowledge of $\{P_0, P_1 ..., P_M\}$,[1] since the power level set of PU is predetermined and could be easily known in advance, say, from standard. However, SU does not know which power level PU is currently working on and wish to make the decision from the received signals x_n.

Before proceeding, we make the following assumptions:

1) The exact noise variance σ^2 is unknown to SU but SU knows the variance would stay in the range of (σ_L^2, σ_R^2).

[1] In conventional binary sensing, SU should also know the unique non-zero power level of PU [10], [11]

2) In order to emphasize only on the effect of the noise uncertainty, we assume $\sqrt{\gamma}$ is known at SU, as did in [4], [10], [11] and many existing noise uncertainty works [2] [9]. The case of channel gain uncertainty will be studied in our future work.

B. Energy Detection

Suppose that SU receives a total number of N samples during the sensing period, and define $\mathbf{x} = [x_1, x_2, \cdots, x_N]^T$, $\mathbf{s} = [s_1, s_2, \cdots, s_N]^T$, $\mathbf{w} = [w_1, w_2, \cdots, w_N]^T$. Due to the independence between $\mathbf{s}$ and $\mathbf{w}$, the likelihood probability density function (pdf) of $\mathbf{x}$ under hypothesis $\mathcal{H}_k$ is given by

$$p(\mathbf{x}|\mathcal{H}_k) = \frac{1}{[\pi(\sigma^2 + P_k\gamma)]^N} \exp\left(-\frac{\|\mathbf{x}\|^2}{\sigma^2 + P_k\gamma}\right). \quad (3)$$

Since the noise variance σ^2 is unknown, we apply the generalized likelihood ratio test (GLRT) method and obtain the maximum likelihood estimation (MLE) of σ^2 under different hypothesis as

$$\begin{cases} \sigma_j^2 = \sigma_R^2, & \text{if } P_j\gamma < T(\mathbf{x}) - \sigma_R^2 \\ \sigma_i^2 = T(\mathbf{x}) - P_i\gamma, & \text{if } T(\mathbf{x}) - \sigma_R^2 \leqslant P_i\gamma \leqslant T(\mathbf{x}) - \sigma_L^2 \\ \sigma_l^2 = \sigma_L^2, & \text{if } P_l\gamma > T(\mathbf{x}) - \sigma_L^2, \end{cases} \quad (4)$$

where $T(\mathbf{x}) \triangleq \frac{1}{N}\sum_{n=0}^{N} |x(n)|^2$ is the average received energy. Substituting (4) into (3) yields:

$$\begin{cases} p(\mathbf{x}|\mathcal{H}_j, \sigma_j^2) = \dfrac{1}{[\pi(P_j\gamma + \sigma_R^2)]^N} \exp\left[-\dfrac{NT(\mathbf{x})}{P_j\gamma + \sigma_R^2}\right] \\ p(\mathbf{x}|\mathcal{H}_i, \sigma_i^2) = \dfrac{1}{(\pi T(\mathbf{x}))^N} \exp(-N) \\ p(\mathbf{x}|\mathcal{H}_l, \sigma_l^2) = \dfrac{1}{[\pi(P_l\gamma + \sigma_L^2)]^N} \exp\left[-\dfrac{NT(\mathbf{x})}{P_l\gamma + \sigma_L^2}\right]. \end{cases} \quad (5)$$

Since the conditional probability density function (pdf) of $\mathbf{x}$ is a function of $T(\mathbf{x})$, we select $T(\mathbf{x})$ as the test statistics.

It can be checked that the second equation in (5) is always larger than the first and the third one. Thus, those power levels P_i that satisfy $T(\mathbf{x}) - \sigma_R^2 \leqslant P_i\gamma \leqslant T(\mathbf{x}) - \sigma_L^2$ will beat others. Let us then define $Y_P \triangleq \{P_k | T(\mathbf{x}) - \sigma_R^2 \leqslant P_k\gamma \leqslant T(\mathbf{x}) - \sigma_L^2\}$ as the set of the power levels that win out. It could be found that all power levels in set Y_P share the same conditional pdf in (5) and hence, these power levels cannot be discriminated when Y_P has more than one elements. We call this special phenomenon in MPTP scenario as *power ambiguity*[2].

Thus, to make successful recognition, the set Y_P should either contain only one element or be empty.

Three cases are presented here based on the number of elements contained in Y_P:

1) Case I: For each $k \in \{0, 1, \cdots, M-1\}$, $\gamma(P_{k+1} - P_{k-1}) < (\sigma_R^2 - \sigma_L^2)$. If set Y_P is empty, there should be

$$\gamma P_k + \sigma_R^2 < T(\mathbf{x}) < \gamma P_{k+1} + \sigma_L^2, \quad (6)$$

but this inequality never holds for true due to the constraint $\gamma(P_{k+1} - P_{k-1}) < (\sigma_R^2 - \sigma_L^2)$. Moreover, if we wish P_k to be the only element in set Y_P, there must be:

$$\gamma P_{k-1} < T(\mathbf{x}) - \sigma_R^2 < \gamma P_k < T(\mathbf{x}) - \sigma_L^2 < \gamma P_{k+1}, \quad (7)$$

i.e.,

$$\gamma P_{k-1} + \sigma_R^2 < T(\mathbf{x}) < \gamma P_{k+1} + \sigma_L^2. \quad (8)$$

However, since $\gamma(P_{k+1} - P_{k-1}) < (\sigma_R^2 - \sigma_L^2)$, (8) cannot hold for true. Hence, there must be more than two elements in set Y_p and power ambiguity happens.

2) Case II: For each k, $\gamma(P_{k+1} - P_k) < (\sigma_R^2 - \sigma_L^2)$, and $\gamma(P_{k+1} - P_{k-1}) > (\sigma_R^2 - \sigma_L^2)$. Similar to Case I, it is easily proved that set Y_P cannot be empty, so we only need to check the situation where set Y_P is nonempty. The condition for P_k being the only element in set Y_P is the same as (8). Different from Case I, the interval $(\gamma P_{k-1} + \sigma_R^2, \gamma P_{k+1} + \sigma_L^2)$ is a valid interval. When $T(\mathbf{x})$ falls in this interval, P_k is considered to be the transmit power level at PU. So we consider $(\gamma P_{k-1} + \sigma_R^2, \gamma P_{k+1} + \sigma_L^2)$ as a potential decision region for P_k. However, the summation of all these potential decision regions for power levels $P_k, k \in \{0, 1, \cdots, M\}$ does not cover every possible value of $T(\mathbf{x})$, for example, $T(\mathbf{x}) \in (\gamma P_{k+1} + \sigma_L^2, \gamma P_k + \sigma_R^2)$ cannot be contained in any potential decision regions. Regions like $(\gamma P_{k+1} + \sigma_L^2, \gamma P_k + \sigma_R^2)$ are named as *blind regions*. When $T(\mathbf{x})$ falls in blind regions, the set Y_P will contain more than one element, and power ambiguity occurs. Since we could not rule out the possibility that $T(\mathbf{x})$ falls into blind regions, the recognition in Case II is not stable.

3) Case III: For each k, $\gamma(P_{k+1} - P_k) > (\sigma_R^2 - \sigma_L^2)$. It is easily known that there is at most one element in set Y_P for Case III. When P_k is the only element in set Y_P, there must be $\gamma P_k + \sigma_L^2 < T(\mathbf{x}) < \gamma P_k + \sigma_R^2$ and this interval is symbolized as $(\Theta_k^{(L)}, \Theta_k^{(R)})$, which is the potential decision region for P_k when set Y_P is nonempty. Note that the summation of $(\Theta_k^{(L)}, \Theta_k^{(R)})$ for all k does not cover all possible values of $T(\mathbf{x})$, which is similar to the situation in Case II.

However, different from Case II, Y_P is empty when $T(\mathbf{x}) \in (\gamma P_k + \sigma_L^2, \gamma P_k + \sigma_R^2)$ in this case. Note that the combination of $(\gamma P_k + \sigma_R^2, \gamma P_{k+1} + \sigma_L^2)$ and $(\gamma P_k + \sigma_L^2, \gamma P_k + \sigma_R^2)$ covers all values of $T(\mathbf{x})$, thus, no blind region exists. Therefore, the decision region for any power level P_k consists of two parts: $(\Theta_k^{(L)}, \Theta_k^{(R)})$ and parts of $(\gamma P_k + \sigma_L^2, \gamma P_k + \sigma_R^2)$ and $(\gamma P_{k-1} + \sigma_L^2, \gamma P_k + \sigma_R^2)$, where $p(\mathbf{x}|\mathcal{H}_k, \sigma_k^2)$ has the largest value. The first part is the potential decision region for P_k when Y_P is nonempty, while the second part is for an empty Y_P.

To compute a united form of decision region, we propose to divide $(\gamma P_k + \sigma_L^2, \gamma P_k + \sigma_R^2)$ and $(\gamma P_{k-1} + \sigma_L^2, \gamma P_k + \sigma_R^2)$ according to which power level has the largest probability of yielding a certain sample $\mathbf{x}$. Note that this division is based on the condition that Y_P is empty, so the recogniton could be simplified to a comparison of hypothesis pair (P_m, P_{m+1}). P_m is the highest power level satisfying $P_j\gamma < T(\mathbf{x}) - \sigma_R^2$, and P_{m+1} is the lowest power satisfying $P_l\gamma > T(\mathbf{x}) - \sigma_L^2$. The

[2]Please note that *power ambiguity* is different from *power mask* effect in [4], where the former is an inherent problem due to noise uncertainty.

ratio of likelihood between $p(\mathbf{x}|\mathcal{H}_{m+1},\sigma_l^2)$ and $p(\mathbf{x}|\mathcal{H}_m,\sigma_j^2)$ is given by:

$$\frac{1}{N}\ln\xi_{m+1,m}(\mathbf{x}) = T(\mathbf{x})\frac{(\sigma_L^2+P_{m+1}\gamma)-(\sigma_R^2+P_m\gamma)}{(\sigma_L^2+P_{m+1}\gamma)(\sigma_R^2+P_m\gamma)} + \ln\frac{\sigma_R^2+P_m\gamma}{\sigma_L^2+P_{m+1}\gamma}. \tag{9}$$

When (9) is greater than 1, P_{m+1} is determined as the transmit power level, otherwise P_m is determined. Let us define the value of $T(\mathbf{x})$ when $\frac{1}{N}\ln\xi_{m+1,m}(\mathbf{x}) = 1$ as θ_m. When $T(\mathbf{x})$ is larger than θ_m, P_{m+1} is determined, otherwise P_m is determined.

The value of θ_m for different m is given as:

$$\theta_k = \begin{cases} 0 & k=0 \\ \dfrac{(\sigma_L^2+P_m\gamma)(\sigma_R^2+P_{m-1}\gamma)}{(\sigma_L^2+P_m\gamma)-(\sigma_R^2+P_{m-1}\gamma)} \times\ln\frac{\sigma_L^2+P_m\gamma}{\sigma_R^2+P_{m-1}\gamma}, & k=1,2,\cdots,M \\ +\infty, & k=M+1. \end{cases} \tag{10}$$

When Y_P is empty, the potential decision region for a certain power level P_k is $(P_k+\sigma_R^2,\theta_{k+1})$ and $(\theta_k,P_k+\sigma_L^2)$. Note that the left border of $(P_k+\sigma_R^2,\theta_{k+1})$ has the same value of right border of $(\Theta_k^{(L)},\Theta_k^{(R)})$, and $\Theta_k^{(L)}$ is the right border of $(\theta_k,P_k+\sigma_L^2)$. Thus, the combination of potential decision regions for P_k when set Y_P is empty and nonempty yields a continuous region: (θ_k,θ_{k+1}). It is apparent that the summation of (θ_k,θ_{k+1}) covers all possible values of $T(\mathbf{x})$, so (θ_k,θ_{k+1}) is considered as the decision region for P_k.

From all discussions above, we know that only when power levels of PU satisfy Case III could the power recognition at SU be successful, i.e., $\gamma(P_{k+1}-P_k) > (\sigma_R^2-\sigma_L^2)$ is the necessary condition for successful recognition.

C. Performance Analysis

When PU is operating under power level P_k, the test statistic $T(x)$ approximately follows Gaussian distribution for a large N [12], i.e.,

$$T(x)|\mathcal{H}_k \sim \mathcal{N}(P_k\gamma+\sigma^2,\frac{1}{N}(P_k\gamma+\sigma^2)^2) \quad \forall\mathcal{H}_k. \tag{11}$$

We then calculate the decision probability that SU makes the decision as hypothesis $\mathcal{H}_i$ while PU is in fact operating under power P_k:

$$\begin{aligned} &\Pr(\mathcal{H}_i|\mathcal{H}_k) \qquad\qquad (12)\\ &=\Pr\{\theta_i < T \leqslant \theta_{i+1}|\mathcal{H}_k\}\\ &=\mathcal{Q}\left[\sqrt{N}\left(\frac{\theta_i}{P_k\gamma+\sigma^2}-1\right)\right]-\mathcal{Q}\left[\sqrt{N}\left(\frac{\theta_{i+1}}{P_k\gamma+\sigma^2}-1\right)\right]. \end{aligned}$$

Note that $\Pr(\mathcal{H}_i|\mathcal{H}_k)$ represents the recognition probability when SU makes correct decision for power levels.

III. Analysis of SNR Wall

Similar to [5], we also observe the SNR wall effect in MPTP scenario with some modified definition of wall. For each P_k, let us define the following two probabilities:

$$P_{HD}^{(k)} = \sum_{i=k+1}^{M}\Pr(\mathcal{H}_i|\mathcal{H}_k) = \mathcal{Q}\left[\sqrt{N}\left(\frac{\theta_{k+1}}{P_k\gamma+\sigma_k^2}-1\right)\right], \tag{13}$$

$$P_{LD}^{(k)} = \sum_{i=0}^{k-1}\Pr(\mathcal{H}_i|\mathcal{H}_k) = 1-\mathcal{Q}\left[\sqrt{N}\left(\frac{\theta_k}{P_k\gamma+\sigma_k^2}-1\right)\right], \tag{14}$$

where $P_{LD}^{(k)}$ represents the probabilities when the recognized powers are lower than the true power level P_k, while $P_{HD}^{(k)}$ represents the probabilities when the recognized powers are higher than the true power level P_k.

Let us denote the minimum requirements for $P_{LD}^{(k)}$ and $P_{HD}^{(k)}$ as two constant values $\mathcal{P}_{LDS}^{(k)}$ and $\mathcal{P}_{HDS}^{(k)}$, respectively. We call the power recognition at SU as *robust* only when the performance meets $P_{LD}^{(k)} \le \mathcal{P}_{LDS}^{(k)}$ and $P_{HD}^{(k)} \le \mathcal{P}_{HDS}^{(k)}$. Since the noise variance σ_k^2 in the considered signal model swings between σ_L^2 and σ_R^2, whether the recognition is robust is determined by whether the worst performance at SU satisfies:

$$\mathcal{P}_{HDS}^{(k)} \ge \max_{\sigma_k^2\in(\sigma_L^2,\sigma_R^2)}\left\{\mathcal{Q}\left[\sqrt{N}\left(\frac{\theta_{k+1}}{P_k\gamma+\sigma_k^2}-1\right)\right]\right\}, \tag{15}$$

$$\mathcal{P}_{LDS}^{(k)} \ge \max_{\sigma_k^2\in(\sigma_L^2,\sigma_R^2)}\left\{1-\mathcal{Q}\left[\sqrt{N}\left(\frac{\theta_k}{P_k\gamma+\sigma_k^2}-1\right)\right]\right\}. \tag{16}$$

Since $\mathcal{Q}\left[\sqrt{N}\left(\frac{\theta_{k+1}}{P_k\gamma+\sigma_k^2}-1\right)\right]$ and $\mathcal{Q}\left[\sqrt{N}\left(\frac{\theta_k}{P_k\gamma+\sigma_k^2}-1\right)\right]$ are both monotonic increasing function of σ_k^2, we further obtain

$$\mathcal{P}_{HDS}^{(k)} \ge \left\{\mathcal{Q}\left[\sqrt{N}(\frac{\theta_{k+1}}{P_k\gamma+\sigma_R^2}-1)\right]\right\}, \tag{17}$$

$$\mathcal{P}_{LDS}^{(k)} \ge \left\{1-\mathcal{Q}\left[\sqrt{N}\left(\frac{\theta_k}{P_k\gamma+\sigma_L^2}-1\right)\right]\right\}. \tag{18}$$

Obviously, the only variable that could be adjusted by SU in (17) and (18) is N, namely, the sensing time. As in many papers [12] [13], increasing sensing time could improve the performance of detection and recognition at SU. To achieve the robust detection, the sensing time N should satisfy

$$N \ge \max\left\{\left[\frac{\mathcal{Q}^{-1}(\mathcal{P}_{HDS})}{\theta_{k+1}/(P_k\gamma+\sigma_R^2)-1}\right]^2,\left[\frac{\mathcal{Q}^{-1}(1-\mathcal{P}_{LDS})}{\theta_k/(P_k\gamma+\sigma_L^2)-1}\right]^2\right\}, \tag{19}$$

We could then compute the lower bound of N from (19), namely, the minimum value of N to achieve the robust recognition. If the lower bound is computed as infinite, then the recognition fails no matter how long the sensing time is. We then claim the recognition is suffering from SNR wall effect, similar to [5].

It is then interesting to derive the conditions under which SNR wall happens and then try to evade such condition in the real design to avoid SNR wall. Since

$\mathcal{Q}(\mathcal{P}_{HDS}^{(k)})$ is a constant, when $\frac{\theta_{k+1}}{(P_k\gamma+\sigma_R^2)}-1$ approaches zero $\left[\frac{\mathcal{Q}^{-1}(\mathcal{P}_{HD})}{\theta_{k+1}/(P_k\gamma+\sigma_R^2)-1}\right]^2$ will approach infinite. Substitute θ_{k+1} by $\frac{(\sigma_L^2+P_k\gamma)(\sigma_R^2+P_{k-1}\gamma)}{(\sigma_L^2+P_k\gamma)-(\sigma_R^2+P_{k-1}\gamma)}\ln\frac{\sigma_L^2+P_k\gamma}{\sigma_R^2+P_{k-1}\gamma}$ in (10), we have

$$\begin{aligned}
&\frac{\theta_{k+1}}{P_k\gamma+\sigma_R^2}-1\\
=&\frac{P_{k+1}\gamma+\sigma_L^2}{(P_{k+1}\gamma+\sigma_L^2)-(P_k\gamma+\sigma_R^2)}\ln\!\left(\frac{P_{k+1}\gamma+\sigma_L^2}{P_k\gamma+\sigma_R^2}\right)-1\\
=&\frac{P_{k+1}\gamma+\sigma_L^2}{((P_{k+1}\gamma-P_k)-(\sigma_R^2-\sigma_L^2)}\\
&\times\ln\left[1+\frac{(P_{k+1}\gamma-P_k)-(\sigma_R^2-\sigma_L^2)}{P_k\gamma+\sigma_R^2}\right]-1. \quad (20)
\end{aligned}$$

Then we let $(P_{k+1}-P_k)\gamma$ drops from infinite to $\sigma_R^2-\sigma_L^2$. For easier explaination, we replace $(P_{k+1}-P_k)\gamma-(\sigma_R^2-\sigma_L^2)$ with A. Note that $\ln(1+x)\sim O(x)$, which means $\ln(1+x)$ and x are infinitesimal of the same order. Thus (20) becomes:

$$\begin{aligned}
&\lim_{(P_{k+1}-P_k)\gamma\to\sigma_R^2-\sigma_L^2}\left\{\frac{P_{k+1}\gamma+\sigma_L^2}{A}\ln\left[1+\frac{A}{P_k\gamma+\sigma_R^2}\right]-1\right\}\\
=&\frac{P_{k+1}\gamma+\sigma_L^2}{A}\frac{A}{P_k\gamma+\sigma_R^2}-1=0. \quad (21)
\end{aligned}$$

It shows that when $(P_{k+1}-P_k)\gamma-(\sigma_R^2-\sigma_L^2)$ approaches zero, the lower bound of N increases till infinite. Thus, (17) can never be met.

Similarly, we can prove that when $(P_k-P_{k-1})\gamma-(\sigma_R^2-\sigma_L^2)$ approaches zeros, $\left[\frac{\mathcal{Q}^{-1}(1-\mathcal{P}_{LD})}{\theta_k/(P_k\gamma+\sigma_L^2)-1}\right]^2$ is infinite too.

Rewrite $(P_k-P_{k-1})\gamma\to(\sigma_R^2-\sigma_L^2)$ as $\Delta\text{SNR}_{(k)}\to\frac{\sigma_R}{\sigma_L}-\frac{\sigma_L}{\sigma_R}$. When $\Delta SNR_{(k)}$ approaches $\frac{\sigma_R}{\sigma_L}-\frac{\sigma_L}{\sigma_R}$, the required sensing time to achieve robust recognition increases till infinite. Thus, $\frac{\sigma_R}{\sigma_L}-\frac{\sigma_L}{\sigma_R}$ is defined as the position of SNR wall.

In subsection B, we reach the conclusion that only when $\gamma(P_{k+1}-P_k)>(\sigma_R^2-\sigma_L^2)$ could the recognition be robust. Define a new variable: $\Delta\text{SNR}_{(k)}\triangleq\frac{(P_{k+1}-P_k)\gamma}{\sigma_R\sigma_L}$. For any k, $\frac{\sigma_R}{\sigma_L}-\frac{\sigma_L}{\sigma_R}$ is the minimum value of $\Delta\text{SNR}_{(k)}$ when recognition is robust. To achieve robust recognition, the set of power levels at PU should comply to both of these two restrictions:

$$\Delta\text{SNR}_{(k)}\geq\frac{\sigma_R}{\sigma_L}-\frac{\sigma_L}{\sigma_R}, \quad (22)$$

$$\Delta\text{SNR}_{(k-1)}\geq\frac{\sigma_R}{\sigma_L}-\frac{\sigma_L}{\sigma_R}. \quad (23)$$

Thus, when either $\Delta\text{SNR}_{(k)}$ or $\Delta\text{SNR}_{(k-1)}$ is lower than SNR wall, the recognition fails.

Let us summarize the effects of SNR wall as follows:

1) When the gap between power levels $\gamma(P_k-P_{k-1})$ or $\gamma(P_{k+1}-P_k)$ approaches SNR wall, the required sensing time for correct recognition increases.
2) When the gap between power levels $\gamma(P_k-P_{k-1})$ or $\gamma(P_{k+1}-P_k)$ is lower than SNR wall, the recognition fails.

Consider PU is transmitting in P_k, Fig. 1 gives three examples to depict the required N to satisfy (18), where $\mathcal{P}_{LDS}$ is set as 0.1. The value of $\frac{\sigma_R}{\sigma_L}$ for three examples differs, i.e.

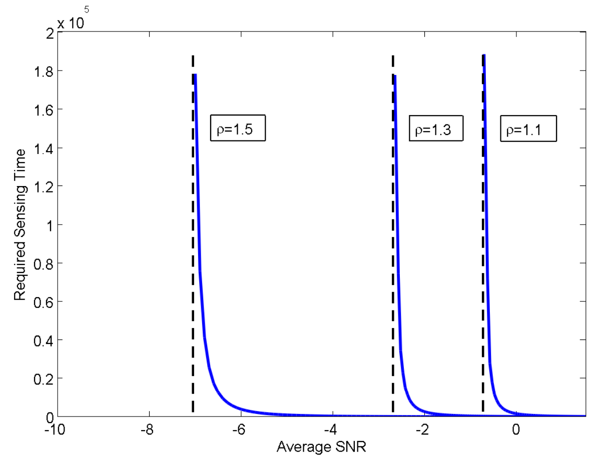

Fig. 1. The required number of samples versus ΔSNR

SNR walls for three examples are on different positions. We see in Fig. 1 that when $\Delta\text{SNR}_{(k)}$ approaches the SNR wall, the required sensing time increases, which complies to the effect of SNR wall.

Remark 1: Although the mathematical expression of SNR wall and the restriction for power levels here are different from classic SNR wall, they have consistent physical sense. SNR wall sets restrictions on the power gap between each power level and its two neighbours. The value of $\frac{\sigma_R}{\sigma_L}$ represents the size of noise uncertainty at SU. Different sizes of noise uncertainty will construct SNR walls on different positions, as shown in Fig.1. If noise uncertainty decreases, the position of SNR wall will be pushed back. When noise variance is a certain value, the SNR wall will be pushed back infinitely, i.e., SNR wall disappears. Traditional CR has only two power levels: $P_0=0$ and $P_1=P$, to evade SNR wall, (P_1-P_0) is attached with restriction, which is a special case for our MPTP scenario.

Remark 2: Since $P_{k+1}-P_k$ differs for different k, it is possible that some power levels could satisfy the requirements (22), (23), while the others cannot. Hence, we could divide the power levels into two groups as shown in Fig. 2, where the power levels on the right side of SNR wall meet (22) and (23), while the power levels on the left side do not. A special case is that gaps between any two neighboring power levels are the same i.e.

$$P_{i+1}-P_i=\text{Constant} \qquad i=0,1,...,M-1. \quad (24)$$

In this case, we could eliminate the subscript of ΔSNR_k since the set $\{\Delta\text{SNR}_k|k\in\{0,1,...,M-1,M\}\}$ has a single value.

IV. Discussions

We have already derived the exact position of SNR wall: $\frac{\sigma_R}{\sigma_L}-\frac{\sigma_L}{\sigma_R}$. For each power level, it has to keep away from the SNR wall to achieve robust recognition. All power levels except P_0 and P_M have to meet the requirement of (17) and (18). For P_0, only (17) is needed, while for P_M, only (18)

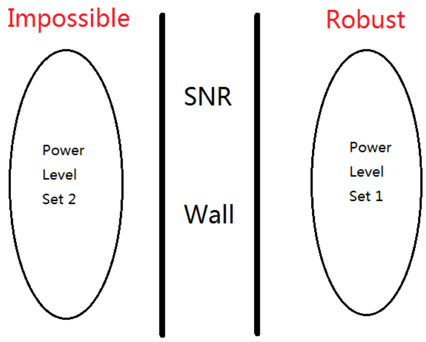

Fig. 2. two sets of power levels separated by ΔSNR wall.

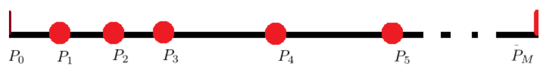

Fig. 3. Uneven distribution of power levels

is needed. Either approaching or penetrating through the SNR wall will invalidate the recognition at SU.

A lot of efforts have been done by researchers to avoid the classic SNR Wall, including using cross-correlation [8], or noise estimation [9] and noise calibration [5]. Nearly all of these works are dedicated to weaken the effects of the noise uncertainty and could be applied here to lower the SNR wall. However, in MPTP scenario, we can simply re-set the values of power levels at PU to achieve better recognition at SU without lowering the SNR wall. From Section III, we know that larger the power gap, weaker the effects of SNR wall, thus, enlarging the gap between power levels could bring better recognition. Large gaps can be achieved by raising the maximum power level that is available at PU. However, in CR systems under MPTP scenario, the maximum power and average power are both settled, which means that the adjustment of power levels is under restriction of energy consumption.

We use the average accurate probability of recognition to represent the recognition performance of the whole system: $\mathrm{P_{dis}} = \sum_{i=0}^{M} \Pr(\mathcal{H}_i|\mathcal{H}_i)\Pr(P_i) = \sum_{i=0}^{M} \Pr(\mathcal{H}_i|\mathcal{H}_i)\Pr(P_i)$. If we know the approximate prior probability of PU transmitting in P_k, the target to achieve best recognition becomes a nonlinear programming problem. The average value and maximum value of P_k for all k serve as the constraint, our target here is to find a set of P_k to achieve the largest $\mathrm{P_{dis}}$, i.e., to achieve the best recognition performance. A basic rule is that power levels with higher prior probability to be chosen should be attributed with larger gaps between its two neighboring power levels.

We use a segment of constant length to illustrate the sets of transmitting power levels. Different power levels are symbolized as nodes on this segment, and the distance between each nodes stands for the gap between power levels.

Fig. 3 shows one possible distribution of power levels. This strategy is suitable for the situation where P_4 is of higher probability than others to be selected by PU. Since the gap between P_4 and P_3 and the gap between P_4 and P_5 are larger than the rest, the recognition for P_4 would bump into the SNR wall later than the rest when noise uncertainty increases.

The optimal strategy for transmitting power levels is out of the range of this paper, and is being studied in the future.

V. Simulation Results

In this section, we provide simulation results of spectrum sensing related to the discussions in section II. Standards for robust recognition $\mathcal{P}_{LDS}^{(k)}$ and $\mathcal{P}_{HDS}^{(k)}$ are both set as 0.1. For simpler simulation, we assume that gaps between two neighboring power levels are the same, i.e. $P_{i+1} - P_i = \mathrm{Constant} \quad i = 0, 1, ..., M-1$.

First we demonstrate the recognition probability of one selected power level, still symbolized as P_k here, versus sample length N in Fig. 4. From Fig. 4, it is seen that the detection performance is a monotonic increasing function of sample length N. When SNR is high enough, even for small N ($N < 500$), the performance is somewhat acceptable.

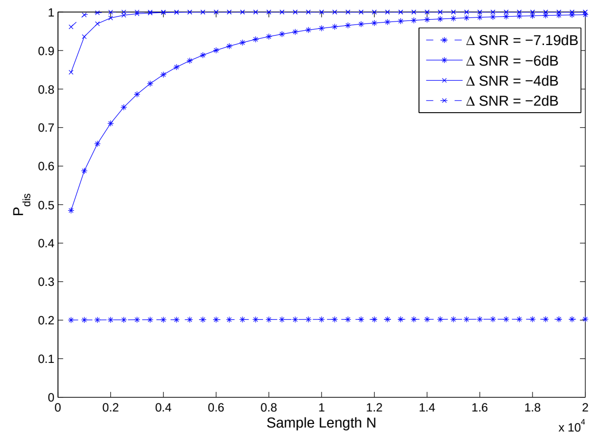

Fig. 4. Probability of discrimination versus sample length N.

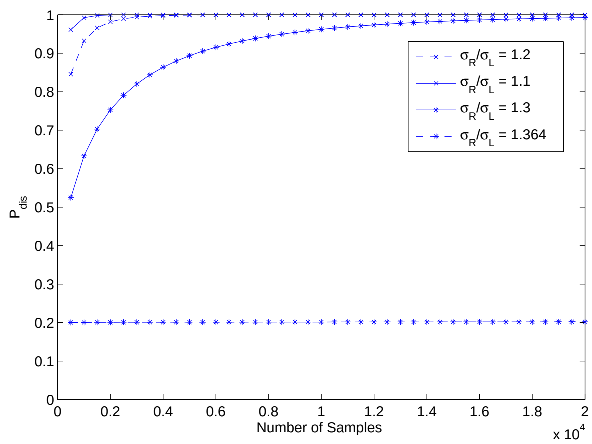

Fig. 5. Probability of discrimination versus sample length N (Stable SNR).

Furthermore, when the sample length increases, the detection performance reaches our target quickly. When SNR decreases but has not dropped below the SNR wall, the detection performance still gets improved when the sensing time increases. However, longer sensing time is needed for robust recognition. When SNR approaches the SNR wall, we see the gain of recognition due to increasing sensing time becomes minor. Specially, when SNR is exactly on the position of SNR wall, we see from Fig. 4 that the recognition performance does not change due to longer sensing time.

We then give SNR a certain value but change the position of SNR wall i.e. the value of $\frac{\sigma_R}{\sigma_L}$. The simulation result is shown in Fig. 5. From Fig. 5, we see similar phenomena as Fig. 4. When noise uncertainty is large enough, the recognition performance does not change due to a longer sensing time, which is the impact of SNR wall.

Hence, we see that when recognition suffers from SNR wall, no matter how long the sensing time is, recognition fails.

VI. Conclusions

In this paper we discussed the spectrum sensing methods under MPTP scenario with noise uncertainty. GLRT methods are adopted to compute decision thresholds for different transmitting power levels. We found that only when gaps between power levels and noise uncertainty satisfy a particular requirement could the recognition at SU be successful. Based on this premise, we further studied the SNR wall effect. The exact position of SNR wall was computed and its effects on the recognition was analyzed. Numerical examples are provided to corroborate the proposed studies.

References

[1] D. Cabric, S. Mishra, and R. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Conf. Record of Conf. on Signals, Syst. and Comput.*, vol. 1, Nov. 2004, pp. 772–776 Vol.1.

[2] A. Sonnenschein and P. Fishman, "Radiometric detection of spread-spectrum signals in noise of uncertain power," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, no. 3, pp. 654–660, Jul. 1992.

[3] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the-user interference channel," *IEEE Trans. Info. Theory*, vol. 54, no. 8, pp. 3425–3441, 2008.

[4] F. Gao, J. Li, T. Jiang, and W. Chen, "Sensing and recognition when primary user has multiple transmit power levels," *IEEE Trans. Signal Process.*, vol. 63, no. 10, pp. 2704–2717, May. 2015.

[5] R. Tandra and A. Sahai, "SNR walls for signal detection," *IEEE J. Sel. Topics in Signal Process.*, vol. 2, no. 1, pp. 4–17, Feb. 2008.

[6] ——, "Fundamental limits on detection in low SNR under noise uncertainty," in *Int. Conf. on Wireless Net., Commun. and Mobile Comp.*, vol. 1, June. 2005, pp. 464–469 vol.1.

[7] H. Poor and D. Looze, "Minimax state estimation for linear stochastic systems with noise uncertainty," in *IEEE Conf. on Decis. and Contr. including the Symp. on Adaptive Process.*, Dec. 1980, pp. 1020–1025.

[8] M. Oude Alink, A. Kokkeler, E. Klumperink, G. Smit, and B. Nauta, "Lowering the SNR wall for energy detection using cross-correlation," *IEEE Trans. Vehicul. Tech.*, vol. 60, no. 8, pp. 3748–3757, Oct. 2011.

[9] A. Mariani, A. Giorgetti, and M. Chiani, "SNR wall for energy detection with noise power estimation," in *IEEE Int. Conf. on Commun. (ICC 2011)*, June. 2011, pp. 1–6.

[10] D. Torrieri, "The radiometer and its practical implementation," in *Military Commun. Conf. (MILCOM 2010)*, Oct. 2010, pp. 304–310.

[11] V. Blaschke, T. Renk, and F. Jondral, "A cognitive radio receiver supporting wide-band sensing," in *IEEE Conf. on Commun. Workshops (ICC Workshops 2008)*, May. 2008, pp. 499–503.

[12] Y.-C. Liang, Y. Zeng, E. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1326–1337, April. 2008.

[13] S. Stotas and A. Nallanathan, "Optimal sensing time and power allocation in multiband cognitive radio networks," *IEEE Trans. Commun.*, vol. 59, no. 1, pp. 226–235, January. 2011.