APP PHY 157 WFY-FX-2

LAB REPORT 2

Fourier Transform Model of Image Formation (Part 1 of 2)

Source code here!

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https://github.com/lovelyandeo/AP157-Computational-Analysis-and-Modeling-in-Physics

Background

Fourier transform is a very useful image processing technique used in a wide array of applications such as image analysis, image filtering, image convolution, etc. How it works is that it basically divides the image into its sine and cosine components, wherein the output of the transformation represents the image in the Fourier domain and the input image is the equivalent in the spatial domain. A particular frequency that appears in the spatial domain image corresponds to each point in the Fourier domain image.

Objectives

In this activity, we demonstrated the properties of Fourier Transform (FT) and its applications by:

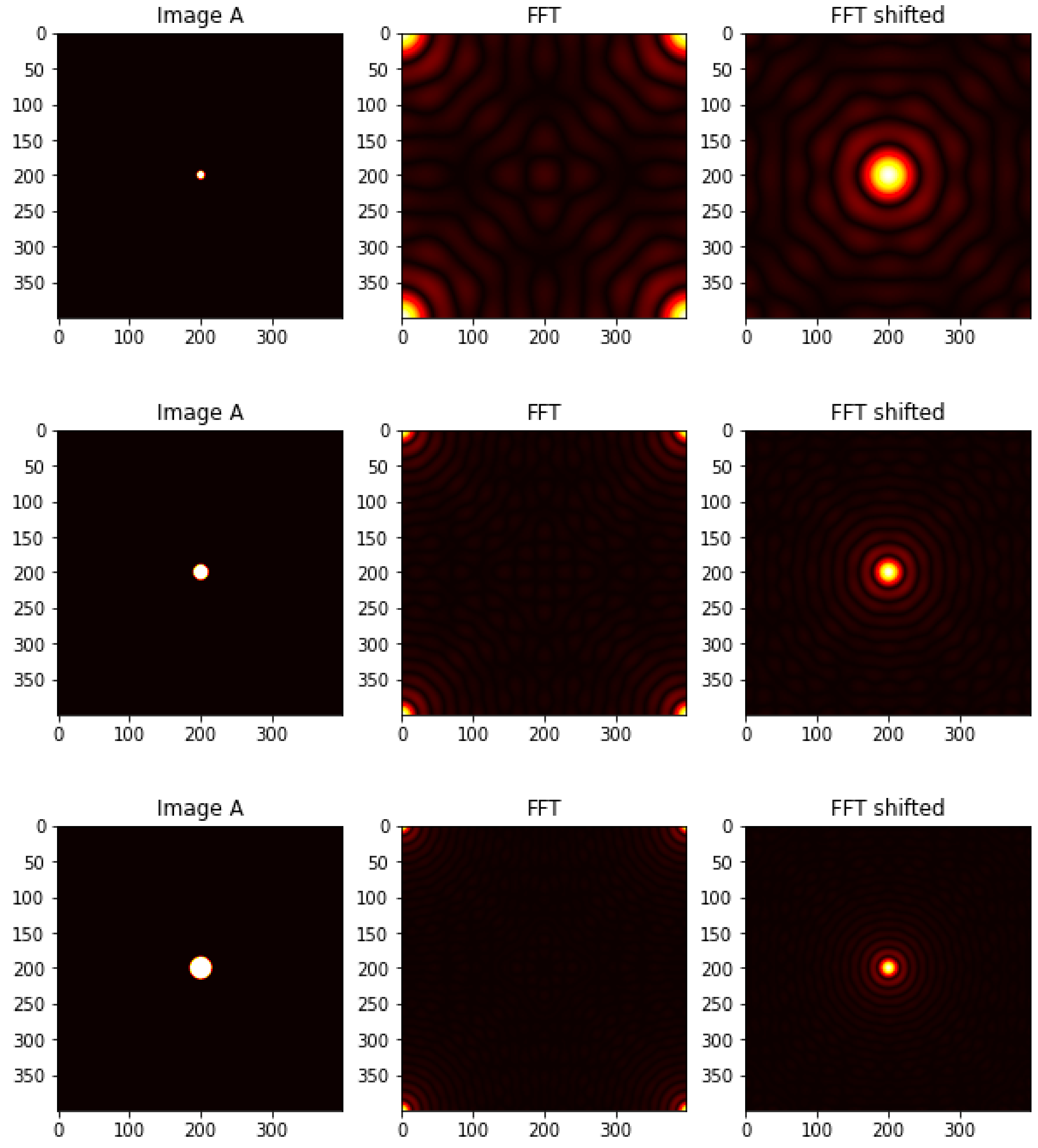
1. Exploring the discrete fourier transform
2. Performing convolution on two images
3. Showing the correlation between images through template matching

Results and Analysis

r = 5

r =10

r =15

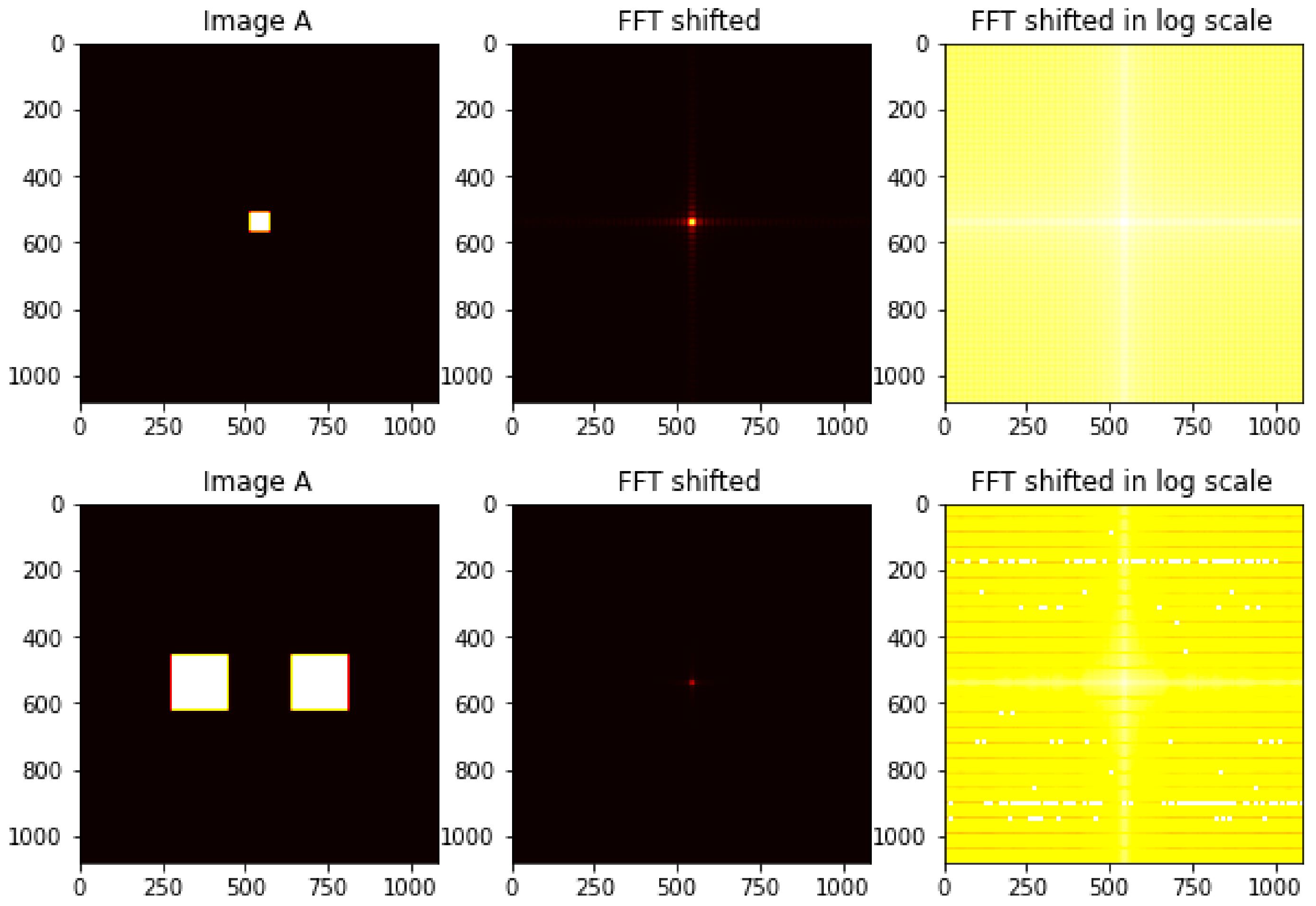


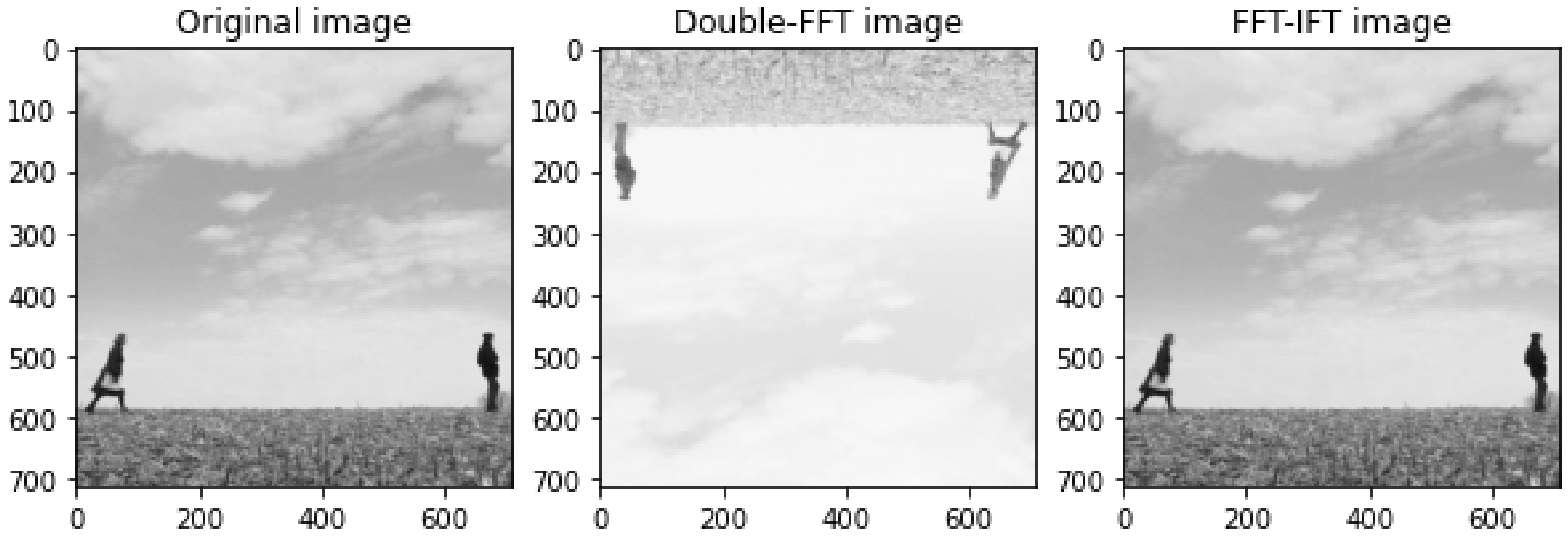
For this activity, an image of a white circle was created against a black background. Then, the discrete Fourier transform was performed on the image, displaying its magnitude as an intensity image. From the figures beside (DFT), it is observed that the intensities are at the corners. As mentioned in the lab manual, it is because the diagonal quadrants are interchanged when fft2 is used, hence, the image was shifted to make the output zero-centered (Shifted).

The radius of the circle was also varied to r = 5, 10, 15 as shown in the figures. But since the intensity at the Fourier domain depends on the frequency of the image in the spatial domain, it was shown that the smaller the circle, the larger the intensity image. and it is because the peak of the FT moves to higher frequencies causing the magnitude of the peak to also increase.

Familiarization with Discrete FT

Here, FT was applied to different images. First, the FT of a single square showed peaks at the center relative to the size and location of the image. Comparing it to image another image, the FT of two squares beside each other showed almost the same output plus additional peaks in the horizontal direction corresponding to the distance between the squares.



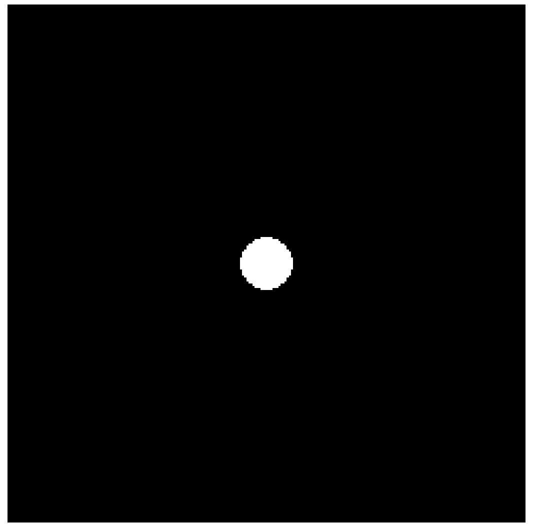


The effects of FT were further explored by performing double FFT on an image and FFT followed by its inverse. Performing double FFT basically removes the low-frequency components and retains the high-frequency ones, and then reverses the image, which is shown in the figure. Performing the inverse after the first FFT will just basically return the same input, which is also shown in the figure.

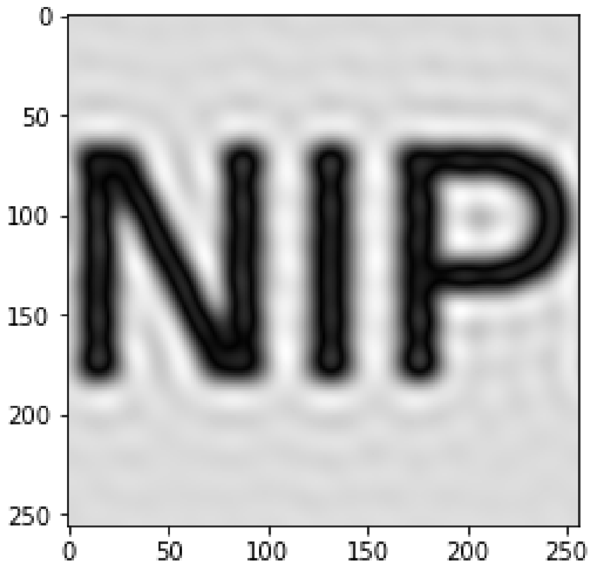
Simulation of an imaging system

In this activity, we demonstrated image convolution using the Fourier transform. In convolution, we basically just performed matrix multiplication of two images to produce a new matrix representing a new image. The figure below shows that when an image is convolved with a circle that has a diameter of 10% of the image width, the output is a blurry version of the original one. The circle here basically serves as the aperture, which is like the opening of the camera that allows the 'light' to pass through.

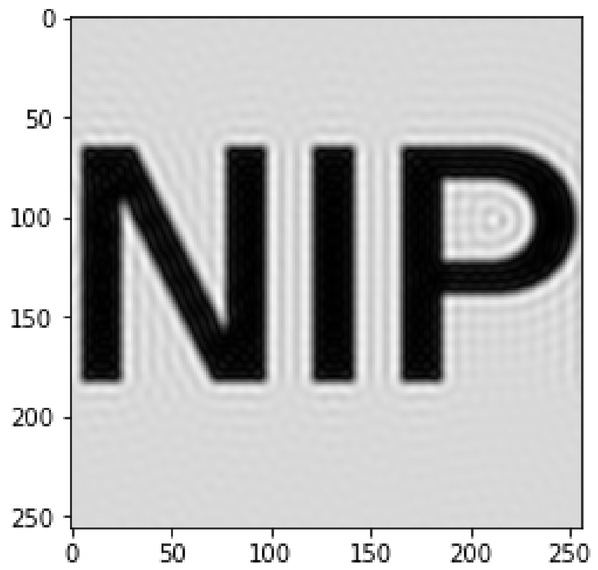
Original



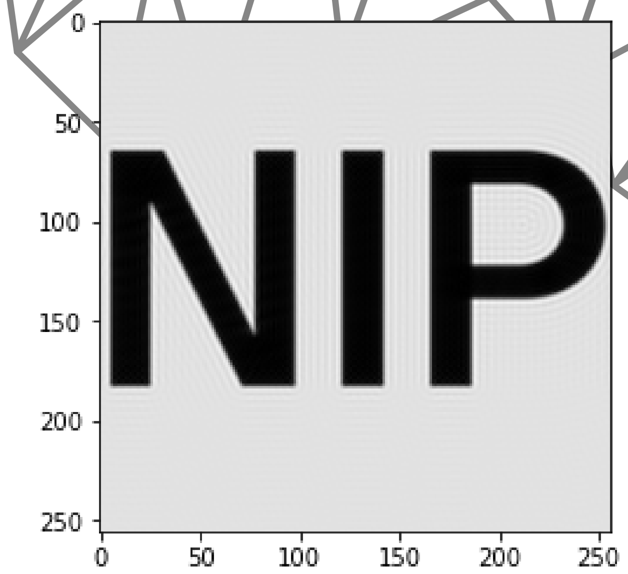
10%



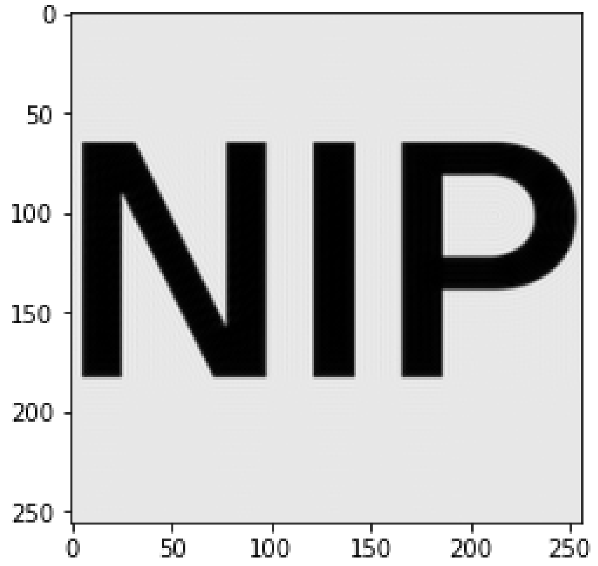
Output



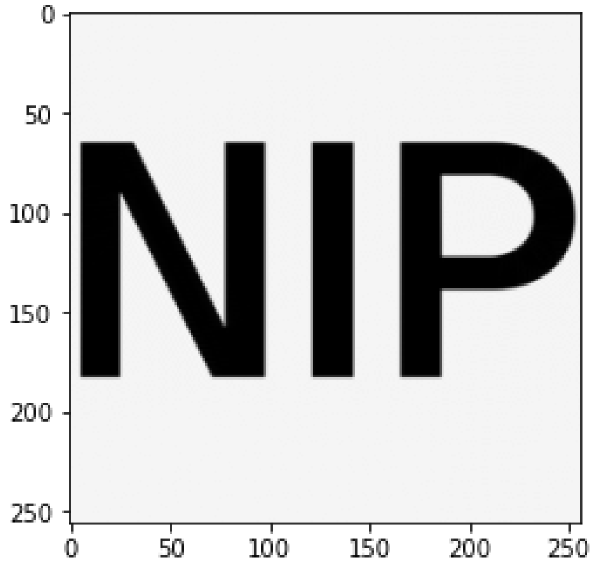
25%



50%



75%

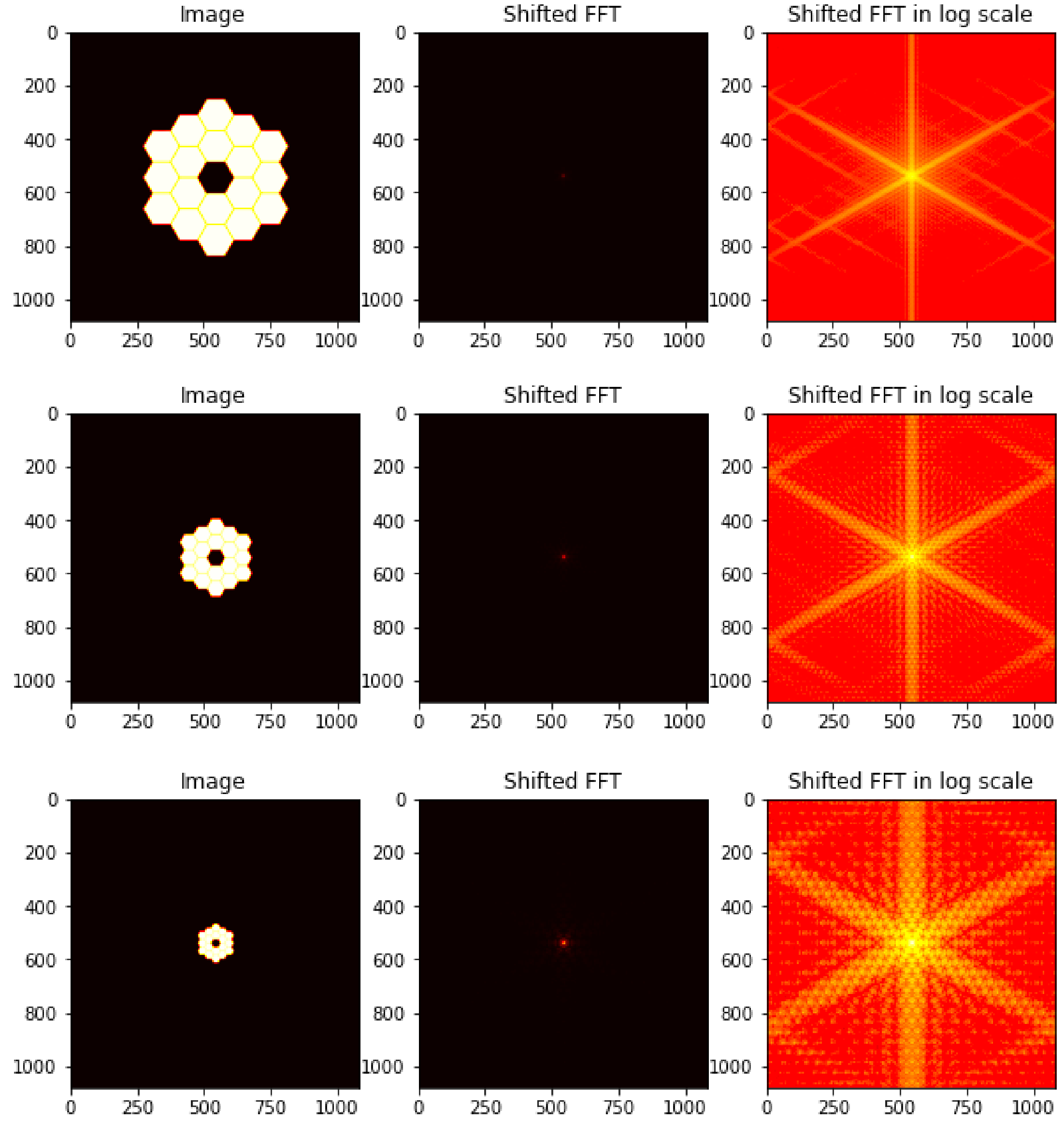


100%

The images above show the output when we vary the size of the circle or the aperture into having a diameter that is 25%, 50%, 75%, and 100% of the image width. As we can see, the image becomes clearer as the size of the aperture increases.

Simulating a star imaged by different telescopes

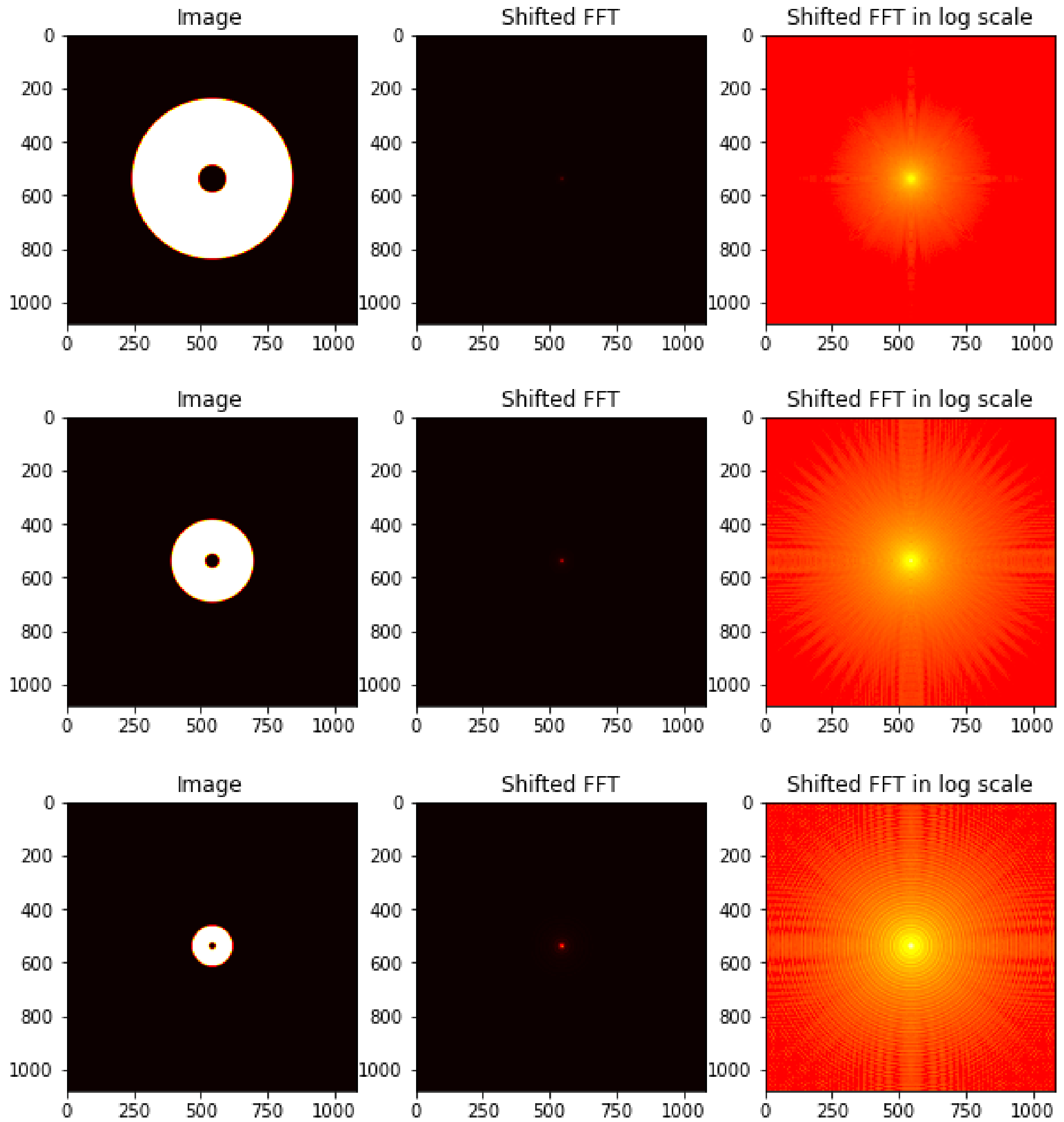
Simulated



James Webb Space Telescope

The image of the star simulated by getting the FT of the JWST mirror was much sharper with six (6) visible streaks/spikes than the star image simulated by getting the FT of the Hubble mirror, with only four (4) hardly visible streaks/spikes. This can also be observed in actual space pictures taken by both telescopes. Engineering-wise, it is because JWST has three struts that produce two spikes each. Whereas Hubble has four struts that only produce one spike each. But in these images formed, the differences between them also had something to do with the shape and size of the mirrors.

Simulated



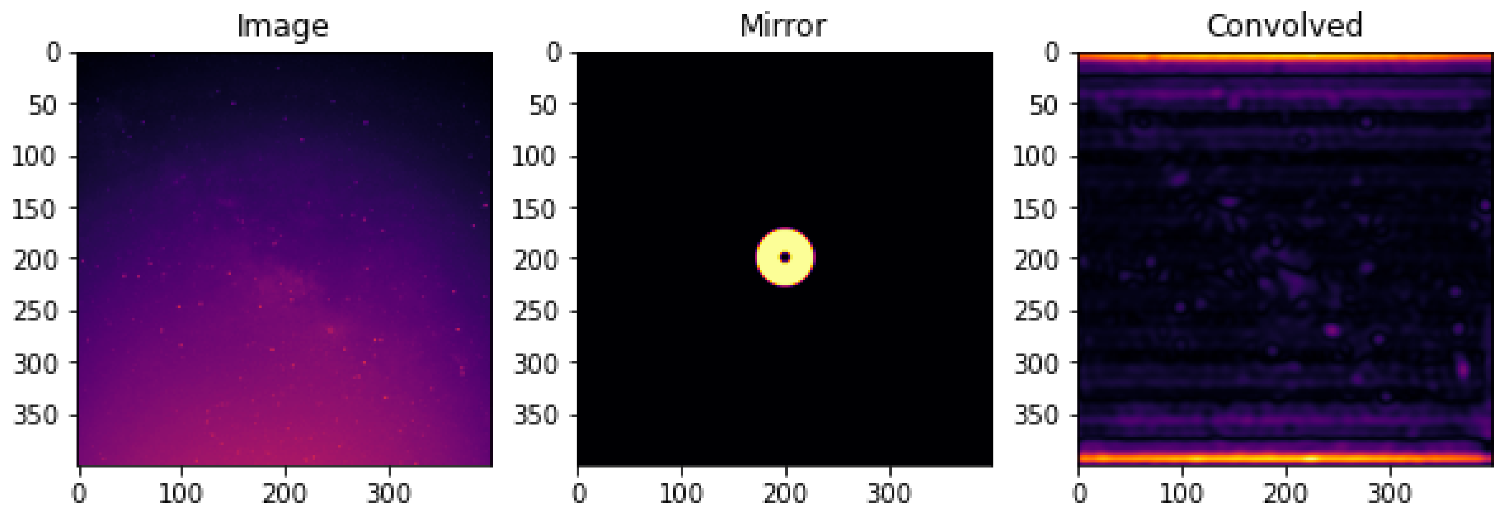
Hubble Space Telescope

Actual images

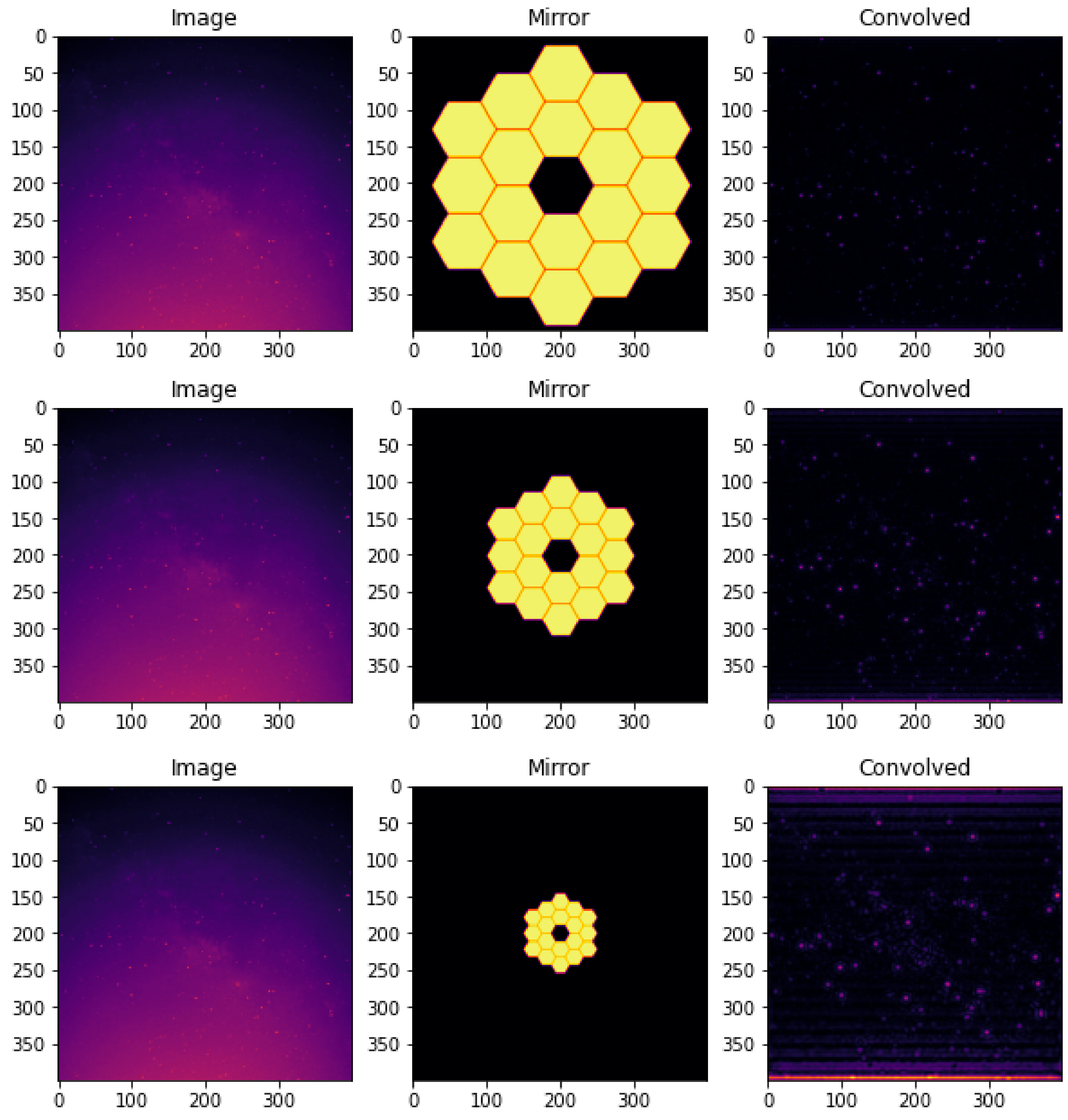
Convolving a raw milky way picture with the JWST mirror (aperture)

Apparently, the same effect was observed when a night sky image was convolved with the JWST mirror image. To be precise, a small 'aperture' resulted in more apparent and sharper stars. The reason for that could be that the stars are small objects in the picture, hence it 'matches' with the smaller aperture better. A cluster of faint stars is also observed in the output.

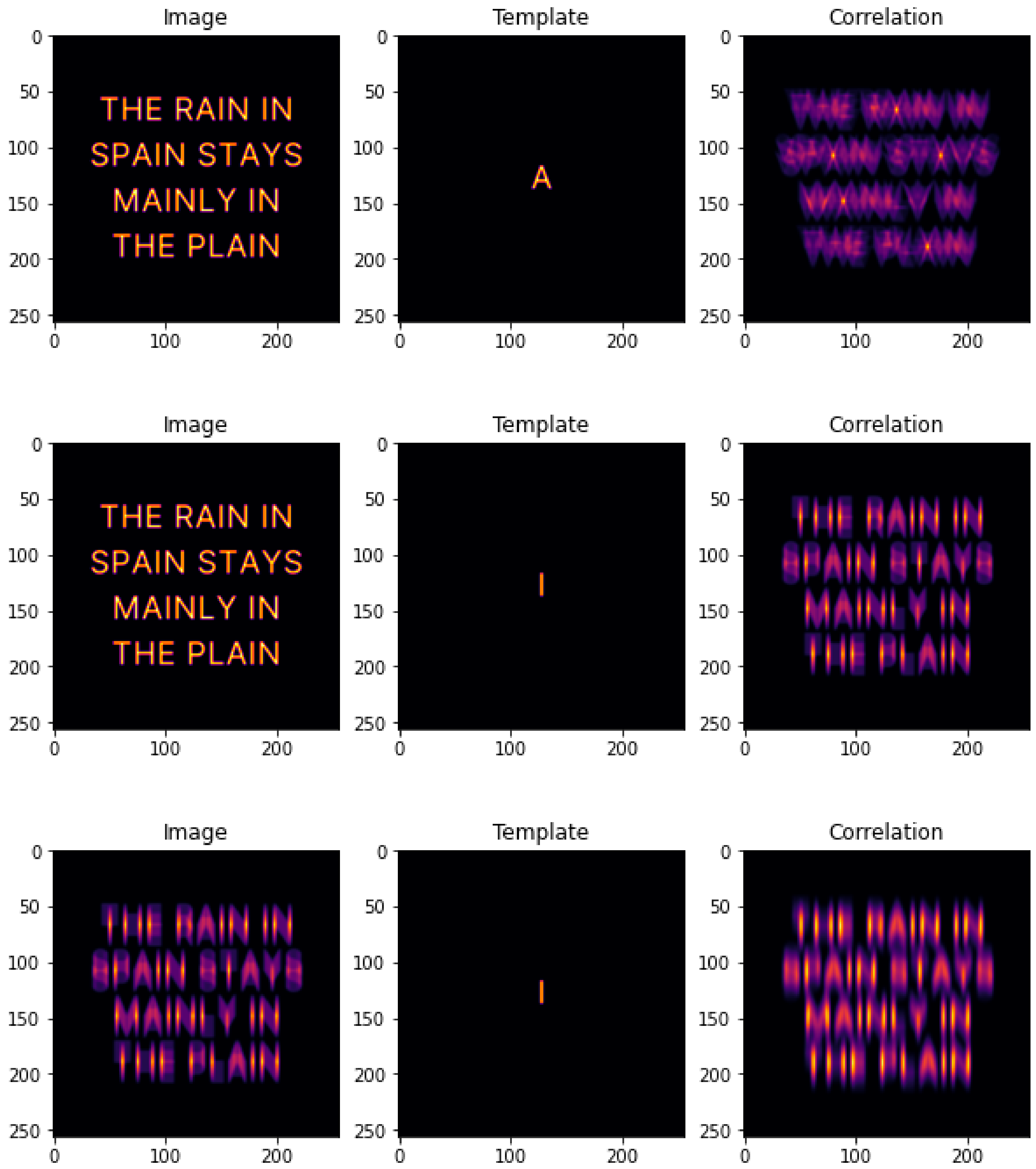
Trying it out with the Hubble mirror, the convolved output image was blurry. It's cool to see that these also somehow depict the capabilities of the actual telescopes in action.



Hubble



JWST

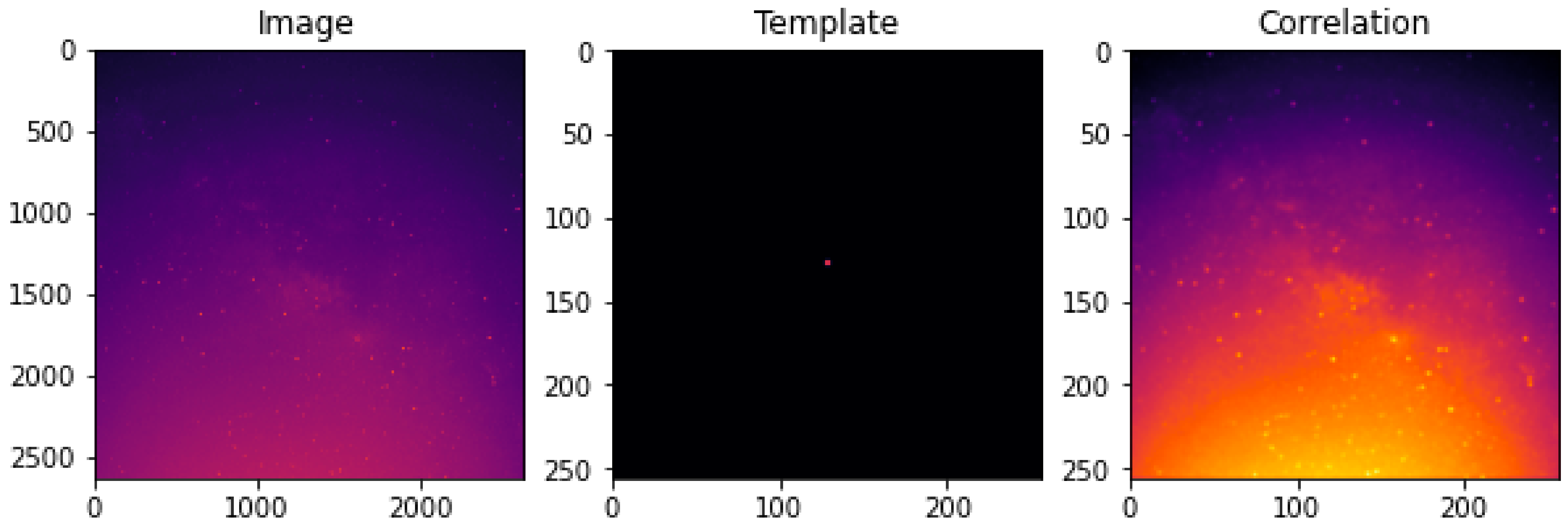


Here, a template was matched to the input image by calculating and showing the degree of similarities between them. To visualize, it's like the template is laid over the input to try and find where it matches best. The bright spots are basically the maximum values at which the template pattern matched best to the input image.

As observed, using the letter "I" as the template results in more maximum values or more matches with the input image, as expected since it's technically just a vertical line that is found on almost all the letters in the input.

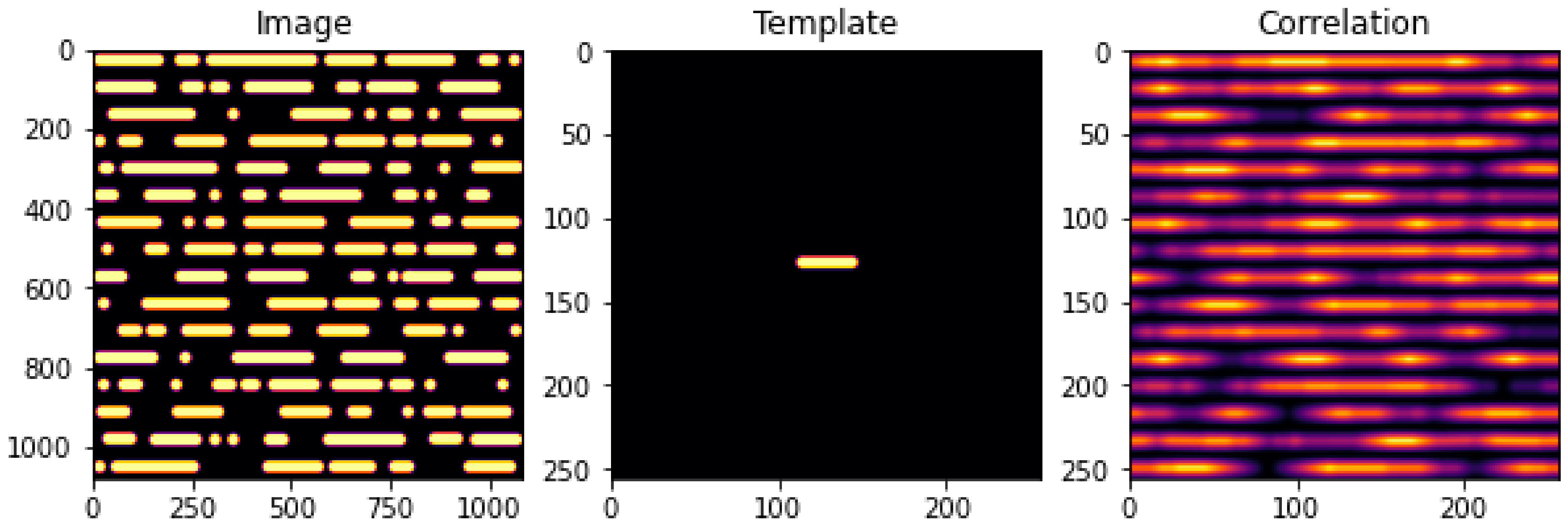
The third row in the figure is just to show that the maximum values are intensified as u correlate the same template to the input over and over again.

Template matching using correlation



Now here, we correlate a template of a single star to the same sky image used in convolution. Comparing the results to that of the convolution, the stars here appear much brighter. The correlation technique was also able to sharpen the very faint stars at the bottom of the input image.

Correlation can be used with other different images and templates, the figure on the right is an example. The output really depends on how similar the template is to the patterns of the image. Since the example template was just a cropped pattern from the input, the maximum correlation values were found in almost the entirety of the input.



Reflection

I really really enjoyed this activity! I have encountered Fourier Transform in our previous subjects but never yet in this light where I actually get to apply it to something. I was also ecstatic to use my first original milky way picture that I took last 2020 when I was doing astrophotography. During that time, I wasn't very skilled in manipulating images so I didn't know how to enhance the image properly or play around with it. And it was such a joy to be able to do it here in this activity. It was also really fulfilling to see and actually understand how FT transforms an image and how convolution and correlation work. Excited for the part two of the application of the FT!

Self-evaluation

100/100

+ 10 bonus points

I am pretty confident that I got the desired results for each part of the activity. I also believe I went above and beyond by applying the techniques to different images. By comparison, I also go to find out which certain factors or parts of the image affect the output

References

Here are the materials and the code templates I used to accomplish this activity:

Soriano, M. (2023). Activity 2. Fourier Transform Model of Image Formation (Part 1 of 2) [Review of Activity 2. Fourier Transform Model of Image Formation (Part 1 of 2)]. https://uvle.upd.edu.ph/pluginfile.php/856970/mod_resource/content/1/FT%20in%20Image%20Processing%20Part1.pdf

Image Transforms - Fourier Transform. (2019). Ed.ac.uk. https://homepages.inf.ed.ac.uk/rbf/HIPR2/fourier.htm

Garner, R. (2015, May 19). NASA's Webb Telescope "Struts" its Stuff. NASA. https://www.nasa.gov/feature/goddard/nasas-webb-strutting-its-stuff-in-new-behind-the-webb-video