Response Spectra

Objectives

1. Define a response spectrum.

2. Give the uses of a response spectrum.

3. Define the types of response spectra.

4. Know how to calculate a velocity and acceleration time history from a displacement response spectrum.

5. Know how to calculate a deterministic response spectrum for a given earthquake magnitude and source distance.

6. Know how to calculate a IBC 2000 spectrum.

1. Definition

The peak or maximum response (acceleration, velocity, displacement) of all possible linear single degree of freedom (SDF) system to a particular component of ground motion for a given level of damping.

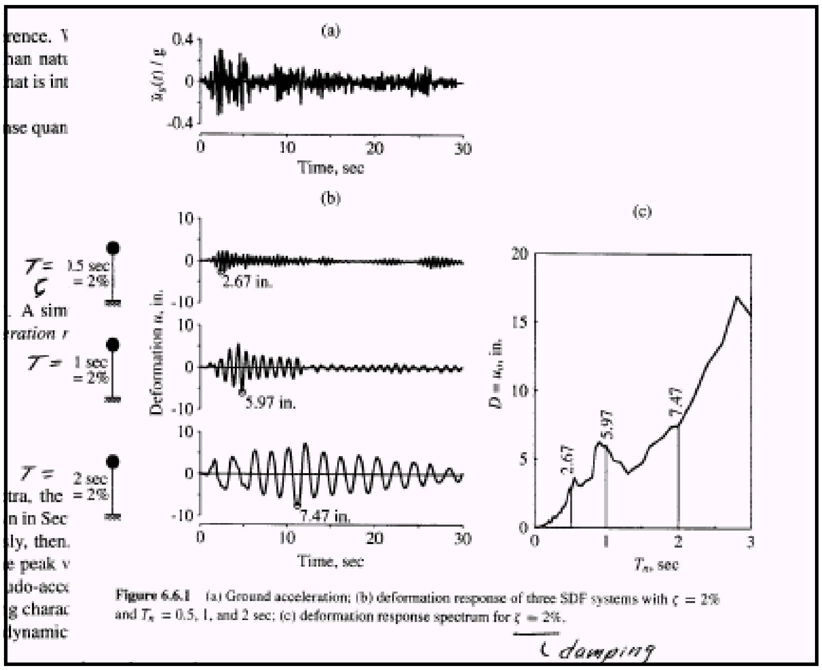

Figure 1. Displacement response spectrum for 2 percent damping (after Chopra).

2. Uses of the response spectrum

   a. The response spectrum provided a convenient and practical way to summarize the frequency content of a given acceleration, velocity or displacement time history.
   b. It provides a practical way to apply the knowledge of structural dynamics to design of structures and development of lateral force requirements in building codes.

3. Types of Response Spectra

a. Response Spectra calculated from actual time histories

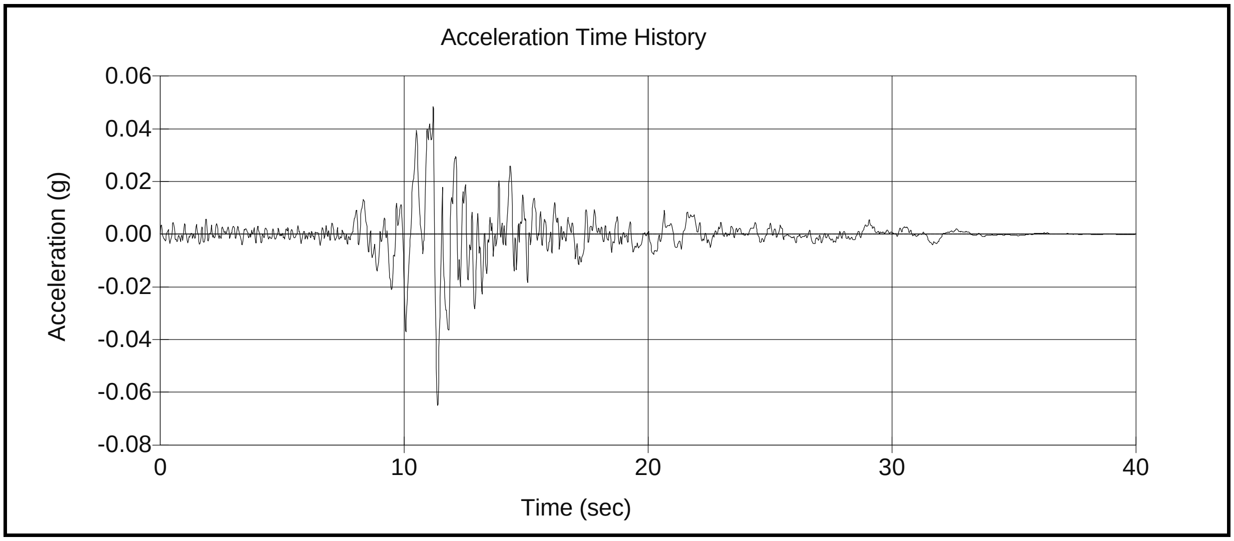

Figure 3. Acceleration Time History for Yerba Buena Island from the 1989 Loma Prieta Earthquake.

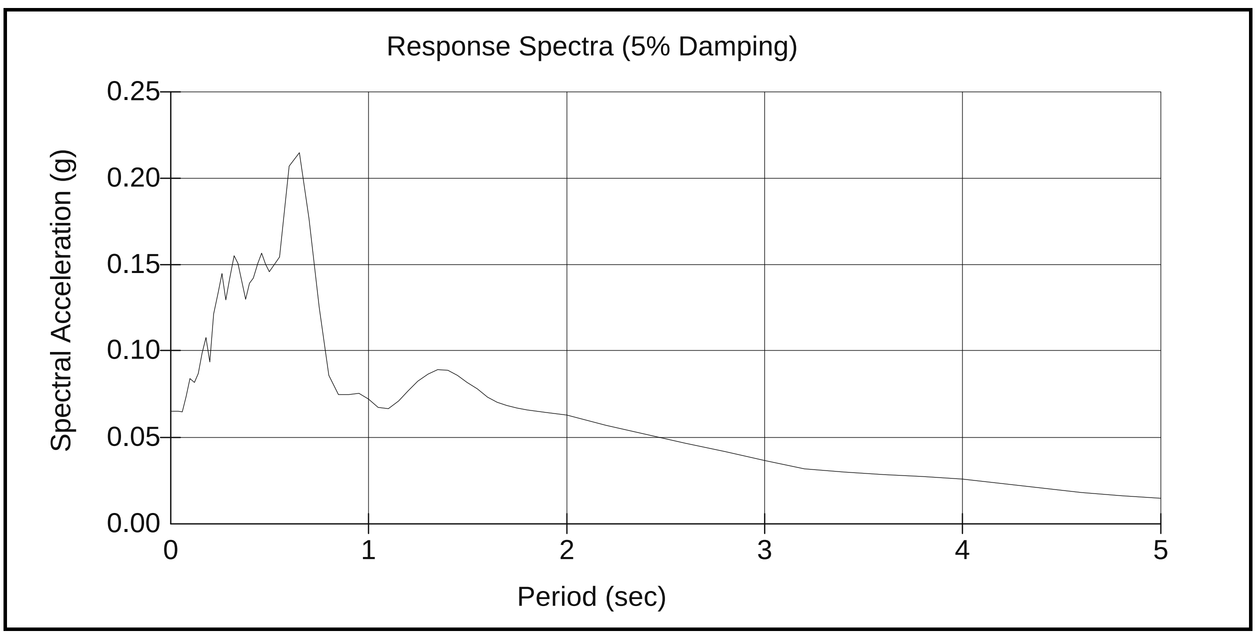

Figure 2. Acceleration Response Spectrum for Yerba Buena Record.

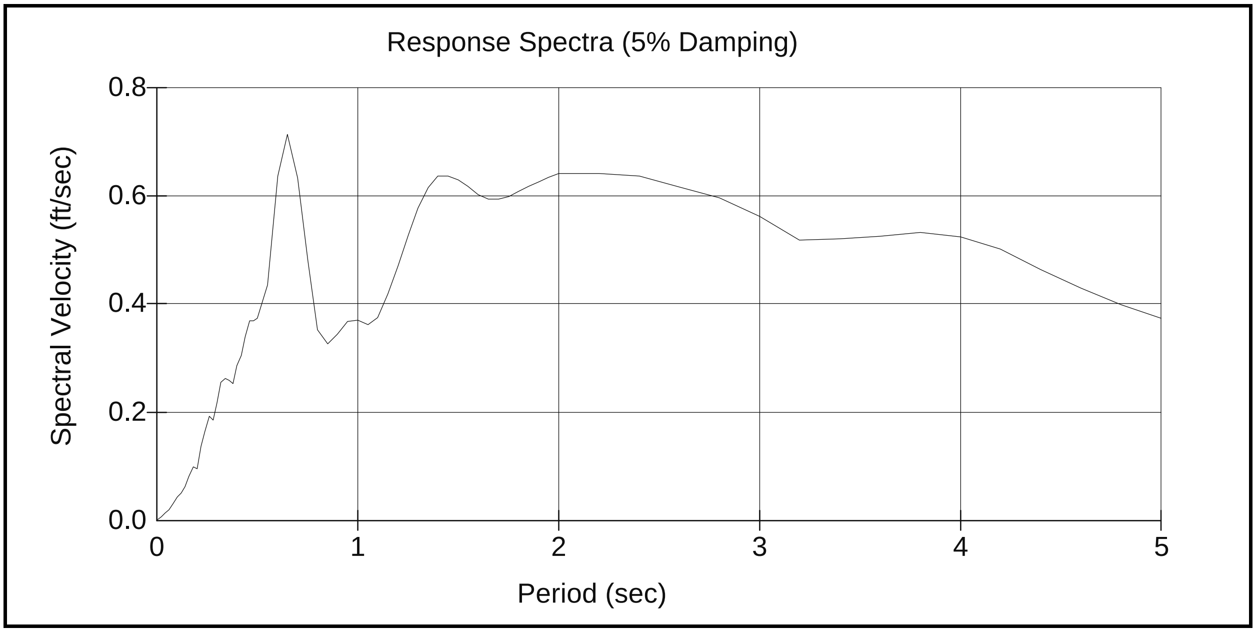

Figure 5. Velocity Response Spectrum for the Yerba Buena Record.

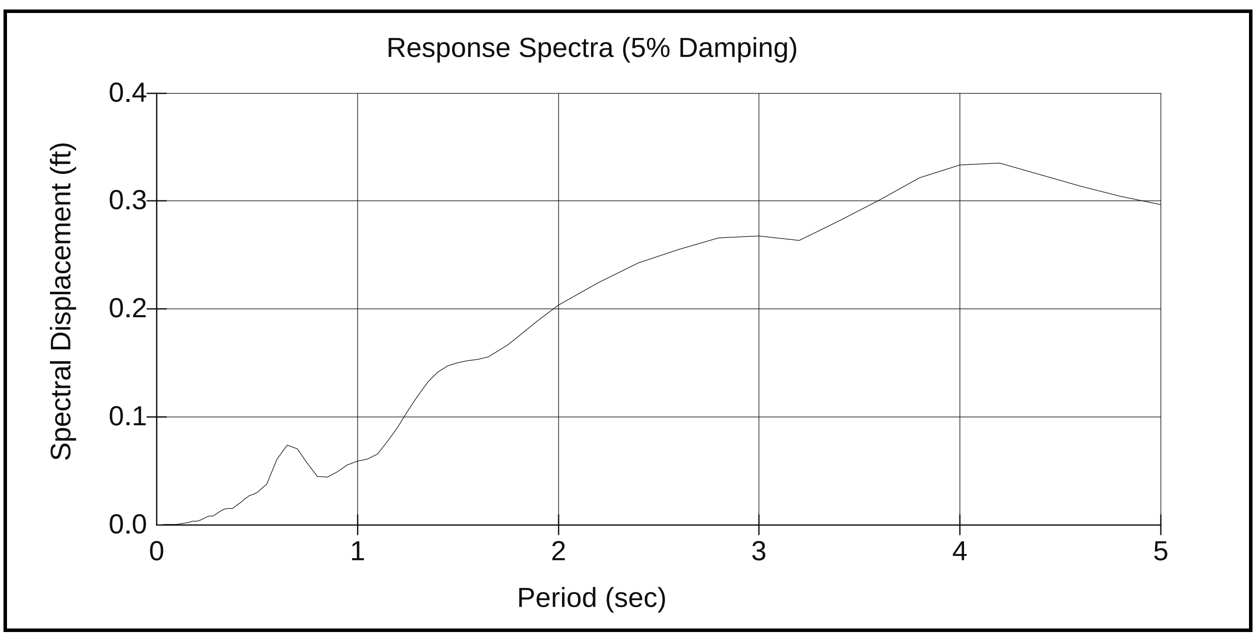

Figure 4. Displacement Response Spectrum for the Yerba Buena Record.

Steps for calculating a response spectrum from a time history

The response spectrum for a given ground motion component (e.g., a(t)) is developed using the following steps:

(1) Obtain the ground motion (a(t)) for an earthquake. Typically the acceleration values should be defined at time steps of 0.02 second, or less.

(2) Select the natural vibration period, T_n, and damping ratio, ξ, for SDOF system. (Usually 5 percent damping is selected.)

(3) Determine the maximum displacement response for a SDOF structure with the selected percent damping for a given period or frequency of vibration.

To do this, you must solve the following differential equation

$$mM^2u/dt^2 + c\,Mu/dt + k\,u = F(t)$$

This can be solved using (see Chopra, Ch. 1, pp. 28-32):

(a) Closed form solution (linear systems)

! Solution is only valid for initial conditions at rest

(b) Duhamel’s integral (linear systems)

! based on treating the periodic motion as a series of short impulses

(c) Frequency Domain method (linear systems)

! Fourier transform

! Inverse Fourier transform

(d) Numerical methods (linear and nonlinear systems)

! Numerical time stepping methods

(4) Repeat step 3 and vary the fundamental period of the structure by changing the mass (m), the stiffness (k), or both. Plot the new results.

Example of Using NONLIN to create a displacement response spectrum

First linear analysis

m = 100 kips
k = 100 kips / in
c = 5 percent of critical damping
earthquake record is Imperial Valley El Centro Record (Impval1.acc)

period = 0.32 s, f = 3.13 Hz, ω = **19.65 rad/s**

max. displacement = **0.700 inches**

Second linear analysis

m = 100 kips
k = 50 kips / in
c = 5 percent of critical damping
earthquake record is Imperial Valley El Centro Record (Impval1.acc)

period = 0.45 s, f = 2.21 Hz, ω = **13.89 rad/s**

max. displacement = **1.677 inches**

Third linear analysis

m = 100 kips
k = 25 kips / in
c = 5 percent of critical damping
earthquake record is Imperial Valley El Centro Record (Impval1.acc)

period = 0.64 s, f = 1.56 Hz, ω = **9.82 rad/s**

max. displacement = **3.051 inches**

Example of Using NONLIN to create a displacement response spectrum

Fourth linear analysis

m = 100 kips
k = 200 kips / in
c = 5 percent of critical damping
earthquake record is Imperial Valley El Centro Record (Impval1.acc)

period = 0.23 s, f = 4.42 Hz, ω = **27.79 rad/s**

max. displacement = **0.359 inches**

Fifth linear analysis

m = 100 kips
k = 400 kips / in
c = 5 percent of critical damping
earthquake record is Imperial Valley El Centro Record (Impval1.acc)

period = 0.16 s, f = 6.25 Hz, ω = **39.30 rad/s**

max. displacement = **0.135 inches**

Sixth linear analysis

m = 100 kips
k = 1000 kips / in
c = 5 percent of critical damping
earthquake record is Imperial Valley El Centro Record (Impval1.acc)

period = 0.10 s, f =0.259 Hz, ω = **62.14 rad/s**

max. displacement = **0.056 inches**

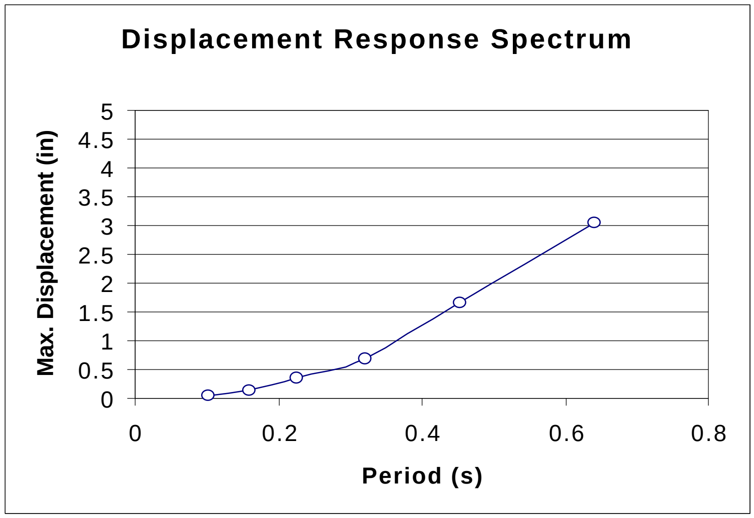

Figure 6. Displacement Response Spectrum for El Centro Record.

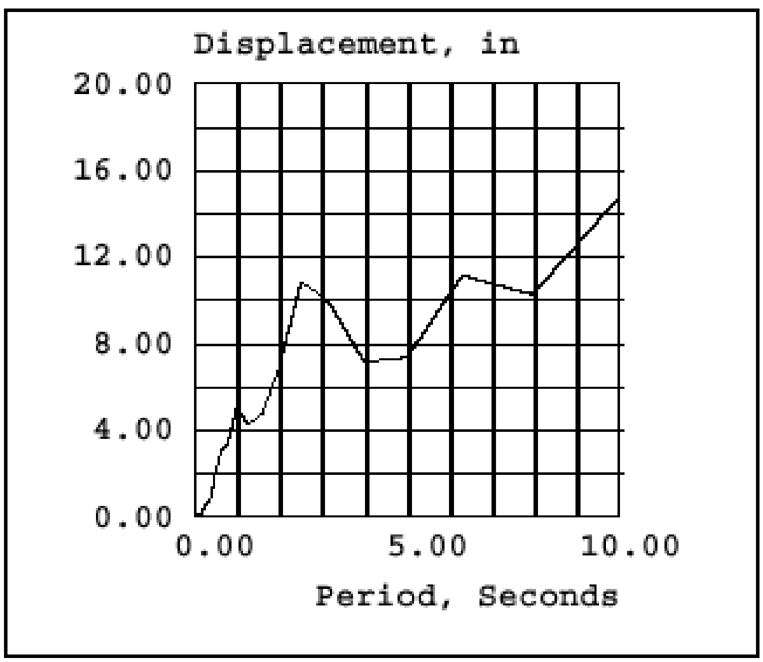

Figure 7. Displacement Response Spectrum from NONLIN.

(5) Calculate the other pseudospectral velocity values using:

$$\mathbf{V} = \mathrm{w}\,\mathbf{D}$$

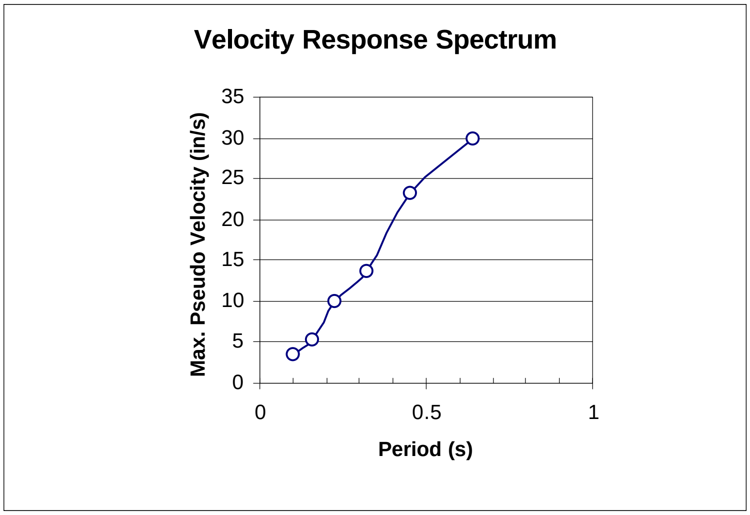

Figure 8. Velocity Response Spectrum for example.

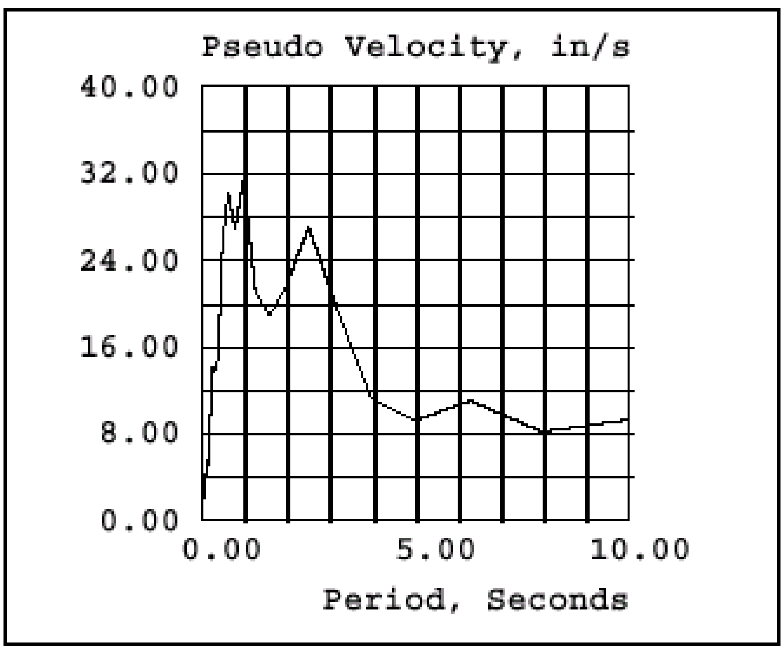

Figure 9. Velocity Response Spectrum from NONLIN

(6) Calculate the pseudoacceleration response values from:

$$\mathbf{A = w^2D}$$

It is common to express A in units of g, thus:

$$\mathbf{A(in\ g) = w^2D/g}$$

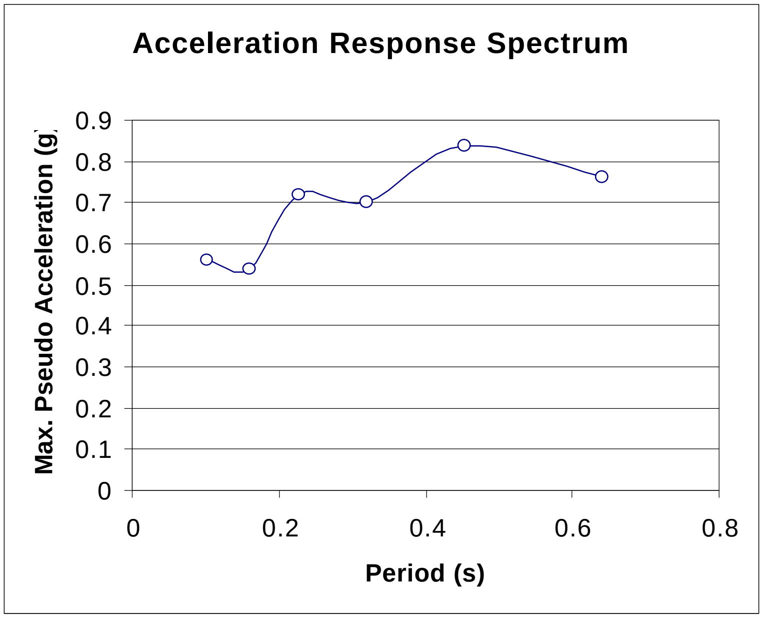

Figure 10. Acceleration Response Spectrum

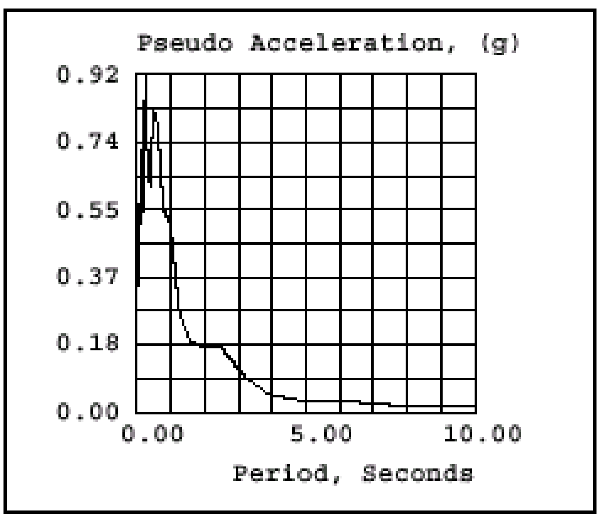

Figure 11. Acceleration Response Spectrum from NONLIN

b. Deterministic Response Spectra from attenuation relations

- ! Deterministic spectrum are usually developed for the maximum credible earthquake (MCE).
- ! The maximum credible earthquake is the largest earthquake possible from the active faults in the region.

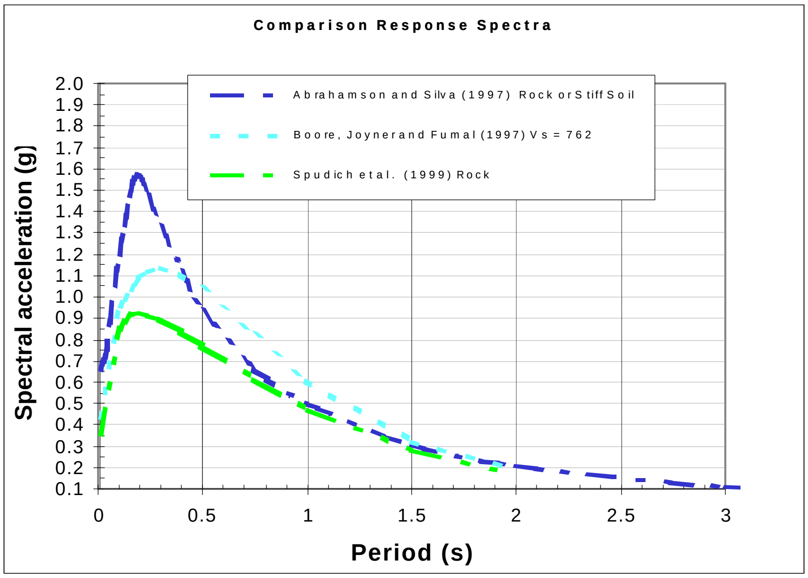

Figure 12. Comparison of Deterministic Rock Spectra for M = 7.0, R = 5 km earthquake from attenuation relations.

c. Probabilistic Spectra for Probabilistic Seismic Hazard Analysis (PSHA)

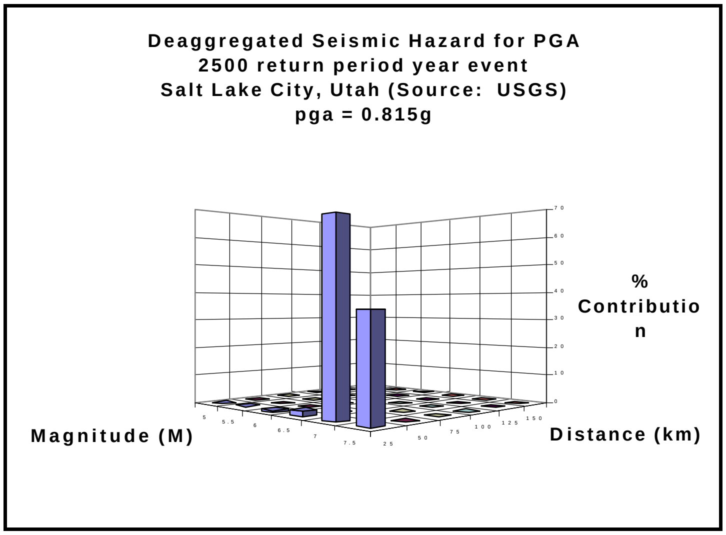

Figure 13. Seismic Hazard and Percent Contribution of M and R pairs for 2500 year return period event (2 percent probability of exceedance in 50 years) for peak ground acceleration.

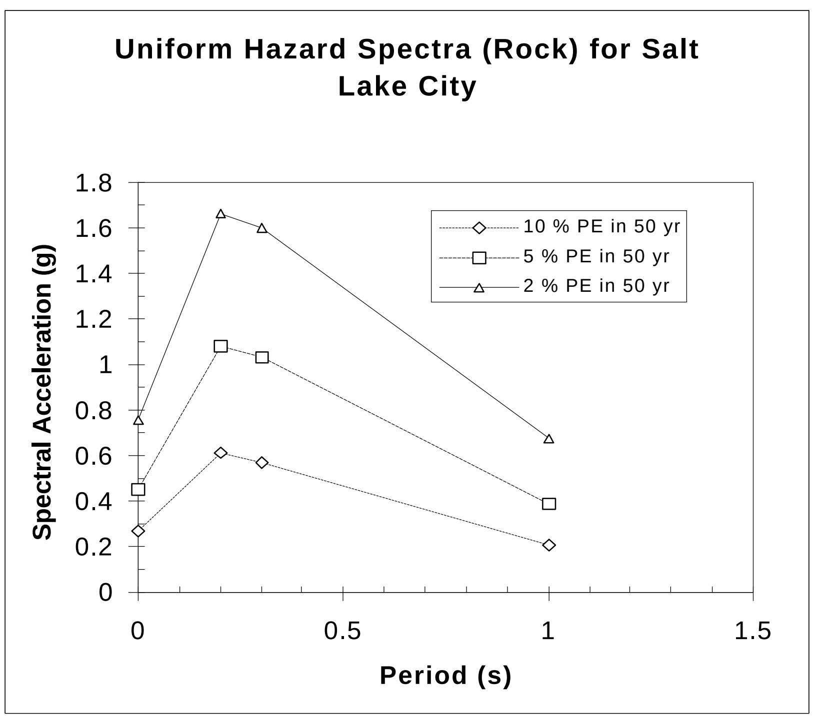

Figure 14. Probabilistic Uniform Hazard Spectra for input zipcode 84115 for USGS.

d. Design Spectra Developed from Building Codes

(See Lecture 6b).

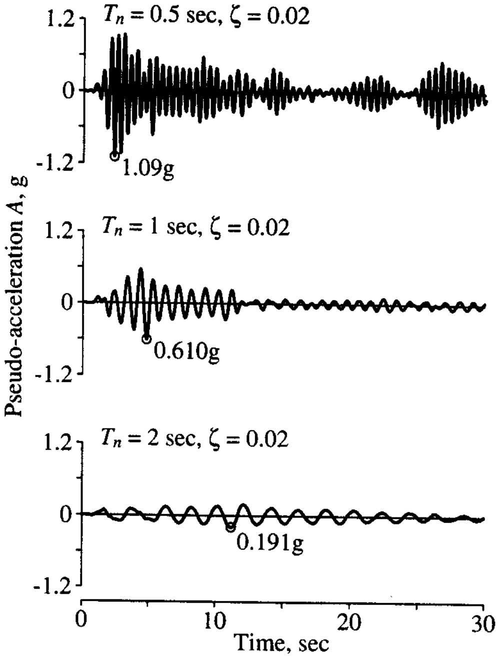

Figure 6.4.3 Pseudo-acceleration response of SDF systems to El Centro ground motion.

where h is the height of the mass above the base. We put Eq. (6.4.2) into these equations to obtain

$$V_b(t) = mA(t) \qquad M_b(t) = hV_b(t) \tag{6.4.4b}$$

If the SDF system is viewed as a mass–spring–damper system (Fig. 6.2.1b), the notion of equivalent static force is not necessary. One can readily visualize that the spring force is given by Eq. (6.4.1).

6.5 RESPONSE SPECTRUM CONCEPT

G. W. Housner was instrumental in the widespread acceptance of the concept of the earthquake response spectrum—introduced by M. A. Biot in 1932—as a practical means of characterizing ground motions and their effects on structures. Now a central concept in earthquake engineering, the response spectrum provides a convenient means to summarize the peak response of all possible linear SDF systems to a particular component of ground motion. It also provides a practical approach to apply the knowledge of structural dynamics to the design of structures and development of lateral force requirements in building codes.

A plot of the peak value of a response quantity as a function of the natural vibration period T_n of the system, or a related parameter such as circular frequency ω_n or cyclic frequency f_n, is called the *response spectrum* for that quantity. Each such plot is for SDF systems having a fixed damping ratio ζ, and several such plots for different values of ζ are included to cover the range of damping values encountered in actual structures. Whether the peak response is plotted against f_n or T_n is a matter of personal preference. We have chosen the latter because engineers prefer to use natural period rather than natural frequency because the period of vibration is a more familiar concept and one that is intuitively appealing.

A variety of response spectra can be defined depending on the response quantity that is plotted. Consider the following peak responses:

$$u_o(T_n, \zeta) \equiv \max_t |u(t, T_n, \zeta)|$$

$$\dot{u}_o(T_n, \zeta) \equiv \max_t |\dot{u}(t, T_n, \zeta)|$$

$$\ddot{u}_o^t(T_n, \zeta) \equiv \max_t |\ddot{u}^t(t, T_n, \zeta)|$$

The *deformation response spectrum* is a plot of u_o against T_n for fixed ζ. A similar plot for $\dot{u}_o$ is the *relative velocity response spectrum*, and for $\ddot{u}_o^t$ is the *acceleration response spectrum.*

6.6 DEFORMATION, PSEUDO-VELOCITY, AND PSEUDO-ACCELERATION RESPONSE SPECTRA

In this section the deformation response spectrum and two related spectra, the pseudo-velocity and pseudo-acceleration response spectra, are discussed. As shown in Section 6.4, only the deformation $u(t)$ is needed to compute internal forces. Obviously, then, the deformation spectrum provides all the information necessary to compute the peak values of deformation $D \equiv u_o$ and internal forces. The pseudo-velocity and pseudo-acceleration response spectra are included, however, because they are useful in studying characteristics of response spectra, constructing design spectra, and relating structural dynamics results to building codes.

6.6.1 Deformation Response Spectrum

Figure 6.6.1 shows the procedure to determine the deformation response spectrum. The spectrum is developed for El Centro ground motion, shown in part (a) of this figure. The time variation of the deformation induced by this ground motion in three SDF systems is presented in part (b). For each system the peak value of deformation $D \equiv u_o$ is determined from the deformation history. (Usually, the peak occurs during ground shaking; however, for lightly damped systems with very long periods the peak response may occur during the free vibration phase after the ground shaking has stopped.) The peak deformations are $D = 2.67$ in. for a system with natural period $T_n = 0.5$ sec and damping ratio $\zeta = 2\%$; $D = 5.97$ in. for a system with $T_n = 1$ sec and $\zeta = 2\%$; and $D = 7.47$ in. for a system with $T_n = 2$ sec and $\zeta = 2\%$. The D value so determined for each system provides one point on the deformation response spectrum; these three values of D are identified in Fig. 6.6.1c. Repeating such computations for a range of values of T_n while keeping ζ constant at 2% provides the deformation response spectrum shown in Fig. 6.6.1c. As we shall show

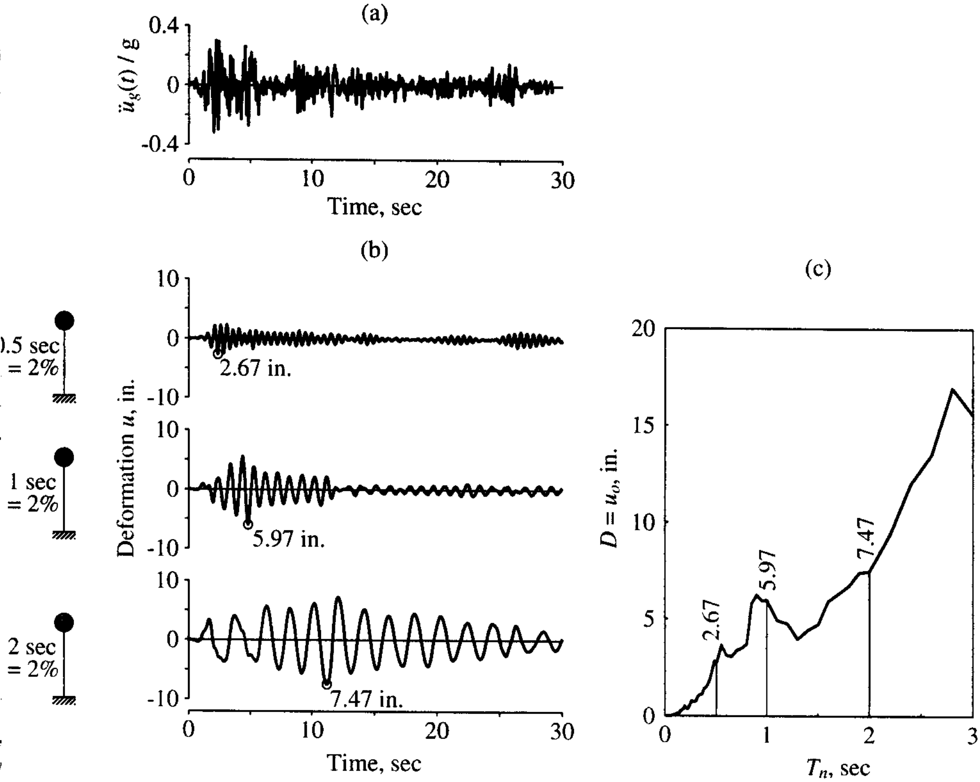

Figure 6.6.1 (a) Ground acceleration; (b) deformation response of three SDF systems with $\zeta = 2\%$ and $T_n = 0.5$, 1, and 2 sec; (c) deformation response spectrum for $\zeta = 2\%$.

(damping

later, the complete response spectrum includes such spectrum curves for several values of damping.

6.6.2 Pseudo-velocity Response Spectrum

Consider a quantity V for an SDF system with natural frequency ω_n related to its peak deformation $D \equiv u_o$ due to earthquake ground motion:

$$\boxed{V = \omega_n D = \frac{2\pi}{T_n} D} \tag{6.6.1}$$

The quantity V has units of velocity. It is related to the peak value of strain energy E_{So} stored in the system during the earthquake by the equation

$$E_{So} = \frac{mV^2}{2} \tag{6.6.2}$$

This relationship can be derived from the definition of strain energy and using Eq. (6.6.1) as follows:

$$E_{So} = \frac{ku_o^2}{2} = \frac{kD^2}{2} = \frac{k(V/\omega_n)^2}{2} = \frac{mV^2}{2}$$

The right side of Eq. (6.6.2) is the kinetic energy of the structural mass m with velocity V, called the *peak relative pseudo-velocity*, or simply peak *pseudo-velocity*. The prefix *pseudo* is used because V is not equal to the peak velocity $\dot{u}_o$, although it has the correct units. We return to this matter in Section 6.12.

The *pseudo-velocity response spectrum* is a plot of V as a function of the natural vibration period T_n, or natural vibration frequency f_n, of the system. For the ground motion of Fig. 6.6.1a the peak pseudo-velocity V for a system with natural period T_n can be determined from Eq. (6.6.1) and the peak deformation D of the same system available from the response spectrum of Fig. 6.6.1c, which has been reproduced in Fig. 6.6.2a. As an example, for a system with $T_n = 0.5$ sec and $\zeta = 2\%$, $D = 2.67$ in.; from Eq. (6.6.1), $V = (2\pi/0.5)2.67 = 33.7$ in./sec. Similarly, for a system with $T_n = 1$ sec and the same ζ, $V = (2\pi/1)5.97 = 37.5$ in./sec; and for a system with $T_n = 2$ sec and the same ζ, $V = (2\pi/2)7.47 = 23.5$ in./sec. These three values of peak pseudo-velocity V are identified in Fig. 6.6.2b. Repeating such computations for a range of values of T_n while keeping ζ constant at 2% provides the pseudo-velocity spectrum shown in Fig. 6.6.2b.

6.6.3 Pseudo-acceleration Response Spectrum

Consider a quantity A for an SDF system with natural frequency ω_n related to its peak deformation $D \equiv u_o$ due to earthquake ground motion:

$$\boxed{A = \omega_n^2 D = \left(\frac{2\pi}{T_n}\right)^2 D} \qquad (6.6.3)$$

The quantity A has units of acceleration and is related to the peak value of base shear V_{bo} [or the peak value of the equivalent static force f_{So}, Eq. (6.4.4a)]:

$$V_{bo} = f_{So} = mA \qquad (6.6.4)$$

This relationship is simply Eq. (6.4.4b) specialized for the time of peak response with the peak value of $A(t)$ denoted by A. The peak base shear can be written in the form

$$\boxed{V_{bo} = \frac{A}{g} w} \qquad (6.6.5)$$

where w is the weight of the structure and g the gravitational acceleration. When written in this form, A/g may be interpreted as the *base shear coefficient* or *lateral force coefficient*. It is used in building codes to represent the coefficient by which the structural weight is multiplied to obtain the base shear.

Observe that the base shear is equal to the inertia force associated with the mass m undergoing acceleration A. This quantity defined by Eq. (6.6.3) is generally different from the peak acceleration $\ddot{u}_o^t$ of the system. It is for this reason that we call A the *peak*

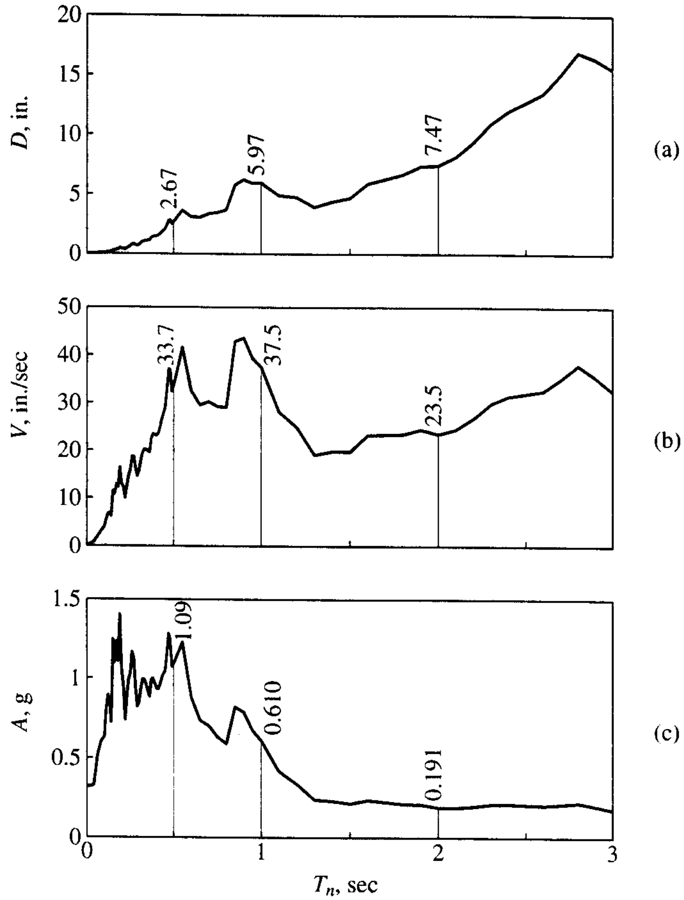

Figure 6.6.2 Response spectra ($\zeta = 0.02$) for El Centro ground motion: (a) deformation response spectrum; (b) pseudo-velocity response spectum; (c) pseudo-acceleration response spectrum.

pseudo-acceleration; the prefix *pseudo* is used to avoid possible confusion with the true peak acceleration $\ddot{u}_o^t$. We return to this matter in Section 6.12.

The *pseudo-acceleration response spectrum* is a plot of A as a function of the natural vibration period T_n, or natural vibration frequency f_n, of the system. For the ground motion of Fig. 6.6.1a the peak pseudo-acceleration A for a system with natural period T_n and damping ratio ζ can be determined from Eq. (6.6.3), and the peak deformation D of the system from the spectrum of Fig. 6.6.2a. As an example, for a system with $T_n = 0.5$ sec and $\zeta = 2\%$, $D = 2.67$ in.; from Eq. (6.6.3), $A = (2\pi/0.5)^2 2.67 = 1.09$g, where g $= 386$ in./sec^2. Similarly, for a system with $T_n = 1$ sec and the same ζ, $A = (2\pi/1)^2 5.97 = 0.610$g; and for a system with $T_n = 2$ sec and the same ζ, $A = (2\pi/2)^2 7.47 = 0.191$g. Note that the same values for A are also available as the peak values of $A(t)$ presented in Fig. 6.4.3. These three values of peak pseudo-acceleration are identified in Fig. 6.6.2c.

Repeating such computations for a range of values of T_n, while keeping ζ constant at 2%, provides the pseudo-acceleration spectrum shown in Fig. 6.6.2c.

6.6.4 Combined *D–V–A* Spectrum

Each of the deformation, pseudo-velocity, and pseudo-acceleration response spectra for a given ground motion contains the same information, no more and no less. The three spectra are simply different ways of presenting the same information on structural response. Knowing one of the spectra, the other two can be obtained by algebraic operations using Eqs. (6.6.1) and (6.6.3).

Why do we need three spectra when each of them contains the same information? One of the reasons is that each spectrum directly provides a physically meaningful quantity. The deformation spectrum provides the peak deformation of a system. The pseudo-velocity spectrum is related directly to the peak strain energy stored in the system during the earthquake; see Eq. (6.6.2). The pseudo-acceleration spectrum is related directly to the peak value of the equivalent static force and base shear; see Eq. (6.6.4). The second reason lies in the fact that the shape of the spectrum can be approximated more readily for design purposes with the aid of all three spectral quantities rather than any one of them alone; see Sections 6.8 and 6.9. For this purpose a combined plot showing all three of the spectral quantities is especially useful. This type of plot was developed for earthquake response spectra, apparently for the first time, by A. S. Veletsos and N. M. Newmark in 1960.

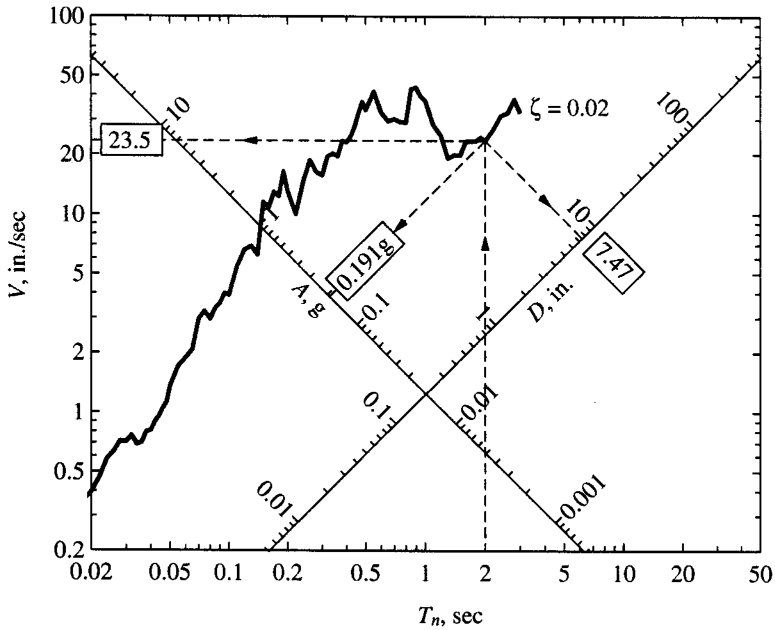

Figure 6.6.3 Combined D–V–A response spectrum for El Centro ground motion; $\zeta = 2\%$.

This integrated presentation is possible because the three spectral quantities are interrelated by Eqs. (6.6.1) and (6.6.3), rewritten as

$$\boxed{\frac{A}{\omega_n} = V = \omega_n D \quad \text{or} \quad \frac{T_n}{2\pi} A = V = \frac{2\pi}{T_n} D} \tag{6.6.6}$$

Observe the similarity between these equations relating D, V, and A and Eq. (3.2.21) for the dynamic response factors R_d, R_v, and R_a for an SDF system subjected to harmonic excitation. Equation (3.2.21) permitted presentation of R_d, R_v, and R_a, all together, on four-way logarithmic paper (Fig. 3.2.8), constructed by the procedure described in Appendix 3 (Chapter 3). Similarly, the graph paper shown in Fig. A6.1 (Appendix 6) with four-way logarithmic scales can be constructed to display D, V, and A, all together. The vertical and horizontal scales for V and T_n are standard logarithmic scales. The two scales for D and A sloping at $+45°$ and $-45°$, respectively, to the T_n-axis are also logarithmic scales but not identical to the vertical scale; see Appendix 3.

Once this graph paper has been constructed, the three response spectra—deformation, pseudo-velocity, and pseudo-acceleration—of Fig. 6.6.2 can readily be combined into a single plot. The pairs of numerical data for V and T_n that were plotted in Fig. 6.6.2b on

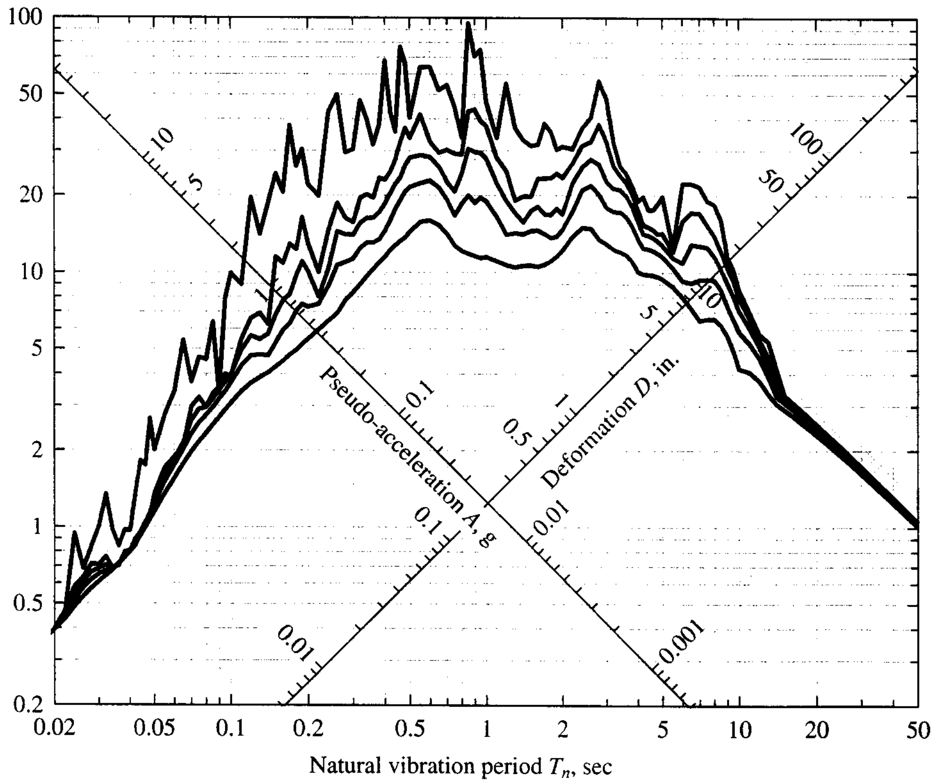

Figure 6.6.4 Combined D–V–A response spectrum for El Centro ground motion; $\zeta = 0$, 2, 5, 10, and 20%.

linear scales are replotted in Fig. 6.6.3 on logarithmic scales. For a given natural period T_n, the D and A values can be read from the diagonal scales. As an example, for $T_n = 2$ sec, Fig. 6.6.3 gives $D = 7.47$ in. and $A = 0.191$g. (Actually, these numbers cannot be read so accurately from the graph; in this case they were available from Fig. 6.6.2.) The four-way plot is a compact presentation of the three—deformation, pseudo-velocity, and pseudo-acceleration—response spectra, for a single plot of this form replaces the three plots of Fig. 6.6.2.

A response spectrum should cover a wide range of natural vibration periods and several damping values so that it provides the peak response of all possible structures. The period range in Fig. 6.6.3 should be extended because tall buildings and long-span bridges, among other structures, may have longer vibration periods (Fig. 2.1.2) and several damping values should be included to cover the practical range of $\zeta = 0$ to 20%. Figure 6.6.4 shows spectrum curves for $\zeta = 0$, 2, 5, 10, and 20% over the period range 0.02 to 50 sec. This, then, is the response spectrum for the north–south component of ground motion recorded at one location during the Imperial Valley earthquake of May 18, 1940. Because the lateral force or base shear for an SDF system is related through Eq. (6.6.5) to A/g, we also plot this normalized pseudo-acceleration spectrum in Fig. 6.6.5. Similarly, because the peak deformation is given by D, we also plot this deformation response spectrum in Fig. 6.6.6.

The response spectrum has proven so useful in earthquake engineering that spectra for virtually all ground motions strong enough to be of engineering interest are now

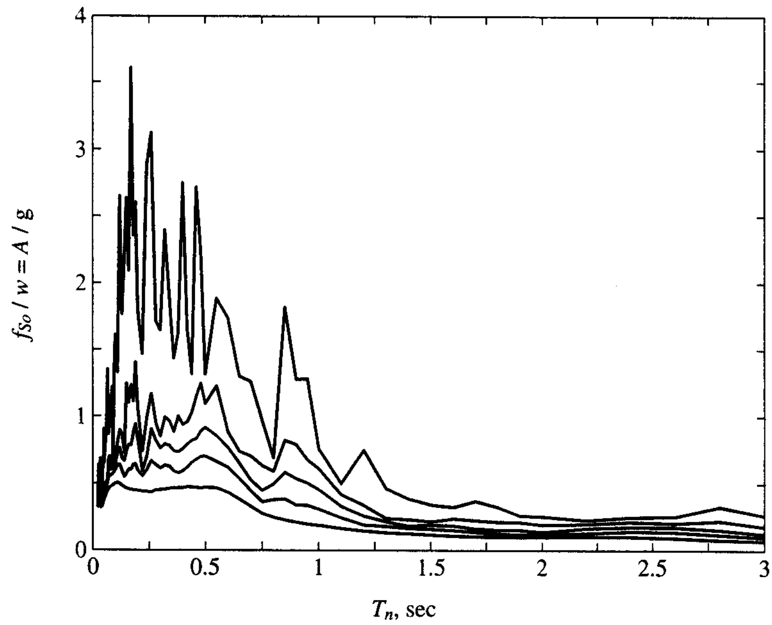

Figure 6.6.5 Normalized pseudo-acceleration, or base shear coefficient, response spectrum for El Centro ground motion; $\zeta = 0$, 2, 5, 10, and 20%.

computed and published soon after they are recorded. Enough of them have been obtained to give us a reasonable idea of the kind of motion that is likely to occur in future earthquakes, and how response spectra are affected by distance to the causative fault, local soil conditions, and regional geology.

6.6.5 Construction of Response Spectrum

The response spectrum for a given ground motion component $\ddot{u}_g(t)$ can be developed by implementation of the following steps:

1. Numerically define the ground acceleration $\ddot{u}_g(t)$; typically, the ground motion ordinates are defined every 0.02 sec.
2. Select the natural vibration period T_n and damping ratio ζ of an SDF system.
3. Compute the deformation response $u(t)$ of this SDF system due to the ground motion $\ddot{u}_g(t)$ by any of the numerical methods described in Chapter 5. [In obtaining the

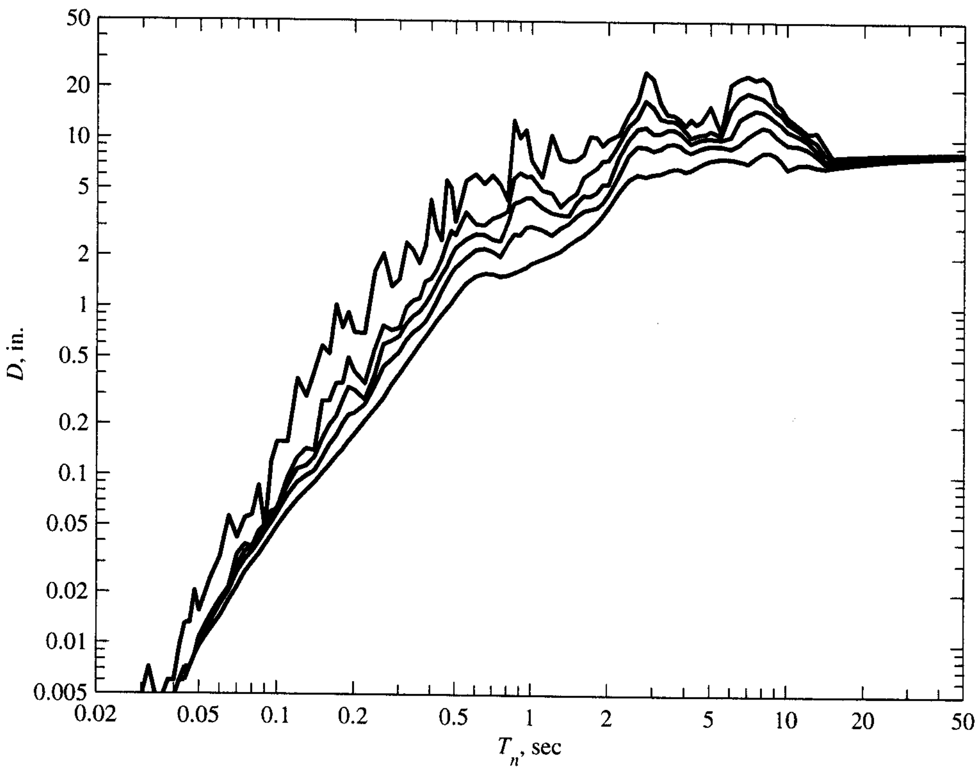

Figure 6.6.6 Deformation response spectrum for El Centro ground motion; ζ = 0, 2, 5, 10, and 20%.

responses shown in Fig. 6.6.1, the exact solution of Eq. (6.2.1) for ground motion assumed to be piecewise linear over every $\Delta t = 0.02$ sec was used; see Section 5.2.]

4. Determine u_o, the peak value of $u(t)$.
5. The spectral ordinates are $D = u_o$, $V = (2\pi/T_n)D$, and $A = (2\pi/T_n)^2 D$.
6. Repeat steps 2 to 5 for a range of T_n and ζ values covering all possible systems of engineering interest.
7. Present the results of steps 2 to 6 graphically to produce three separate spectra like those in Fig. 6.6.2 or a combined spectrum like the one in Fig. 6.6.4.

Considerable computational effort is required to generate an earthquake response spectrum. A complete dynamic analysis to determine the time variation (or history) of the deformation of an SDF system provides the data for one point on the spectrum corresponding to the T_n and ζ of the system. Each curve in the response spectrum of Fig. 6.6.4 was produced from such data for 112 values of T_n unevenly spaced over the range $T_n = 0.02$ to 50 sec.

Example 6.1

Derive equations for and plot deformation, pseudo-velocity, and pseudo-acceleration response spectra for ground acceleration $\ddot{u}_g(t) = \dot{u}_{go}\delta(t)$, where $\delta(t)$ is the Dirac delta function and $\dot{u}_{go}$ is the increment in velocity, or the magnitude of the acceleration impulse. Only consider systems without damping.

Solution

1. *Determine the response history.* The response of an SDF system to $p(t) = \delta(t - \tau)$ is available in Eq. (4.1.6). Adapting that solution to $p_{\text{eff}}(t) = -m\ddot{u}_g(t) = -m\dot{u}_{go}\delta(t)$

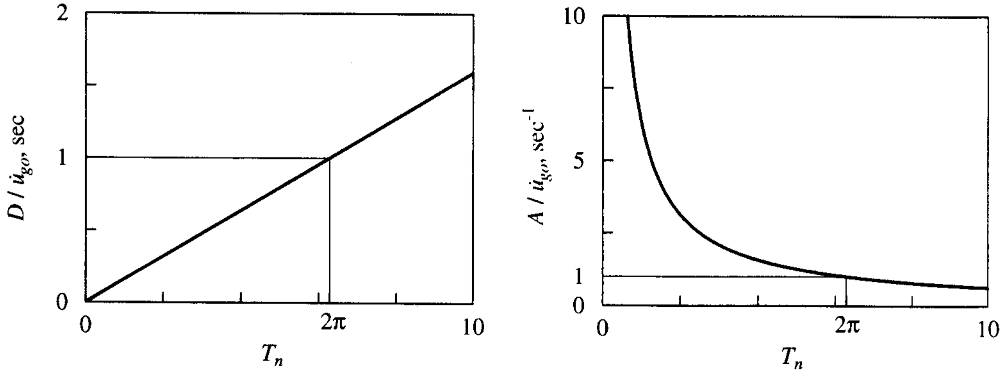

Figure E6.1

gives

$$u(t) = -\frac{\dot{u}_{go}}{\omega_n} \sin \omega_n t \tag{a}$$

The peak value of $u(t)$ is

$$u_o = \frac{\dot{u}_{go}}{\omega_n} \tag{b}$$

2. *Determine the spectral values.*

$$D \equiv u_o = \frac{\dot{u}_{go}}{\omega_n} = \frac{\dot{u}_{go}}{2\pi} T_n \tag{c}$$

$$V = \omega_n D = \dot{u}_{go} \qquad A = \omega_n^2 D = \frac{2\pi \dot{u}_{go}}{T_n} \tag{d}$$

Two of these response spectra are plotted in Fig. E6.1.

6.7 PEAK STRUCTURAL RESPONSE FROM THE RESPONSE SPECTRUM

If the response spectrum for a given ground motion component is available, the peak value of deformation or of an internal force in any linear SDF system can be determined readily. This is the case because the computationally intensive dynamic analyses summarized in Section 6.6.5 have already been completed in generating the response spectrum. Corresponding to the natural vibration period T_n and damping ratio ζ of the system, the values of D, V, or A are read from the spectrum, such as Fig. 6.6.6, 6.6.4, or 6.6.5. Now all response quantities of interest can be expressed in terms of D, V, or A and the mass or stiffness properties of the system. In particular, the peak deformation of the system is

$$u_o = D = \frac{T_n}{2\pi} V = \left(\frac{T_n}{2\pi}\right)^2 A \tag{6.7.1}$$

and the peak value of the equivalent static force f_{So} is [from Eqs. (6.6.4) and (6.6.3)]

$$\boxed{f_{So} = kD = mA} \tag{6.7.2}$$

Static analysis of the one-story frame subjected to lateral force f_{So} (Fig. 6.7.1) provides the internal forces (e.g., shears and moments in columns and beams). This involves application of well-known procedures of static structural analysis, as will be illustrated later

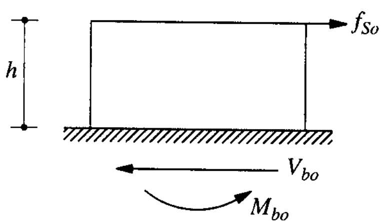

Figure 6.7.1 Peak value of equivalent static force.

by examples. We emphasize again that no further dynamic analysis is required beyond that necessary to determine $u(t)$. In particular, the peak values of shear and overturning moment at the base of the one-story structure are

$$\boxed{V_{bo} = kD = mA \qquad M_{bo} = hV_{bo}} \tag{6.7.3}$$

We note that only one of these response spectra—deformation, pseudo-velocity, or pseudo-acceleration—is sufficient for computing the peak deformations and forces required in structural design. For such applications the velocity or acceleration spectra (defined in Section 6.5) are not required, but for completeness we discuss these spectra briefly at the end of this chapter.

Example 6.2

A 12-ft-long vertical cantilever, a 4-in.-nominal-diameter standard steel pipe, supports a 5200-lb weight attached at the tip as shown in Fig. E6.2. The properties of the pipe are: outside diameter, $d_o = 4.500$ in., inside diameter $d_i = 4.026$ in., thickness $t = 0.237$ in., and second moment of cross-sectional area, $I = 7.23\ \text{in}^4$, elastic modulus $E = 29{,}000$ ksi, and weight = 10.79 lb/foot length. Determine the peak deformation and bending stress in the cantilever due to the El Centro ground motion. Assume that $\zeta = 2\%$.

Solution The lateral stiffness of this SDF system is

$$k = \frac{3EI}{L^3} = \frac{3(29 \times 10^3)7.23}{(12 \times 12)^3} = 0.211 \text{ kip/in.}$$

The total weight of the pipe is $10.79 \times 12 = 129.5$ lb, which may be neglected relative to the lumped weight of 5200 lb. Thus

$$m = \frac{w}{g} = \frac{5.20}{386} = 0.01347 \text{ kip-sec}^2/\text{in.}$$

The natural vibration frequency and period of the system are

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.211}{0.01347}} = 3.958 \text{ rad/sec} \qquad T_n = 1.59 \text{ sec}$$

From the response spectrum curve for $\zeta = 2\%$ (Fig. E6.2b), for $T_n = 1.59$ sec, $D = 5.0$ in. and $A = 0.20$g. The peak deformation is

$$u_o = D = 5.0 \text{ in.}$$

The peak value of the equivalent static force is

$$f_{So} = \frac{A}{g}w = 0.20 \times 5.2 = 1.04 \text{ kips}$$

The bending moment diagram is shown in Fig. E6.2d with the maximum moment at the base = 12.48 kip-ft. Points A and B shown in Fig. E6.2e are the locations of maximum bending stress:

$$\sigma_{\max} = \frac{Mc}{I} = \frac{(12.48 \times 12)(4.5/2)}{7.23} = 46.5 \text{ ksi}$$

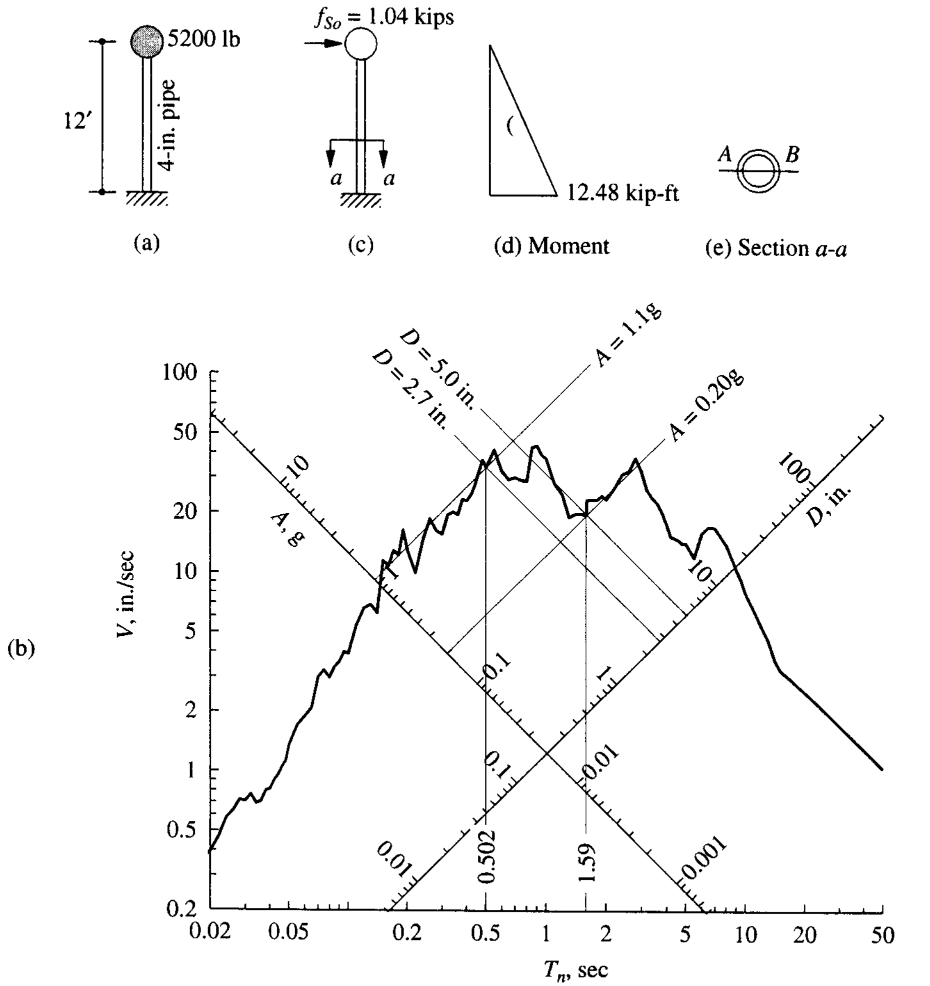

Figure E6.2

Thus, $\sigma = +46.5$ ksi at A and $\sigma = -46.5$ ksi at B, where $+$ denotes tension. The algebraic signs of these stresses are irrelevant because the direction of the peak force is not known, as the pseudo-acceleration spectrum is, by definition, positive.

Example 6.3

The stress computed in Example 6.2 exceeded the allowable stress and the designer decided to increase the size of the pipe to an 8-in.-nominal standard steel pipe. Its properties are $d_o = 8.625$ in., $d_i = 7.981$ in., $t = 0.322$ in., and $I = 72.5$ in^4. Comment on the advantages and disadvantages of using the bigger pipe.

Solution

$$k = \frac{3(29 \times 10^3)72.5}{(12 \times 12)^3} = 2.112 \text{ kips/in.}$$

$$\omega_n = \sqrt{\frac{2.112}{0.01347}} = 12.52 \text{ rad/sec} \qquad T_n = 0.502 \text{ sec}$$

From the response spectrum (Fig. E6.2b): $D = 2.7$ in. and $A = 1.1g$. Therefore,

$$u_o = D = 2.7 \text{ in.}$$

$$f_{So} = 1.1 \times 5.2 = 5.72 \text{ kips}$$

$$M_{\text{base}} = 5.72 \times 12 = 68.64 \text{ kip-ft}$$

$$\sigma_{\max} = \frac{(68.64 \times 12)(8.625/2)}{72.5} = 49.0 \text{ ksi}$$

Using the 8-in.-diameter pipe decreases the deformation from 5.0 in. to 2.7 in. However, contrary to the designer's objective, the bending stress increases slightly.

This example points out an important difference between the response of structures to earthquake excitation and to a fixed value of static force. In the latter case, the stress would decrease, obviously, by increasing the member size. In the case of earthquake excitation, the increase in pipe diameter shortens the natural vibration period from 1.59 sec to 0.50 sec, which for this response spectrum has the effect of increasing the equivalent static force f_{So}. Whether the bending stress decreases or increases by increasing the pipe diameter depends on the increase in section modulus, I/c, and the increase or decrease in f_{So}, depending on the response spectrum.

Example 6.4

A small one-story reinforced concrete building is idealized for purposes of structural analysis as a massless frame supporting a total dead load of 10 kips at the beam level (Fig. E6.4a). The frame is 24 ft wide and 12 ft high. Each column and the beam has a 10-in.-square cross section. Assume that the Young's modulus of concrete is 3×10^3 ksi and the damping ratio for the building is estimated as 5%. Determine the peak response of this frame to the El Centro ground motion. In particular, determine the peak lateral deformation at the beam level and plot the diagram of bending moments at the instant of peak response.

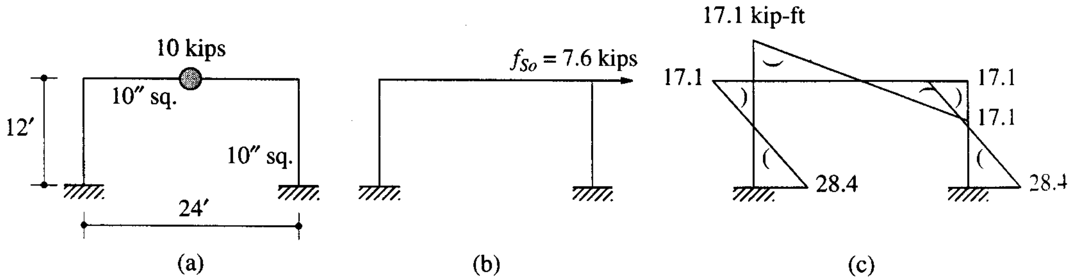

Figure E6.4 (a) Frame; (b) equivalent static force; (c) bending moment diagram.

Solution The lateral stiffness of such a frame was calculated in Chapter 1: $k = 96EI/7h^3$, where EI is the flexural rigidity of the beam and columns and h is the height of the frame. For this particular frame,

$$k = \frac{96(3 \times 10^3)(10^4/12)}{7(12 \times 12)^3} = 11.48 \text{ kips/in.}$$

The natural vibration period is

$$T_n = \frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{\frac{10/386}{11.48}} = 0.30 \text{ sec}$$

For $T_n = 0.3$ and $\zeta = 0.05$, we read from the response spectrum of Fig. 6.6.4: $D = 0.67$ in. and $A = 0.76$g. Peak deformation: $u_o = D = 0.67$ in. Equivalent static force: $f_{So} = (A/g)w = 0.76 \times 10 = 7.6$ kips. Static analysis of the frame for this lateral force, shown in Fig. E6.4b, gives the bending moments that are plotted in Fig. E6.4c.

Example 6.5

The frame of Example 6.4 is modified for use in a building to be located on sloping ground (Fig. E6.5). The beam is now made much stiffer than the columns and can be assumed to be rigid. The cross sections of the two columns are 10 in. square, as before, but their lengths are 12 ft and 24 ft, respectively. Determine the base shears in the two columns at the instant of peak response due to the El Centro ground motion. Assume the damping ratio to be 5%.

Solution

1. *Compute the natural vibration period.*

$$k = \frac{12(3 \times 10^3)(10^4/12)}{(12 \times 12)^3} + \frac{12(3 \times 10^3)(10^4/12)}{(24 \times 12)^3}$$
$$= 10.05 + 1.26 = 11.31 \text{ kips/in.}$$

$$T_n = 2\pi\sqrt{\frac{10/386}{11.31}} = 0.30 \text{ sec}$$

2. *Compute the shear force at the base of the short and long columns.*

$$u_o = D = 0.67 \text{ in.,} \qquad A = 0.76\text{g}$$

$$V_{\text{short}} = k_{\text{short}}u_o = (10.05)0.67 = 6.73 \text{ kips}$$

$$V_{\text{long}} = k_{\text{long}}u_o = (1.26)0.67 = 0.84 \text{ kip}$$

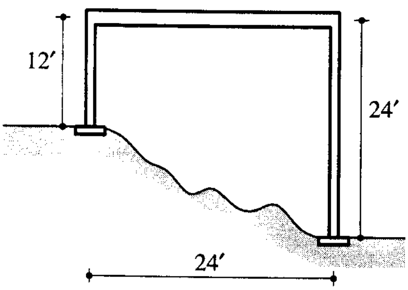

Figure E6.5

Observe that both columns go through equal deformation. Undergoing equal deformations, the stiffer column carries a greater force than the flexible column; the lateral force is distributed to the elements in proportion to their relative stiffnesses. Sometimes this basic principle has, inadvertently, not been recognized in building design, leading to unanticipated damage of the stiffer elements.

Example 6.6

For the three-span box-girder bridge of Example 1.3, determine the base shear in each of the six columns of the two bents due to El Centro ground motion applied in the longitudinal direction. Assume the damping ratio to be 5%.

Solution The weight of the bridge deck was computed in Example 1.3: $w = 6919$ kips. The natural period of longitudinal vibration of the bridge was computed in Example 2.2: $T_n = 0.573$ sec. For $T_n = 0.573$ sec and $\zeta = 0.05$, we read from the response spectrum of Fig. 6.6.4: $D = 2.591$ in. and $A = 0.807g$.

All the columns have the same stiffness and they go through equal deformation $u_o = D = 2.591$ in. Thus, the base shear will be the same in all columns, which can be computed in one of two ways: The total equivalent static force on the bridge is [from Eq. (6.6.5)]

$$f_{so} = 0.807 \times 6919 = 5584 \text{ kips}$$

Base shear for one column, $V_b = 5584 \div 6 = 931$ kips. Alternatively, the base shear in each column is

$$V_b = k_{\text{col}} u_o = 4313 \times \frac{2.591}{12} = 931 \text{ kips}$$

6.8 RESPONSE SPECTRUM CHARACTERISTICS

We now study the important properties of earthquake response spectra. Figure 6.8.1 shows the response spectrum for El Centro ground motion together with u_{go}, $\dot{u}_{go}$, and $\ddot{u}_{go}$, the peak values of ground acceleration, ground velocity, and ground displacement, respectively, identified in Fig. 6.1.4. To show more directly the relationship between the response spectrum and the ground motion parameters, the data of Fig. 6.8.1 have been presented again in Fig. 6.8.2 using normalized scales: D/u_{go}, $V/\dot{u}_{go}$, and $A/\ddot{u}_{go}$. Figure 6.8.3 shows one of the spectrum curves of Fig. 6.8.2, the one for 5% damping, together with an idealized version shown in dashed lines; the latter will provide a basis for constructing smooth design spectra directly from the peak ground motion parameters (see Section 6.9). Based on Figs. 6.8.1 to 6.8.3, we first study the properties of the response spectrum over various ranges of the natural vibration period of the system separated by the period values at a, b, c, d, e, and f: $T_a = 0.035$ sec, $T_b = 0.125$, $T_c = 0.5$, $T_d = 3.0$, $T_e = 10$, and $T_f = 15$ sec. Subsequently, we identify the effects of damping on spectrum ordinates.

For systems with very short period, say $T_n < T_a = 0.035$ sec, the peak pseudo-acceleration A approaches $\ddot{u}_{go}$ and D is very small. This trend can be understood based on physical reasoning. For a fixed mass, a very short period system is extremely stiff or essentially rigid. Such a system would be expected to undergo very little deformation and its mass would move rigidly with the ground; its peak acceleration should be approximately