Minimum-Norm Differential Beamforming for Linear Array with Directional Microphones

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Abstract

Among different differential beamforming approaches, the minimum-norm one has received much attention as it maximizes the white noise gain(WNG). WNG measures the robustness of beamformer. But in practice, the conventional minimum-norm differential beamforming with omnidirectional elements still suffers in low white-noise-gain at the low frequencies. The major contributions of this paper are as follows: First, we extend the existing work by presenting a new solution with the use of the directional microphone elements, and show clearly the connection between the conventional beamforming and the proposed beamforming. Second, through the derivation as well as simulations, we show the proposed solution brings noticeable improvement in WNG at the low frequencies when the null positions of the directional elements coincide with the null-constraints of minimum norm solution.

Index Terms: linear microphone array, differential beamformer, directional microphone

1. Introduction

Microphone array with a beamforming algorithm can provide a multi-channel speech enhancement solution for many far-field sound capture devices in adverse environments. Beamforming performs as a spatial filter to capture the target signal from the desired direction and attenuate the interferences from the undesired direction [1]. Generally, the beamforming algorithms can be categorized into two groups: adaptive beamformer and fixed beamformer. As an adaptive beamformer, the minimum variance distortionless response(MVDR) beamformer plays a pivotal role since it has no distortion on the target speech in principle. In practical real-time systems, generalized sidelobe canceller (GSC) with a fixed blocking matrix [2] or adaptive blocking matrix [3] is an efficient approach to implement MVDR. In the GSC structure, the fixed beamformer is an essential part. As compared to adaptive beamfomers, fixed beamformers are more robust since they are not involved with the adaptation process.

In the area of fixed beamforming, differential beamforming or differential microphone array (DMA) has received a lot of attention recently, since DMA possesses a few advantages over some traditional beamformers [4, 5, 6, 7, 8]; firstly, it can form a relatively frequency-invariant beampattern, thus appropriate for the broadband speech processing; secondly, it has the potential to obtain a large directivity factor (DF) with small and compact aperture. On the other hand, it is noted that omni-directional microphone elements are commonly utilized in the DMA designs and the WNG at low frequencies tends to be relatively low which would cause the well-known white noise amplification in

practice [6, 7].

Directional microphone elements have been shown advantageous in beamforming design according to some previous work [9, 10, 11]. A 4-element cardioid microphone array is employed in Microsoft Kinect [10]. Motivated by their work, the authors proposed to utilize the directional microphone elements in the design of DMA for a uniform circular array (referred thereafter as circular differential directional microphone array or CDDMA) to achieve a better performance in terms of WNG and DF [12]. This paper is to extend the idea to the design of minimum-norm differential beamforming for linear array with directional microphone, i.e., linear differential directional microphone array (LDDMA). Different from circular array, linear microphone array has a unique geometric property that all of the directional microphones can point at the same direction. Through the derivation, we show neatly the connection between the conventional beamforming and the proposed beamforming. And the relation between the acoustics of elements and design constraints of beamforming are studied. The directional elements can not only improve the DI of the array but also boost the WNG at low frequencies.

The rest of paper is organized as follows. First, the directional microphone is introduced. The signal model is briefly described. We then study our proposal and compare it to conventional approach. Lastly, some conclusions are drawn.

2. Signal Model

Microphone can be categorized by acoustic directionality into two classes: omni-directional microphone that captures the sound with equal gain from all directions and the directional microphone that predominantly does it from some specific directions. A comprehensive study on different types of microphones can be found in the Eargle's book [14]. Mathematically, the directional pattern of a first-order directional microphone can be expressed as $u(p, \theta, \alpha) = p + (1-p)\cos(\theta - \alpha)$, where θ is the sound incident angle, α is the steering direction of the microphone element and p defines the property of the directional microphone [14]; for instance, it makes the well-known cardioid pattern when $p = 0.5$ and dipole pattern when $p = 0$.

In this paper, we consider a uniform linear array (ULA) with M first-order directional microphones as illustrated in Fig.1, where the inter-element spacing is denoted as δ and all the directional microphones are pointing in the same direction of α. We assume that a plane wave impinges on the array with an incident angle of θ, the steering vector is then given by:

$$\mathbf{d}(\omega, \theta, \alpha) = [p + (1-p)\cos(\theta - \alpha)]$$
$$[1 \; e^{-j\omega\delta\cos\theta/c} \; \cdots \; e^{-j\omega(M-1)\delta\cos\theta/c}]^T, \quad (1)$$
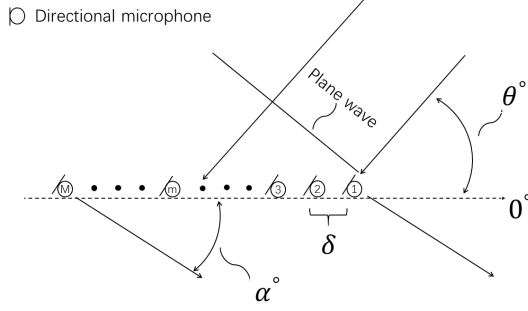
○ Directional microphone

Figure 1: *Uniform linear array with directional microphones*

where the superscript ^{T} is the transpose operator, $j = \sqrt{-1}$ is the imaginary unit, $\omega = 2\pi f$ is the angular frequency, and f is the temporal frequency. For comparison, the conventional steering vector for an array with omni-directional elements is recalled as:

$$\mathbf{a}(\omega, \theta) = [1 \; e^{-j\omega\delta\cos\theta/c} \; \cdots \; e^{-j\omega(M-1)\delta\cos\theta/c}]^T. \quad (2)$$

By examining the equations (1) and (2), the directional microphone steering vector $\mathbf{d}(\omega, \theta, \alpha)$ can be reformulated as:

$$\mathbf{d}(\omega, \theta, \alpha) = u(p, \theta, \alpha)\,\mathbf{a}(\omega, \theta), \quad (3)$$

where $u(p, \theta, \alpha) = [p + (1 - p)\cos(\theta - \alpha)]$ is the magnitude response of first-order directional microphone.

Beamforming process can be viewed as a spatial filter to capture the signal from desired look direction and attenuate the signal from undesired direction by applying a complex weight vector:

$$\mathbf{h}(\omega) = [H_1(\omega) \; H_2(\omega) \; \cdots \; H_M(\omega)]^T. \quad (4)$$

Given the signal model, the beamformer exhibits a distortion-less response in the desired look direction $\theta_{desired}$; in other directions, the beamformer shows a certain distortion on the response, i.e.,

$$\mathbf{d}^H(\omega, \theta, \alpha)\mathbf{h}(\omega) \begin{cases} = 1, & \text{if } \theta = \theta_{desired} \\ < 1, & \text{if } \theta \neq \theta_{desired} \end{cases}, \quad (5)$$

where the superscript ^{H} is the conjugate-transpose operator. In order to achieve distortionless response of original microphone signal, it is noted that pointing direction of the directional microphones is identical to 'look direction' of beamformer ($\alpha = \theta_{desired}$) in the following text.

3. Performance Measure

For a fixed beamformer, three widely-used performance measurements are beampattern, white noise gain and directivity factor. Beampattern illustrates the directional sensitivity of a beamformer to a plane wave impinging on the array from the incident angle θ (see Fig.1) and is mathematically defined as

$$\mathcal{B}[\mathbf{h}(\omega), \theta, \alpha] = \mathbf{h}^H(\omega)\,\mathbf{d}(\omega, \theta, \alpha). \quad (6)$$

In this paper, the power pattern, i.e., $|\mathcal{B}[\mathbf{h}(\omega), \theta, \alpha]|^2$, is utilized to demonstrate the performance.

WNG is a most convenient way to evaluate the robustness for the sensitivity of a beamformer to some of its imperfections,

such as sensor noise, position errors, etc [15]. WNG can be given by:

$$\mathcal{W}[\mathbf{h}(\omega)] = 1/[\mathbf{h}^H(\omega)\mathbf{h}(\omega)]. \quad (7)$$

Usually, tradeoff has to be made between WNG and DF in practical design. The DF is defined as the ratio between the response power in the desired steering direction and the power averaged over all directions[13], i.e., DF is computed as

$$\mathcal{DF}[\mathbf{h}(\omega)] = \frac{1}{\int_0^\pi d\phi \int_0^{2\pi} d\theta \sin\phi \, |\mathcal{B}[\mathbf{h}(\omega), \theta, \phi, \alpha]|^2} \quad (8)$$

where θ is the azimuth angle, the ϕ is the elevation angle, and $\mathcal{B}[\mathbf{h}(\omega), \theta, \phi, \alpha]$ is the beampattern in the spherical coordinate system, defined as:

$$\mathcal{B}[\mathbf{h}(\omega), \theta, \phi, \alpha] = \mathbf{h}^H(\omega)\mathbf{d}(\omega, \theta, \phi, \alpha) \quad (9)$$

where the vector $\mathbf{d}(\omega, \theta, \phi, \alpha)$ is:

$$\mathbf{a}(\omega, \theta, \phi, \alpha) = [p + (1 - p)\cos(\theta - \alpha)\sin\phi]$$
$$[1 \; e^{-j\omega\delta\cos\theta\sin\phi\,/c} \; \cdots \; e^{-j\omega(M-1)\delta\cos\theta\sin\phi\,/c}]^T. \quad (10)$$

In the following section, DF will be calculated numerically according to Eqn.(8). Lastly, directivity index (DI) is defined as: $\mathcal{DI}[\mathbf{h}(\omega)] = 10 * log10(\mathcal{DF}[\mathbf{h}(\omega)])$.

4. Proposed Method

To design the differential beamforming for the proposed array, we formulated that the problem as a set of linear system equations[4] as below:

$$\mathbf{R}(\omega, \boldsymbol{\theta}, \alpha)\mathbf{h}(\omega) = \mathbf{c}_{\boldsymbol{\theta}}, \quad (11)$$

where the constraint matrix $\mathbf{R}(\omega, \boldsymbol{\theta}, \alpha)$ of size $N \times M$ is given by

$$\mathbf{R}(\omega, \boldsymbol{\theta}, \alpha) = \begin{bmatrix} \mathbf{d}^H(\omega, \theta_1, \alpha) \\ \mathbf{d}^H(\omega, \theta_2, \alpha) \\ \vdots \\ \mathbf{d}^H(\omega, \theta_N, \alpha) \end{bmatrix}, \quad (12)$$

where $\mathbf{d}^H(\omega, \theta_n, \alpha), n = 1, 2, \cdots, N$, is the steering vector of length M as defined in (1), and

$$\boldsymbol{\theta} = [\theta_1 \; \cdots \; \theta_N]^T, \quad (13)$$

$$\mathbf{c}_{\boldsymbol{\theta}} = [c_{\theta_1}, \cdots, c_{\theta_N}]^T, \quad (14)$$

are the vectors of size N containing the design parameters of the beamformer. $\theta_1, \cdots, \theta_N$ usually define the desired or null directions, and $c_{\theta_1}, \cdots, c_{\theta_N}$ are the corresponding response for these directions, e.g., given $c_{\theta_1} = 1$, the steering vector at θ_1 follows $\mathbf{d}^H(\omega, \theta_1, \alpha)\mathbf{h}(\omega) = 1$, which means that θ_1 equals the look direction α of the proposed beamformer that produces the distortionless output for the sound coming from this direction. The remaining $c_{\theta_i} = 0$ for $i = 2, 3, ..., N$ are usually set to zero, i.e., these θ_i decides the nulls of the beampattern. Thus $\mathbf{c}_{\boldsymbol{\theta}}$ describes the null constraints of differential beamforming, i.e.:

$$c_{\theta_i} = \begin{cases} 1, & \text{if } \theta_i = \alpha \\ 0, & \text{if } \theta_i \neq \alpha \end{cases}. \quad (15)$$

In order to achieve the maximum WNG for robust beamforming, we take the well-known minimum-norm solution in

[4, 7, 16, 17] to solve our linear system equations as shown in (11), thus the proposed LDDMA beamformer is obtained by:

$$\mathbf{h}_{lddma}(\omega) = \mathbf{R}^H(\omega, \boldsymbol{\theta}, \alpha)[\mathbf{R}(\omega, \boldsymbol{\theta}, \alpha)\mathbf{R}^H(\omega, \boldsymbol{\theta}, \alpha)]^{-1}\mathbf{c}_{\boldsymbol{\theta}}. \quad (16)$$

Based on (11) and (16), we can see that the properties of the beamformer are determined by the null constraints vector $\mathbf{c}_{\boldsymbol{\theta}}$. For comparison, we recall the conventional LDMA beamformer with the minimum-norm solution based on the omnidirectional microphone signal model in [18] as:

$$\mathbf{h}_{ldma}(\omega) = \mathbf{A}^H(\omega, \boldsymbol{\theta})[\mathbf{A}(\omega, \boldsymbol{\theta})\mathbf{A}^H(\omega, \boldsymbol{\theta})]^{-1}\mathbf{c}_{\boldsymbol{\theta}}, \quad (17)$$

where $\mathbf{A}(\omega, \boldsymbol{\theta})$ is the conventional far-field steering vectors for omnidirectional microphones defined as below:

$$\mathbf{A}(\omega, \boldsymbol{\theta}) = \begin{bmatrix} \mathbf{a}^H(\omega, \theta_1) \\ \vdots \\ \mathbf{a}^H(\omega, \theta_N) \end{bmatrix}. \quad (18)$$

The difference between the proposed beamformer and conventional LDMA beamformer is reflected in $\mathbf{R}(\omega, \theta, \alpha)$ and $\mathbf{A}(\omega, \boldsymbol{\theta})$. Combining (3), (12) and (18), we obtain:

$$\mathbf{R}(\omega, \boldsymbol{\theta}, \alpha) = \mathbf{U}(p, \boldsymbol{\theta}, \alpha)\mathbf{A}(\omega, \boldsymbol{\theta}), \quad (19)$$

where $\mathbf{U}(p, \boldsymbol{\theta}, \alpha)$ is called the directional microphone response matrix and expressed as:

$$\mathbf{U}(p, \boldsymbol{\theta}, \alpha) = \text{diag}\left(u(p, \theta_1, \alpha), \cdots, u(p, \theta_N, \alpha)\right), \quad (20)$$

then the proposed beamformer can be reformulated as below by combining (16) and (19):

$$\mathbf{h}_{lddma}(\omega) = \mathbf{A}^H(\omega, \boldsymbol{\theta})\mathbf{U}^H(p, \boldsymbol{\theta}, \alpha)$$
$$[\mathbf{U}(p, \boldsymbol{\theta}, \alpha)\mathbf{A}(\omega, \boldsymbol{\theta})\mathbf{A}^H(\omega, \boldsymbol{\theta})\mathbf{U}^H(p, \boldsymbol{\theta}, \alpha)]^{-1}\mathbf{c}_{\boldsymbol{\theta}}. \quad (21)$$

This equation shows neatly the relationship between the solutions of conventional LDMA and proposed LDDMA, i.e., LD-DMA extends LDMA by introducing another degree of freedom, i.e., $\mathbf{U}(p, \boldsymbol{\theta}, \alpha)$. As an example to introduce the null position of directional microphone, the first-order cardioid microphone are chosen to form the array with $p = 0.5$, then $u(p, \theta, \alpha) = [0.5 + 0.5 * \cos(\theta - \alpha)]$, the null position θ_{null} of diretional microphone is $\theta_{null} = \alpha + \pi$. Given null position of the directional elements is different from each null constraints of minimum-norm solution, i.e., $\theta_{null} \notin \boldsymbol{\theta}$, each elements in $\mathbf{U}(p, \boldsymbol{\theta}, \alpha)$ would be a nonzero number such that $\mathbf{U}(p, \boldsymbol{\theta}, \alpha)$ becomes a $N \times N$ square matrix of full rank, then the proposed beamformer can be simplified as below:

$$\mathbf{h}_{lddma}(\omega)\big|_{\theta_{null} \notin \boldsymbol{\theta}} = \mathbf{A}^H(\omega, \boldsymbol{\theta})\mathbf{U}^H(p, \boldsymbol{\theta}, \alpha)[\mathbf{U}^H(p, \boldsymbol{\theta}, \alpha)]^{-1}$$
$$[\mathbf{A}(\omega, \boldsymbol{\theta})\mathbf{A}^H(\omega, \boldsymbol{\theta})]^{-1}\mathbf{U}^{-1}(p, \boldsymbol{\theta}, \alpha)\mathbf{c}_{\boldsymbol{\theta}}$$
$$= \mathbf{A}^H(\omega, \boldsymbol{\theta})[\mathbf{A}(\omega, \boldsymbol{\theta})\mathbf{A}^H(\omega, \boldsymbol{\theta})]^{-1}$$
$$\mathbf{U}^{-1}(p, \boldsymbol{\theta}, \alpha)\mathbf{c}_{\boldsymbol{\theta}}. \quad (22)$$

Based on the representation of $\mathbf{c}_{\boldsymbol{\theta}}$ in (15), and the directional microphone has the unity response at desired direction α, i.e., $u(p, \theta_i, \alpha)\big|_{\theta_i = \alpha} = 1$, we can easily obtain:

$$\mathbf{U}^{-1}(p, \boldsymbol{\theta}, \alpha)\mathbf{c}_{\boldsymbol{\theta}} = \mathbf{c}_{\boldsymbol{\theta}}. \quad (23)$$

Combining (22) and (23), we obtain:

$$\mathbf{h}_{lddma}(\omega)\big|_{\theta_{null} \notin \boldsymbol{\theta}} = \mathbf{A}^H(\omega, \boldsymbol{\theta})[\mathbf{A}(\omega, \boldsymbol{\theta})\mathbf{A}^H(\omega, \boldsymbol{\theta})]^{-1}\mathbf{c}_{\boldsymbol{\theta}}$$
$$= \mathbf{h}_{ldma}(\omega). \quad (24)$$

Now we can see that LDMA is just a special case of LD-DMA when the microphone response matrix $\mathbf{U}(p, \boldsymbol{\theta}, \alpha)$ is full rank matrix. In another word, when null position of the directional elements is different from each null constraints of beamforming design, the minimum norm differential beamformer weights for proposed array with directional microphones are identical to the beamformer weights for conventional array with omni-directional elements.
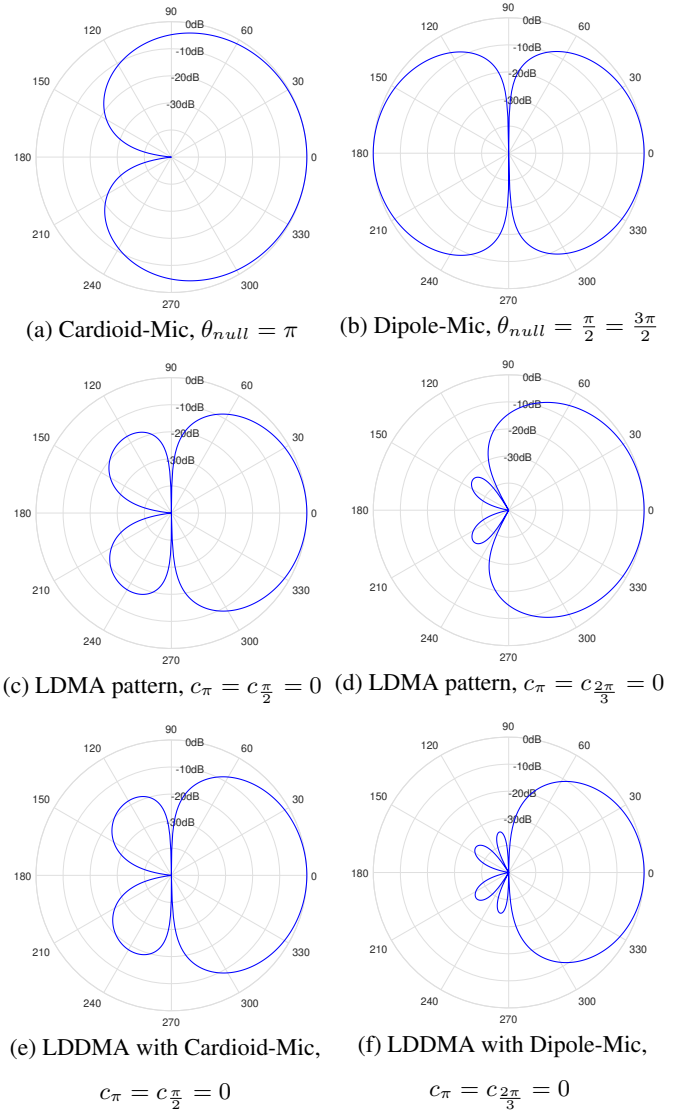
5. Simulations

(a) Cardioid-Mic, $\theta_{null} = \pi$ (b) Dipole-Mic, $\theta_{null} = \frac{\pi}{2} = \frac{3\pi}{2}$

(c) LDMA pattern, $c_\pi = c_{\frac{\pi}{2}} = 0$ (d) LDMA pattern, $c_\pi = c_{\frac{2\pi}{3}} = 0$

(e) LDDMA with Cardioid-Mic,   (f) LDDMA with Dipole-Mic,

$\quad c_\pi = c_{\frac{\pi}{2}} = 0$ $\quad c_\pi = c_{\frac{2\pi}{3}} = 0$

Figure 2: *The beampatterns at 1kHz of directional microphones, LDMA beamformers and proposed LDDMA beamformers*

In this section, we will study the properties of the proposed

LDDMA beamformers through the numerical simulation using measures of beampattern, WNG and DI defined in section 3. They will be compared to conventional LDMA. In the remaining part of the paper, we assume the desired 'look' direction α is 0 degree, i.e., $c_0 = 1$. And the microphone array is constructed with six elements and uniform inter-element spacing of one centimeter. In the simulation, signal models of directional elements are cardioid microphone and dipole microphone, which are two most commercially used first-order directional microphone product.

The beampatterns at 1kHz of cardioid microphone, dipole microphone, LDMA beamformers and proposed LDDMA beamformers are shown in Fig.2. First, through Fig.2.(a,c,e), we can see if the null position $\theta_{null} = \pi$ of directional microphone is one of the null constraints $c_\pi = c_{\frac{2\pi}{3}} = 0$, the beampattern of LDDMA would be identical to the conventional LDMA under same null constraints design. On the other hand, through Fig.2.(b,d,f), when the null position $\theta_{null} = \frac{\pi}{2} = \frac{3\pi}{2}$ of directional microphone is different from all of the null constraints $c_\pi = c_{\frac{2\pi}{3}} = 0$, the LDDMA beamformer weights will be identical to the LDMA beamformer weights (proved in Section 4), therefore when we apply the LDMA beamformer weights on the array with directional elements, the beampattern of LDDMA is obtained by the pattern multiplication of directional microphone and conventional LDMA.
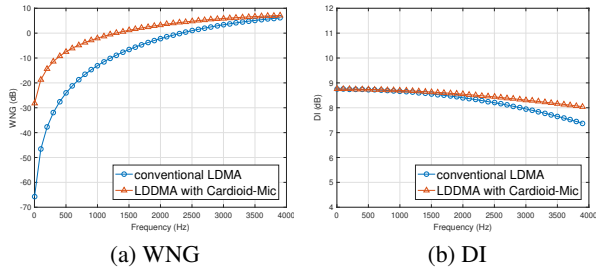

(a) WNG

(b) DI

Figure 3: *WNG and DI comparison for same null constraints of beamforming design ($c_\pi = c_{\frac{\pi}{2}} = 0$): conventional LDMA and LDDMA with Cardioid-Mic*
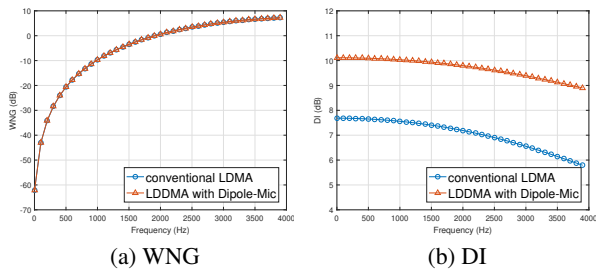

(a) WNG

(b) DI

Figure 4: *WNG and DI comparison for same null constraints of beamforming design ($c_\pi = c_{\frac{2\pi}{3}} = 0$): conventional LDMA and LDDMA with Dipole-Mic*

In Fig.3, we compare proposed LDDMA using cardioid microphones to the conventional LDMA in terms of WNG and DI for the same designs of null constraints $c_\pi = c_{\frac{\pi}{2}} = 0$. This set of WNG and DI corresponds to the beampattern of Fig.2.(a,c,e), where the null position of microphone coincides with one of the

null constraints of beamforming design. From Fig.3a, we can see the LDDMA has same beampattern as LDMA at 1 kHz, but the WNG of differential beamformer can be boosted significantly using the array of directional microphones, about 10-20 dB improvement. Fig.3b indicates that LDDMA has more flat DI than LDMA, which means the LDDMA can achieve more frequencies-invariant beampattern.

In Fig.4, we compare proposed LDDMA using dipole microphones to the conventional LDMA in terms of WNG and DI for the same designs of null constraints $c_\pi = c_{\frac{2\pi}{3}} = 0$. This set of WNG and DI corresponds to the beampattern of Fig.2.(b,d,f), where the null-postion is not related to the null constraints of beamforming design. From Fig.4a, we can see the LDDMA have totally same WNG as the LDMA, since the beamformers have same weights. From Fig.4b, we can see the LDDMA achieves significantly higher DI than LDDMA over the frequencies. Because the beampattern of LDDMA is obtained through the pattern multiplication of LDMA beamformer and dipole microphone, LDDMA will have 'narrower' beam such that higher directivity.
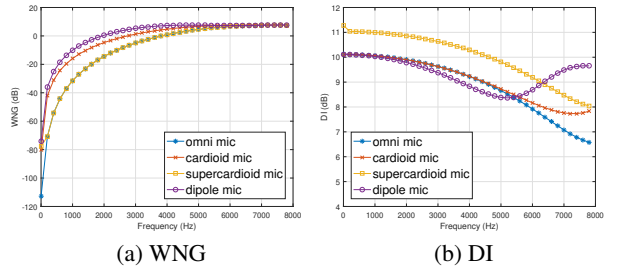

(a) WNG

(b) DI

Figure 5: *WNG and DI comparison for different types of microphones of proposed LDDMA*

Fig.5 compares the proposed LDDMA using different types of first-order microphones in terms of WNG and DI under same design of $c_\pi = c_{\frac{\pi}{2}} = c_{\frac{2\pi}{3}} = 0$. Specifically, we use omni, cardioid, super-cardoid and dipole by setting p to 1, 0.5, 0.37 and 0, respectively[14]. We can see that the use of super-cardioid microphone leads to the highest DI at the low and middle frequecies, whereas the dipole brings a slight boost to WNG over the cardioid microphone. Generally speaking, they all exhibit some advantage over the omni-directional.

6. Conclusions

In this paper, we incorporated an additional degree of freedom, i.e., acoustic characteristic of the array elements into the design of minimum-norm differential beamforming. Intuitively, the directional microphone could offer better directivity to boost the DI of entire microphone array. Through the derivation and simulation, we investigate the relation between the null position of directional elements and null constraints of beamformer. Given the null position of the directional elements coincides with one of the null-constraints of minimum norm differential beamforming, the proposed solution holds the similar DI but improve the WNG at low frequencies significantly. Given the null position of the directional elements is not related to the null-constraints of minimum norm differential beamforming, the acoustics of microphones have no impact on the proposed beamformer weights that is derived to be identical to the convention one.

7. References

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