



Prosody of Case Markers in Urdu

Benazir Mumtaz, Massimiliano Canzi, Miriam Butt

University of Konstanz, Germany

firstname.lastname@uni-konstanz.de

Abstract

This paper studies the prosody of case clitics in Urdu, for which various different claims exist in the literature. We conducted a production experiment and controlled for effects potentially arising from the phonetics of the case clitics, the syntactic function they express and clausal position. We find that case clitics are incorporated into the prosodic phrase of the noun and that they become part of the overall LH contour found on accentual phrases in Urdu/Hindi. We also find some differences across case type and position which we tie to information structural effects.

Index Terms: prosody, pitch, duration, intonation, LH contour, case, word order, Urdu/Hindi, information structure, clitics.

1. Introduction

As part of a project to develop more natural sounding speech for Text-to-Speech (TTS) for Urdu¹ we are working on improving our basic linguistic understanding of the speech prosody of Urdu/Hindi. In this paper we focus on Urdu case markers in relation to determining boundaries of prosodic phrasing.

The basic phrasal accent in Urdu/Hindi has been established to be some form of LH, where the H functions as a boundary tone for the accentual phrase [1, 2, 3, 4, 5]. We follow [4] and [5] in positing L*H as the basic phrasal accent. In terms of prosodic phrasing, there seem to be at least two basic levels in Urdu/Hindi: i) accentual phrase (AP) and ii) intonational phrase [2, 4], but the size of the AP remains unclear, especially with respect to case markers, which have the morphophonological status of clitics [6]. Generally it seems that each content word receives an AP and each AP is characterized by an L*H with the H indicating the right boundary. However, case markers have been claimed to have their own LH intonational contour [7] while still being phrased together with the AP of the noun they are attached to. If this is true, the question of how to reliably identify the right boundary of AP arises.

Indeed, the data reported in the literature seems to show variation in the realization of pitch contours on the case markers [8, 9]. For example, take the ergative [ne] and the dative [ko] in the sentence ‘Rahul gave medicine to (his) mother.’ in Figure 1 from [10]. The pitch contour seems to peak with an H at the end of *Rahul* before the case marker [ne], while in the second NP, the H aligns with the case marker [ko] following *ma*.

In our own work on developing an annotated speech corpus for Urdu TTS [11], we furthermore observed that the genitive NP in phrases such as ‘Rahul’s mother’ seem to carry an L*HL contour rather than the usual L*H. This raises the question whether genitives are not only syntactically different, but also prosodically different from other case marked NPs.

On the other hand, the observed variation might also be due to artefacts caused by acoustic factors such as vowel height and degree of sonority (e.g., ergative [ne] vs. genitive [ki]).

¹<https://tech.cle.org.pk/services/speech/tts>

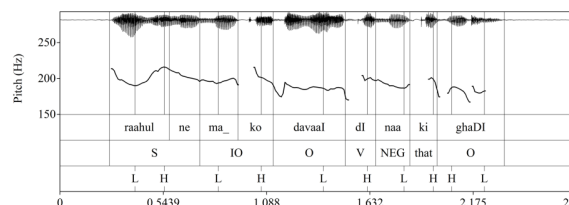


Figure 1: Variation in H boundary alignment on NPs from [10]

We conducted a carefully controlled production experiment which varied the factors of case type, function and position and found that the AP high boundary tone aligns with the case markers, regardless of case type and NP position. This stands in contrast to the findings/observations of previous studies and indicates that the case clitics form part of the AP with the preceding noun. We did find an increased pitch excursion associated with more emphasized NPs and we also observed that the duration of the case markers varied with position in the clause. We present and discuss these findings in more detail below.

2. Urdu Intonation, Case and Word Order

2.1. Urdu/Hindi Intonation and Word Order

Urdu/Hindi² intonation is characterized by a series of repeated rising contours which seem to be characteristic of South Asian languages [2, 3]. The minimal consensus on Urdu/Hindi is that each content word except the clause final one (this receives a fall) carries a rising contour (LH) [1, 2, 4, 12]. However, the precise distribution of this LH remains the subject of debate. [1, 3] and [4] associate an L tone with a stressed syllable followed by phrasal tone (Ha), whereas [2] and [13] interpret the f0 rise as a pair of phrase tones (Lp and Hp), finding no systematic alignment of the low (Lp) with a particular prominent (or stressed) syllable, in part because lexical stress is claimed to be difficult to identify in Urdu/Hindi. [5] investigates lexical stress in Urdu and shows that difficulties only arise in cases of stress clash. We therefore adopt the L*H analysis.

Default word order in Urdu/Hindi is SOV, but all major constituents may scramble. Word order variation is tied to information structure [14, 15, 16, 17]. The default position for focus is immediately preverbal and this position typically exhibits a steep pitch excursion or post-focal compression [10]. The default position for topics is clause initial and we have observed that this position generally also shows a larger pitch excursion.

[2, 12] and [18] have investigated the interplay of prosody with focus and word order. They found that the basic prosodic and tonal structure (i.e. LH) remained unchanged in all focus structures (subject focus, object focus, wide focus, corrective focus, selective focus) in SOV vs. OSV order in Urdu/Hindi.

²Urdu is the national language of Pakistan, Hindi is an official language of India. They are structurally almost identical.

To date, prosodic studies on Urdu/Hindi have mainly targeted an understanding of focus. Detailed examinations of the clause-initial position and how different case markers play out in different positions have not been carried out as far as we know. For instance, subjects are default topics, but if an ergative is found on an NP in a non-initial position, then this is a marked case of a non-topic subject. Similarly, objects appear to be in focus by default [17], so when an accusative object appears clause-initially, this is a marked situation.

2.2. Case in Urdu/Hindi

Urdu/Hindi case markers have been innovated from about 1100 CE on and operate on top of a morphological direct/oblique distinction [6]. Urdu/Hindi has five core cases along with some locatives. The nominative is phonologically null but the other cases are overt: ergative [ne], dative/accusative [ko], instrumental/comitative [se], genitive [ka/ki/ke]. The genitive is the only case which inflects (for gender, number and obliqueness).

Case markers in Urdu/Hindi always appear postnominally and are therefore often confused with affixes. However, evidence from several tests (i.e. coordination, stress and ordering with respect to other clitics) shows that case markers are attached to phrases and must be analyzed as clitics [6]. [6] further argue that case markers represent independent terminal nodes as a functional head of KP (KaseP): KP → NP K. We adopt this.

Given that Ks in Urdu/Hindi are clitics and prosodically dependent on nouns, one would assume that all KPs form one AP and are aligned with one L*H contour [19]. However, this has been disputed [7], with Ks asserted to have their own LH contour. Furthermore, since genitives form a syntactic constituent with their head noun (i.e. *John's car*), one might expect the two NPs in a genitive to share one overall LH contour. However, [4] observed that in constituents containing a genitive, both nouns carry an L*Ha tone — indicating that there are two APs. Example (1) from [20] illustrates that unlike English, genitive KPs in Urdu/Hindi can occur outside of the constituent they modify. They can be scrambled freely within a clause along with the other major constituents.

- (1) **gaṛi** naḍja=ne us=ki bazar=me dekʰi
car.F.Sg.Nom Nadya=Erg his=Gen.F.Sg market=Loc saw.F.Sg
'His car, Nadya saw in the market'

The separation of the modifying KP from its head further confirms that both phrases within a genitive construction can exist as two separate prosodic entities.

3. Methods

3.1. Materials

We designed a production experiment that focused on Ks which showed phonetic and syntactic contrasts that we could exploit. The ergative [ne] and comitative [se] share the same vowel, but represent syntactically very different functions: ergatives only appear on subjects, the comitative on obliques. The dative/accusative [ko] and the genitives [ka, ki, ke] share the consonant, but differ in vowel height and reflect different syntactic functions (objects vs. possessors). We also included the phonologically null nominative forms. Each case marker was combined with mono-, di- and trisyllabic common Urdu names. Five names per syllable size were generated, see Table 1. All of the names have a CV syllable structure and end in the same type of vowel /a/ across all conditions.

Table 1: Target names for mono-, di- and trisyllabic words

Condition: size	Names
Mono syl. words	bīa, rīa, d̪īa, ʃa, zīa ³
Disyl. words	rīma, mīna, sīma, sara, mona
Trisyl. words	gəzala, soṃera, həlīma, memuna, məḍiḥa

All target names (NPs and KPs) are contained in declarative sentences with the structure shown in (2). Based on previous experience, we placed an adverb in the immediately preverbal position in order to avoid confounding issues with focus that arise when arguments appear immediately preverbally [5, 17]. The position of the adverb remained constant, as did the clause-final position of the simple verb.

- (2) KP/NP KP/NP AdvP V

A sample list of declarative sentences for each case using the disyllabic target name [rīma] is given in Table 2.

Table 2: Sentences with case and clitic information using the disyllabic target item [rīma]

Case	Clitic	Sentence
Erg	ne	[rīma=ne] meri nəsiḥət əksər māni. Rīma often agreed to my advice.
Acc	ko	[rīma=ko] meri səḥli əndər lai. My friend brought Rīma inside.
Com ⁴	se	[rīma=se] meri səḥli pərsō mīli. My friend met Rīma yesterday.
Gen	ka/ki/ke	[rīma=ke māmū] pile xərbuze bəḥər ləe Rīma's uncle brought yellow melons outside.
Nom	∅	[rīma] meri əlməri pərsō lai. Rīma brought my cupboard yesterday.

We test each case in clause-initial vs. clause medial position, with the genitive functioning slightly differently as it is syntactically contained within a larger KP/NP. However, recall that the genitive KP can scramble independently and forms its own prosodic phrase [4, 20], so the position of the genitive actually does not add any variability to the design.

3.2. Participants

Seven speakers (F = 6; Age \bar{X} = 21, SD = 2) born and raised in Lahore, Pakistan participated in this study for a small payment. All participants were multilingual and spoke the same regional language (Punjabi) besides Urdu. We are aware that participants' regional language may influence Urdu but it is difficult to find monolingual speakers in Pakistan. Most of the literate population speaks Urdu and at least one regional language in their daily routine. According to the last census of Pakistan, 46.17% of the population uses Punjabi in the household [22]. This study used Urdu speakers who only have Punjabi as their regional language. Participants were unaware of the purpose of the study. None reported hearing or speaking disorders.

³Lexical stress in Urdu/Hindi is sensitive to syllabic weight and the word-final mora is analyzed as extrametrical [5, 21]. Therefore, participants might treat monosyllabic words as unstressed. To avoid this, we selected diphthongs for the monosyllabic names.

⁴The [se] is highly polysemous. It may mark instrumentals, sources or comitatives. In this study, only the comitative [se] is employed.

3.3. Procedure

Voice samples were recorded in a sound-attenuated booth at a sampling rate of 48KHz using Praat [23]. All the recordings were conducted at the University of Engineering and Technology, Lahore, Pakistan. The stimuli (174 sentences, 30 fillers) were presented to participants as a slide presentation written in Urdu script. Participants were instructed to speak out the sentence displayed on the screen as naturally as possible. In case of disfluency or mispronunciation, they were asked to produce the sentence again. The item presentation order was pseudo-randomized with the constraint that two sentences belonging to the same type of case would be separated by one filler (mathematical equations, counting, or dates) and one other type of case. The average duration of the experiment was 80 minutes.

3.4. Data treatment

A total of 1218 sentences (5 case types x 15 items x 2 position conditions x 7 speakers)⁵ were recorded. Fourteen sentences were rejected due to mispronunciation and glottalization. The remaining sentences (1204) were manually annotated at word, tone and phrase levels using standard annotation criteria [24] and Praat [23]. The tone label was marked in the middle of each vowel in all of the phrases using the scheme established by [4]. We found a total of 1192 APs. As shown in Table 3, L*H constituted the overwhelming majority and so we only subjected this pattern to further statistical analysis.

Table 3: Tonal structure within KP in overall data

	L*H	L*	L*HL	H*L
Count	1146	26	18	2
Percentage	96.1%	2.2%	1.5%	0.2%

We calculated a series of linear-mixed effects regression models (LMM) for multiple outcome variables: phrase duration, f0 excursion, f0-minimum (L*), and the f0-maximum (H). The case type (7 distinctions: Erg, Com, Acc, Nom, Gen-ka, Gen-ke, Gen-ki), NP/KP position (2 possibilities: clause-initial and clause-medial), and noun size (3 types: mono-, di- and trisyllabic) were selected as fixed effects (using the packages lme4 and lmerTest in R, [25], [26]). Participants and items were added as crossed random effects. Since the nominative is phonologically null it was excluded from the duration model. For the f0 excursion, L*, and H models we converted f0 into semitones. We report f0 results for two categories: a) nominative (phonologically empty) vs. non-nominative; b) Ks that are phonetically similar but syntactically different: Acc [ko] vs. Gen [ka/ki/ke] and Com [se] vs. Erg [ne].

4. Results

4.1. Phrase duration

The LMM model with duration as the outcome variable showed a significant interaction between case type and case position [$F(5,969.67) = 2.60, p < .05$]. To investigate the nature of the interaction, post-hoc pair-wise comparisons were conducted using the emmeans function in R with Tukey correction [27]. Results showed no significant difference between Acc [ko] and Gen [ka/ki/ke] in initial position [$p > 0.3$]. However, we found

⁵Note: The genitive has three versions, so for each of the [ka/ke/ki] we only used three names from each size condition.

Acc to be significantly longer than Gen-ka [$\beta = .042, p < .0001$], Gen-ki [$\beta = .024, p = .01$] and Gen-ke phrases [$\beta = .025, p = .001$] in medial position. Moreover, Com [se] was significantly longer than Erg [ne] [$\beta = -0.02, p < .005$] in initial position. But no significant difference in duration was found between Com and Erg in medial position [$p = 0.9$]. We also obtained a significant main effect of size [$F(2,43.45) = 11.87, p < .001$]. The trisyllabic KPs had a significantly longer duration than the di- [$p < .001$] and monosyllabic KPs [$p < .0001$]. No significant difference in duration was found between mono- and disyllabic KPs [$p = 0.15$]. Recall that the monosyllables contain diphthongs.

4.2. f0 excursion

The results of the f0 excursion model also showed a significant interaction between case type and position [$F(1,1059.42) = 5.75, p < .0001$]. We found no main effect of size on f0 range [$p = 0.1$], confirming the phrasal nature of the L*H contour. Post-hoc pair-wise analyses were carried out on the significant interaction between case type and position. In both initial and medial position, the f0 range between L and H was significantly narrower for the nominatives. In initial position, significant differences in pitch excursion size were noticed between Nom & Acc [$\beta = -0.70, p = .01$], Nom & Com [$\beta = -0.69, p = .01$], and Nom & Gen-ke [$\beta = -0.73, p = .02$]. In medial position, significant differences in pitch excursion size were observed between Nom & Com [$\beta = -1.19, p < .0001$], and Nom & all Gen phrases [$p < .0001$ for all comparison], see Figure 2.

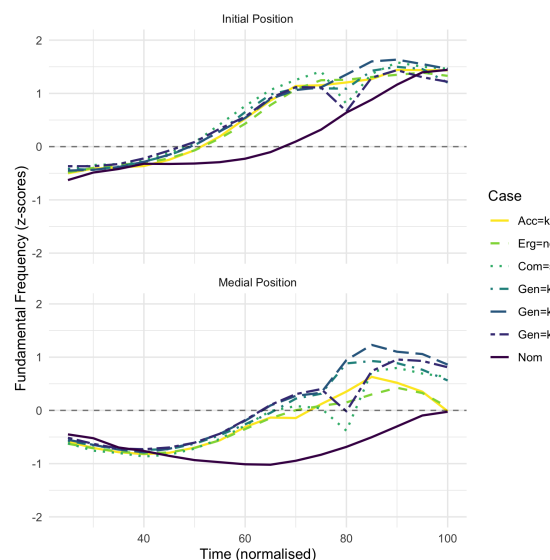


Figure 2: F0 contour: nominative vs. non-nominative phrases at sentence initial & medial position.

No significant differences in f0 range were found across the KPs in clause-initial position [$p > 0.3$ for all comparisons]. However, in the medial position the Com showed significantly wider f0 range than Erg [$\beta = -0.81, p = 0.0001$] (Figure 3: left column) and all types of Gen showed a wider f0 range than Acc [$p < 0.05$ for all comparisons] (Figure 3: right column).

4.3. Fundamental frequency: L* and H

To investigate whether the difference in f0 excursion is due to the low tone (L*) or high boundary tone (H), we calculated

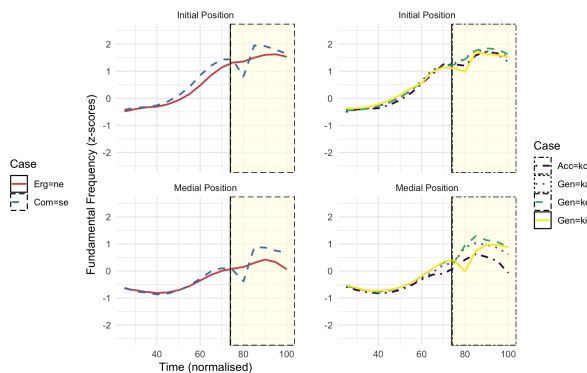


Figure 3: *F0 contours: Erg-[ne] vs. Com-[se] (left columns) and Acc-[ko] vs. Gen-[ka/ki/ke] (right columns) at clause-initial & medial position. The black vertical solid line indicates the median of the point where clitics start and nouns end.*

the f_0 values for L^* and H . These were labelled manually in the middle of the vowel. Results of the L^* model suggest a main effect of case type [$F_{(6,964.62)} = 3.212, p < .01$], case position [$F_{(1,960.32)} = 528.97, p < .0001$], as well as significant interaction between case position and size [$F_{(1,961.56)} = 3.64, p < .0001$]. The output of pair-wise comparisons for case type showed that L^* was significantly lower with Nom as compared to Erg [$\beta = 0.32, p < 0.01$]. However, no significant differences for L^* across non-nominatives were observed (Erg vs. Com [$p = 0.9$], and Acc vs. Gen [$p > 0.4$ for all comparisons]).

The H model results were similar to the f_0 excursion results. We thus do not report them here. We did find that in the H model all Gen phrases have a significantly higher f_0 than the Acc [$p < 0.0001$]. This is especially noteworthy with respect to the Gen-[ka], where the vowel has an intrinsic lower f_0 than the Acc-[ko], but where the f_0 is significantly higher f_0 than that of Acc-[ko] [$\beta = -0.87, p < 0.001$].

5. Discussion

The H tone appears on the clitic in 96% of our data. This indicates very strongly that clitics are incorporated into the noun's AP, as would be expected [19]. We find no evidence for a separate LH contour on the case clitics in either of the two positions, across types of clitics and size of KPs. We also find no difference in overall contours between genitives and other KPs, indicating that genitives work prosodically just like other KPs, despite their syntactic status as modifiers.

We do find differences in either duration or f_0 depending on case type and position. For example, we find significant differences between Gen & Acc. Recall that we compared these specifically because we could control for possible effects due to vowel height. The phonetics of the case clitics are shown to make no difference, case type and position are the relevant factors. We find a difference in the pitch excursion size in the clause-medial position, with a higher pitch excursion on the LH for Gen as compared to Acc. This might be due to a continuation rise on the Gen, anticipating its head noun. This analysis fits in well with the attendant shorter duration of Gen vs. Acc in medial position, which may be due to the syntactic status of the Gen as an embedded phrase.

The absence of such a durational difference in clause-initial position could be due to the connection of this position with

topicalization. We generally found an interaction between type of case and word order. All non-nominative phrases in the topic position have a steeper rise as compared to phrases in the medial position. The latter are all realized with what one might call a scooped rise (Figure 3). The KPs in the clause-initial topic position have a steeper rise and an increased pitch span, but we found no differences across case types (Acc vs. Gen, Erg vs. Com). This behavior of topic phrases is in line with what has been previously established for focus [2, 9, 12].

We do find effects that can be explained with respect to non-default positioning. For example, the non-default Com in initial position has longer duration than the default Erg. In medial position, the non-default phrases instead appear with a quicker fall, as can be seen with the Erg (default position is initial) and the Acc (default position is immediately preverbal) in Figure 3.

Finally, we would like to point out an interesting finding with respect to case clitics vs. the phonologically null nominative. As can be seen in Figure 2, on clitics the H tone begins going down at the very end of the clitics, presumably in anticipation of the next LH. However, we do not find this pattern with nominatives. In our data the H tone on nominatives was mostly fully realized on the initial syllable of the next word. This needs to be investigated further, but could indicate that the case clitics represent a clear indicator for the prosodic boundaries, and that rhythmic principles might instead apply in the absence of case. It would be interesting to explore whether pluralized nominatives behave in line with KPs or with singular nominatives. Plural morphology comes at the very end of a noun and so in principle also constitutes reliable morphosyntactic information for the determination of prosodic boundaries.

6. Conclusions

We presented a production experiment to investigate the prosodic properties of case markers in Urdu/Hindi. We investigated potential effects of the phonetic material of the case markers in interaction with length of the noun and NP/KP position and type. Our data shows that the H tone of the basic L^*H contour found on APs always aligns with the case clitics. This indicates that the clitics have indeed been integrated into the prosodic phrase of the noun. We were careful to rule out confounding factors possibly engendered by focus, but did find differences between clause-initial and clause-medial position that we ascribe to the initial position being associated with topics. We also found differences in duration and pitch excursion across case type and position that we analyze as marking KPs in non-default positions. Finally, we found differences between the basic L^*H intonational contour on nominatives (scooped contour) vs. non-nominatives (steep rise).

7. Acknowledgements

We thank the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) for funding within project BU 1806/9-2 “Information Structure and Questions in Urdu/Hindi” of the FOR 2111 *Questions at the Interfaces* and the DAAD (German Academic Exchange Office) for funding as part of a German-Pakistan Cooperation scheme. We thank Saira Bano for assistance with the manual annotation and Tina Bögel and Ashwini Deo for comments and valuable discussions.

8. References

- [1] J. D. Harnsberger, "Towards an intonational phonology of Hindi," Master's thesis, University of Florida, 1994.
- [2] C. Féry, "The intonation of Indian languages: An areal phenomenon," in *Problematizing Language Studies: Festschrift for Ramakant Agnihotri*, I. Hasnain and S. Chaudhury, Eds. Akar Publishers, 2010, pp. 288–312.
- [3] S. ud Dowla Khan, "The intonation of South Asian languages: towards a comparative analysis," *Formal Approaches to South Asian Languages*, vol. 6, pp. 1–23, 2016.
- [4] S. Urooj, B. Mumtaz, and S. Hussain, "Urdu intonation," *Journal of South Asian Linguistics*, vol. 10, pp. 3–22, 2019.
- [5] B. Mumtaz, T. Bögel, and M. Butt, "Lexical Stress in Urdu," *Proceedings of Interspeech 2020*, pp. 1888–1892, 2020.
- [6] M. Butt and T. H. King, "The status of case," in *Clause Structure in South Asian Languages*, V. Dayal and A. Mahajan, Eds. Berlin: Springer Verlag, 2005.
- [7] F. Jabeen, "Prosody and Word Order: Prominence Marking in Declaratives and Wh-questions in Urdu/Hindi," Ph.D. dissertation, Universität Konstanz, Konstanz, 2019.
- [8] A. Sengar and R. Mannell, "A Preliminary study of Hindi intonation," in *Proceedings of the 14th Australasian International Conference on Speech Science and Technology*, 2012, pp. 149–152.
- [9] F. Jabeen, T. Bögel, and M. Butt, "Variable prosodic realization of verb focus in Urdu," in *Speech Prosody 2016*, 2016, pp. 731–735.
- [10] F. Kügler, "Post-focal compression as a prosodic cue for focus perception in Hindi," *Journal of South Asian Linguistics*, vol. 10, pp. 38–59, 2020.
- [11] B. Mumtaz, S. Urooj, S. Hussain, and W. Habib, "Stress annotated Urdu speech corpus to build female voice for TTS," in *2015 International Conference Oriental COCODA held jointly with 2015 Conference on Asian Spoken Language Research and Evaluation (O-COCODA/CASLRE)*. IEEE, 2015, pp. 13–20.
- [12] U. Patil, G. Kentner, A. Gollrad, F. Kügler, C. Féry, and S. Vasisht, "Focus, word order and intonation in Hindi," *Journal of South Asian Linguistics*, vol. 1, no. 1, 2008.
- [13] C. Féry, P. Pandey, and G. Kentner, "The prosody of focus and givenness in Hindi and Indian English," *Studies in Language. International Journal sponsored by the Foundation "Foundations of Language"*, vol. 40, no. 2, pp. 302–339, 2016.
- [14] V. Gambhir, "Syntactic Restrictions and Discourse Functions of Word Order in Standard Hindi," Ph.D. dissertation, University of Pennsylvania, Philadelphia, 1981.
- [15] M. Butt and T. H. King, "Structural topic and focus without movement," *Online Proceedings of LFG*, 1996.
- [16] A. Kidwai, *XP-adjunction in Universal Grammar: Scrambling and binding in Hindi-Urdu*. Oxford: Oxford University Press, 2000.
- [17] M. Butt, F. Jabeen, and T. Bögel, "Verb Cluster Internal Wh-Phrases in Urdu: Prosody, Syntax and Semantics/Pragmatics," *Linguistic Analysis*, vol. 40, no. 3–4, 2016.
- [18] A. Choudhury and E. Kaiser, "Comparing languages and comparing focus types: Insights from an interactive task," in *Talk presented at Prosody and Information Structure workshop, University of Stuttgart*, 2016.
- [19] E. O. Selkirk, "The prosodic structure of function words," in *Papers in Optimality Theory*, J. N. Beckmann, L. W. Dickey, and S. Urbanczyk, Eds. University of Massachusetts: Department of Linguistics, 1995.
- [20] T. Bögel and M. Butt, "Possessive clitics and ezafe in Urdu," *Morphosyntactic categories and the expression of possession*, vol. 199, no. 291, pp. 86–129, 2013.
- [21] S. Hussain, "Phonetic correlates of stress in Urdu," Ph.D. dissertation, Northwestern University, 1997.
- [22] S. S. Hashmi and M. Sultan, "Population trends and rates of population growth in Pakistan: assessment of preliminary results of the 1998 census," *The Pakistan Development Review*, pp. 495–506, 1998.
- [23] P. Boersma and D. Weenink, "Praat: doing phonetics by computer [computer program, Version 5.3.56]," 2013, available at <http://www.praat.org/> [retrieved 15.09.2013].
- [24] A. Turk, S. Nakai, and M. Sugahara, "Acoustic segment durations in prosodic research: a practical guide," in *Methods in Empirical Prosody Research*, S. Sudhoff, D. Lenertová, R. Meyer, S. Papert, P. Augurzky, I. Mleinek, N. Richter, and J. Schliesser, Eds. Berlin, New York: De Gruyter, 2006, pp. 1–28.
- [25] D. Bates, M. Mächler, B. Bolker, and S. Walker, "Fitting linear mixed-effects models using lme4," *arXiv preprint arXiv:1406.5823*, 2014.
- [26] A. Kunzetsova, P. Brockhoff, and R. Christensen, "lmerTest package: tests in linear mixed effect models," *J Stat Softw*, vol. 82, pp. 1–26, 2017.
- [27] R. Lenth, H. Singmann, J. Love, P. Buerkner, and M. Herve, "Emmeans: Estimated marginal means, aka least-squares means," *R package version*, vol. 1, no. 1, p. 3, 2018.