

# Notes of the Introduction To Algorithms

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**Part I**

**Foundations**



## **Chapter 1**

# **The Role of Algorithms in Computing**

## 1.1 Algorithms

### Exercises

1.1-1 Give a real-world example that requires sorting or a real-world example that requires computing a convex hull.

**Answer:** One example that requires sorting is that teachers will sort our scores after the exam.

1.1-2 Other than speed, what other measures of efficiency might one use in a real-world setting ?

**Answer:** cost, space, manpower, material resources. In different cases, each can be the key of measures of efficiency.

**Reference:** <https://www.quora.com/Other-than-speed-what-other-measures-of-efficiency-might-one-use-in-a-real-world-setting>

1.1-3 Select a data structure that you have seen previously, and discuss its strengths and limitations.

**Answer:** Array  
strengths: access directly  
limitations: costs lot when insert or delete

1.1-4 How are the [shortest-path](#) and [traveling-salesman](#) problems given [similar](#)? How they are [different](#)?

**Answer:**

1.1-5 Come up with a real-world problem in which only the best solution will do. Then come up with one in which a solution that is "approximately" the best is good enough.

**Answer:**

## 1.2 Algorithms as a technology



## **Chapter 2**

# **Getting Started**



## **Chapter 3**

# **Growth of Functions**



## **Chapter 4**

# **Divide-and-conquer**



## **Chapter 5**

# **Probabilistic Analysis and Randomized Algorithms**





## **Part II**

# **Sorting and Order Statistics**



## **Chapter 6**

# **Heapsort**



**Part III**

**Data Structures**



## **Part IV**

# **Advanced Design and Analysis Techniques**





## **Part V**

# **Advanced Data Structures**



**Part VI**

**Graph Algorithms**



## **Chapter 7**

# **Minimum Spanning Tree**

## 7.1 Notes

- (i) There maybe more than one MST in a forest.
- (ii) The number of all the edges in the MST is equal to  $V - 1$ .

## 7.2 Growing a minimum spanning tree

### 7.2.1 Definition

**A**

A is a subset of some minimum spanning tree.

#### Safe edge

Safe edge is a edge that add to A and A is also a subset of some minimum spanning tree.

### 7.2.2 Generic-MST

**GENERIC-MST**( $G, \omega$ )

```

1  $A = \emptyset$ 
2 while A does not form a spanning tree
3   find an edge( $\mu, v$ ) that is safe edge for A
4    $A = A \cup (\mu, v)$ 
5 return A
```

**Initialization:** After line 1, the set A trivially satisfies the loop invariant.

**Maintenance:** The loop in lines 2-4 maintains the invariant by adding only safe edges.

**Termination:** All edges added to A are in a minimum spanning tree, and so the set A returned in line 5 must be a minimum spanning tree.

### 7.2.3 Theorem

Let  $G = (V, E)$  be a connected, undirected graph with a real-valued weight function  $\omega$  defined on  $E$ . Let  $A$  be a subset of  $E$  that is included in some minimum spanning tree for  $G$ , let  $(S, V - S)$  be any cut of  $G$  that respects  $A$ , and let  $(u, v)$  be a light edge crossing  $(S, V - S)$ . Then, edge  $(u, v)$  is **safe** for  $A$ .

**Part VII**

**Selected Topics**





## **Part VIII**

# **Appendix: Mathematical Background**

