1 Matrix Derivatives

1.1 $\mathbf{tr}AB = \mathbf{tr}BA$

Let A be a m-by-n matrix, let B be a n-by-m matrix.

$$trAB = \sum_{i=1}^{m}(AB)_{ii} = \sum_{i=1}^{m}(\sum_{j=1}^{n} A_{ij}B_{ji}) \tag{1}$$

$$trBA = \sum_{i=1}^{n}(BA)_{ii} = \sum_{i=1}^{n}(\sum_{j=1}^{m} B_{ij}A_{ji}) \tag{2}$$

$$\begin{aligned} trBA &= \sum_{i=1}^{n}(\sum_{j=1}^{m} B_{ij}A_{ji}) \\ &= \sum_{j=1}^{m}(\sum_{i=1}^{n} B_{ij}A_{ji}) \\ &= \sum_{j=1}^{m}(\sum_{i=1}^{n} A_{ji}B_{ij}) \\ &= \sum_{i=1}^{m}(\sum_{j=1}^{n} A_{ij}B_{ji}) \\ &= trAB \end{aligned} \tag{3}$$