

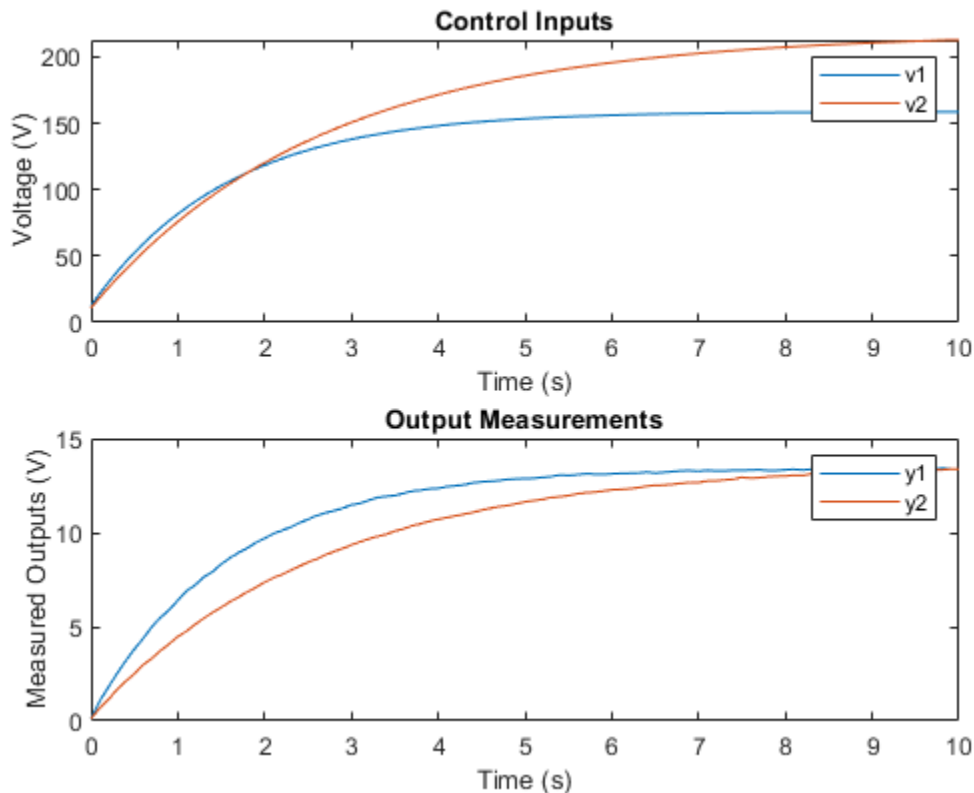
# REPORT

**A) Implement using actual calculations, the unconstrained MPC with prediction horizon and control horizon chosen with some heuristics for the following case:**

1. Controlled Variables are  $h_1$ ,  $h_2$  when all states are measured for a set-point for  $[h_1 \ h_2]$  of  $[13.4 \ 13.7]$

**Solution:** The matlab code for implementing the unconstrained MPC for controlling  $h_1$  and  $h_2$  using the state-space model is submitted as a separate file. The simulation is working, and the control inputs guide the system to the desired set points for  $h_1=13.4$  cm and  $h_2=13.7$  cm.

The plot is given below



2. Give justification for the heuristics

**Solution:** 1. Prediction Horizon( $N_p$ )

- Chosen value in code:  $N_p=5$
- Heuristic Explanation:

- The prediction horizon should be long enough to capture the significant dynamics of the system. For this system:
  - The settling time is observed to be approximately 10 seconds based on the dynamics of  $h_1$  and  $h_2$
  - Given a sampling time  $T_s=0.1s$ ,  $N_p=5$  corresponds to a look-ahead period of 0.5s, which provides the controller with sufficient foresight to optimize the control trajectory effectively.
- A longer  $N_p$  might add unnecessary computational burden without significantly improving performance.

## 2. Control horizon ( $N_c$ )

- Chosen Value in Code:  $N_c=3$
- Heuristic Explanation:
  - The control horizon determines how many future control actions are optimized
    - A smaller  $N_c$  reduces computational complexity and ensures smoother control inputs.
    - $N_c=3$  is a good balance because it allows the controllers to apply aggressive corrections during the initial steps while relying on the system's natural dynamics to stabilize.
    - Choosing  $N_c=3$  also reflects that the control effort doesn't need to change significantly after the initial adjustments.

## 3. Weighing Matrix for control effort ( $R_{mpc}$ )

- Chosen Value in Code:  $R_{mpc}=0.5.I$
- Heuristic Explanation:
  - This value penalise excessive changes in the control inputs
  - A lower value (eg.  $R_{mpc}<0.1$ ) would make the control inputs more aggressive but could lead to instability
  - A higher value (eg:  $R_{mpc}>1$ ) would over-penalise the control inputs, making the system slower to respond to set-point changes.

## 3. Analyze closed loop stability of the unconstrained MPC system at two places:

### Poles at the First Move:

[1.0000,1.0000,0.9405,0.9642,-0.0419,-0.0159,-0.0333,-0.0111]

### Poles at Stabilization:

[1.0000,1.0000,0.9405,0.9642,-0.0419,-0.0159,-0.0333,-0.0111]

### Observations:

- **Stable Poles:** All poles except the repeated 1.0 are within the unit circle, indicating stability.
- **Steady Dynamics:** The poles remain unchanged from the first move to stabilization. This implies the controller effectively neutralizes the error on, leading to consistent dynamics
- **Dominant Poles:** The real parts of the poles near 0.94 and 0.96 dominate the system's response, reflecting slightly slower settling times but stable behaviour.

### Comments:

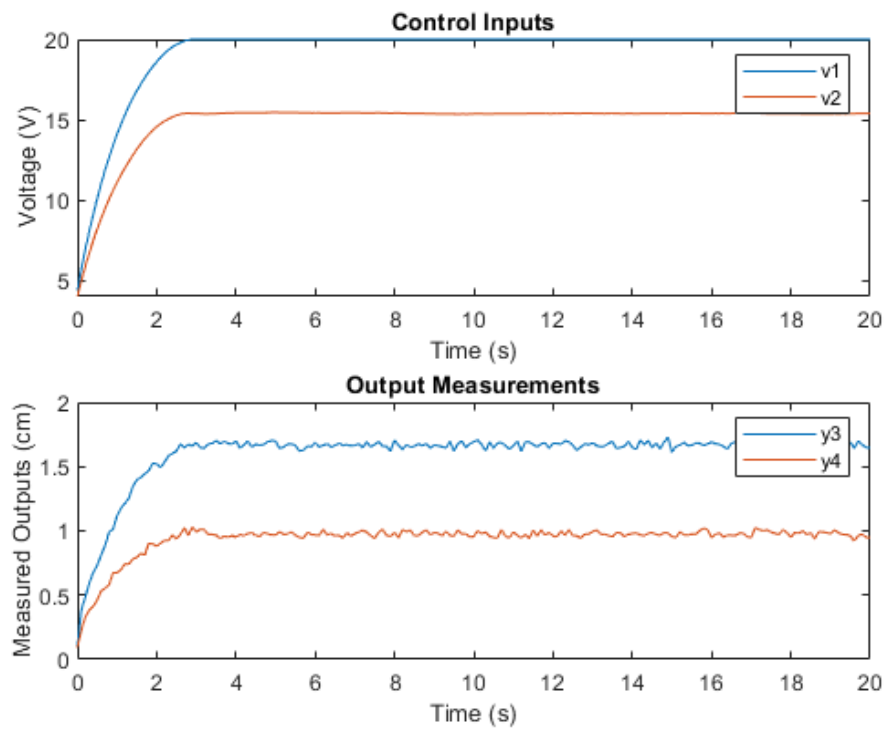
- The repeated poles at 1 suggest potential unobservable or uncontrollable modes, which may not affect the outputs due to the design of the MPC controller.

## B) Implement Constraint MPC to control

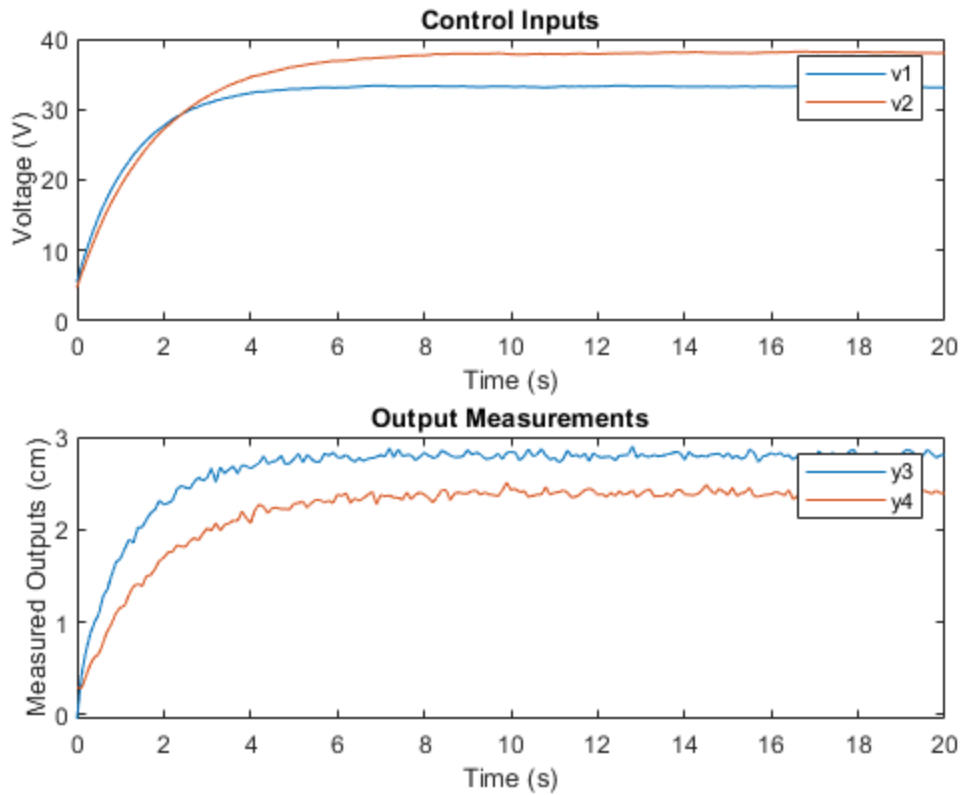
a) Control  $h_3, h_4$  when  $h_1$  and  $h_2$  are measured; set point for  $[h_3 \ h_4]$  is  $[2.8 \ 2.4]$

The matlab code for implementing the constrained MPC for controlling  $h_3$  and  $h_4$  using the state-space model is submitted as a separate file. The simulation is working, and the control inputs guide the system to the desired set points for  $h_3=2.8$  cm and  $h_2=2.4$  cm.

The plots is given below



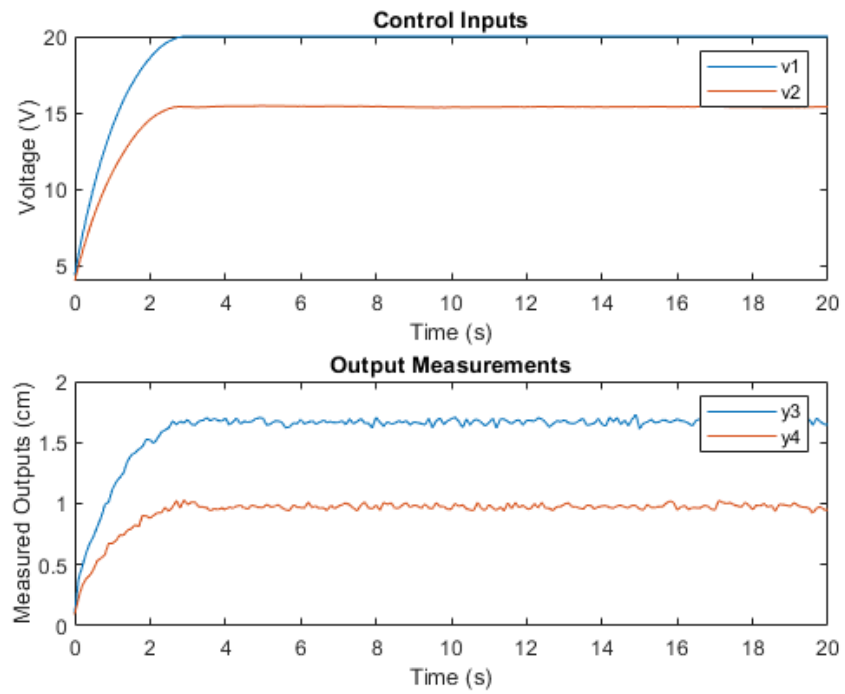
It can be seen that with given  $U$  limits, the controller is unable to take  $h_3$  and  $h_4$  to set points. Below plot shows the tracking of  $h_3$  and  $h_4$  when the  $U$  max limit was increased to 50.



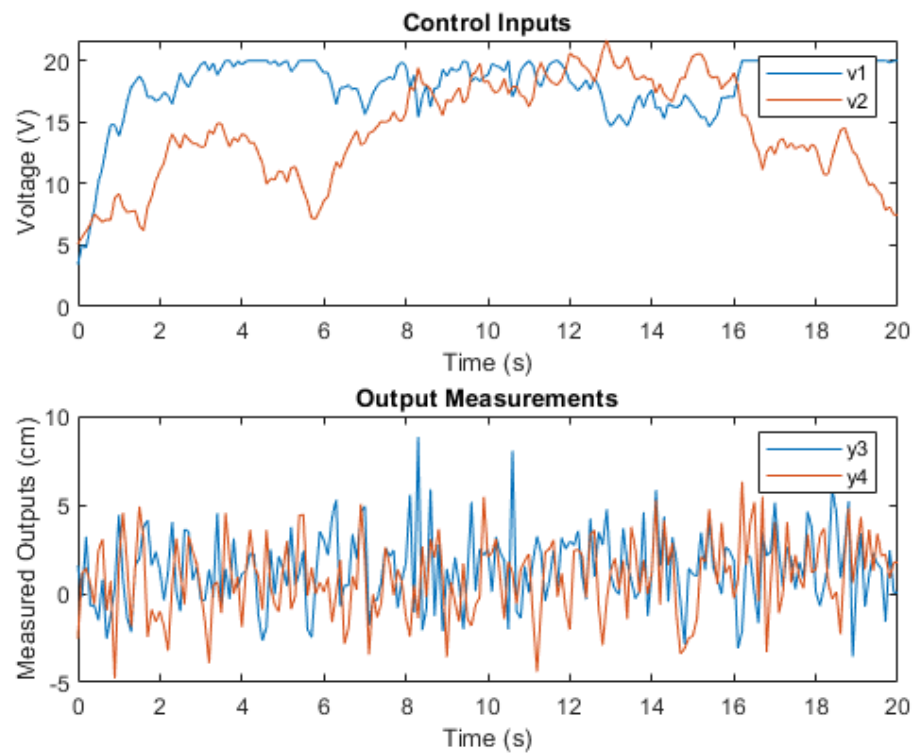
**b) Analyze how Kalman filter performance affect MPC performance by experimenting with Kalman gain parameters.**

We will modify the process noise covariance  $Q$  and the measurement noise covariance matrix  $R$  to simulate different levels of confidence in the process model and sensor measurements. The kalman filter provides states estimates used by the MPC for predictions. If these estimates are noisy or biased, the MPC control inputs will not accurately achieve the desired outputs.

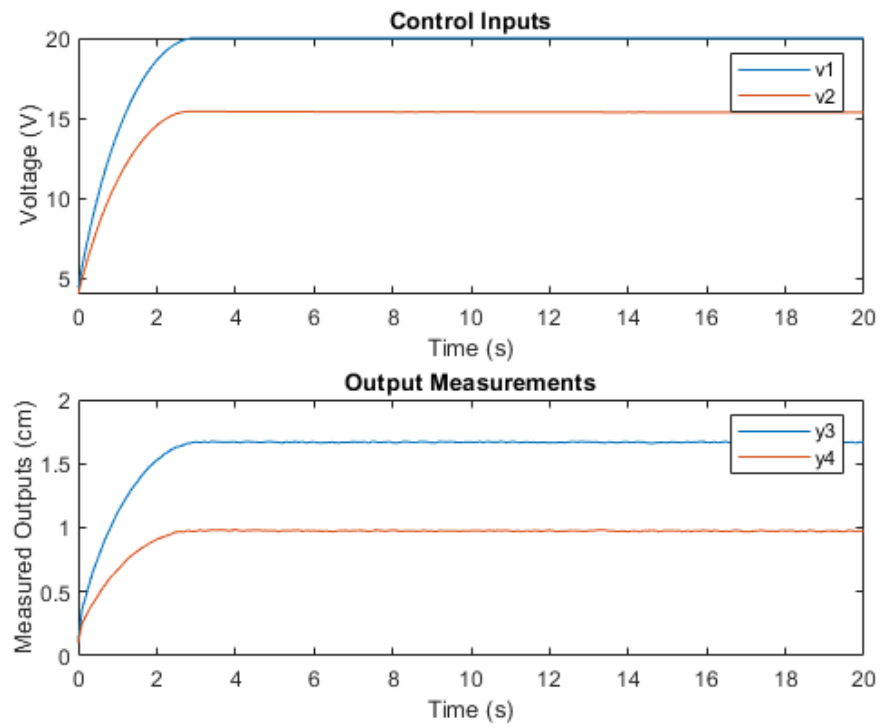
- $Q = 0.1 \cdot \text{eye}(4)$  and  $R = 20 \cdot \text{eye}(2)$



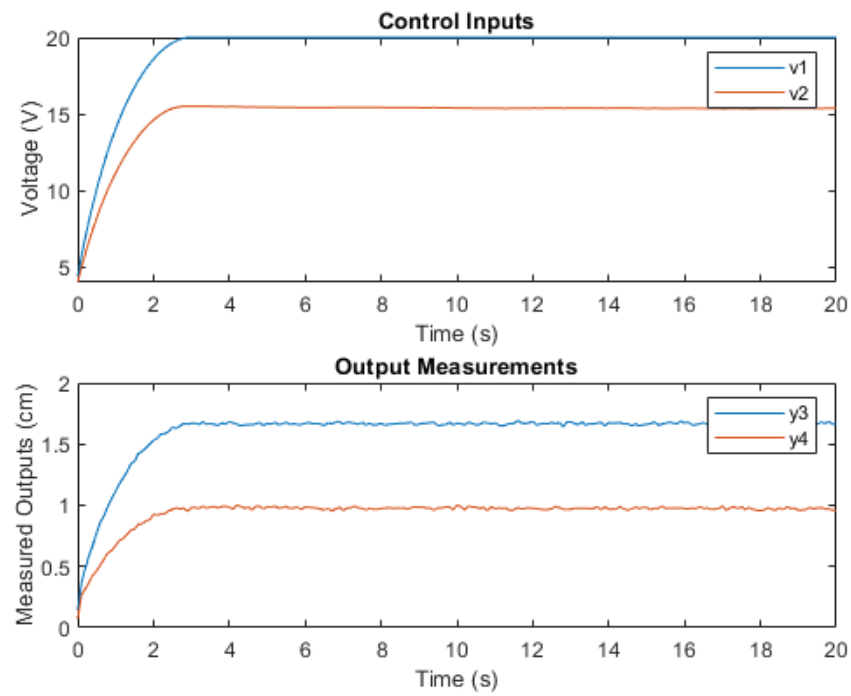
- $Q = 10 \cdot \text{eye}(4)$  and  $R = 10 \cdot \text{eye}(2)$



- $Q = 0.01 \cdot \text{eye}(4)$  and  $R = 10 \cdot \text{eye}(2)$



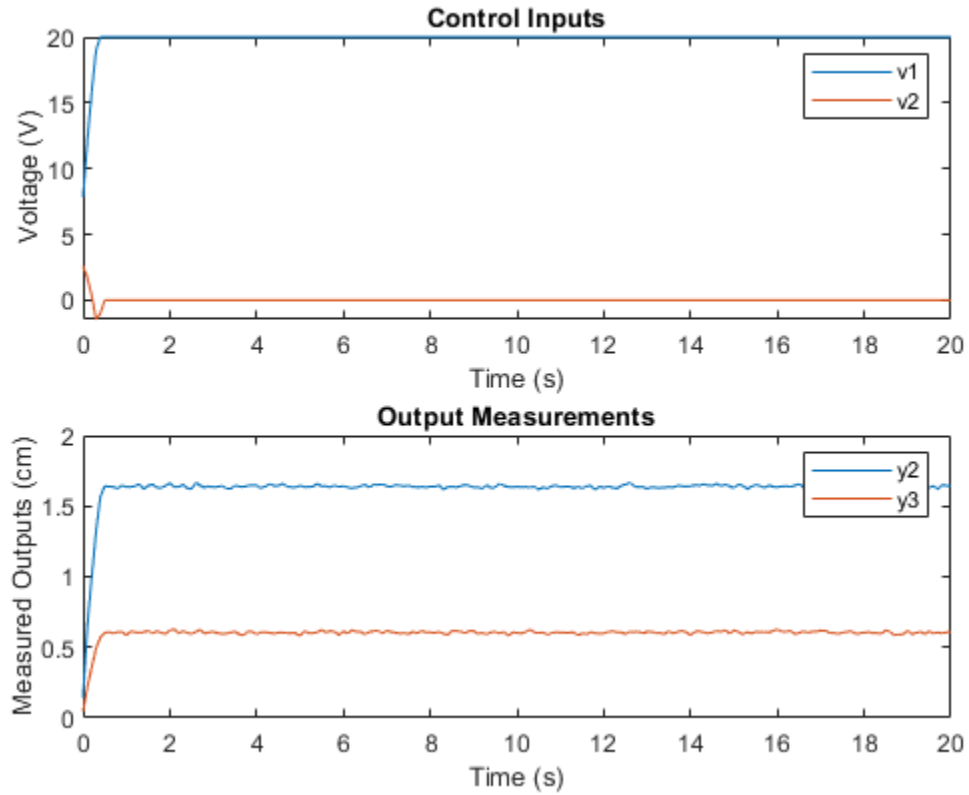
- $Q = 0.1 \cdot \text{eye}(4)$  and  $R = 100 \cdot \text{eye}(2)$



### C) Implement Constraint MPC such that it can be used to control

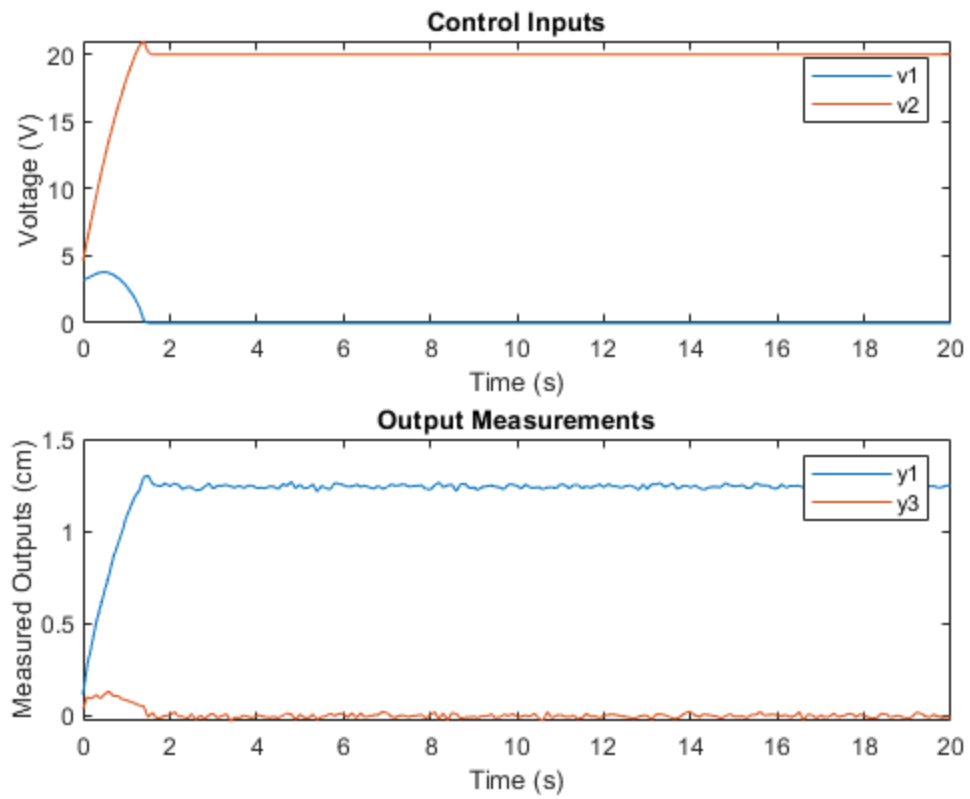
- a)  $h_2, h_3$  when  $h_1, h_4$  are measured with set-point for  $[h_2 \ h_3]$  as  $[13.7 \ 2.8]$

Plot for this case is given below



- b)  $h_1, h_3$  when  $h_2, h_4$  are measured with set-point for  $[h_1 \ h_3]$  as  $[13.7 \ 2.4]$





It can be seen from the plots that MPC cannot achieve set point tracking in both the above cases