**CSCI 2150L**

**Session 14 Topics:**

**11/5/2014**

**Fall 2014**

1. Newton’s form of interpolating polynomial was the topic of discussion this week. As a starting point let’s assume we are provided with the following set of points.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  | … |  |
|  |  |  |  | … |  |

The values were obtained at the corresponding points and we have a total of points in the table. The values are distinct i.e. for. With these points we would be able to construct an approximation of the function in a form of a polynomial of degree.

Using Newton form we construct the polynomial starting with one point and then gradually taking into consideration more number of points to add refinement to the previously estimated approximation. Some initial approximation are listed below.

and so on.

In the process of gradually estimating the polynomial with higher degree we are using coefficients … etc. These coefficients associate with them a multiplicative term whose length increases along with the subscript of coefficient. These terms for any coefficient can be denoted as and formally written as follows.

As coefficient doesn’t have any such term associated with it we have to assume

As we can observe from above any is a polynomial of degree and thus has roots. This is reflected in the following figure which plots some initial for. Number of roots can be found by observing how many time has intersected the line.

Using these newton polynomials we can formally express the final approximated polynomial as follows.

Notice that the degree of this polynomial is as we are given points in the original table.

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Fig.1 First few newton polynomial[Courtesy: Numerical Mathematics and Computing, Kincaid and Cheney, Sixth edition.]

We can observe from the equations above that apart from the first coefficient determination of other coefficients are non-trivial as they depend increasingly on the previous values. However, solving first few coefficients from the equations listed as a, b, c as follows help find a pattern for all the coefficients.

If we denote certain quantities in the equation using the following notations we can find a pattern in the coefficient equations. These are represented as follows.

The expressions in the expressions above are called divided differences. As we observed in the above equations one divided difference can be expressed in terms of divided difference computed in the previous stage. This calculation is shown in the table below.



Fig. 2: Divided difference table [Courtesy: Numerical Mathematics and Computing, Kincaid and Cheney, Sixth edition.]

Generally divided difference can be expressed recursively as follows.

From the table we can easily find the coefficients as they are the first element of each column of the table i.e.

Thus and so on. Using these formulations we find all the coefficients and subsequently the following approximation of the function is formed based on the provided points in the original table.