**CSCI 2150L**

**Session 15 Topics:**

**11/17/2014**

**Fall 2014**

1. In this session we discussed how to estimate first derivatives of a function via interpolation polynomials. For this purpose three variations of this approximate first derivative was presented. These formulae have distinct form depending on the number of points considered for the approximation.
2. The first derivative approximation considered two points and is derived as follows. For a function *f* a polynomial approximation considering two points would be

[The degree of this approximation is indicated by its subscript. Refer to previous handouts for interpolation expressions.]

As is an approximation of then it’ll be fair to say that it can also approximate the first derivative of i.e. .

From the formula of above we can derive

1. Let’s assume and is very close to each other such that if then for very small *h*. Subsequently we can approximate derivative of as
2. Another variation to this approach is to consider and where *x* is the point at which derivative is being calculated. For this case the approximate derivative would be
3. For the third formula we considered four points around point *x* where the derivative is being determined. A function from four point can be approximated using a polynomial of degree 3 as follows.

Differentiating the formula above we obtain approximate derivative of a function considering four points as follows.

Consider the distribution where

, .

Substituting these expressions in the derivative formula above we obtain the following approximate formula.

D3

In order to test the relative merit of these expressions we experimented with different values of h which determines the spacing between the points considered. We found that when the spacing is large all the formulae gives poor approximation to the exact derivative though margin of error is less with the formula considering four points.

However when the spacing is small enough the D3 gives highest accuracy where D2 is demonstrated to be better than D1.

1. Next we considered the problem of approximating integral of a function within a given lower and a upper bound. The problem of finding an integral can be visualized as finding the area under the curve *f(x)* between lower and upper bound a and b. In order to visualize the problem in terms of computation we can divide the area under the curve into small trapezoid as depicted in the following figure.

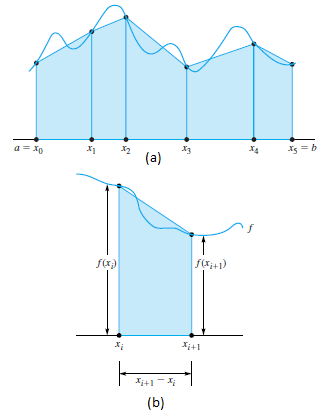


Fig. 1: Area under a curve divided into many trapezoids. [Courtesy: Numerical Mathematics and Computing, Kincaid and Cheney, Sixth edition.]

In Fig. 1(a) the area under the curve is divided into five trapezoids having different heights (Height of the trapezoid is the subinterval on x axis.). Now generally this area can be divided into many such trapezoids of equal height and each trapezoid can be indicated by the subinterval of x axis it covers. For e.g. if we consider a range [a, b] of *x* within which the integral would be computed then we can divide the range as partition. Moreover as we consider uniform spacing between subsequent points on *x* axis then.

One such trapezoid between interval is shown in Fig. 1(b). The area of this trapezoid would be

Thus adding all the area of such trapezoids found within the integral bound we can approximate the area under the curve as follows.

As the subintervals are uniformly spaced let’s assume for. Subsequently the integral approximation could be stated as follows.

Equation *I3* is an optimized form of *I2* where we don’t have to evaluate function *f* for all the points twice. [Convince yourself that both *I2* and *I3* are equivalent.]