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ME - 1203

ASSIGNMENT 2

1. $F(s) = \frac{1}{s(s^2+2s+2)}$

$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+2}\right\} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$

$1 = A(s^2+2s+2) + s(Bs+C)$

IF $s = 0$;

$1 = A(2) + 0$

$A = \frac{1}{2}$

SUBSTITUTE THE VALUE OF A:

$\left[1 = \frac{1}{2}(s^2+2s+2) + Bs^2 + Cs\right] 2$

$2 = s^2 + 2s + 2Bs^2 + 2Cs$

$B = -\frac{1}{2}$, $C = -1$

$\mathcal{L}^{-1}\left\{\frac{1/2}{s} - \frac{(1/2 s + 1)}{s^2+2s+2}\right\}$

① 1/2

② $\frac{1}{2} = \frac{(s+1)+1}{(s^2+2s+1)+1}$

FORMULA:

$\mathcal{L}^{-1}\left[\frac{(s+a)+w}{(s+a)^2+w^2}\right] = e^{-at}[\cos wt + \sin wt]\, u(t) = \frac{(s+1)+1}{(s+1)^2+1}$, $a = -1$, $w = \sqrt{1} = 1$

$\frac{1}{2} e^{-t}(\cos t + \sin t)\, u(t)$

$f(t) = \frac{1}{2} - \frac{1}{2} e^{-t}(\cos t + \sin t)$

$$\boxed{f(t) = \frac{1}{2}\left[1 - e^{-t}(\cos t + \sin t)\right]}$$

2. $F(s) = \frac{5(s+2)}{s^2(s+1)(s+3)}$

$\mathcal{L}^{-1}\left\{\frac{5(s+2)}{s^2(s+1)(s+3)}\right\} = \frac{A}{s^2} + \frac{B}{s+1} + \frac{C}{s+3} + \frac{D}{s}$

$5(s+2) = A(s+1)(s+3) + B(s^2)(s+3) + C(s^2)(s+1) + D(s)(s+1)(s+3)$

IF $s = 0$; IF $s = -1$; IF $s = -3$

$10 = 3A + 0 + 0 + 0$ $5 = 0 + 2B + 0 + 0$ $-5 = 0 + 0 - 18C + 0$

$A = \frac{10}{3}$ $B = \frac{5}{2}$ $C = \frac{5}{18}$

IF $s = -2$; SUBSTITUTE A, B, C

$0 = \frac{10}{3} + 10 - \frac{10}{9} + 2D$

$0 = \frac{50}{9} + 2D$

$\left[2D = -\frac{50}{9}\right]\frac{1}{2}$

$D = -\frac{25}{9}$

$$\mathcal{L}^{-1}\left\{\frac{10/3}{s^3}+\frac{5/2}{s+1}+\frac{5/18}{s+3}-\frac{25/9}{s}\right\}$$

$$f(t)=\frac{10t}{3}+\frac{5e^{-t}}{2}+\frac{5e^{-3t}}{18}-\frac{25}{9}$$

3.

$$F(s)=\frac{s^4+2s^3+3s^2+4s+5}{s(s+1)}$$

$$\begin{array}{r|l} s^2+s & s^2+s+2 \\ \hline -s^4+s^3 & \\ \hline & s^3+3s^2+4s+5 \\ & -s^3\ \ s^2 \\ \hline & 2s^2+4s+5 \\ & -2s^2+2s \\ \hline & 2s+5 \end{array}$$

$$\mathcal{L}^{-1}\left\{s^2+s+2+\frac{2s+5}{s^2+s}\right\}$$

① y''

② y'

③ $2\delta(t)$

④ $\mathcal{L}^{-1}\left\{\frac{2s+5}{s^2+s}\right\}=\frac{A}{s}+\frac{B}{s+1}$

$2s+5=A(s+1)+Bs$

IF $s=-1$;

$3=0-B$

$B=-3$

IF $s=0$;

$5=A+0$

$A=5$

$$\mathcal{L}^{-1}\left\{\frac{5}{s}-\frac{3}{s+1}\right\}=5(1)+[-3e^{-t}]$$

$$=5-3e^{-t}$$

$$f(t)=y''+y'+2\delta(t)+5-3e^{-t}$$