

ASSIGNMENT 1

1. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING

$$1. \mathcal{L} [3 - e^{-3t} + 5 \sin 2t] = F(s)$$

$$3\mathcal{L} \{1\} = 3 \left(\frac{1}{s} \right) = \frac{3}{s}$$

$$\mathcal{L} \{e^{-3t}\} = \frac{1}{s+3}$$

$$5\mathcal{L} \{\sin 2t\} = 5 \left(\frac{2}{s^2 + 2^2} \right) = \frac{10}{s^2 + 4}$$

$$F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2 + 4}$$

$$2. \mathcal{L} [3 + 12t + 42t^3 - 3e^{2t}] = F(s)$$

$$3\mathcal{L} \{1\} = 3 \left(\frac{1}{s} \right) = \left(\frac{3}{s} \right)$$

$$12\mathcal{L} \{t\} = 12 \left(\frac{1}{s^2} \right) = \left(\frac{12}{s^2} \right)$$

$$42\mathcal{L} \{t^3\} = 42 \left(\frac{3!}{s^{3+1}} \right) = 42 \left(\frac{6}{s^4} \right) = \frac{252}{s^4}$$

$$3\mathcal{L} \{e^{2t}\} = 3 \left(\frac{1}{s-2} \right) = \frac{3}{s-2}$$

$$F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}$$

$$3. \mathcal{L} [(t+1)(t+2)] = F(s)$$

$$\mathcal{L} [t^2 + 3t + 2] = F(s)$$

$$\mathcal{L} \{t^2\} = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$$

$$3\mathcal{L} \{t\} = 3 \left(\frac{1}{s^2} \right) = \frac{3}{s^2}$$

$$2\mathcal{L} \{1\} = 2 \left(\frac{1}{s} \right) = \frac{2}{s}$$

$$F(s) = \frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}$$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

$$1. \mathcal{L}^{-1} \left[\frac{8 - 3s + s^2}{s^3} \right] = f(t)$$

$$\mathcal{L}^{-1} \left[\frac{8}{s^3} - \frac{3}{s^2} + \frac{1}{s} \right] = f(t)$$

$$1\mathcal{L} \left\{ \frac{2}{s^3} \right\} = 1(t^2) u(t) = 4t^2 u(t)$$

$$3\mathcal{L}^{-1} \left\{ \frac{1}{s^2} \right\} = 3(t) u(t) = 3t u(t)$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} = u(t)$$

$$\boxed{f(t) = (4t^2 - 3t + 1) u(t)}$$

$$2. \mathcal{L}^{-1} \left[\frac{5}{s-2} - \frac{4s}{s^2+9} \right] = f(t)$$

$$\mathcal{L}^{-1} \left[\frac{5}{s-2} \right] - \mathcal{L}^{-1} \left[\frac{4s}{s^2+9} \right] = f(t)$$

$$5\mathcal{L}^{-1} \left\{ \frac{1}{s-2} \right\} = 5(e^{2t}) u(t) = 5e^{2t} u(t)$$

$$4\mathcal{L}^{-1} \left\{ \frac{s}{s^2+9} \right\} = 4(\cos 3t) u(t) = 4\cos 3t u(t)$$

$$\boxed{f(t) = (5e^{2t} - 4\cos 3t) u(t)}$$

$$3. \mathcal{L}^{-1} \left[\frac{7}{s^2+6} \right] = f(t)$$

$$f(t) = \frac{7}{\sqrt{6}} \mathcal{L}^{-1} \left\{ \frac{\sqrt{6}}{s^2+6} \right\}$$

$$= \frac{7}{\sqrt{6}} (\sin \sqrt{6}) u(t)$$

$$= \left[\frac{7}{\sqrt{6}} \sin \sqrt{6} u(t) \right] \left[\frac{\sqrt{6}}{\sqrt{6}} \right]$$

$$\boxed{f(t) = \frac{7\sqrt{6}}{6} \sin \sqrt{6} u(t)}$$