Homework 4: Response of a Helical Spring in Discrete Elastic Rod

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I. Introduction: Problem Setup and Input Conditions

This homework covers the modeling a helical coil as a 3-D Discrete Elastic Rod (DER) and quantify its axial stiffness from dynamic relaxations to steady states. The stiffness is determined by varying end load for a given geometry, this process is repeated for each helix diameter to determine the stiffness trends. This trend is then compared against a linear model and the trend from the simulation data and given trend line matches.

Helix Geometry, Load, and Simulation Parameters

- Wire diameter: $d = 0.002\,\mathrm{m}$
- Mean coil diameter: $D = 0.04\,\mathrm{m}$
- Helix radius: $R = D/2$
- Pitch per turn: $p = d$
- Number of turns: $N = 5$
- Axial length: $L_{\text{axial}} = Np$
- Arc length per turn: $L_{\text{turn}} = \sqrt{(2\pi R)^2 + p^2}$
- Total contour length: $L = NL_{\text{turn}}$
- Young's modulus: $E = 10\,\mathrm{MPa}$
- Poisson ratio: $\nu = 0.5$
- Shear modulus: $G = \dfrac{E}{2(1+\nu)}$
- Cross-section area (circular wire): $A = \dfrac{\pi d^2}{4}$
- Second moment of area: $I = \dfrac{\pi d^4}{64}$
- Polar moment of inertia: $J = \dfrac{\pi d^4}{32}$
- Time Step: $\Delta t = 0.06\,\mathrm{s}$
- Total Time: $\Delta t = 10\,\mathrm{s}$
- Characteristic Force: $F_{\text{char}} = \dfrac{EI}{L_{\text{axial}}^2}$
- Axial displacement: $\delta_z(t) = z_{\text{end}}(t) - z_{\text{end}}(0)$
- Force–displacement law: $F = k\,\delta_z^*$
- Textbook spring constant: $k_{\text{text}} = \dfrac{Gd^4}{8ND^3}$

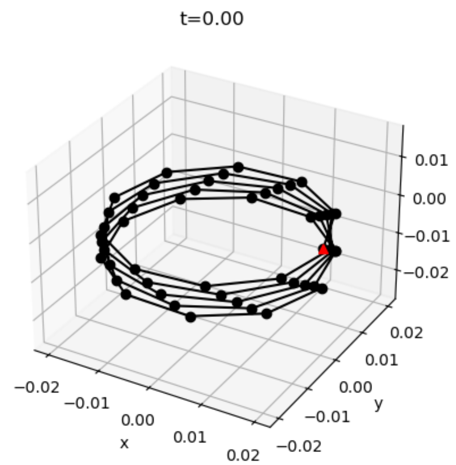

Fig. 1: Initial State

Figure 1 shows the starting configuration of the system. A constant tensile force F at the last node and the spring deforms over time and reaches a new steady state. The deformation depends on stiffness, which in turn depends on geometry. The stiffness for a given geometry is obtained by running the DER simulation through a range of loads. This process can be repeated to find an array stiffness that correspond to the diameter sweep.

II. Results

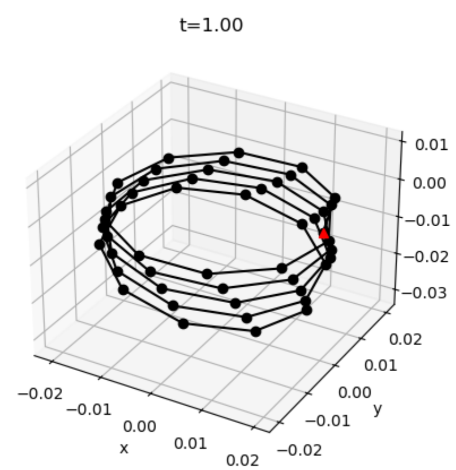

Fig. 2: Helix Shape at t = 1s

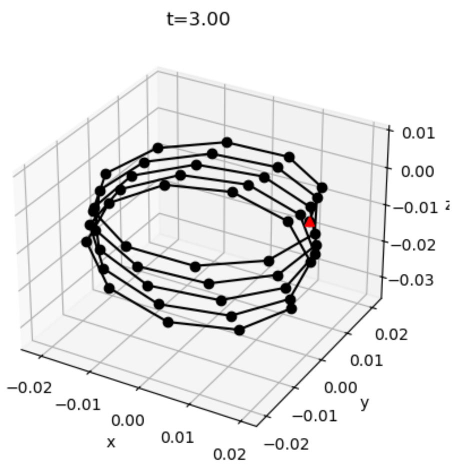

Fig. 3: Helix Shape at t = 3s

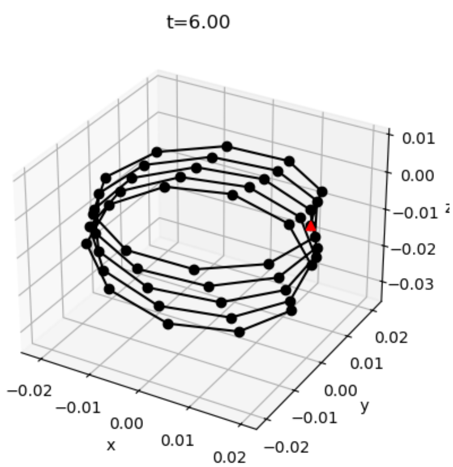

Fig. 4: Helix Shape at t = 6s

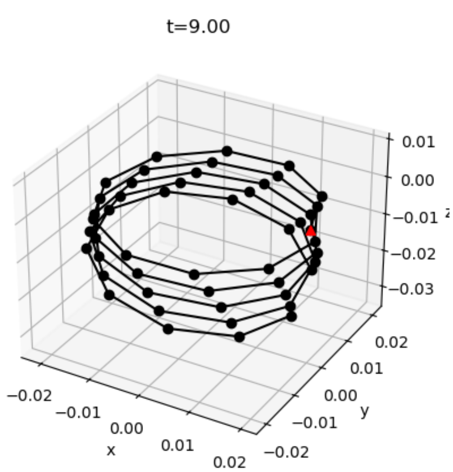

Fig. 5: Helix Shape at t = 9s

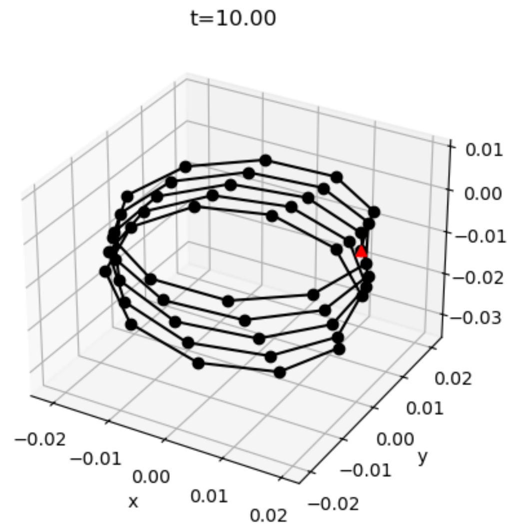

Fig. 6: Helix Shape at t = 10s

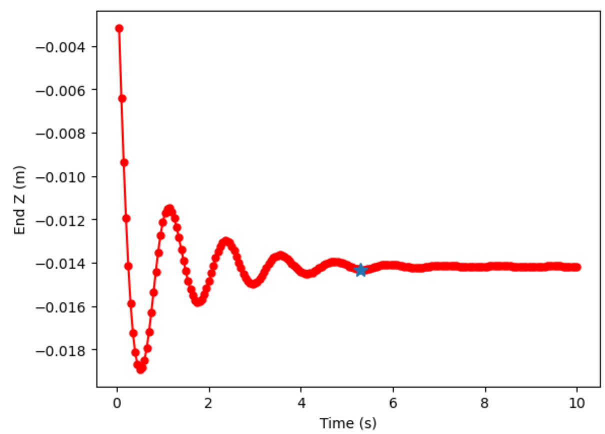

Fig. 7: End Point Location with Time

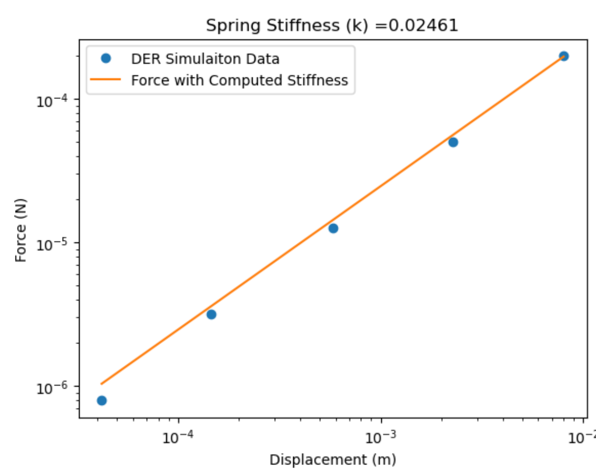

Fig. 8: Spring Stiffness = 0.024 (N/m)

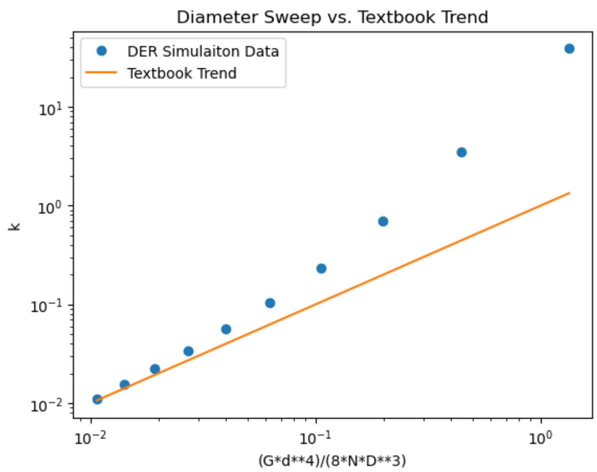

Fig. 9: Diameter Sweep Stiffness Trends

III. Discussion of Results

In Figure 7, the steady state is reached just over 5 seconds for a Δt of 0.05. The condition used for this was percent change of value under 0.002%. This number was obtained with trial and error to land at a point where the graph oscillations died down. This condition is applicable for the rest of the cases ran in this homework. A better approach to determine steady state can be a smarter method that takes more patterns into account instead of trial and error to set an initial limiter percentage.

As the helix diameter D increases, the spring becomes more flexible and the axial stiffness k decreases. This is observed in the stiffness data obtained from DER simulation and the textbook trend. Although the trend matches, the values diverge as the diameter decreases. The DER simulation data has higher stiffness values for small diameters and breaks from the near linear trend seen for the higher diameter cases.

DER simulation accuracy in the context of tolerance, convergence, and other forms of numerical errors could play a role but this is likely a difference from a theoretical model and a computational model of the actual system. In Figure 8, the stiffness value obtained can be impacted by the number of points chosen and numerical accuracy limits of convergence for very small loads. In that isolated case, there could be higher error but broadly the trend aligns in Figure 9.