Homework 5: Deformation of a Clamped Thin Beam Using a Plate Model

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I. Introduction: Problem Setup and Method

This homework covers the static deformation simulation of a thin, rectangular beam under its own weight using the discrete plate model and compare your numerical results against the classical Euler-Bernoulli beam theory prediction.

Simulation Parameters and Geometry

Beam dimensions:
length $l = 0.1$ m,
width $w = 0.01$ m,
thickness $h = 0.002$ m.

Material parameters:
Young's modulus $Y = 10^7$ Pa,
density $\rho = 1000\,\text{kg/m}^3$.

Section properties (for Euler–Bernoulli comparison)
Cross-sectional area: $A = wh$.
Second moment of area: $I = \dfrac{wh^3}{12}$.
Distributed load from gravity: $q = \rho A g$.
Euler–Bernoulli Beam Theory: $\delta_{\text{EB}} = \frac{ql^4}{8YI}$.

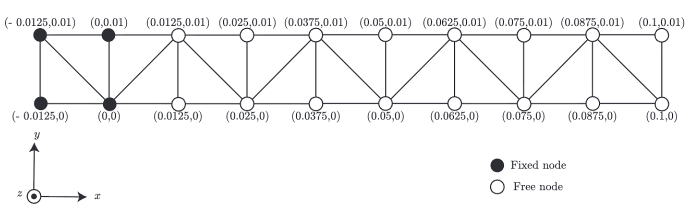

Fig. 1: Beam Discrete Geometry

Figure 1 shows the starting configuration of the system. Gravity is the only force acting on the system. The length of the free portion of the plate is l = 0.1 m. The left edge of the plate (at x= 0) is fully clamped: all displacement and rotation components are fixed. The rest of the beam is free to deform under it's down weight.

II. Results and Discussion

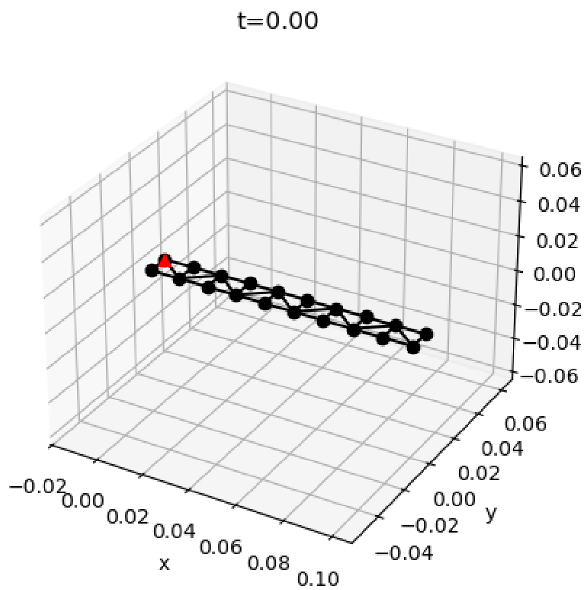

Fig. 2: Initial State

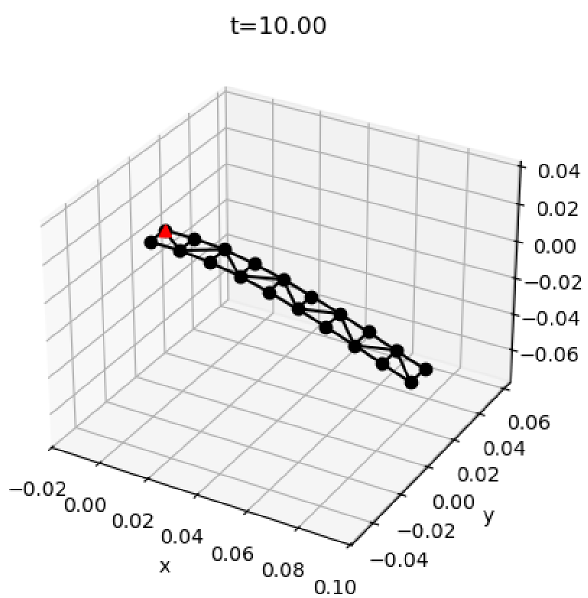

Fig. 3: Final State

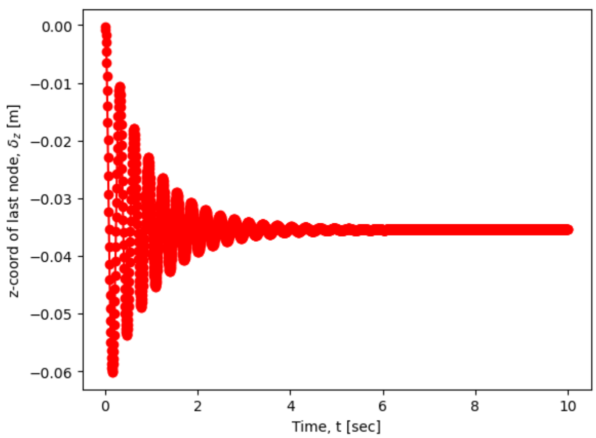

Fig. 4: Displacement Over Time

Discrete Plate Model: -0.035415 m

Euler–Bernoulli Beam Theory: $\delta_{\mathrm{EB}} = \frac{ql^4}{8YI}. = -0.03675$ m

Difference From Theory: 3.63%

Overall, the results of the simulation match very well with the Euler Beam Theory. They are likely to diverge more if the load is higher than this loading case.