network *blockmodeling*

introduction to *network analysis in Python* (*NetPy*)

Lovro Šubelj
University of Ljubljana
10th Dec 2019

blockmodeling *overview*

graph partitioning [KL70, Fie73]

blockmodeling [LW71, WR83]

stochastic block models [Pei15]

communities [GN02]

overlapping communities [PDFV05]

link communities [EL09, ABL10]

---

* assortative & disassortative equivalence blockmodeling

blockmodeling *equivalence*

— *standard equivalence blockmodeling* [DBF05]

- define *node similarity* as *(structural) equivalence*

$$\sigma_{ij} \sim |\Gamma_i \cap \Gamma_j|$$

1. *blockmodeling* by *(hierarchical) clustering* $\mathcal{O}(n^2)$
2. return *block model* at desired *clustering resolution*

javax adjacency matrix

javax block model

‡ `javax.swing`, `javax.management`, `javax.naming`, `javax.print`, `javax.xml`, `javax.lang` etc.

stochastic models

introduction to *network analysis in Python* (*NetPy*)

Lovro Šubelj
University of Ljubljana
10th Dec 2019

stochastic *models*

- *random graph model* $G(n, m)$ for *network links* m [ER59]
- *configuration model* $G(\{k\})$ for *node degrees* $\{k\}$ [NSW01]
- *exponential* p^**-model* $G(n, \{\langle x \rangle\})$ for *any expectations* $\{\langle x \rangle\}$

- *stochastic block model* $G(\{C\})$ for *node clusters* $\{C\}$ [HLL83]
- *hierarchical model* $G(H)$ for *node hierarchy* H [CMN08]

* assortative & disassortative stochastic block models

stochastic $G(\{C\})$ *model*

- $G(\{C\},\{p\})$ *stochastic block model* [HLL83]
- *link* between i and j *placed with probability* $p_{c_i c_j}$
  - $m_{c_i c_j}$ is *number of links* between C_i and C_j
  - $M_{c_i c_j}$ is *maximum* $m_{c_i c_j}$ hence $n_i n_j$ or $\binom{n_i}{2}$

$$\mathrm{P}(G|\{C\},\{p\}) = \prod_{i\leq j} p_{c_i c_j}^{A_{ij}}(1-p_{c_i c_j})^{1-A_{ij}} = \prod_{c_i \leq c_j} p_{c_i c_j}^{m_{c_i c_j}}(1-p_{c_i c_j})^{M_{c_i c_j}-m_{c_i c_j}}$$

- *maximum likelihood* $G(\{C\})$ *block model*
  - $\frac{m_{c_i c_j}}{M_{c_i c_j}}$ is *maximum likelihood estimate* for $p_{c_i c_j}$

$$\mathcal{L}(G|\{C\}) = \log \mathrm{P}(G|\{C\}) = \sum_{c_i \leq c_j} m_{c_i c_j} \log \frac{m_{c_i c_j}}{M_{c_i c_j}-m_{c_i c_j}} + M_{c_i c_j} \log \frac{M_{c_i c_j}-m_{c_i c_j}}{M_{c_i c_j}}$$

see graph-tool implementation

stochastic *overview*

stochastic block models [HLL83]

corrected block models [KN11]

principled block models [Pei15]

hierarchical models [CMN08, ŠB14]

role models [RW07, NL07, GSPA07]

image graphs [ŠB12]

† overlapping & corrected models also known as mixture & mixed membership models

blockmodeling *references*

Yong-Yeol Ahn, James P. Bagrow, and Sune Lehmann.
Link communities reveal multiscale complexity in networks.
Nature, 466(7307):761–764, 2010.

Aaron Clauset, Cristopher Moore, and M. E. J. Newman.
Hierarchical structure and the prediction of missing links in networks.
Nature, 453(7191):98–101, 2008.

Patrick Doreian, Vladimir Batagelj, and Anuska Ferligoj.
Generalized Blockmodeling.
Cambridge University Press, Cambridge, 2005.

T. S. Evans and R. Lambiotte.
Line graphs, link partitions and overlapping communities.
Phys. Rev. E, 80(1):016105, 2009.

P. Erdős and A. Rényi.
On random graphs I.
Publ. Math. Debrecen, 6:290–297, 1959.

M. Fiedler.
Algebraic connectivity of graphs.
Czech. Math. J., 23:298–305, 1973.

M. Girvan and M. E. J Newman.
Community structure in social and biological networks.
P. Natl. Acad. Sci. USA, 99(12):7821–7826, 2002.

Roger Guimerà, Marta Sales-Pardo, and Luis A. N. Amaral.
Classes of complex networks defined by role-to-role connectivity profiles.
Nat. Phys., 3(1):63–69, 2007.

blockmodeling *references*

Paul W. Holland, Kathryn Blackmond Laskey, and Samuel Leinhardt.
Stochastic blockmodels: First steps.
Soc. Networks, 5(2):109–137, 1983.

Brian W. Kernighan and S. Lin.
An efficient heuristic procedure for partitioning graphs.
Bell Sys. Tech. J., 49(2):291–308, 1970.

Brian Karrer and M. E. J Newman.
Stochastic blockmodels and community structure in networks.
Phys. Rev. E, 83(1):016107, 2011.

F. Lorrain and H. C. White.
Structural equivalence of individuals in social networks.
J. Math. Sociol., 1(1):49–80, 1971.

M. E. J Newman and E. A Leicht.
Mixture models and exploratory analysis in networks.
P. Natl. Acad. Sci. USA, 104(23):9564–9569, 2007.

M. E. J. Newman, S. H. Strogatz, and D. J. Watts.
Random graphs with arbitrary degree distributions and their applications.
Phys. Rev. E, 64(2):026118, 2001.

Gergely Palla, Imre Derényi, Illes Farkas, and Tamas Vicsek.
Uncovering the overlapping community structure of complex networks in nature and society.
Nature, 435(7043):814–818, 2005.

Tiago P. Peixoto.
Model selection and hypothesis testing for large-scale network models with overlapping groups.
Phys. Rev. X, 5(1):011033, 2015.

blockmodeling *references*

J. Reichardt and D. R. White.
Role models for complex networks.
Eur. Phys. J. B, 60(2):217–224, 2007.

Lovro Šubelj and Marko Bajec.
Ubiquitousness of link-density and link-pattern communities in real-world networks.
Eur. Phys. J. B, 85(1):32, 2012.

Lovro Šubelj and Marko Bajec.
Group detection in complex networks: An algorithm and comparison of the state of the art.
Physica A, 397:144–156, 2014.

D. R. White and K. P. Reitz.
Graph and semigroup homomorphisms on networks of relations.
Soc. Networks, 5(2):193–234, 1983.