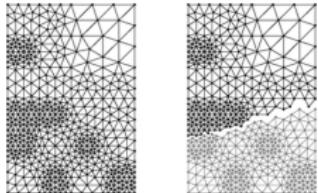


# network *blockmodeling*

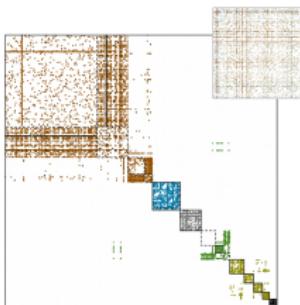
introduction to *network science in Python* (*NetPy*)

Lovro Šubelj  
University of Ljubljana  
14th Dec 2021

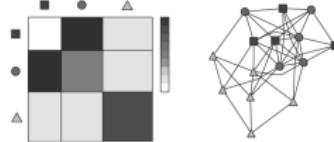
# blockmodeling *overview*



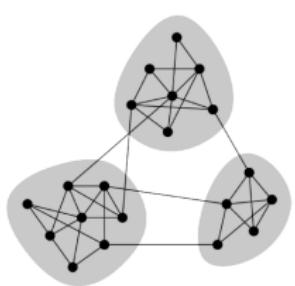
## graph partitioning [KL70, Fie73]



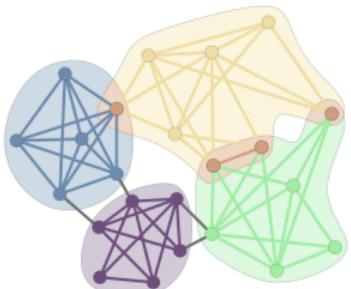
*blockmodeling* [LW71, WR83]



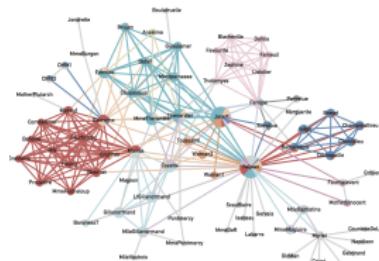
## *stochastic block models* [Pei15]



*communities* [GN02]



## *overlapping communities* [PDFV05]

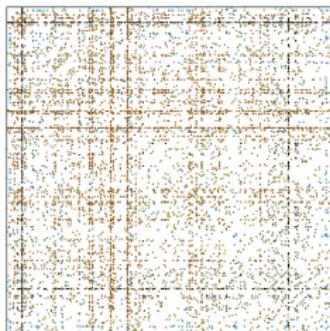


*link communities* [EL09, ABL10]

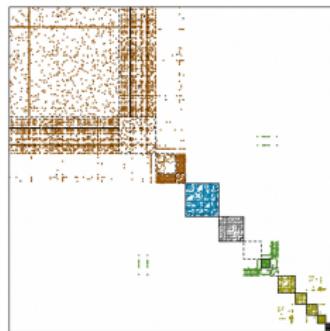
\* assortative & disassortative equivalence blockmodeling

# blockmodeling *equivalence*

- standard equivalence blockmodeling [DBF05]
  - define *node similarity* as (*structural*) equivalence
$$\sigma_{ij} \sim |\Gamma_i \cap \Gamma_j|$$
- 1. *blockmodeling* by (*hierarchical*) clustering  $\mathcal{O}(n^2)$
- 2. return *block model* at desired *clustering resolution*



javax adjacency matrix



javax block model

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‡ javax.swing, javax.management, javax.naming, javax.print, javax.xml, javax.lang etc.

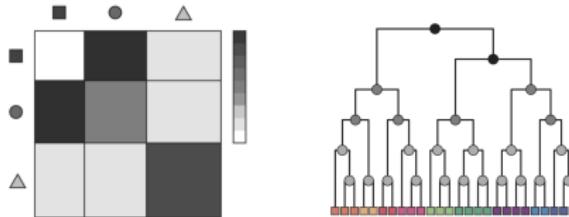
# *stochastic* models

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## stochastic *models*

- random graph model  $G(n, m)$  for network links  $m$  [ER59]
- configuration model  $G(\{k\})$  for node degrees  $\{k\}$  [NSW01]
- exponential  $p^*$ -model  $G(n, \{\langle x \rangle\})$  for any expectations  $\{\langle x \rangle\}$
- stochastic block model  $G(\{C\})$  for node clusters  $\{C\}$  [HLL83]
- hierarchical model  $G(H)$  for node hierarchy  $H$  [CMN08]



---

\* assortative & disassortative stochastic block models

# stochastic $G(\{C\})$ model

- $G(\{C\}, \{p\})$  stochastic block model [HLL83]
- link between  $i$  and  $j$  placed with probability  $p_{c_i c_j}$

—  $m_{c_i c_j}$  is number of links between  $C_i$  and  $C_j$

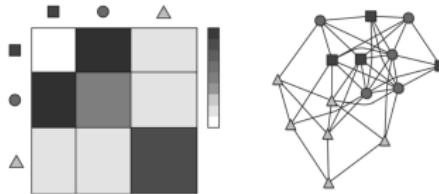
—  $M_{c_i c_j}$  is maximum  $m_{c_i c_j}$  hence  $n_i n_j$  or  $\binom{n_i}{2}$

$$P(G|\{C\}, \{p\}) = \prod_{i \leq j} p_{c_i c_j}^{A_{ij}} (1 - p_{c_i c_j})^{1 - A_{ij}} = \prod_{c_i \leq c_j} p_{c_i c_j}^{m_{c_i c_j}} (1 - p_{c_i c_j})^{M_{c_i c_j} - m_{c_i c_j}}$$

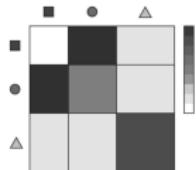
- maximum likelihood  $G(\{C\})$  block model

—  $\frac{m_{c_i c_j}}{M_{c_i c_j}}$  is maximum likelihood estimate for  $p_{c_i c_j}$

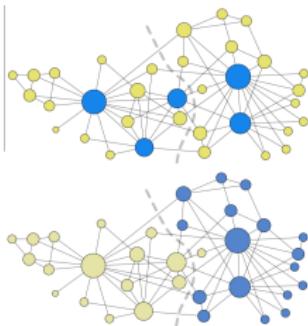
$$\mathcal{L}(G|\{C\}) = \log P(G|\{C\}) = \sum_{c_i \leq c_j} m_{c_i c_j} \log \frac{m_{c_i c_j}}{M_{c_i c_j} - m_{c_i c_j}} + M_{c_i c_j} \log \frac{M_{c_i c_j} - m_{c_i c_j}}{M_{c_i c_j}}$$



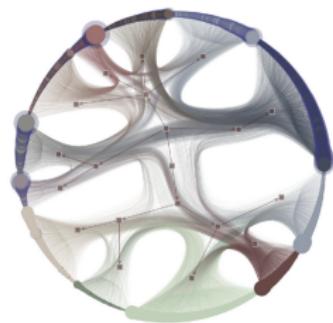
# stochastic overview



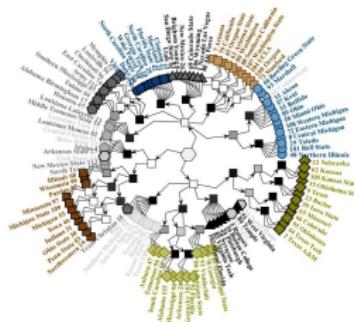
stochastic block models [HLL83]



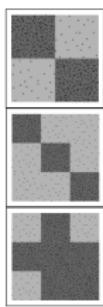
corrected block models [KN11]



principled block models [Pei15]



hierarchical models [CMN08, ŠB14]



role models [RW07, NL07, GSPA07]

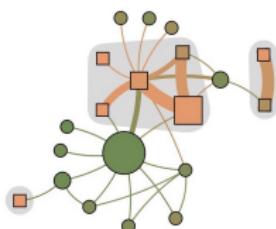
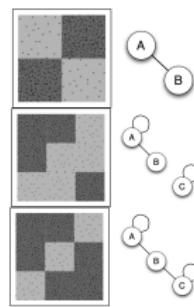


image graphs [ŠB12]

†

overlapping & corrected models also known as mixture & mixed membership models

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