network *clustering*

introduction to *network science in Python* (*NetPy*)

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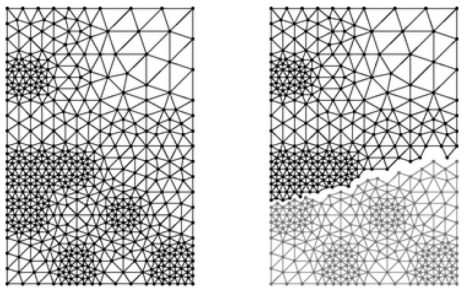
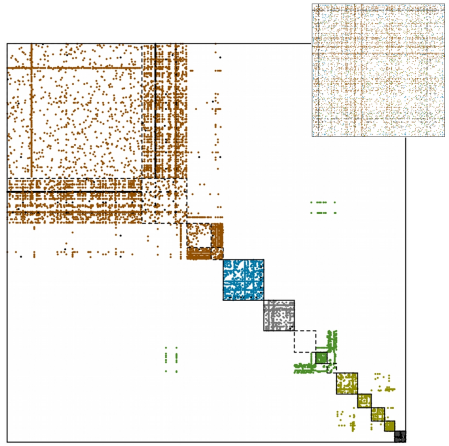
14th Dec 2021

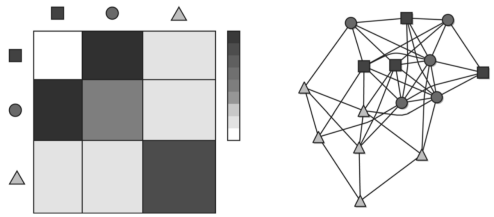
clustering *overview*



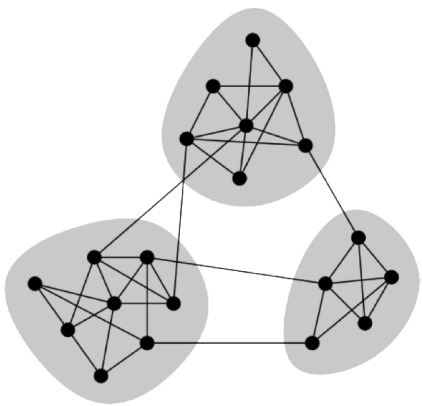
graph partitioning [KL70, Fie73]



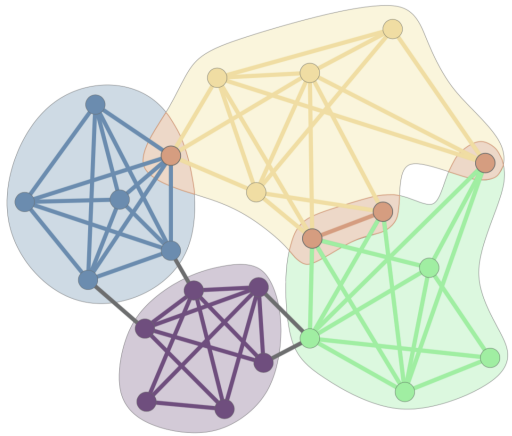
blockmodeling [LW71, WR83]



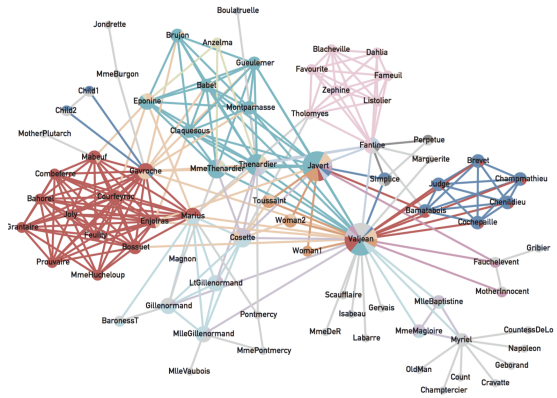
stochastic block models [Pei15]



communities [GN02]



overlapping communities [PDFV05]



link communities [EL09, ABL10]

community detection

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community *divisive*

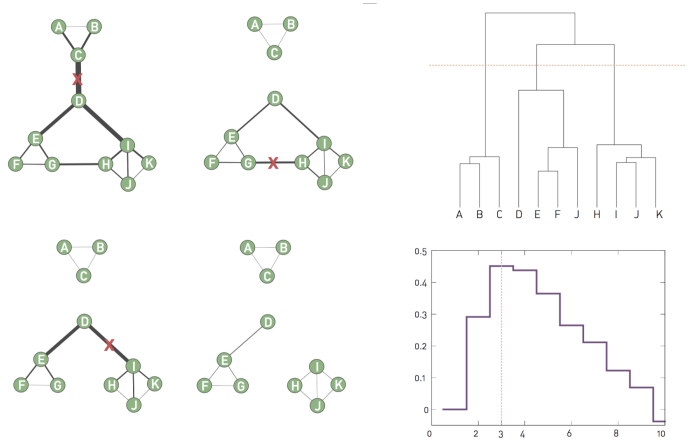
— Girvan-Newman *hierarchical clustering* [GN02]

– define *node dissimilarity* as *link betweenness*

$$\sigma_{ij} = \sum_{st \notin \{i,j\}} \frac{g_{st}^{ij}}{g_{st}}$$

1. top-down *divisive hierarchical clustering* $\mathcal{O}(nm^2)$
2. cut *cluster dendrogram* at *maximum modularity*

$$Q = \frac{1}{2m} \sum_{ij} (A_{ij} - \frac{k_i k_j}{2m}) \delta_{c_i c_j}$$

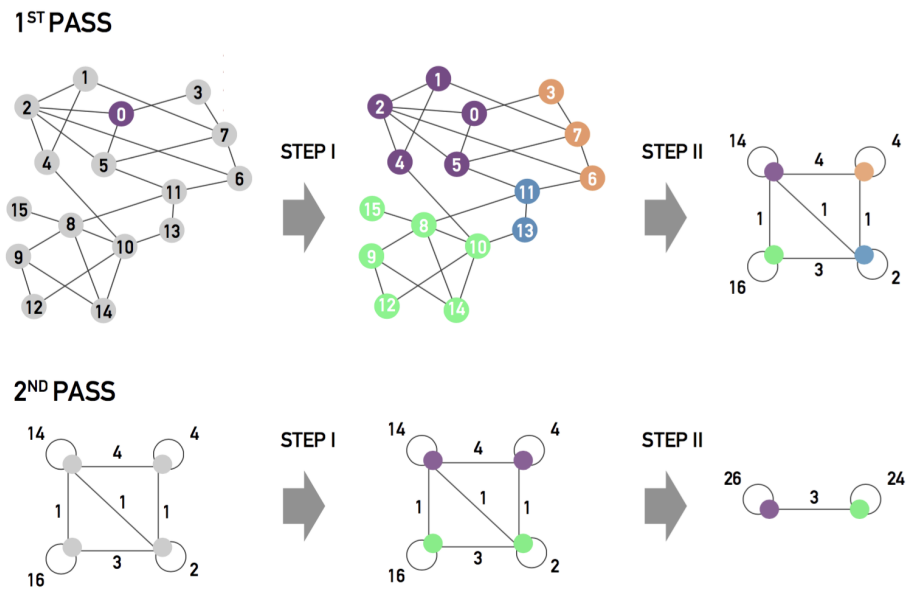


community *modularity*

— Louvain *modularity optimization* [BGLL08]

1. set *node community* by *modularity optimization* $\mathcal{O}(cm)$
2. *aggregate community nodes into supernodes* and repeat 1.
3. return *community structure maximizing modularity*

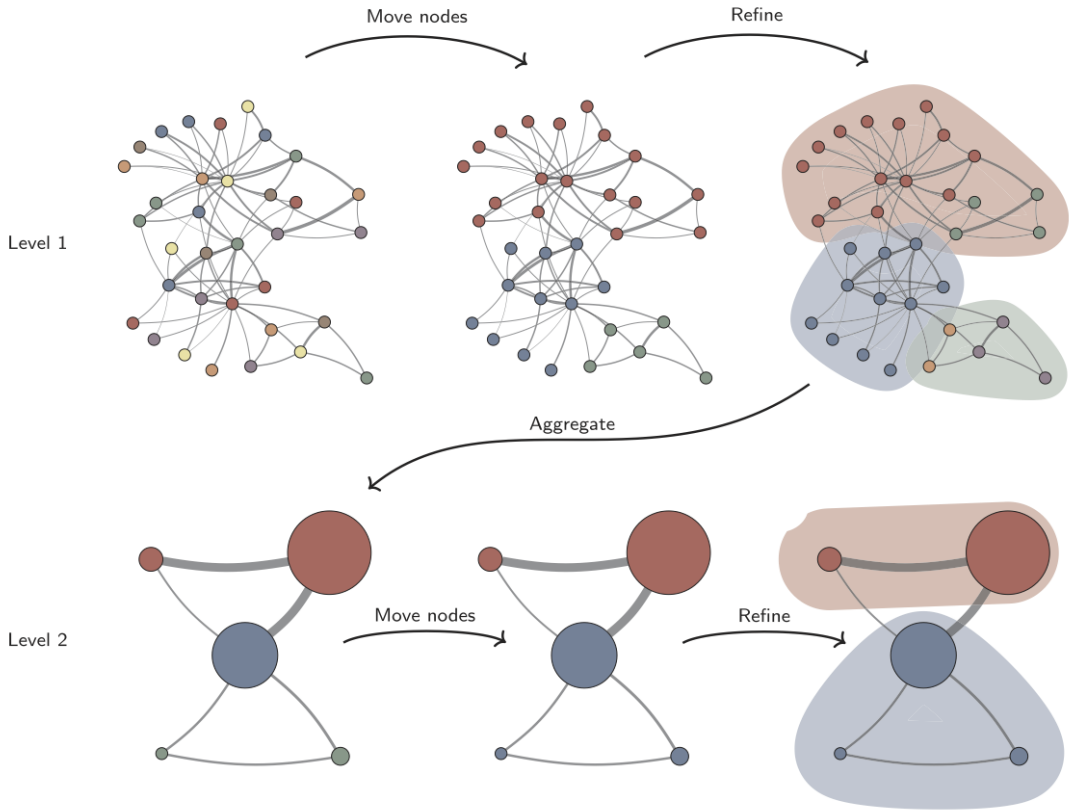
$$Q = \frac{1}{2m} \sum_{ij} (A_{ij} - \frac{k_i k_j}{2m}) \delta_{c_i c_j}$$



community *modularity*

— Leiden *modularity optimization* [TWVE19]

x. *improved* Louvain *algorithm* with *quality guarantees*

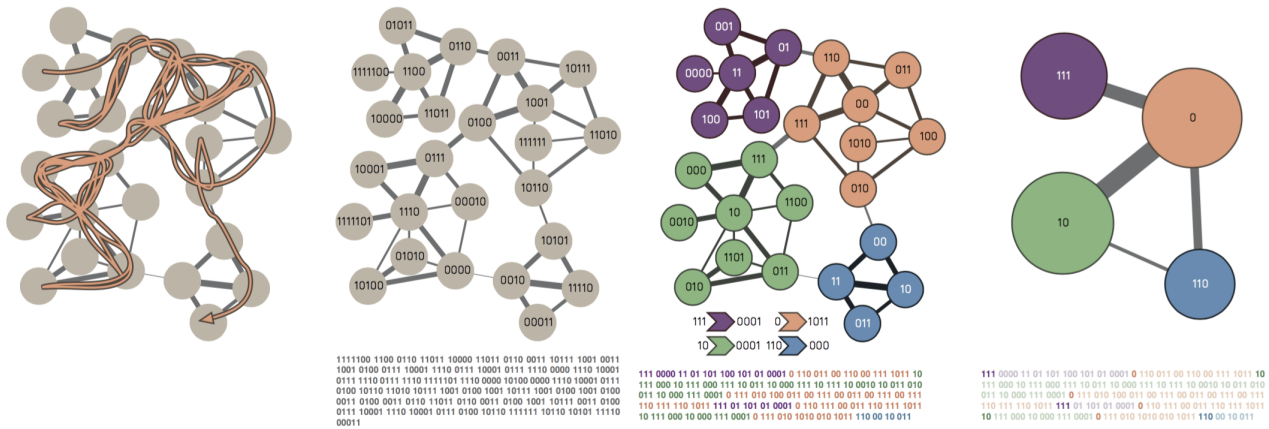


community *map equation*

— Infomap *map equation compression* [RB08]

1. set *node community* by *optimal coding* $\mathcal{O}(m \log m)$
2. *compress community nodes into supernodes* and repeat 1.
3. return *community structure maximizing map equation*

$$\mathcal{L} = \sum_i p_{i\curvearrowright} H(\widetilde{\mathcal{C}}) + \sum_i p_{i\hookleftarrow} H(\mathcal{C}_i)$$

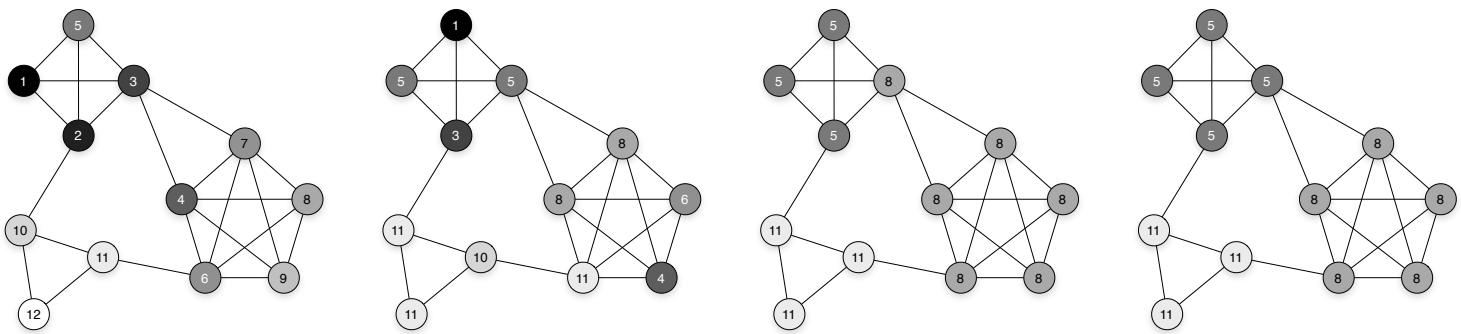


community *propagation*

— Raghavan *label propagation* [RAK07, ŠB11]

1. set *node community* by *neighbors frequency* $\mathcal{O}(cm)$
2. *randomly shuffle nodes* and repeat 1. *until convergence*
3. return *community structure* as *connected components*

$$\forall i : c_i = \arg\max_c \sum_j A_{ij}\delta_{c_j c}$$



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