## graph theory o *network science*

introduction to network science in Python (NetPy)

Lovro Šubelj University of Ljubljana 14th Dec 2021

## history graph theory

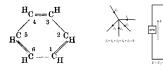
1736 seven *bridges of Königsberg* [Eul36] (Leonhard Euler)

1800s travelling salesman problem (William Hamilton)



1845 electrical circuit laws (Gustav Kirchhoff)

1857 chemical structure theory (August Kekulé)



## history operations research

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1956 shortest paths (Edsger Dijkstra)
1956 minimum spanning tree (Joseph Kruskal)
1956 maximum flow/minimum cut (Ford & Fulkerson)
1956 signed graph theory [CH56] (Cartwright & Harary)
1959 random graph theory [ER59] (Erdős & Rényi)
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## history sociometry

1934 children sociograms [Mor34] (Jacob Moreno)



1967 *small-world* experiment [Mil67] (Stanley Milgram) 1970 university *karate club* [Zac77] (Wayne Zachary)





1973 strength of *weak ties* [Gra73] (Mark Granovetter) 1977 measures of *centrality* [Fre77] (Linton Freeman)

#### revolution data

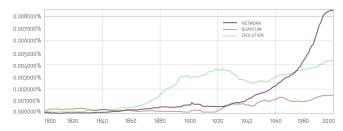
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< 2000 small graphs 10^2-10^3 nodes
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pprox 2000 communication networks  $10^5$ - $10^8$  nodes

 $\approx$  2005 online social networks 10<sup>8</sup> nodes

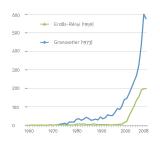
today  $Facebook\ graph > 10^9\ users$ 

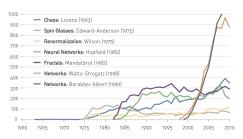
today  $Web graph > 10^{12}$  pages



#### revolution models

- 1959 random graph models [ER59]
- 1973 valued graphs models [Gra73]
- 1998 *small-world network* structure [WS98]
- 1999 scale-free network structure [BA99]





## revolution language

"A key discovery of network science is that the architecture of networks emerging in various domains of science, nature, and technology are similar to each other, a consequence of being governed by the same organizing principles. Consequently we can use a common set of tools to explore these systems."

Albert-László Barabási

"Networks are ideal structures to describe problems of organized complexity."

César A. Hidalgo

"I think the 21st century will be the century of complexity."

Stephen Hawking

#### network *science*

# problem understanding real networks

#### means

study of network properties design of mathematical models implementation of efficient algorithms

## goals

network structure and evolution nodes, links, fragments, clusters, layers, network network dynamics and processes spreading, diffusion, epidemics

## network analysis





text mining



computer vision



network analysis

### history references



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