

link *bridging*

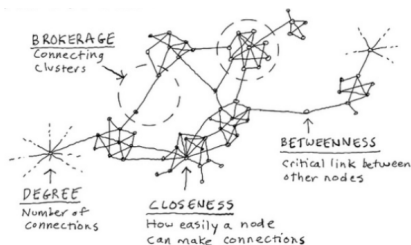
introduction to *network science in Python* (*NetPy*)

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bridging *measures*

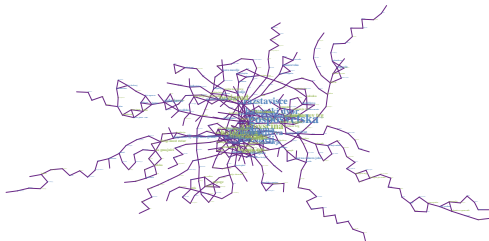
which *links* are most *important*?

- *link bridging measures* for (*un*)*directed* networks
 - *betweenness-based* centrality [Fre77, FBW91, New05]
- *link embeddedness measures* for (*un*)*directed* networks
 - *topological overlap* measures [RSM⁺02, OSH⁺07, dNMB11]



networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 2.72$ connections
- *giant component* 95.4% nodes (6 components)
- “*small-world*” with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- “*scale-free*” with $\gamma = 2.43$ for cutoff $k_{min} = 2$



* reduced to largest connected component of simple undirected graph

bridging *betweenness*

important *links* are *between other nodes*

- for (*un*)*directed* *G* *link betweenness* σ [Fre77] of $\{i, j\}$ is
 - g_{st} is number of *shortest paths* *between* *s* and *t*
 - g_{st}^{ij} is number of *such shortest paths* *through* $\{i, j\}$

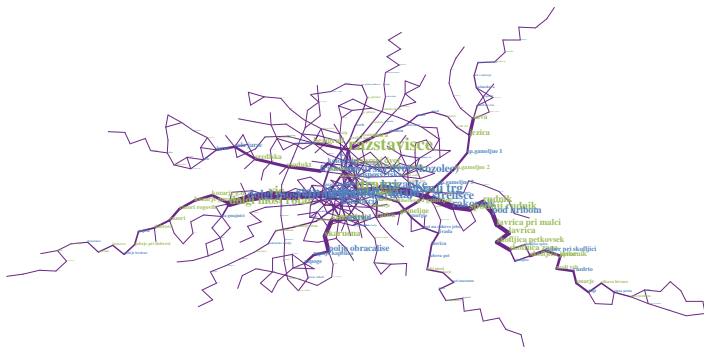
$$\sigma_{ij} = \sum_{st \notin \{i, j\}} \frac{g_{st}^{ij}}{g_{st}}$$

- σ considers *only shortest paths* [FBW91, New05]



networkology *betweenness*

- *link betweenness* σ in partial LPP network[†]
- *highest* $\sigma_{ij} = 0.176n^2$ link is {*Vič, Stan in dom*}



[†] reduced to largest connected component of simple undirected graph

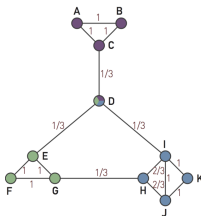
bridging *embeddedness*

important *links* are *embedded between nodes*

— for *undirected* G *link embeddedness*[‡] θ [OSH⁺07] of $\{i, j\}$ is

– Γ_i is set of *neighbors* or *neighborhood* of i

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{|\Gamma_i \cup \Gamma_j|} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \quad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$



[‡] θ better known as topological overlap index/weight

networkology *embeddedness*

bridging *overview*

which *links* are most *important*?

1 IA												18 VIIIA												
1	DC																						2	EC
	Degree Centrality																							Eigenvector Centrality
2	BC	CC											13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA						3	PR
	Betweenness Centrality	Closeness Centrality											SC	C _{COEF}	C _{COEF} ⁻¹	MNC	EC _{COEF}							PageRank
3	RL	IC											CC _{COEF}	SC ₀	LAC	DMNC	SEC _{COEF}						4	LR
	Range-Linked Betweenness	Information Centrality											old Subgraph Centrality	old Subgraph Centrality	loc. avg. Connectivity	dens. max. length comp.	sum of EC _{COEF}							LeaderRank
4	BN	RC	IG	DC _{0x}	BC _{0x}	CC _{0x}	EC _{0x}	KS _{0x}	PR _{0x}	IG _{0x}	RC _{0x}	DC _{0x}	BC _{0x}	SC ₁	KL	COC _{COEF}	PEC _{COEF}					5	KS	
	Betweenness Centrality	Radiality Centrality	Integration											new Subgraph Centrality	Clique Level	cooper. weight CO _{COEF}	PCC + EC _{COEF}						KatzRank	
5	RWBC	RWCC	CC _{2,3,4}	EC _{0x0x}	PR _{0x0x}	KS _{0x0x}	CC _{0x0x}	RC _{0x0x}	IG _{0x0x}	DC _{0x0x}	BC _{0x0x}	CC _{0x0x}	KS _{0x0x}	SC ₂	β	SC ₂	NC					6	EC ₃	
	RandomWalk Betweenness	RandomWalk Closeness	2,3,4-localized CC											Bipartivity	2-localized SC	Neighborhood Centrality	2-localized EC							
6	σ	ECC	WDC	DC _{0x0x}	CC _{0x0x}	BC _{0x0x}	KS _{0x0x}	PR _{0x0x}	IG _{0x0x}	DC _{0x0x}	BC _{0x0x}	CC _{0x0x}	KS _{0x0x}	SC ₃	LI	LI	EC ₃					7	EC ₄	
	sigma Centrality	Eccentricity	Weighted Degree											3-localized SC	Lobby Index	4-localized EC	4-localized EC							
7	BC _{2,3,4}	ECC ⁻¹	SDC	DC _{0x0x0x}	CC _{0x0x0x}	BC _{0x0x0x}	KS _{0x0x0x}	PR _{0x0x0x}	IG _{0x0x0x}	DC _{0x0x0x}	BC _{0x0x0x}	CC _{0x0x0x}	KS _{0x0x0x}	SC ₄	EC ₄									
	2,3,4-localized-BC	Inverse Eccentricity	Sphere Degree Centrality											4-localized SC										

Z	max	
C		Hybrid
Name		

22	23	24	25	26	27	117
FC	FD	US	DIS	ASS	DAM	UC
Functional Centrality	Functional Diversity	UniScore	Pairwise Dis-connectivity	Assortative Mixing	Damage	United compl. Centrality

28	29	30	31	13	116	118
EI	CM	NαC	MC	HGI	HYP	HC
Essentiality Index	Complexity Measure	Normalized α Centrality	Modular Centrality	Hungry Graph Information	Hyperbolic Index	Harmonic Centrality

Betweenness-based

Distance-based

Linear Combinations

Subgraph-based

Clustering Coefficient-based

Edge Clustering Coefficient-based

Spectral-based

Miscellaneous

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bridging *references*



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bridging *references*



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