graph theory o *network science*

introduction to network science in Python (NetPy)

Lovro Šubelj University of Ljubljana 3rd Dec 2022

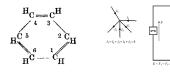
history graph theory

1736 seven *bridges of Königsberg* [Eul36] (Leonhard Euler) 1800s *travelling salesman* problem (William Hamilton)



1845 electrical circuit laws (Gustav Kirchhoff)

1857 chemical structure theory (August Kekulé)



history operations research

```
1956 shortest paths (Edsger Dijkstra)
1956 minimum spanning tree (Joseph Kruskal)
1956 maximum flow/minimum cut (Ford & Fulkerson)
1956 signed graph theory [CH56] (Cartwright & Harary)
1959 random graph theory [ER59] (Erdős & Rényi)
```

history sociometry

1934 children sociograms [Mor34] (Jacob Moreno)



1970 university karate club [Zac77] (Wayne Zachary)





1967 small-world experiment [Mil67] (Stanley Milgram)

1973 strength of weak ties [Gra73] (Mark Granovetter)

1977 measures of *centrality* [Fre77] (Linton Freeman)

revolution data

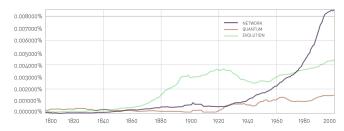
```
< 2000 small graphs 10^2-10^3 nodes
```

pprox 2000 communication networks 10^5 - 10^8 nodes

 \approx 2005 online social networks 10⁸ nodes

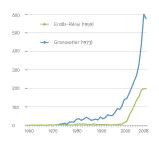
today Facebook graph > 10⁹ users

today $Web graph > 10^{12}$ pages



revolution models

- 1959 random graph models [ER59]
- 1973 valued graphs models [Gra73]
- 1998 *small-world network* structure [WS98]
- 1999 scale-free network structure [BA99]





revolution language

"A key discovery of network science is that the architecture of networks emerging in various domains of science, nature, and technology are similar to each other, a consequence of being governed by the same organizing principles. Consequently we can use a common set of tools to explore these systems."

Albert-László Barabási

"Networks are ideal structures to describe problems of organized complexity."

César A. Hidalgo

"I think the 21st century will be the century of complexity."

Stephen Hawking

network science

problem understanding real networks

means

study of network properties design of mathematical models implementation of efficient algorithms

goals

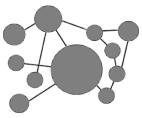
network structure and evolution nodes, links, fragments, clusters, layers, network network dynamics and processes spreading, diffusion, epidemics

network analysis











history references



A.-L. Barabási and R. Albert.

Emergence of scaling in random networks. *Science*, 286(5439):509–512, 1999.



A.-L. Barabási.

Network Science.

Cambridge University Press, Cambridge, 2016.



Dorwin Cartwright and Frank Harary.

Structural balance: A generalization of Heider's theory.

Psychological Review, 63(5):277–293, 1956.



David Easley and Jon Kleinberg.

Networks, Crowds, and Markets: Reasoning About a Highly Connected World. Cambridge University Press, Cambridge, 2010.



P. Erdős and A. Rényi.

On random graphs I.

Publ. Math. Debrecen, 6:290-297, 1959.



Leonhard Euler.

Solutio problematis ad geometriam situs pertinentis.

Comment. Academiae Sci. I. Petropolitanae, 8:128-140, 1736.



L. Freeman.

A set of measures of centrality based on betweenness. *Sociometry*, 40(1):35–41, 1977.



Mark S. Granovetter.

The strength of weak ties.

Am. J. Sociol., 78(6):1360-1380, 1973.

history references



César A. Hidalgo.

Disconnected, fragmented, or united? A trans-disciplinary review of network science. *Appl. Netw. Sci.*, 1:6, 2016.



Stanley Milgram.

The small world problem. Psychol. Today, 1(1):60–67, 1967.



J. L. Moreno.

Who Shall Survive? Beacon House, Beacon, 1934.



Mark E. J. Newman.

Networks: An Introduction.
Oxford University Press, Oxford, 2010.



D. J. Watts and S. H. Strogatz.

Collective dynamics of 'small-world' networks.

Nature, 393(6684):440-442, 1998.



Wayne W. Zachary.

An information flow model for conflict and fission in small groups.

J. Anthropol. Res., 33(4):452-473, 1977.