

link *bridging*

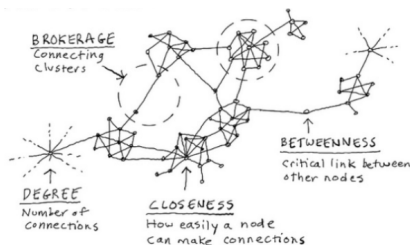
introduction to *network science in Python* (*NetPy*)

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# bridging *measures*

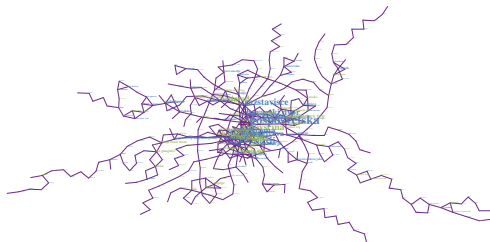
which *links* are most *important*?

- *link bridging measures* for (un)directed networks
  - *betweenness-based* centrality [Fre77, FBW91, New05]
- *link embeddedness measures* for (un)directed networks
  - *topological overlap* measures [RSM<sup>+</sup>02, OSH<sup>+</sup>07, dNMB11]



# networkology *LPP*

- partial *LPP public bus transport network*\*
- $n = 416$  bus stops with  $\langle k \rangle = 2.72$  connections
- *giant component* 95.4% nodes (6 components)
- “*small-world*” with  $\langle C \rangle = 0.09$  and  $\langle d \rangle = 14.26$
- “*scale-free*” with  $\gamma = 2.43$  for cutoff  $k_{min} = 2$



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\* reduced to largest connected component of simple undirected graph

# bridging *betweenness*

important *links* are *between other nodes*

- for (*un*)*directed* *G* *link betweenness*  $\sigma$  [Fre77] of  $\{i, j\}$  is
  - $g_{st}$  is number of *shortest paths between* *s* and *t*
  - $g_{st}^{ij}$  is number of *such shortest paths through*  $\{i, j\}$

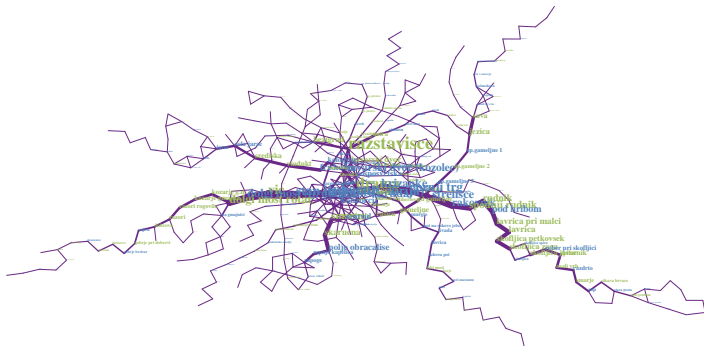
$$\sigma_{ij} = \sum_{st \notin \{i, j\}} \frac{g_{st}^{ij}}{g_{st}}$$

- $\sigma$  considers *only shortest paths* [FBW91, New05]



# networkology *betweenness*

- *link betweenness*  $\sigma$  in partial LPP network<sup>†</sup>
- *highest*  $\sigma_{ij} = 0.176n^2$  link is {*Vič, Stan in dom*}



<sup>†</sup>reduced to largest connected component of simple undirected graph

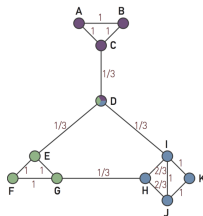
# bridging *embeddedness*

important *links* are *embedded between nodes*

— for *undirected*  $G$  *link embeddedness*<sup>‡</sup>  $\theta$  [OSH<sup>+</sup>07] of  $\{i, j\}$  is

–  $\Gamma_i$  is set of *neighbors* or *neighborhood* of  $i$

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{|\Gamma_i \cup \Gamma_j|} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \quad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$



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<sup>‡</sup>  $\theta$  better known as topological overlap index/weight

networkology *embeddedness*

# bridging *overview*

which *links* are most *important*?

1 IA												18 VIIIA													
1	DC																						2	EC	
	Degree Centrality																							Eigenvector Centrality	
2	BC	CC											13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA						3	PR	
	Betweenness Centrality	Closeness Centrality											SC	C <sub>coef</sub>	C <sub>coef</sub> <sup>-1</sup>	MNC	EC <sub>coef</sub>							PageRank	
3	RL	IC											11 IB	12 IIB											
	Range-Linked Betweenness	Information Centrality											SC <sub>ox</sub>	C <sub>coef</sub> <sub>ox</sub>	C <sub>coef</sub> <sub>ox</sub> <sup>-1</sup>	MNC <sub>ox</sub>	EC <sub>coef</sub> <sub>ox</sub>							LeaderRank	
4	BN	RC	IG	DC <sub>ox</sub>	BC <sub>ox</sub>	CC <sub>ox</sub>	EC <sub>ox</sub>	KS <sub>ox</sub>	PR <sub>ox</sub>	IG <sub>ox</sub>	RC <sub>ox</sub>	DC <sub>ox</sub>	BC <sub>ox</sub>	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA							
	Betweenness Centrality	Radiality Centrality	Integration											SC <sub>ox</sub>	C <sub>coef</sub> <sub>ox</sub>	C <sub>coef</sub> <sub>ox</sub> <sup>-1</sup>	MNC <sub>ox</sub>	EC <sub>coef</sub> <sub>ox</sub>					KS		
5	RWBC	RWCC	CC <sub>2,3,4</sub>	EC <sub>2,3,4</sub>	PR <sub>2,3,4</sub>	KS <sub>2,3,4</sub>	CC <sub>2,3,4</sub>	RC <sub>2,3,4</sub>	IG <sub>2,3,4</sub>	DC <sub>2,3,4</sub>	BC <sub>2,3,4</sub>	EC <sub>2,3,4</sub>	KS <sub>2,3,4</sub>	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA							
	RandomWalk Betweenness	RandomWalk Closeness	2,3,4-localized CC											SC <sub>2,3,4</sub>	C <sub>coef</sub> <sub>2,3,4</sub>	C <sub>coef</sub> <sub>2,3,4</sub> <sup>-1</sup>	MNC <sub>2,3,4</sub>	EC <sub>coef</sub> <sub>2,3,4</sub>					KS		
6	$\sigma$	ECC	WDC	DC <sub>2,3,4</sub>	CC <sub>2,3,4</sub>	KS <sub>2,3,4</sub>	CC <sub>2,3,4</sub>	RC <sub>2,3,4</sub>	IG <sub>2,3,4</sub>	DC <sub>2,3,4</sub>	BC <sub>2,3,4</sub>	EC <sub>2,3,4</sub>	KS <sub>2,3,4</sub>	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA							
	sigma Centrality	Eccentricity	Weighted Degree											SC <sub>2,3,4</sub>	C <sub>coef</sub> <sub>2,3,4</sub>	C <sub>coef</sub> <sub>2,3,4</sub> <sup>-1</sup>	MNC <sub>2,3,4</sub>	EC <sub>coef</sub> <sub>2,3,4</sub>					KS		
7	BC <sub>2,3,4</sub>	ECC <sup>-1</sup>	SDC	DC <sub>2,3,4</sub>	CC <sub>2,3,4</sub>	KS <sub>2,3,4</sub>	CC <sub>2,3,4</sub>	RC <sub>2,3,4</sub>	IG <sub>2,3,4</sub>	DC <sub>2,3,4</sub>	BC <sub>2,3,4</sub>	EC <sub>2,3,4</sub>	KS <sub>2,3,4</sub>	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA							
	2,3,4-localized-BC	Inverse Eccentricity	Sphere Degree Centrality											SC <sub>2,3,4</sub>	C <sub>coef</sub> <sub>2,3,4</sub>	C <sub>coef</sub> <sub>2,3,4</sub> <sup>-1</sup>	MNC <sub>2,3,4</sub>	EC <sub>coef</sub> <sub>2,3,4</sub>					KS		

z

m

C

Name

Hybrid

22	FC	23	FD	24	US	25	DIS	26	ASS	27	DAM	117	UC
	Functional Centrality		Functional Diversity		UniScore		Pairwise Disconnectedness		Assortative Mixing		Damage		United Comp. Centrality

28	EI	29	CM	30	N $\alpha$ C	31	MC	13	HGI	116	HYP	118	HC
	Essentiality Index		Complexity Measure		Normalized $\alpha$ Centrality		Modular Centrality		Hungry Graph Information		Hyperbolic Index		Harmonic Centrality

Betweenness-based

Distance-based

Linear Combinations

Subgraph-based

Clustering Coefficient-based

Edge Clustering Coefficient-based

Spectral-based

Miscellaneous

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# bridging *references*



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