

node *centrality*

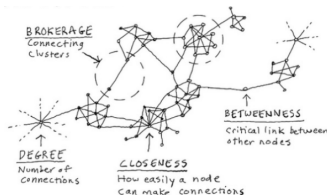
introduction to *network science in Python* (*NetPy*)

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# centrality *measures*

which *nodes* are most *important*?

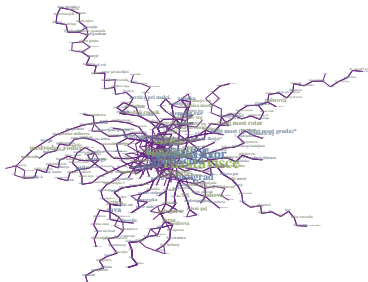
- *node centrality measures* for (*un*)*directed* networks
  - *clustering coefficients* [WS98, SV05, dNMB05]
  - *distance-based* centrality [Fre77, FBW91, New05]
  - *spectral analysis* centrality [Kat53, Bon87, BP98]



- *link analysis algorithms* for *directed* networks

# networkology *LPP*

- partial *LPP public bus transport network*\*
- $n = 416$  bus stops with  $\langle k \rangle = 5.62$  connections
- *giant component* 95.4% nodes (6 components)
- “*small-world*” with  $\langle C \rangle = 0.09$  and  $\langle d \rangle = 14.26$
- “*scale-free*” with  $\gamma = 2.62$  for cutoff  $k_{min} = 5$



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\* reduced to largest connected component

## centrality *clustering*

important *nodes* are *strongly embedded*

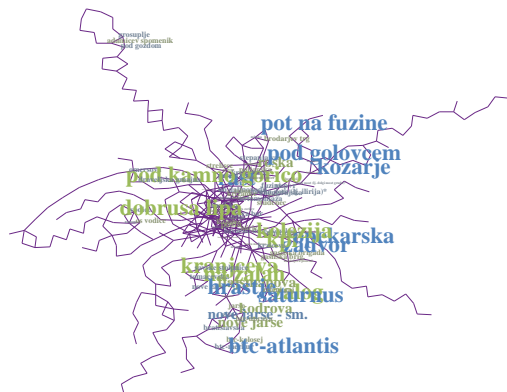
- for *undirected*  $G$  *clustering coefficient*  $C$  [WS98] of  $i$  is
  - $t_i$  is number of *linked neighbors* or *triangles* of  $i$

$$C_i = \frac{2t_i}{k_i(k_i-1)} \quad C_i = 0 \text{ for } k_i \leq 1$$

- $C$  fails for *hub nodes* in *scale-free* networks [dNMB05]

# networkology *clustering*

- *clustering coefficient*  $C$  in partial LPP network<sup>†</sup>
- *highest*  $C_i = 1.0$  nodes are *Na Žalah etc.* with  $k_i = 2$



<sup>†</sup> reduced to simple undirected graph

## centrality *closeness*

important *nodes* are *close to other* nodes

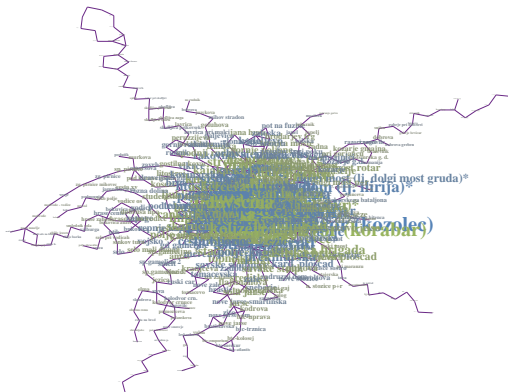
- for (*un*)*directed*  $G$  *closeness centrality*  $\ell^{-1}$  [New10] of  $i$  is
  - $d_{ij}$  is (*un*)*directed distance* between  $i$  and  $j$
  - $d_{ij} = \infty$  for nodes in *different components*

$$\ell_i^{-1} = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

- $\ell^{-1}$  spans *small range* in *small-world* networks

# networkology *closeness*

- *closeness centrality*  $\ell^{-1}$  in partial LPP network<sup>‡</sup>
- *highest*  $\ell_i^{-1} = 0.208$  node is *Gospovetska* with  $k_i = 14$



<sup>‡</sup> reduced to simple undirected graph

## centrality *betweenness*

important *nodes* are *bridges between other* nodes

- for (*un*)*directed*  $G$  *betweenness centrality*  $\sigma$  [Fre77] of  $i$  is
  - $g_{st}$  is number of *shortest paths between*  $s$  and  $t$
  - $g_{st}^i$  is number of *such shortest paths through*  $i$

$$\sigma_i = \frac{1}{n^2} \sum_{st} \frac{g_{st}^i}{g_{st}}$$

- $\sigma$  considers *only shortest paths* [FBW91, New05]





# centrality *degrees*

important *nodes* are *linked by many* nodes

- for *undirected*  $G$  *degree centrality*  $d$  of  $i$  is

$$d_i = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i}{n-1}$$

- in *directed*  $G$  *in-degree centrality*  $d^{in}$  of  $i$  is

$$d_i^{in} = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i^{in}}{n-1}$$

- in *directed*  $G$  *out-degree centrality*  $d^{out}$  of  $i$  is

$$d_i^{out} = \frac{1}{n-1} \sum_{j \neq i} A_{ji} = \frac{k_i^{out}}{n-1}$$



# centrality *eigenvector*

important *nodes* are *linked by important* nodes

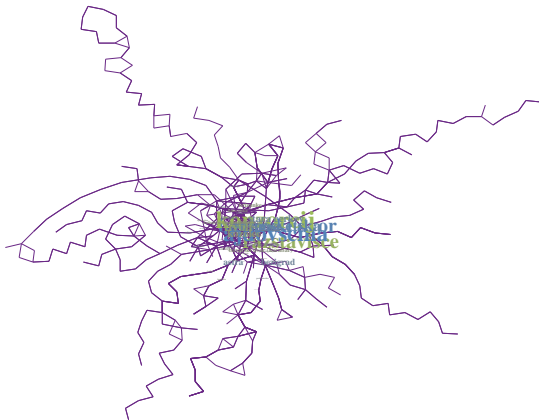
- for (*un*)*directed*  $G$  *eigenvector centrality*  $e$  [Bon87] of  $i$  is
  - $e$  is *leading eigenvector*  $v_1$  of  $A$  with *eigenvalue*  $\lambda_1^{-1}$

$$e_i = \lambda_1^{-1} \sum_j A_{ij} e_j$$

- in *directed*  $G$   $e = 0$  for  $k^{in} = 0$  *nodes etc.*

networkology *eigenvector*

- *eigenvector centrality*  $e$  in partial LPP network
- *highest*  $e_j = 0.082$  node is *Konzorcij* with  $k_j = 30$



## centrality *Katz*

*nodes* get small amount of *importance* for free

- for (un)directed  $G$  *Katz centrality*  $z$  [Kat53] of  $i$  is
  - $\alpha$  and  $\beta$  are *appropriate positive constants*

$$z_i = \alpha \sum_j A_{ij} z_j + \beta$$

- for *convenience*  $\beta = 1$  whereas  $\alpha < \lambda_1^{-1}$ 
  - $\lambda_1$  is *leading eigenvalue* of  $A$  for *eigenvector*  $v_1$

# centrality *PageRank*

*nodes distribute equal amount of importance*

- for (un)directed  $G$  *PageRank centrality*  $p$  [BP98] of  $i$  is
  - $\alpha$  and  $\beta$  are *appropriate positive constants*

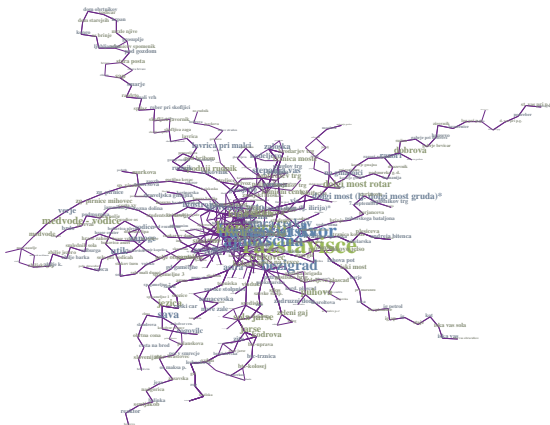
$$p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j} + \beta$$

- for *convenience*  $\beta = \frac{1-\alpha}{n}$  whereas  $\alpha = 0.85$
- $p$  probability of *random surfer with teleports*

see PageRank algorithm *NetLogo* demo

# networkology *PageRank*

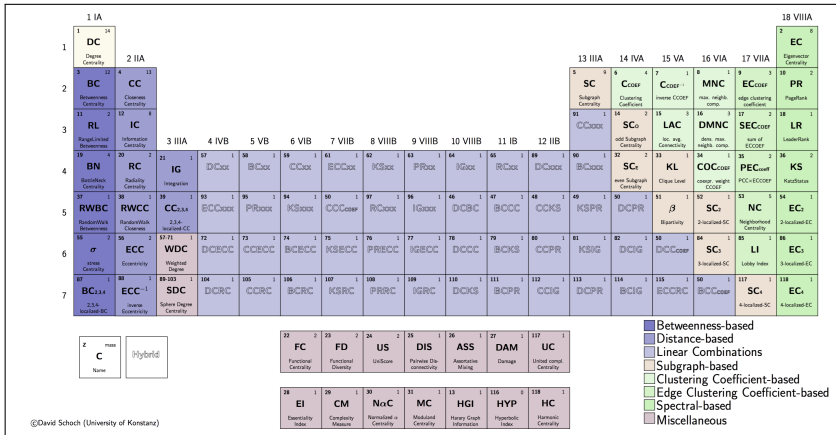
- *PageRank centrality*  $p$  in partial LPP network
- *highest*  $p_i = 0.011$  node is *Razstavišče* with  $k_i = 41$





centrality *overview*

which *nodes* are most *important*?



# centrality *references*



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# centrality *references*



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