# link *bridging*

introduction to *network science in Python* (*NetPy*)

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bridging *measures*

which *links* are most *important*?

— *link bridging measures* for (*un*)*directed* networks
  – *betweenness-based* centrality [Fre77, FBW91, New05]

— *link embeddedness measures* for (*un*)*directed* networks
  – *topological overlap* measures [RSM⁺02, OSH⁺07, dNMB11]
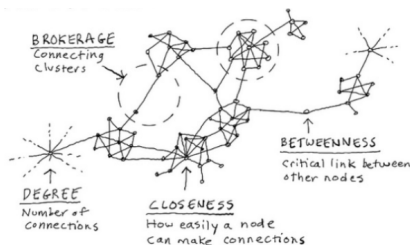
networkology *LPP*

— partial *LPP public bus transport network*[*]
— $n = 416$ bus stops with $\langle k \rangle = 2.72$ connections
— *giant component* 95.4% nodes (6 components)
— "*small-world*" with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
— "*scale-free*" with $\gamma = 2.43$ for cutoff $k_{min} = 2$

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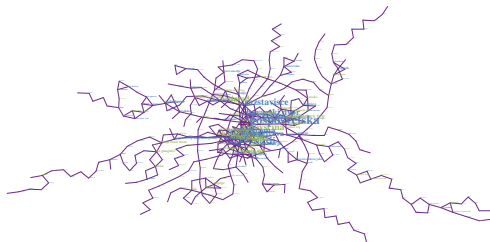
[*]reduced to largest connected component of simple undirected graph

bridging *betweenness*

important *links* are *between other nodes*

— for (*un*)*directed G link betweenness* σ [Fre77] of $\{i,j\}$ is
  – g_{st} is number of *shortest paths between s* and *t*
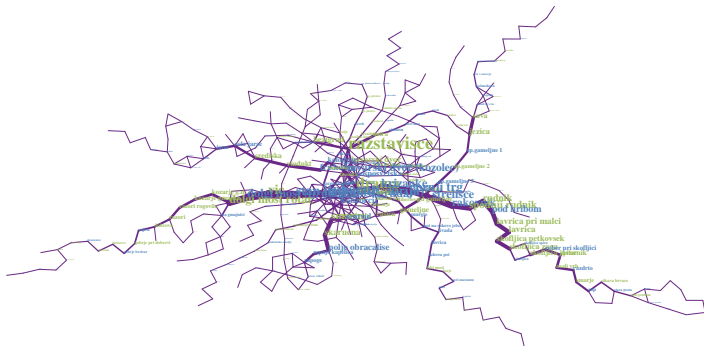 – g_{st}^{ij} is number of *such shortest paths through* $\{i,j\}$

$$\sigma_{ij} = \sum_{st \notin \{i,j\}} \frac{g_{st}^{ij}}{g_{st}}$$

— σ considers *only shortest paths* [FBW91, New05]

networkology *betweenness*

— *link betweenness σ* in partial LPP network[†]
— *highest $\sigma_{ij} = 0.176n^2$* link is {*Vič, Stan in dom*}

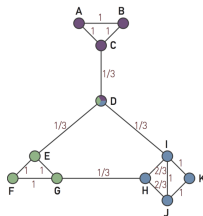[†]reduced to largest connected component of simple undirected graph

bridging *embeddedness*

important *links* are *embedded between nodes*

— for *undirected G link embeddedness*[‡] θ [OSH[+]07] of $\{i, j\}$ is

– Γ_i is set of *neighbors* or *neighborhood* of i

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{|\Gamma_i \cup \Gamma_j|} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \qquad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$

[‡] θ better known as topological overlap index/weight

networkology *embeddedness*

— *link embeddedness θ in partial LPP network*[§]

— *highest $\theta_{ij} = 1.0$ links are {Zalog, Saturnus} etc.*

[§]reduced to largest connected component of simple undirected graph

bridging *overview*

which *links* are most *important*?

bridging *references*

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bridging *references*

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