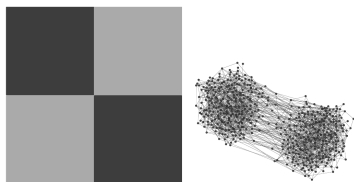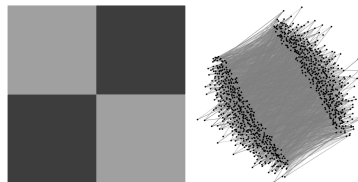# *core-periphery* structure

introduction to *network science in Python* (*NetPy*)

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4th July 2024
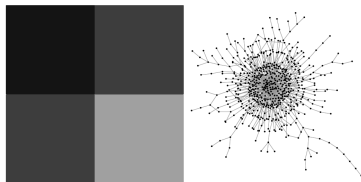
core-periphery *block model*
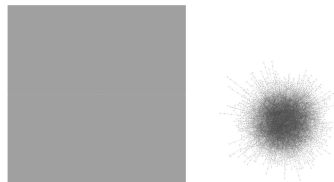

community block model [GN02]

disassortative (bipartite) block model [NL07]

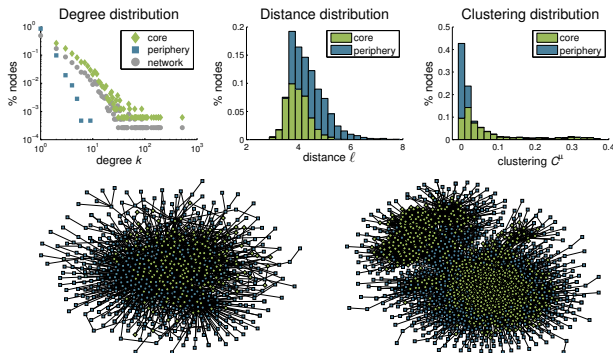*core-periphery block model* [Sei83]

random block model [ER59]

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*origin of core-periphery structure in international relations (e.g. trade)

core-periphery *structure*

— *core*/*periphery* *nodes* have *higher*/*lower* *degrees k*
— *core*/*periphery* *nodes* are on *shorter*/*longer* *distances ℓ*
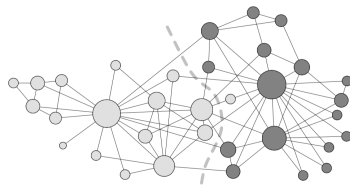— *core*/*periphery* *nodes* have *higher*/*lower* *clustering C*

core-periphery *SBM*

— $G(\{C_1, C_2\}, \{p_{11}, p_{12}, p_{22}\})$ *stochastic block model* [HLL83]
  – n_i is *size* of *cluster* C_i & p_{ij} is *link density* between C_i and C_j

— *density-based core-periphery* structure for $p_{11} \gg p_{12} \gg p_{22}$
— *lookalike core-periphery* for $n_1 p_{11} \gg 1$, $n_1 p_{12} \ll 1$, $n_2 p_{22} \approx 1$
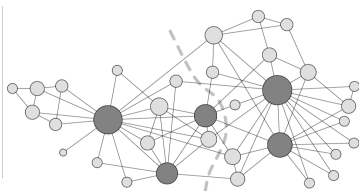
non-corrected block model $p_{11} > p_{12} > p_{22}$ *degree-corrected block model $p_{11} \approx p_{22} > p_{12}$*

core-periphery *k-cores*

— *k-cores* are *subgraphs of nodes* with $\geq k$ *neighbors* [Sei83]

remove nodes with degree $< k$ until no such node remains [BZ11]

— *k-shells* are *nodes of k-cores* that are *not in $k + 1$-cores*

— *k-cores* are *nested* while *k-shells* form *decomposition*
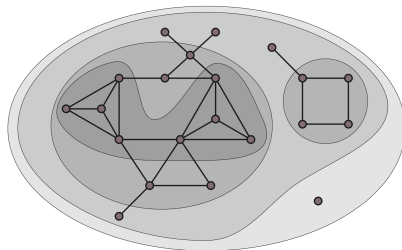
1-cores are connected components w/o isolates & *k*-cores can be disconnected

core-periphery *nestedness*
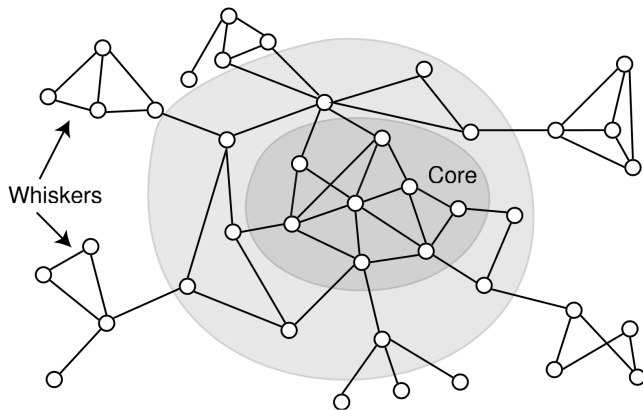
nested cores & whiskers communities [LLDM09, YL13]

core-periphery *references*

V. Batagelj and M. Zaveršnik.
An O(m) algorithm for cores decomposition of networks.
Adv. Data Anal. Classif., 5(2):129–145, 2011.

P. Erdős and A. Rényi.
On random graphs I.
Publ. Math. Debrecen, 6:290–297, 1959.

M. Girvan and M. E. J Newman.
Community structure in social and biological networks.
P. Natl. Acad. Sci. USA, 99(12):7821–7826, 2002.

Paul W. Holland, Kathryn Blackmond Laskey, and Samuel Leinhardt.
Stochastic blockmodels: First steps.
Soc. Networks, 5(2):109–137, 1983.

Jure Leskovec, Kevin J Lang, Anirban Dasgupta, and Michael W Mahoney.
Community structure in large networks: Natural cluster sizes and the absence of large well-defined clusters.
Internet Math., 6(1):29–123, 2009.

Tilen Marc and Lovro Šubelj.
Convexity in complex networks.
Netw. Sci., 6(2):176–203, 2018.

M. E. J Newman and E. A Leicht.
Mixture models and exploratory analysis in networks.
P. Natl. Acad. Sci. USA, 104(23):9564–9569, 2007.

Stephen B. Seidman.
Network structure and minimum degree.
Soc. Networks, 5(3):269–287, 1983.

J. Yang and Jure Leskovec.
Overlapping community detection at scale: A nonnegative matrix factorization approach.
In *Proceedings of the ACM International Conference on Web Search and Data Mining*, pages 587–596, Rome, Italy, 2013.