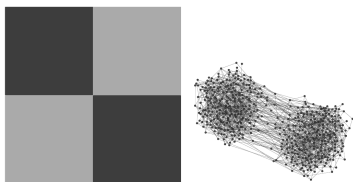


core-periphery structure

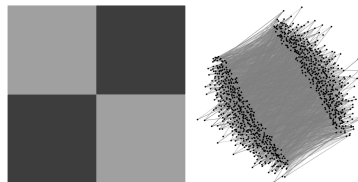
introduction to *network science in Python* (*NetPy*)

Lovro Šubelj
University of Ljubljana
26th September 2024

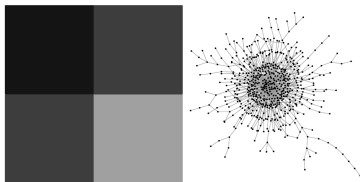
core-periphery *block model*



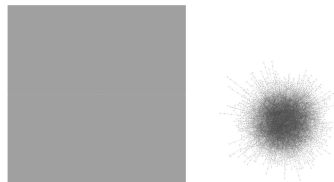
community block model [GN02]



disassortative (bipartite) block model [NL07]



core-periphery block model [Sei83]

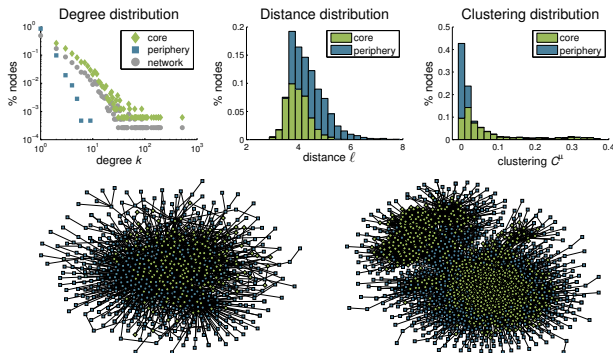


random block model [ER59]

* origin of core-periphery structure in international relations (e.g. trade)

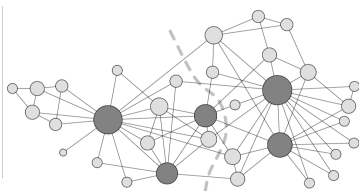
core-periphery *structure*

- *core/periphery nodes* have *higher/lower degrees* k
- *core/periphery nodes* are on *shorter/longer distances* ℓ
- *core/periphery nodes* have *higher/lower clustering* C^i

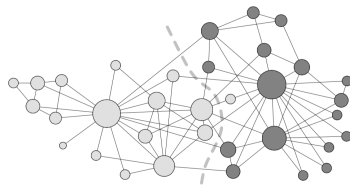


core-periphery *SBM*

- $G(\{C_1, C_2\}, \{p_{11}, p_{12}, p_{22}\})$ *stochastic block model* [HLL83]
 - n_i is *size* of *cluster* C_i & p_{ij} is *link density* between C_i and C_j
- *density-based core-periphery* structure for $p_{11} \gg p_{12} \gg p_{22}$
- *lookalike core-periphery* for $n_1 p_{11} \gg 1, n_1 p_{12} \ll 1, n_2 p_{22} \approx 1$



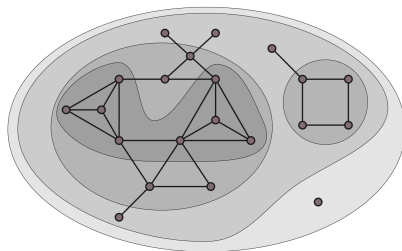
non-corrected block model $p_{11} > p_{12} > p_{22}$



degree-corrected block model $p_{11} \approx p_{22} > p_{12}$

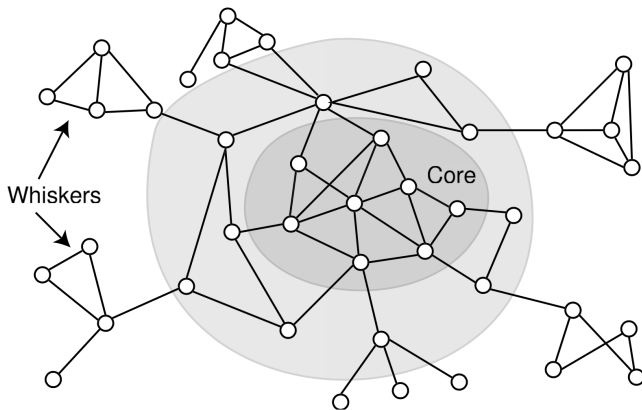
core-periphery *k*-cores

- *k*-cores are subgraphs of nodes with $\geq k$ neighbors [Sei83]
remove nodes with degree $< k$ until no such node remains [BZ11]
- *k*-shells are nodes of *k*-cores that are not in $k + 1$ -cores
- *k*-cores are nested while *k*-shells form decomposition



1-cores are connected components without isolates & *k*-cores can be disconnected

core-periphery *nestedness*



nested cores & whiskers communities [LLDM09, YL13]

core-periphery *references*



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core-periphery *references*



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