

node *centrality*

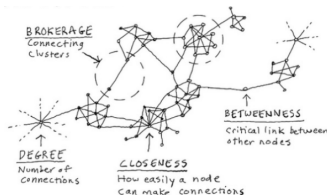
introduction to *network science in Python* (*NetPy*)

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centrality *measures*

which *nodes* are most *important*?

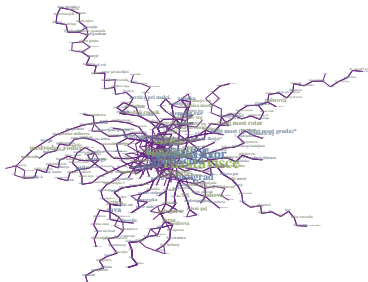
- *node centrality measures* for (*un*)*directed* networks
 - *clustering coefficients* [WS98, SV05, dNMB05]
 - *geodesic-based* measures [Fre77, FBW91, New05]
 - *spectral analysis* measures [Kat53, Bon87, BP98]



- *link analysis algorithms* primarily for *directed* networks

networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 5.62$ connections
- *giant component* 95.4% nodes (6 components)
- “*small-world*” with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- “*scale-free*” with $\gamma = 2.62$ for cutoff $k_{min} = 5$



* reduced to largest connected component

centrality *clustering*

important *nodes* are *strongly embedded*

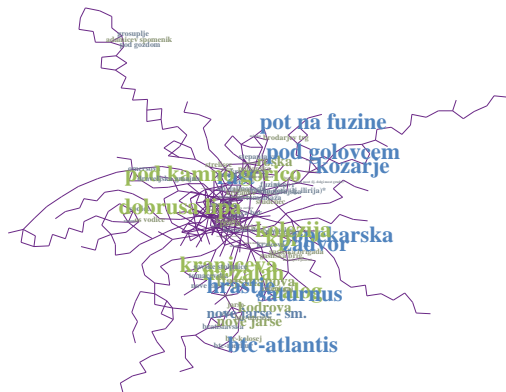
- for *undirected* G *clustering coefficient* C [WS98] of i is
 - t_i is number of *linked neighbors* or *triangles* of i

$$C_i = \frac{2t_i}{k_i(k_i-1)} \quad C_i = 0 \text{ for } k_i \leq 1$$

- C fails for *hub nodes* in *scale-free* networks [dNMB05]

networkology *clustering*

- *clustering coefficient* C in partial LPP network[†]
- *highest* $C_i = 1.0$ nodes are *Na Žalah etc.* with $k_i = 2$



[†] reduced to simple undirected graph

centrality *closeness*

important *nodes* are *close to other* nodes

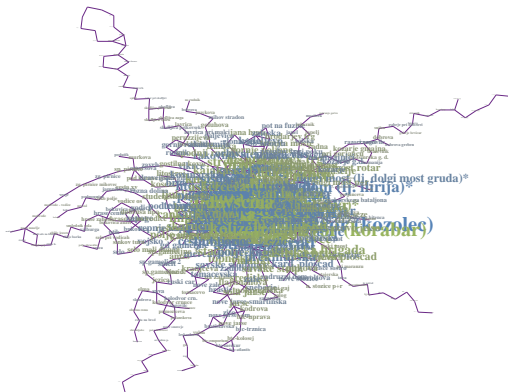
- for (*un*)*directed* G *closeness centrality* ℓ^{-1} [New10] of i is
 - d_{ij} is (*un*)*directed distance* between i and j
 - $d_{ij} = \infty$ for nodes in *different components*

$$\ell_i^{-1} = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

- ℓ^{-1} spans *small range* in *small-world* networks

networkology *closeness*

- *closeness centrality* ℓ^{-1} in partial LPP network[‡]
- *highest* $\ell_i^{-1} = 0.208$ node is *Gospodsvetska* with $k_i = 14$



[‡] reduced to simple undirected graph

centrality *betweenness*

important *nodes* are *bridges between other* nodes

- for (*un*)*directed* G *betweenness centrality* σ [Fre77] of i is
 - g_{st} is number of *shortest paths between* s and t
 - g_{st}^i is number of *such shortest paths through* i

$$\sigma_i = \frac{1}{n^2} \sum_{st} \frac{g_{st}^i}{g_{st}}$$

- σ considers *only shortest paths* [FBW91, New05]

centrality *degrees*

important *nodes* are *linked by many* nodes

- for *undirected* G *degree centrality* d of i is

$$d_i = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i}{n-1}$$

- in *directed* G *in-degree centrality* d^{in} of i is

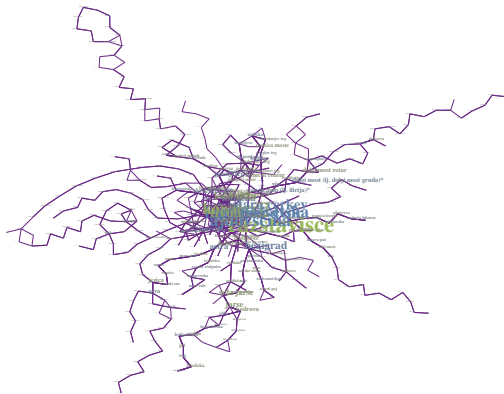
$$d_i^{in} = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i^{in}}{n-1}$$

- in *directed* G *out-degree centrality* d^{out} of i is

$$d_i^{out} = \frac{1}{n-1} \sum_{j \neq i} A_{ji} = \frac{k_i^{out}}{n-1}$$

networkology *degrees*

- *degree centrality* d in partial LPP network
- *highest* $d_i = 0.099$ node is *Razstavišče* with $k_i = 41$
- *highest* d_i node is *Razstavišče* with $k_i^{in} = 20$ and $k_i^{out} = 21$



centrality *eigenvector*

important *nodes* are *linked by important* nodes

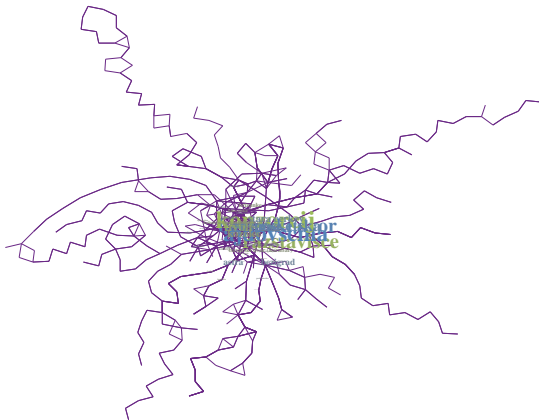
- for (*un*)*directed* G *eigenvector centrality* e [Bon87] of i is
 - e is *leading eigenvector* v_1 of A with *eigenvalue* λ_1^{-1}

$$e_i = \lambda_1^{-1} \sum_j A_{ij} e_j$$

- in *directed* G $e = 0$ for $k^{in} = 0$ *nodes etc.*

networkology *eigenvector*

- *eigenvector centrality* e in partial LPP network
- *highest* $e_j = 0.082$ node is *Konzorcij* with $k_j = 30$



centrality *Katz*

nodes *get* small amount of *importance* *for free*

- for (*un*)*directed* *G* *Katz centrality* *z* [Kat53] of *i* is
 - α and β are *appropriate positive constants*

$$z_i = \alpha \sum_j A_{ij} z_j + \beta$$

- for *convenience* $\beta = 1$ whereas $\alpha < \lambda_1^{-1}$
 - λ_1 is *leading eigenvalue* of *A* for *eigenvector* v_1

centrality *PageRank*

nodes distribute equal amount of *importance*

- for (un)directed G *PageRank centrality* p [BP98] of i is
 - α and β are *appropriate positive constants*

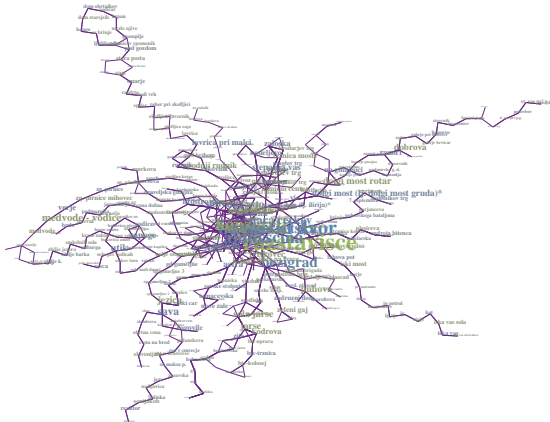
$$p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j} + \beta$$

- for *convenience* $\beta = \frac{1-\alpha}{n}$ whereas $\alpha = 0.85$
- p probability of *random surfer with teleports*

see PageRank algorithm *NetLogo* demo

networkology *PageRank*

- *PageRank centrality* p in partial LPP network
- *highest* $p_i = 0.011$ node is *Razstavišče* with $k_i = 41$



centrality *overview*

which *nodes* are most *important*?

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