graph theory o *network science*

introduction to network science in Python (NetPy)

Lovro Šubelj University of Ljubljana 4th July 2024

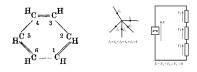
history graph theory

1736 seven *bridges of Königsberg* [Eul36] (Leonhard Euler) 1800s *travelling salesman* problem (William Hamilton)



1845 electrical circuit laws (Gustav Kirchhoff)

1857 chemical structure theory (August Kekulé)



history operations research

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1956 shortest paths (Edsger Dijkstra)
1956 minimum spanning tree (Joseph Kruskal)
1956 maximum flow/minimum cut (Ford & Fulkerson)
1956 signed graph theory [CH56] (Cartwright & Harary)
1959 random graph theory [ER59] (Erdős & Rényi)
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history sociometry

1934 children sociograms [Mor34] (Jacob Moreno)



1970 university karate club [Zac77] (Wayne Zachary)





1967 *small-world* experiment [Mil67] (Stanley Milgram) 1973 strength of *weak ties* [Gra73] (Mark Granovetter)

1973 Strength of Weak ties [Gra75] (Mark Granovetter)

1977 measures of *centrality* [Fre77] (Linton Freeman)

revolution data

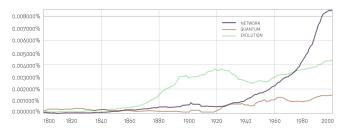
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< 2000 small graphs 10^2-10^3 nodes
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pprox 2000 communication networks 10^5 - 10^8 nodes

 \approx 2005 online social networks 10⁸ nodes

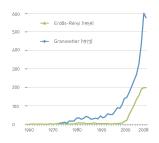
today Facebook graph > 10⁹ users

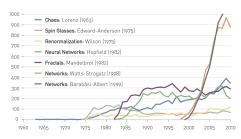
today $Web graph > 10^{12}$ pages



revolution models

- 1959 random graph models [ER59]
- 1973 valued graph models [Gra73]
- 1998 *small-world network* structure [WS98]
- 1999 scale-free network structure [BA99]





revolution language

"A key discovery of network science is that the architecture of networks emerging in various domains of science, nature, and technology are similar to each other, a consequence of being governed by the same organizing principles. Consequently we can use a common set of tools to explore these systems."

Albert-László Barabási

"Networks are ideal structures to describe problems of organized complexity."

César A. Hidalgo

"I think the 21st century will be the century of complexity."

Stephen Hawking

network *science*

problem understanding real networks

means

study of network properties design of mathematical models implementation of efficient algorithms

goals

network structure and evolution nodes, links, fragments, clusters, layers, network network dynamics and processes spreading, diffusion, epidemics

network *analysis*





text mining



computer vision



network analysis

history references



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