link analysis

introduction to *network science in Python* (*NetPy*)

Lovro Šubelj
University of Ljubljana
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link *analysis*

which *web pages* are most *important*?

- *node centrality measures* for (*un*)*directed* networks
- *link analysis algorithms* primarily for *directed web graphs*
  - Google *search ranking PageRank* [BP98, PBMW99]
  - hyperlink-induced *topic search HITS* [Kle99]

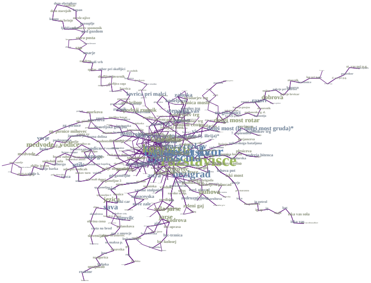
Sergey Brin

Lawrence Page

Jon Kleinberg

analysis *LPP*

- corrected *LPP public bus transport network**
- $n = 408$ bus stops with $\langle k \rangle = 5.73$ connections
- *giant component* 95.3% nodes (6 components)
- "*small-world*" with $\langle C \rangle = 0.10$ and $\langle d \rangle = 14.43$
- "*scale-free*" with $\gamma = 2.60$ for cutoff $k_{min} = 5$



*reduced to largest connected component

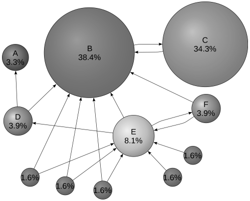
link *PageRank*

ranking algorithm for *web page importance*

- for *directed* G *PageRank rank* p [BP98] of i is
  - α is *positive constant* traditionally set $\alpha = 0.85$

$$p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j^{out}} + \frac{1-\alpha}{n}$$

- p_i probability *random surfer with teleports* lands on i



analysis *PageRank*

- *PageRank ranks p* in corrected LPP network
- *highest p* nodes are *Razstavišče* and *Ajdovščina*

#	**bus stop**	k_i	p_i
1	Razstavišče	43	0.010601
2	Ajdovščina	36	0.007694
3	Bežigrad	23	0.007161
4	Bavarski dvor	30	0.007013
5	Konzorcij	30	0.006884
6	Gosposvetska	30	0.006527
7	Stara cerkev	26	0.005485
8	Sava	12	0.005165
9	Tobačna	22	0.005136
10	Kino Šiška	18	0.004907
11	Medvode	4	0.004853
12	Tivoli	26	0.004838

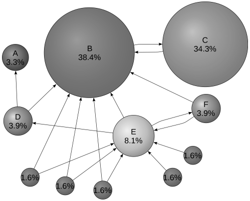
link *random walk with restarts*

ranking algorithm for *web page similarity*

- for *directed* G *random walk rank* w [TFP06] for t of i is
  - α is *positive constant* traditionally set $\alpha = 0.85$

$$w_i = \alpha \sum_j A_{ij} \frac{w_j}{k_j^{out}} + (1 - \alpha)\delta_{it}$$

- w_i probability *random surfer with teleport* t lands on i



analysis *random walk with restarts*

- *random walk ranks* *w* in corrected LPP network
- *highest* *w* nodes for *Razstavišče* and *Hajdrihova*

#	bus stop	k_i	w_i
1	Razstavišče	43	0.236115
2	Bavarski dvor	30	0.065124
3	Bezigrad	23	0.057260
4	Astra	16	0.047765
5	Ajdovščina	36	0.040099
6	Kozolec	10	0.038384
7	Gosposvetska	30	0.030981
8	Konzorcij	30	0.020278
9	Bavarski dvor	8	0.019262
10	Polje	10	0.014254
11	Stadion	8	0.013294
12	Topniška	8	0.013235

#	bus stop	k_i	w_i
1	Hajdrihova	14	0.201318
2	Tobačna	22	0.091186
3	Ilirija	12	0.051714
4	Stara cerkev	26	0.046825
5	Tabor	10	0.038395
6	Vič	16	0.034478
7	Avtomontaža	6	0.030372
8	Stan in dom	4	0.030296
9	Kino Šiška	18	0.028569
10	Tivoli	26	0.028180
11	Glince	8	0.027528
12	Na klancu	10	0.023836

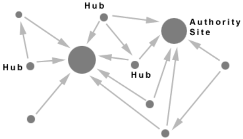
link *HITS*

ranking algorithm for *web hubs & authorities*

- for *directed* G *hub & authority ranks* h & a [Kle99] of i
  - h is *eigenvector* of $A^T A$ with *eigenvalue* $(\alpha\beta)^{-1}$
  - a is *eigenvector* of AA^T with *eigenvalue* $(\alpha\beta)^{-1}$
  - α and β are *appropriate positive constants*

$$h_i = \alpha \sum_j A_{ji} a_j \qquad a_i = \beta \sum_j A_{ij} h_j$$

- a measures *content* and h measures *table of content*
- $a = 0$ for $k^{in} = 0$ *nodes* and $h = 0$ for $k^{out} = 0$ *nodes*



analysis *HITS*

- *hub & authority ranks h & a* in corrected LPP network
- *highest h* node is *Ajdovščina* and *highest a* node is *Konzorcij*

#	**bus stop**	k_i	h_i
1	Ajdovščina	36	0.715370
2	Razstavišče	43	0.455771
3	Tivoli	26	0.286178
4	Drama	23	0.256027
5	Gosposvetska	30	0.175142
6	Bavarski dvor	30	0.129155
7	Pošta	9	0.111497
8	Kolodvor	4	0.090644
9	Konzorcij	30	0.083028
10	Tavčarjeva	7	0.069477
11	Kozolec	10	0.068749
12	Stara cerkev	26	0.064760

#	**bus stop**	k_i	a_i
1	Konzorcij	30	0.656745
2	Bavarski dvor	30	0.512119
3	Gosposvetska	30	0.235790
4	Kozolec	10	0.224651
5	Bežigrad	23	0.176839
6	Astra	16	0.172509
7	Stara cerkev	26	0.172482
8	Ajdovščina	36	0.161840
9	Razstavišče	43	0.110391
10	Tivoli	26	0.106024
11	Bavarski dvor	8	0.096486
12	Kolizej	4	0.088636

link *references*

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