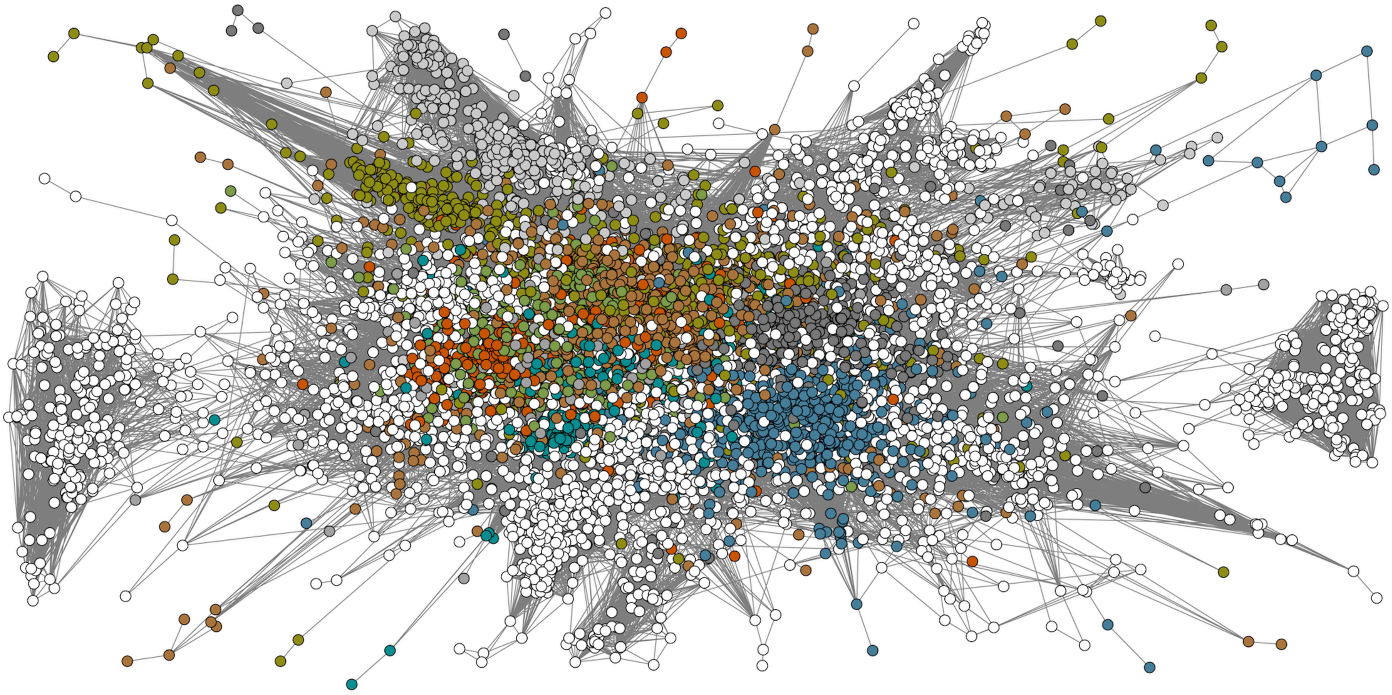
Node position in IMDb actors collaboration network

You are given **IMDb actors collaboration network** in Pajek format (imdb.net). Your task is to find the **most important actors** according to different measures of node centrality. You can either use the methods provided by your library or implement the algorithms yourself.



I. Degree centrality and clustering coefficients

1. Find the **most important actors according to degree centrality** $d_i = \frac{k_i}{n-1}$, where n is the number of nodes and k_i is the degree of node i. Which actors have the highest d (e.g. Hollywood, international, unknown)?

   Computational complexity is linear $\mathcal{O}(n)$ and applicable to any network that fits in your memory.

2. Find the **most important actors according to clustering coefficient** $C_i = \frac{2t_i}{k_i(k_i-1)}$, where k_i is the degree of node i and t_i is the number of triangles including node i. Which actors have the highest C (e.g. Hollywood, international, unknown)?

   Computational complexity is superlinear $\mathcal{O}(m\langle k\rangle)$ and applicable to all but the largest networks.

3. *(tentative)* Find the **most important actors according to μ-corrected clustering coefficient**

$C_i^\mu = \frac{2t_i}{k_i \mu}$, where k_i is the degree of node i, t_i is the number of triangles including node i and μ is an appropriate constant (i.e. maximum number of triangles over a single link). Which actors have the highest C^μ (e.g. Hollywood, international, unknown)?

II. Eigenvector centrality and PageRank algorithm

1. *(tentative)* Find the **most important actors according to eigenvector centrality** $e_i = \lambda_1^{-1} \sum_j A_{ij} e_j$, where A is the adjacency matrix and λ_1 is a normalizing constant. Which actors have the highest e (e.g. Hollywood, international, unknown)?

2. Find the **most important actors according to PageRank score** $p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j} + \frac{1-\alpha}{n}$, where A is the adjacency matrix, n is the number of nodes, k_i is the degree of node i and α is the damping factor set to 0.85. Which actors have the highest p (e.g. Hollywood, international, unknown)?

   Computational complexity is $\approx$ linear $\mathcal{O}(m)$ and applicable to any network that fits in your memory.

```
input   graph G, precision ϵ
output  eigenvector centrality E
E ← array of ones
do
    U ← array of zeros
    for nodes i ∈ N do
        for neighbors j ∈ Γ_i do
            U[i] ← U[i] + E[j]
    u ← ||U||
    for nodes i ∈ N do
        U[i] ← U[i] · n/u
    Δ ← ||E − U||
    E ← U
while Δ > ϵ
return E
```

```
input   graph G, damping α, precision ϵ
output  PageRank ranks P
P ← array of n^-1-s
do
    U ← array of zeros
    for nodes i ∈ N do
        for predecessors j ∈ Γ_i^in do
            U[i] ← U[i] + P[j] · α/k_j^out
    u ← ||U||
    for nodes i ∈ N do
        U[i] ← U[i] + (1 − u)/n
    Δ ← ||P − U||
    P ← U
while Δ > ϵ
return P
```

III. Closeness and betweenness centrality

1. *(tentative)* Find the **most important actors according to closeness centrality** $\ell_i^{-1} = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$, where n is the number of nodes and d_{ij} is the distance between nodes i and j. Which actors have the highest ℓ^{-1} (e.g. Hollywood, international, unknown)?

2. Find the **most important actors according to betweenness centrality** $\sigma_i = \frac{1}{n^2} \sum_{st} \frac{g_{st}^i}{g_{st}}$, where n is the number of network nodes, g_{st} is the number of shortest paths between nodes s and t, and g_{st}^i is the number of such paths through node i. Which actors have the highest σ (e.g. Hollywood, international, unknown)?

   Computational complexity is quadratic $\mathcal{O}(nm)$ and applicable only to medium sized networks.