core-periphery structure

introduction to *network science in Python* (*NetPy*)

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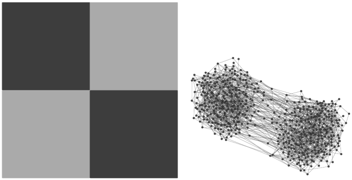
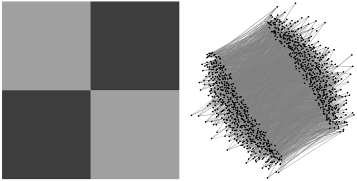
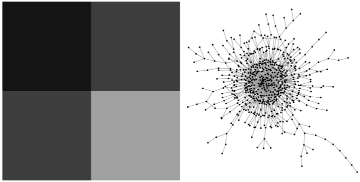
core-periphery *block model*



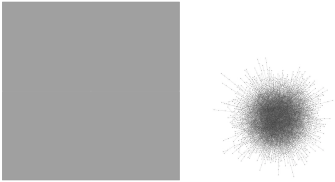
community block model [GN02]



disassortative (bipartite) block model [NL07]



core-periphery block model [Sei83]



random block model [ER59]

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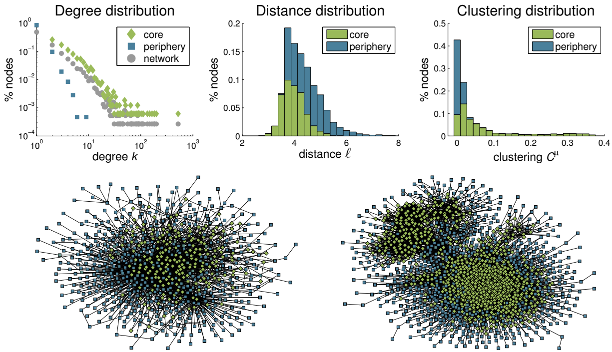
*origin of core-periphery structure in international relations (e.g. trade)

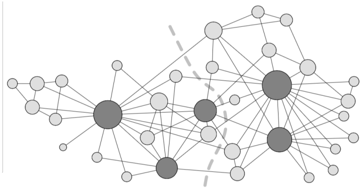
core-periphery *structure*

- *core*/*periphery* *nodes* have *higher*/*lower* *degrees* k
- *core*/*periphery* *nodes* are on *shorter*/*longer* *distances* ℓ
- *core*/*periphery* *nodes* have *higher*/*lower* *clustering* C

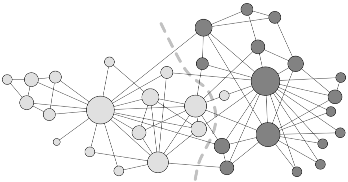


core-periphery *SBM*

- $G(\{C_1, C_2\}, \{p_{11}, p_{12}, p_{22}\})$ *stochastic block model* [HLL83]
  - n_i is *size* of *cluster* C_i & p_{ij} is *link density* between C_i and C_j
- *density-based core-periphery* structure for $p_{11} \gg p_{12} \gg p_{22}$
- *lookalike core-periphery* for $n_1 p_{11} \gg 1$, $n_1 p_{12} \ll 1$, $n_2 p_{22} \approx 1$



non-corrected block model $p_{11} > p_{12} > p_{22}$



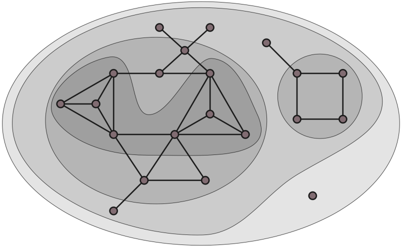
degree-corrected block model $p_{11} \approx p_{22} > p_{12}$

core-periphery *k-cores*

- *k-cores* are *subgraphs of nodes* with $\geq k$ *neighbors* [Sei83]

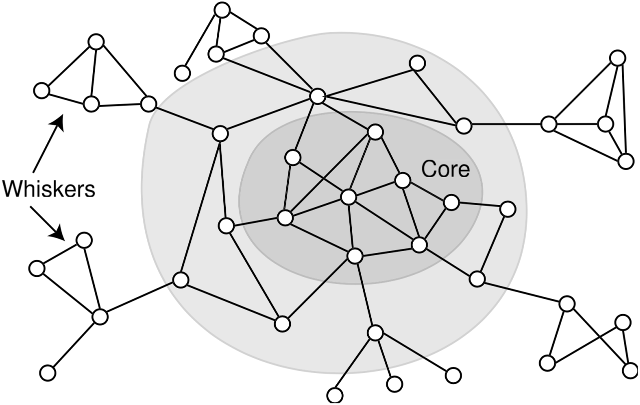
  remove nodes with degree $< k$ until no such node remains [BZ11]

- *k-shells* are *nodes of k-cores* that are *not in $k+1$-cores*
- *k-cores* are *nested* while *k-shells* form *decomposition*



1-cores are connected components without isolates & k-cores can be disconnected

core-periphery *nestedness*



nested cores & whiskers communities [LLDM09, YL13]

core-periphery *references*

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core-periphery *references*

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