small-world networks

introduction to *network analysis* (*ina*)

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small-world *documentary*

connected

the power of six degrees

documentary on small-world and scale-free networks

letters to nature

Collective dynamics of 'small-world' networks

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Networks of coupled dynamical systems have been used to model biological oscillators[1–4], Josephson junction arrays[5], excitable media[6], neural networks[7–9], spatial games[10], genetic control networks[11] and many other self-organizing systems. Ordinarily, the connection topology is assumed to be either completely regular or completely random. But many biological, technological and social networks lie somewhere between these two extremes. Here we explore simple models of networks that can be tuned through this middle ground: regular networks 'rewired' to introduce increasing amounts of disorder. We find that these systems can be highly clustered, like regular lattices, yet have small characteristic path lengths, like random graphs. We call them 'small-world' networks, by analogy with the small-world phenomenon[12,13] (popularly known as six degrees of separation[14]). The neural network of the worm Caenorhabditis elegans, the power grid of the western United States, and the collaboration graph of film actors are shown to be small-world networks. Models of dynamical systems with small-world coupling display enhanced signal-propagation speed, computational power, and synchronizability. In particular, infectious diseases spread more easily in small-world networks than in regular lattices.

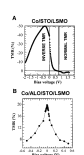

Fig. 2 Rise dependence of the TMR ratio in (A) Co/STO/LSMO and (B) Co/AlO/STO/LSMO tunnel junctions.

REPORTS

Emergence of Scaling in Random Networks

Albert-László Barabási* and Réka Albert

Systems as diverse as genetic networks or the World Wide Web are best described as networks with complex topology. A common property of many large networks is that the vertex connectivities follow a scale-free power-law distribution. This feature was found to be a consequence of two generic mechanisms: (i) networks expand continuously by the addition of new vertices, and (ii) new vertices attach preferentially to sites that are already well connected. A model based on these two ingredients reproduces the observed stationary scale-free distributions, which indicates that the development of large networks is governed by robust self-organizing phenomena that go beyond the particulars of the individual systems.

letters to nature

Error and attack tolerance of complex networks

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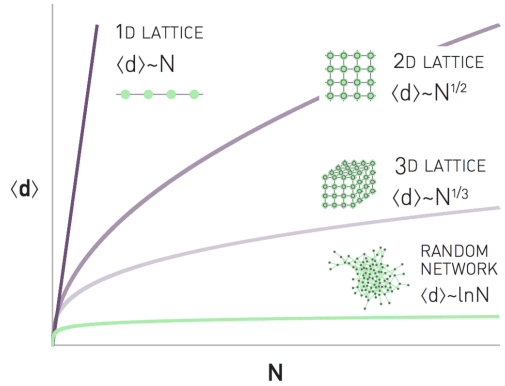
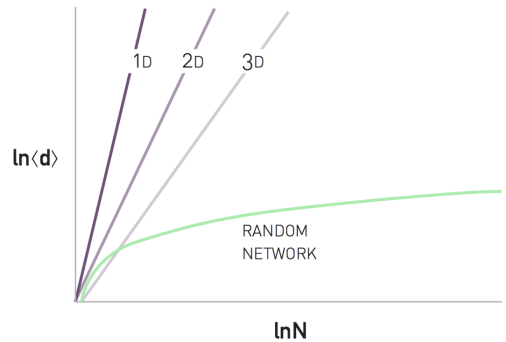
Many complex systems display a surprising degree of tolerance against errors. For example, relatively simple organisms grow, persist and reproduce despite drastic pharmaceutical or environmental interventions, an error tolerance attributed to the robustness of the underlying metabolic network[1]. Complex communication networks[2] display a surprising degree of robustness: although key components regularly malfunction, local failures rarely lead to the loss of the global information-carrying ability. The stability of these and other complex systems[3,4] is often attributed to the redundant wiring of the functional web defined by the systems' components. Here we demonstrate that error tolerance is not shared by all redundant systems: it is displayed only by a class of inhomogeneously wired networks,

[WS98] [BA99] [AJB00]

small-world *phenomenon*

- for *regular lattices*
  - *high clustering* $\langle C \rangle \gg 0$
  - long distances $\langle d \rangle \simeq n^{1/D}$
- in *random graphs* [ER59]
  - low clustering $\langle C \rangle = \frac{\langle k \rangle}{n-1}$
  - *short distances* $\langle d \rangle \simeq \frac{\ln n}{\ln \langle k \rangle}$
- real *small-world networks* [WS98]
  - *high clustering* $\langle C \rangle \gg 0$
  - *short distances* $\langle d \rangle \simeq \frac{\ln n}{\ln \langle k \rangle}$





- $\langle d \rangle = 4.74$ for *Facebook* friendships [BBR+12] while $\frac{\ln n}{\ln \langle k \rangle} = 3.98$
- $\langle C \rangle = 0.61$ for *Facebook* social circles [NL12] while $\rho < 10^{-6}$

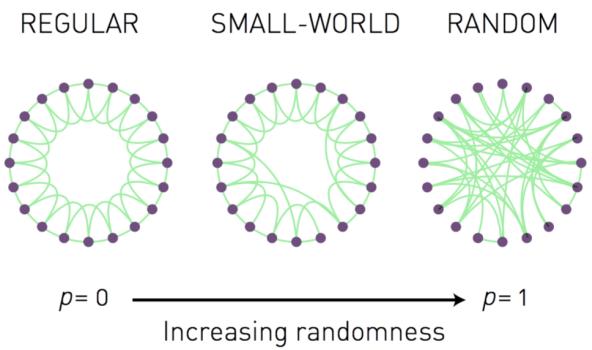
small-world *model*

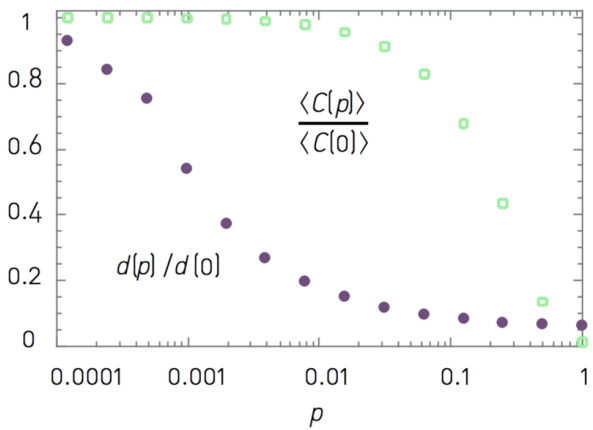
— $G(n, k, p)$ *small-world* model [WS98]

— *randomly rewire* $pnk/2$ *links* of *regular lattice*

— *conceptually interesting* but *practically inapplicable*

- for *some* p *small-world* with $\langle d \rangle \simeq \frac{\ln n}{\ln \langle k \rangle}$ and $\langle C \rangle \gg 0$
- for $p = 1$ *random graph* with $\langle d \rangle \simeq \frac{\ln n}{\ln \langle k \rangle}$
- for $p = 0$ *regular lattice* with $C = \frac{3(k-2)}{4(k-1)}$





see small-world model NetLogo demo

small-world *networks*

— *clustering coefficient* $\langle C \rangle$ in *real small-world* networks

— *average distance* $\langle d \rangle$ in *real small-world* networks

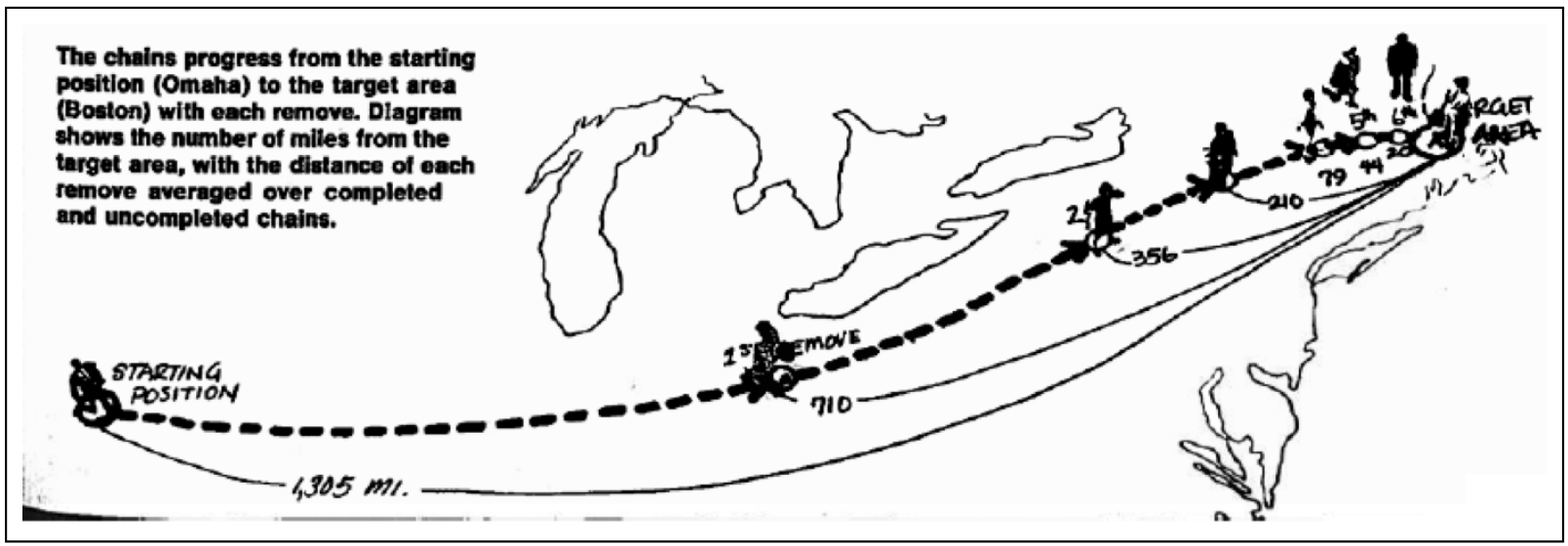
network	n	$\langle C \rangle$	$\gg \frac{\langle k \rangle}{n-1}$	$\langle d \rangle$	$\approx \frac{\ln n}{\ln \langle k \rangle}$
southern women [DGG41]	32	0.000	0.179	2.31	2.02
karate club [Zac77]	34	0.571	0.139	2.41	2.31
American football [GN02]	115	0.403	0.094	2.51	2.00
Java dependencies [ŠB11]	1368	0.497	0.012	2.21	2.59
Facebook circles [ML12]	4039	0.606	0.011	3.69	2.20
physics collaboration [New01]	36 458	0.657	0.000	5.50	4.68
Enron e-mails [LLDM09]	36 692	0.497	0.001	3.39	3.51
Internet map [HJJ+03]	75 885	0.160	0.000	5.83	5.01
actors collaboration [BA99]	382 219	0.780	0.000	≈ 3.6	2.94
physics citation [ŠFB14]	438 943	0.227	0.000	≈ 5.0	4.23
patent citation [HJT01]	3 774 768	0.076	0.000	≈ 8.1	6.98
Facebook snowball [Fer12]	8 217 272	0.019	0.000	≈ 6.8	14.23

small-world *experiments*

— 6 *degrees of separation* in *letter* passing as $\langle d \rangle = 6.2$ [Mil67]

— 4/7 *degrees of separation* in *e-mail* communication [DMW03]

— 4 *degrees of separation* on *Facebook* as $\langle d \rangle = 4.74$ [BBR+12]



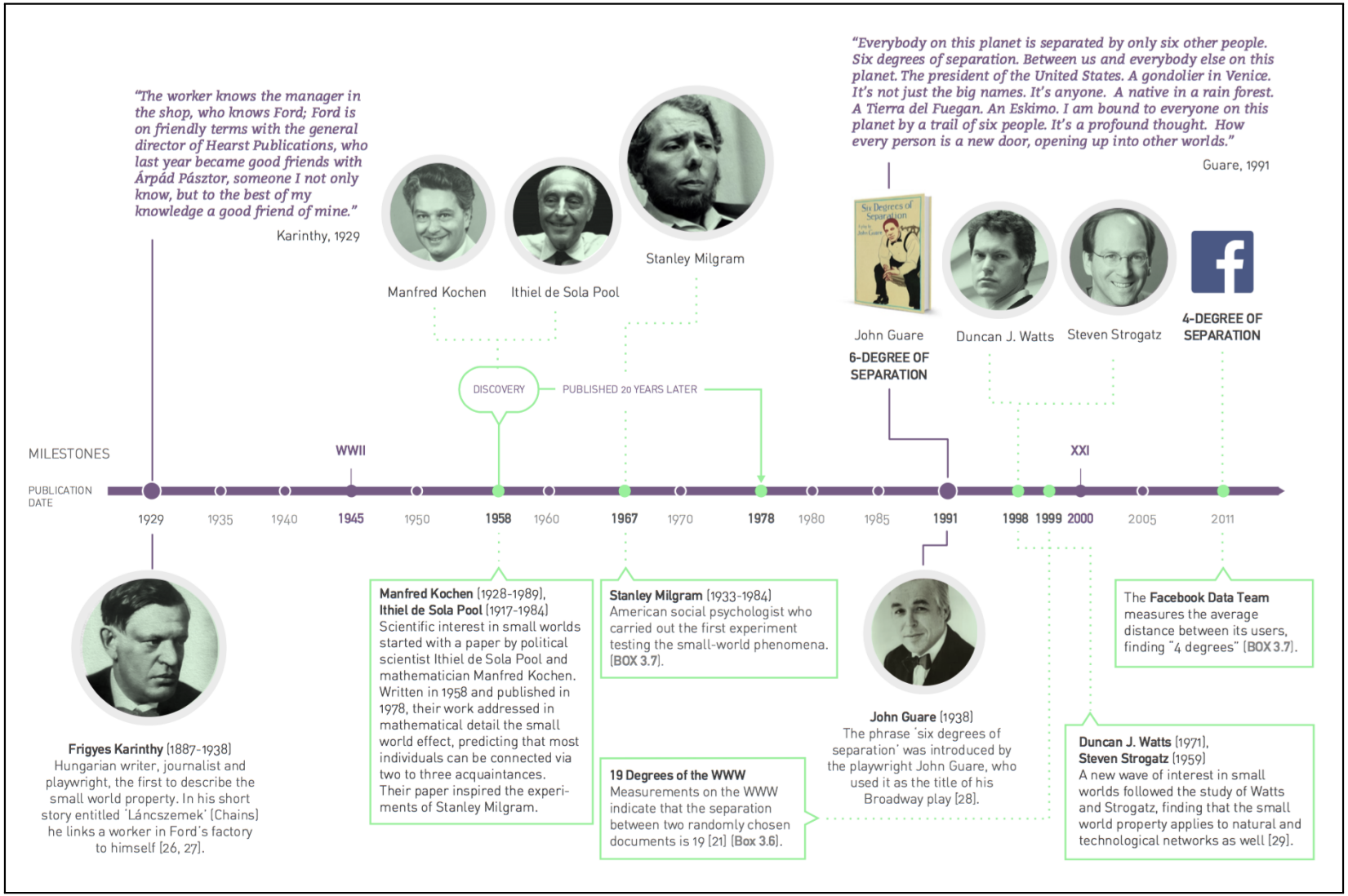
— the *strength* (*weakness*) of *weak* (*strong*) *ties* [Gra73]

small-world *navigation*

does existence of short paths imply

navigable small-world by *decentralized search*? [Kle00]

small-world *history*



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