

node *centrality*

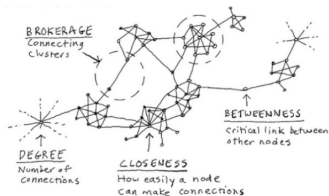
introduction to *network analysis* (*ina*)

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centrality *measures*

which *nodes* are most *important*?

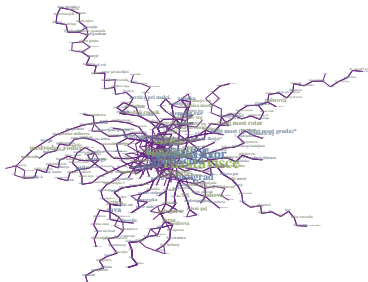
- *node centrality measures* for (*un*)*directed* networks
 - *clustering coefficients* [WS98, SV05, dNMB05]
 - *distance-based* measures [Fre77, FBW91, New05]
 - *spectral analysis* measures [Kat53, Bon87, BP98]
 - *fragment-based* measures [MSOI⁺02, Prž07, EK15]



- *link analysis algorithms* primarily for *directed* networks

networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 5.62$ connections
- *giant component* 95.4% nodes (6 components)
- “small-world” with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- “scale-free” with $\gamma = 2.62$ for cutoff $k_{min} = 5$



* reduced to largest connected component

centrality *clustering*

important *nodes* are *strongly embedded*

- for *undirected* G *clustering coefficient* C [WS98] of i is
 - t_i is number of *linked neighbors* or *triangles* of i

$$C_i = \frac{2t_i}{k_i(k_i-1)} \quad C_i = 0 \text{ for } k_i \leq 1$$

- ω -*corrected clustering coefficient* C^ω [SV05] of i is
 - ω_i is *maximum possible* t_i with *respect to* $\{k\}$

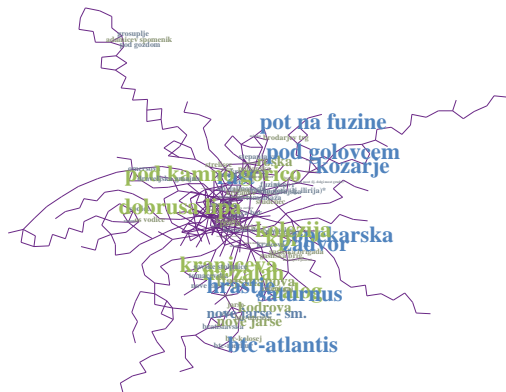
$$C_i^\omega = \frac{t_i}{\omega_i} \quad C_i^\omega = 0 \text{ for } \omega_i = 0$$

- μ -*corrected clustering coefficient* C^μ [dNMB05] of i is
 - μ is *maximum* number of *triangles* over *links*

$$C_i^\mu = \frac{2t_i}{k_i\mu} \quad C_i^\mu = 0 \text{ for } k_i = 0$$

networkology *clustering*

- *clustering coefficient* C in partial LPP network[†]
- *highest* $C_i = 1.0$ nodes are *Na Žalah etc.* with $k_i = 2$



[†] reduced to simple undirected graph

centrality *closeness*

important *nodes* are *close to other* nodes

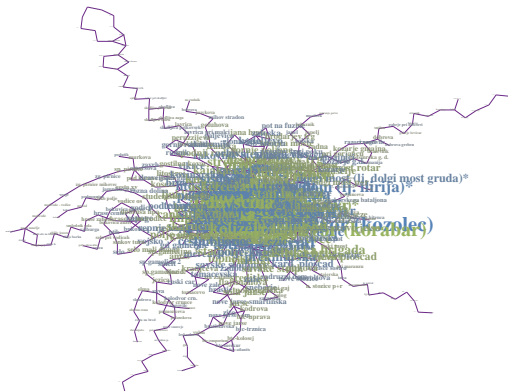
- for (*un*)*directed* G *closeness centrality* ℓ^{-1} [New10] of i is
 - d_{ij} is (*un*)*directed distance* between i and j
 - $d_{ij} = \infty$ for nodes in *different components*

$$\ell_i^{-1} = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

- ℓ^{-1} spans *small range* in *small-world* networks

networkology *closeness*

- *closeness centrality* ℓ^{-1} in partial LPP network[§]
- *highest* $\ell_i^{-1} = 0.208$ node is *Gospodsvetska* with $k_i = 14$



[§] reduced to simple undirected graph

centrality *betweenness*

important *nodes* are *bridges between other* nodes

- for (*un*)*directed* G *betweenness centrality* σ [Fre77] of i is
 - g_{st} is number of *shortest paths between* s and t
 - g_{st}^i is number of *such shortest paths through* i

$$\sigma_i = \frac{1}{n^2} \sum_{st} \frac{g_{st}^i}{g_{st}}$$

- σ considers *only shortest paths* [FBW91, New05]
- σ mixes *local centers* with *global bridges* [JMK⁺16]

centrality *degrees*

important *nodes* are *linked by many* nodes

- for *undirected* G *degree centrality* d of i is

$$d_i = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i}{n-1}$$

- in *directed* G *in-degree centrality* d^{in} of i is

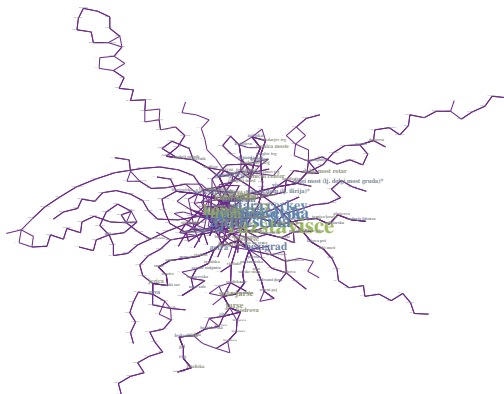
$$d_i^{in} = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i^{in}}{n-1}$$

- in *directed* G *out-degree centrality* d^{out} of i is

$$d_i^{out} = \frac{1}{n-1} \sum_{j \neq i} A_{ji} = \frac{k_i^{out}}{n-1}$$

networkology *degrees*

- *degree centrality* d in partial LPP network
- *highest* $d_i = 0.099$ node is *Razstavišče* with $k_i = 41$
- *highest* d_i^{in} node is *Razstavišče* with $k_i^{in} = 20$ and $k_i^{out} = 21$



centrality *eigenvector*

important *nodes* are *linked by important* nodes

- for (*un*)*directed* G *eigenvector centrality* e [Bon87] of i is
 - v and λ are *eigenvectors* and *eigenvalues* of A
 - e is *proportional* to *leading eigenvector* v_1

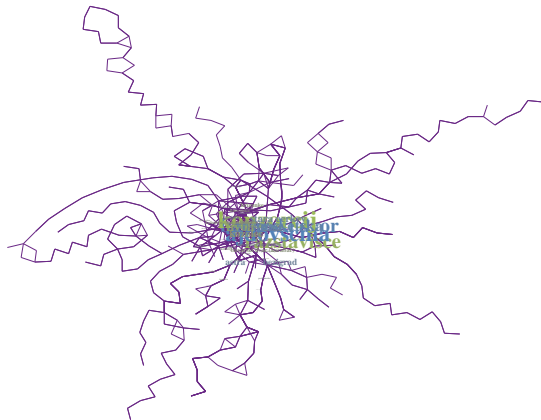
$$e(t) = A^t e(0) = A^t \sum_i C_i v_i = \sum_i C_i \lambda_i^t v_i = \lambda_1^t \sum_i C_i \left[\frac{\lambda_i}{\lambda_1} \right]^t v_i \rightarrow C_1 \lambda_1^t v_1$$

$$e_i = \lambda_1^{-1} \sum_j A_{ij} e_j$$

- in *directed* G $e = 0$ for $k^{in} = 0$ *nodes etc.*

networkology *eigenvector*

- *eigenvector centrality* e in partial LPP network
- *highest* $e_j = 0.082$ node is *Konzorcij* with $k_j = 30$



centrality *Katz*

nodes get small amount of *importance* for free

- for (un)directed G *Katz centrality* z [Kat53] of i is
 - α and β_i are some *positive constants*

$$z_i = \alpha \sum_j A_{ij} z_j + \beta_i$$

- for *convenience* $\beta_i = 1$ whereas $\alpha < \lambda_1^{-1}$
 - λ_1 is *leading eigenvalue* of A

centrality *PageRank*

nodes distribute equal amount of *importance*

- for (un)directed G *PageRank centrality* p [BP98] of i is
 - α and β_i are some *positive constants*

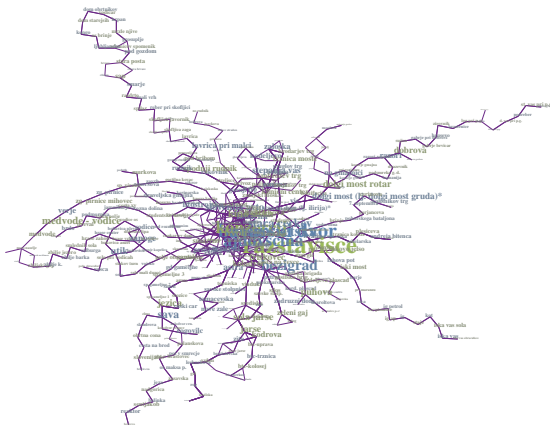
$$p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j^{\text{out}}} + \beta_i$$

- for *convenience* $\beta_i = \frac{1-\alpha}{n}$ whereas $\alpha = 0.85$

see PageRank algorithm *NetLogo* demo

networkology *PageRank*

- *PageRank centrality* p in partial LPP network
- *highest* $p_i = 0.011$ node is *Razstavišče* with $k_i = 41$



centrality *overview*

which *nodes* are most *important*?

| 1 IA | | | | | | | | | | | | 2 IIA | | | | | | | | | | | | 3 IIIA | | | | | | | | | | | | 4 IVB | | | | | | | | | | | | 5 VB | | | | | | | | | | | | 6 VIB | | | | | | | | | | | | 7 VIIB | | | | | | | | | | | | 8 VIIIB | | | | | | | | | | | | 9 VIIIB | | | | | | | | | | | | 10 VIIIB | | | | | | | | | | | | 11 IB | | | | | | | | | | | | 12 IIB | | | | | | | | | | | | 13 IIIA | | | | | | | | | | | | 14 IVA | | | | | | | | | | | | 15 VA | | | | | | | | | | | | 16 VIA | | | | | | | | | | | | 17 VIIA | | | | | | | | | | | | 18 VIIIA | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 1 DC Degree Centrality | | | | | | | | | | | | 2 BC Betweenness Centrality | | | | | | | | | | | | 3 CC Closeness Centrality | | | | | | | | | | | | 4 RL Rangef, Ined Centrality | | | | | | | | | | | | 5 IC Information Centrality | | | | | | | | | | | | 6 BN Betweenness Centrality | | | | | | | | | | | | 7 RC Radiality Centrality | | | | | | | | | | | | 8 IG Integration | | | | | | | | | | | | 9 DCox DCox | | | | | | | | | | | | 10 BCox BCox | | | | | | | | | | | | 11 CCox CCox | | | | | | | | | | | | 12 ECox ECox | | | | | | | | | | | | 13 KSCox KSCox | | | | | | | | | | | | 14 CCox CCox | | | | | | | | | | | | 15 RCox RCox | | | | | | | | | | | | 16 DCox DCox | | | | | | | | | | | | 17 SC Subgraph Centrality | | | | | | | | | | | | 18 CCoef Clustering Coefficient | | | | | | | | | | | | 19 CCoef ⁻¹ Inverse CCoef | | | | | | | | | | | | 20 MNC max. neigh. comp. | | | | | | | | | | | | 21 ECoef edge clustering coefficient | | | | | | | | | | | | 22 PR PageRank | | | | | | | | | | | | 23 EC Eigenvector Centrality | | | | | | | | | | | |
| 19 BN Betweenness Centrality | | | | | | | | | | | | 20 RC Radiality Centrality | | | | | | | | | | | | 21 IG Integration | | | | | | | | | | | | 22 CCox CCox | | | | | | | | | | | | 23 BCox BCox | | | | | | | | | | | | 24 CCox CCox | | | | | | | | | | | | 25 ECox ECox | | | | | | | | | | | | 26 KSCox KSCox | | | | | | | | | | | | 27 CCox CCox | | | | | | | | | | | | 28 RCox RCox | | | | | | | | | | | | 29 DCox DCox | | | | | | | | | | | | 30 SC Subgraph Centrality | | | | | | | | | | | | 31 CCoef Clustering Coefficient | | | | | | | | | | | | 32 CCoef ⁻¹ Inverse CCoef | | | | | | | | | | | | 33 MNC max. neigh. comp. | | | | | | | | | | | | 34 ECoef edge clustering coefficient | | | | | | | | | | | | 35 PR PageRank | | | | | | | | | | | | 36 EC Eigenvector Centrality | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 37 RWBC RandomWalk Betweenness | | | | | | | | | | | | 38 RWCC RandomWalk Closeness | | | | | | | | | | | | 39 CC _{3,3,4} 2,3,4-Isolated-CC | | | | | | | | | | | | 40 ECox ECox | | | | | | | | | | | | 41 BCox BCox | | | | | | | | | | | | 42 KSCox KSCox | | | | | | | | | | | | 43 CCox CCox | | | | | | | | | | | | 44 RCox RCox | | | | | | | | | | | | 45 IGox IGox | | | | | | | | | | | | 46 DCBC DBCC | | | | | | | | | | | | 47 BCC BCC | | | | | | | | | | | | 48 CCKS CCKS | | | | | | | | | | | | 49 KSPR KSPR | | | | | | | | | | | | 50 DCPR DCPR | | | | | | | | | | | | 51 β Bipartity | | | | | | | | | | | | 52 SC ₂ 2-localized SC | | | | | | | | | | | | 53 NC Neighborhood Centrality | | | | | | | | | | | | 54 EC ₂ 2-localized EC | | | | | | | | | | | | 55 EC ₃ 3-localized EC | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 56 σ sigma Centrality | | | | | | | | | | | | 57 ECC Eccentrality | | | | | | | | | | | | 58 WDC Weighted Degree | | | | | | | | | | | | 59 DCECC DCECC | | | | | | | | | | | | 60 COECC COECC | | | | | | | | | | | | 61 BCECC BCECC | | | | | | | | | | | | 62 KSECC KSECC | | | | | | | | | | | | 63 PRECC PRECC | | | | | | | | | | | | 64 GECC GECC | | | | | | | | | | | | 65 DCC DCC | | | | | | | | | | | | 66 BCKS BCKS | | | | | | | | | | | | 67 CCCR CCPR | | | | | | | | | | | | 68 KSG KSG | | | | | | | | | | | | 69 DCIG DCIG | | | | | | | | | | | | 70 DCCoef DCCoef | | | | | | | | | | | | 71 SC ₃ 3-localized SC | | | | | | | | | | | | 72 LI Lobley Index | | | | | | | | | | | | 73 EC ₃ 3-localized EC | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 87 BC _{3,3,4} 2,3,4-Isolated-SC | | | | | | | | | | | | 88 ECC ⁻¹ Inverse Eccentrality | | | | | | | | | | | | 89 SDC Sphere Degree Centrality | | | | | | | | | | | | 90 DCR DCR | | | | | | | | | | | | 91 CCRC CCRC | | | | | | | | | | | | 92 BCRC BCRC | | | | | | | | | | | | 93 KRC KRC | | | | | | | | | | | | 94 PRRC PRRC | | | | | | | | | | | | 95 GRC GRC | | | | | | | | | | | | 96 DCKS DCKS | | | | | | | | | | | | 97 BCRP BCRP | | | | | | | | | | | | 98 CCIG CCIG | | | | | | | | | | | | 99 DCPR DCPR | | | | | | | | | | | | 100 ECCRC ECCRC | | | | | | | | | | | | 101 BCCoef BCCoef | | | | | | | | | | | | 102 SC ₄ 4-localized SC | | | | | | | | | | | | 103 EC ₄ 4-localized EC | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

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| 22 | 2 | 23 | 2 | 24 | 2 | 25 | 1 | 26 | 1 | 27 | 1 | 28 | 1 | 29 | 1 | 30 | 1 | 31 | 1 | 32 | 1 | 33 | 1 | 34 | 1 | 35 | 1 | 36 | 1 | 37 | 1 | 38 | 1 | 39 | 1 | 40 | 1 | 41 | 1 | 42 | 1 | 43 | 1 | 44 | 1 | 45 | 1 | 46 | 1 | 47 | 1 | 48 | 1 | 49 | 1 | 50 | 1 | 51 | 1 | 52 | 1 | 53 | 1 | 54 | 1 | 55 | 1 | 56 | 1 | 57 | 1 | 58 | 1 | 59 | 1 | 60 | 1 | 61 | 1 | 62 | 1 | 63 | 1 | 64 | 1 | 65 | 1 | 66 | 1 | 67 | 1 | 68 | 1 | 69 | 1 | 70 | 1 | 71 | 1 | 72 | 1 | 73 | 1 | 74 | 1 | 75 | 1 | 76 | 1 | 77 | 1 | 78 | 1 | 79 | 1 | 80 | 1 | 81 | 1 | 82 | 1 | 83 | 1 | 84 | 1 | 85 | 1 | 86 | 1 | 87 | 1 | 88 | 1 | 89 | 1 | 90 | 1 | 91 | 1 | 92 | 1 | 93 | 1 | 94 | 1 | 95 | 1 | 96 | 1 | 97 | 1 | 98 | 1 | 99 | 1 | 100 | 1 | 101 | 1 | 102 | 1 | 103 | 1 | 104 | 1 | 105 | 1 | 106 | 1 | 107 | 1 | 108 | 1 | 109 | 1 | 110 | 1 | 111 | 1 | 112 | 1 | 113 | 1 | 114 | 1 | 115 | 1 | 116 | 1 | 117 | 1 | 118 | 1 | 119 | 1 | 120 | 1 | 121 | 1 | 122 | 1 | 123 | 1 | 124 | 1 | 125 | 1 | 126 | 1 | 127 | 1 | 128 | 1 | 129 | 1 | 130 | 1 | 131 | 1 | 132 | 1 | 133 | 1 | 134 | 1 | 135 | 1 | 136 | 1 | 137 | 1 | 138 | 1 | 139 | 1 | 140 | 1 | 141 | 1 | 142 | 1 | 143 | 1 | 144 | 1 | 145 | 1 | 146 | 1 | 147 | 1 | 148 | 1 | 149 | 1 | 150 | 1 | 151 | 1 | 152 | 1 | 153 | 1 | 154 | 1 | 155 | 1 | 156 | 1 | 157 | 1 | 158 | 1 | 159 | 1 | 160 | 1 | 161 | 1 | 162 | 1 | 163 | 1 | 164 | 1 | 165 | 1 | 166 | 1 | 167 | 1 | 168 | 1 | 169 | 1 | 170 | 1 | 171 | 1 | 172 | 1 | 173 | 1 | 174 | 1 | 175 | 1 | 176 | 1 | 177 | 1 | 178 | 1 | 179 | 1 | 180 | 1 | 181 | 1 | 182 | 1 | 183 | 1 | 184 | 1 | 185 | 1 | 186 | 1 | 187 | 1 | 188 | 1 | 189 | 1 | 190 | 1 | 191 | 1 | 192 | 1 | 193 | 1 | 194 | 1 | 195 | 1 | 196 | 1 | 197 | 1 | 198 | 1 | 199 | 1 | 200 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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