network *representations*

introduction to *network analysis* (*ina*)

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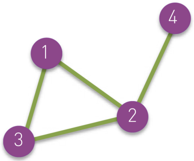
network *representations*



undirected graph

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

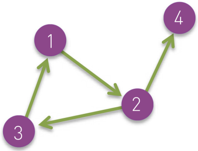
adjacency matrix

1: $[2,3]$
2: $[1,3,4]$
3: $[1,2]$
4: $[2]$

adjacency list

$\{1,2\}$
$\{1,3\}$
$\{2,3\}$
$\{2,4\}$

edge list



directed graph

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

adjacency matrix

$[3]$:1: $[2]$
$[1]$:2: $[3,4]$
$[2]$:3: $[1]$
$[2]$:4: $[\ ]$

adjacency list

$(1,2)$
$(2,3)$
$(2,4)$
$(3,1)$

edge list

[*] adjacency list can be implemented with maps or trees & edge list does not represent isolated nodes

network *representations*

— *adjacency matrix* for elegant *analytical derivations*

most derivations based on matrix representation[†]

— *adjacency list* for efficient *algorithms implementation*

ideal complexity since most algorithms require incidence[†]

— *edge list* for efficient *network storing/manipulation*

easy editing since each edge stored only once

[†] some derivations can also be based on adjacency list & some algorithms require edge list

network *structures*

- *edge list edges data structure* complexity

data structure	link manipulation	random node	random link
array	none	$\mathcal{O}(m)$	$\mathcal{O}(1)$
array list	addition	$\mathcal{O}(m)$	$\mathcal{O}(1)$
hash map	any	$\mathcal{O}(m)$	$\mathcal{O}(m)$

- *adjacency list nodes data structure* complexity

data structure	node manipulation	random node	random link
array	none	$\mathcal{O}(1)$	$\mathcal{O}(m)$
array list	addition	$\mathcal{O}(1)$	$\mathcal{O}(m)$
hash map	any	$\mathcal{O}(n)$	$\mathcal{O}(m)$

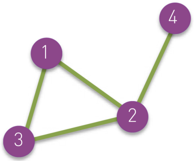
- *adjacency list neighbors data structure* complexity

data structure	link manipulation	node incidence	random neighbor
array	none	$\mathcal{O}(k)$	$\mathcal{O}(1)$
array list	addition	$\mathcal{O}(k)$	$\mathcal{O}(1)$
hash map	any	$\approx \mathcal{O}(1)$	$\mathcal{O}(k)$
tree map	any	$\mathcal{O}(\log k)$	$\mathcal{O}(k)$

- *hash maps* for *construction* and *arrays* for *analysis*
- use *directed adjacency list* with *undirected flag*

[‡] random link selection equivalent to random node selection by degree

network *formats*



undirected graph

```
# undirect
1 2
1 3
2 3
2 4
```

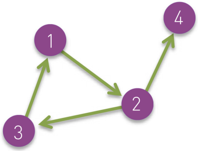
edge list

```
*vertices 4
1 "1"
2 "2"
3 "3"
4 "4"
*edges
1 2
1 3
2 3
2 4
```

Pajek format

```
# undirect
# 0 "1"
# 1 "2"
# 2 "3"
# 3 "4"
#
0 1
0 2
1 2
1 3
```

my format



directed graph

```
# directed
1 2
2 3
2 4
3 1
```

edge list

```
*vertices 4
1 "1"
2 "2"
3 "3"
4 "4"
*arcs
1 2
2 3
2 4
3 1
```

```
# directed
# 0 "1"
# 1 "2"
# 2 "3"
# 3 "4"
#
0 1
1 2
1 3
2 0
```

§ ad-hoc edge list and Pajek format most popular & other formats include GML, GraphML and JSON

network *data*

- easily obtained from *online sources*
- already present in many *standard datasets*
- *personal web pages* of network researchers
- popular *network repositories/collections*
  - **Network Catalogue and Repository** [Netzschleuder]
  - Colorado Index of Complex Networks [ICON]
  - Stanford Network Analysis Project [SNAP]
  - Koblenz Network Collection [KONECT]
  - Open Graph Benchmark [OGB]
  - Network Repository [NetRepo]
  - Pajek datasets [Pajek]

network *software*

- most popular *Python* *libraries*
  - igraph [https://igraph.org]
  - **NetworkX** [https://networkx.org]
  - graph-tool [https://graph-tool.skewed.de]
  - Snap.py [https://snap.stanford.edu/snappy]
  - Pajek [http://mrvar.fdv.uni-lj.si/pajek]

- most popular *network* *software*
  - **Gephi** [https://gephi.org]
  - visone [https://visone.ethz.ch]
  - Pajek [http://mrvar.fdv.uni-lj.si/pajek]