link *bridging*

introduction to *network analysis* (*ina*)

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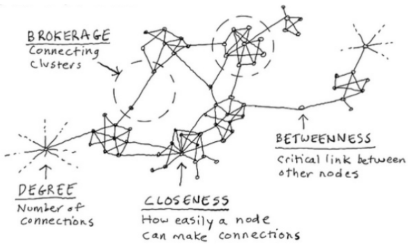
bridging *measures*

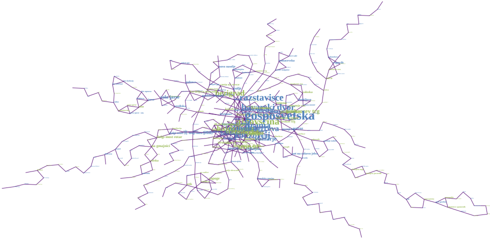
which *links* are most *important*?

- *link bridging measures* for (*un*)*directed* networks
  - *betweenness-based* centrality [Fre77, FBW91, New05]
- *link embeddedness measures* for (*un*)*directed* networks
  - *topological overlap* measures [RSM+02, OSH+07, dNMB11]



networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 2.72$ connections
- *giant component* 95.4% nodes (6 components)
- *"small-world"* with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- *"scale-free"* with $\gamma = 2.43$ for cutoff $k_{min} = 2$



*reduced to largest connected component of simple undirected graph

bridging *betweenness*

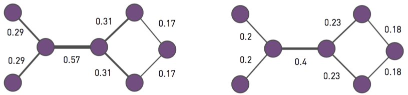
important *links* are *between other nodes*

— for (*un*)*directed* G *link betweenness* σ [Fre77] of $\{i,j\}$ is

- g_{st} is number of *geodesic paths between* s and t
- g_{st}^{ij} is number of *such geodesic paths through* $\{i,j\}$

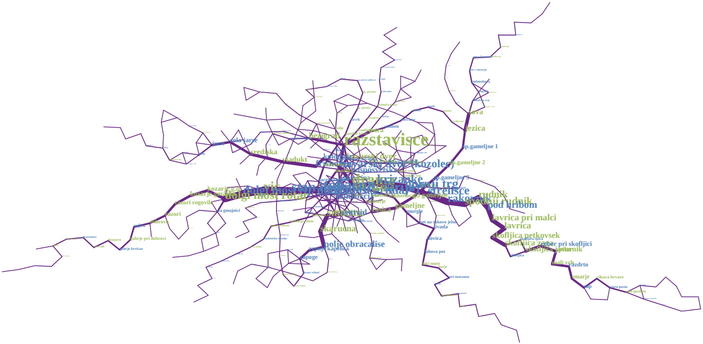
$$\sigma_{ij} = \sum_{st \notin \{i,j\}} \frac{g_{st}^{ij}}{g_{st}}$$

— σ considers *only geodesic paths* [FBW91, New05]



networkology *betweenness*

- *link betweenness* σ in partial LPP network[†]
- *highest* $\sigma_{ij} = 0.176n^2$ link is $\{$*Vič, Stan in dom*$\}$



[†] reduced to largest connected component of simple undirected graph

bridging *bridgeness*

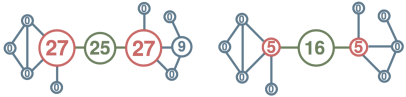
important *links* are *bridges between nodes*

— for (*un*)*directed* G *link bridgeness* $\widetilde{\sigma}$ [JMK+16] of $\{i,j\}$ is

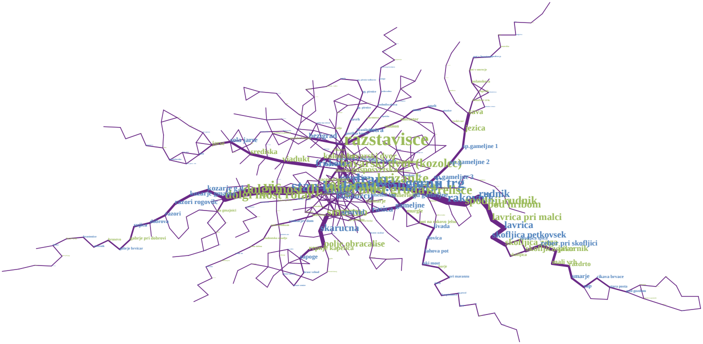
- g_{st} is number of *geodesic paths between* s and t
- g_{st}^{ij} is number of *such geodesic paths through* $\{i,j\}$

$$\widetilde{\sigma}_{ij} = \sigma_{ij} - \sum_{st \in \Gamma_i \cup \Gamma_j} \frac{g_{st}^{ij}}{g_{st}} = \sum_{st \notin \Gamma_i \cup \Gamma_j} \frac{g_{st}^{ij}}{g_{st}}$$

— σ mixes *local centers* with *global bridges* [JMK+16]



- *link bridgeness* $\widetilde{\sigma}$ in partial LPP network‡
- *highest* $\widetilde{\sigma}_{ij} = 0.169n^2$ link is {*Vič, Stan in dom*}



‡reduced to largest connected component of simple undirected graph

bridging *embeddedness*

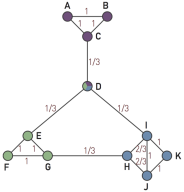
important *links* are *embedded between nodes*

- for *undirected* G *link embeddedness*§ θ [OSH+07] of $\{i,j\}$ is
  - Γ_i is set of *neighbors* or *neighborhood* of i

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \qquad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$

- μ-*corrected link embeddedness* $\widetilde{\theta}$ [dNMB11] of $\{i,j\}$ is
  - μ is *maximum* number of *triangles* over *links*

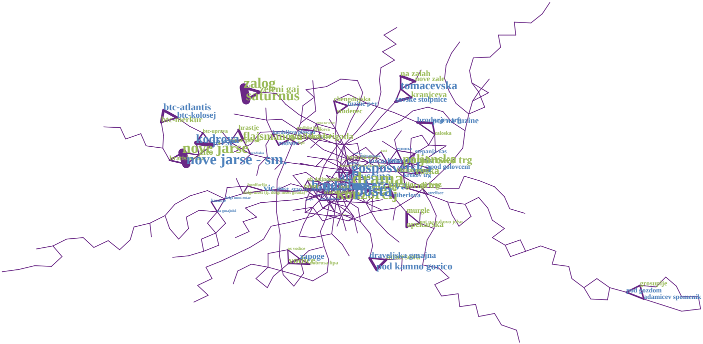
$$\widetilde{\theta}_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{\mu + \max(k_i, k_j) - 1 - |\Gamma_i \cap \Gamma_j|}$$



§ θ & $\widetilde{\theta}$ better known as topological overlap indices/weights

networkology *embeddedness*

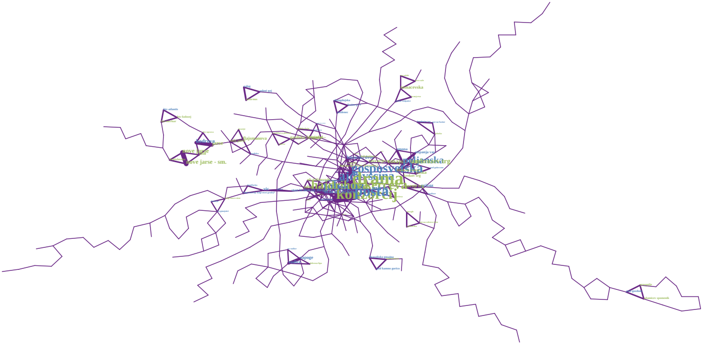
- *link embeddedness* θ in partial LPP network¶
- *highest* $\theta_{ij} = 1.0$ links are $\{$*Zalog*, *Saturnus*$\}$ *etc.*



¶reduced to largest connected component of simple undirected graph

networkology *μ-embeddedness*

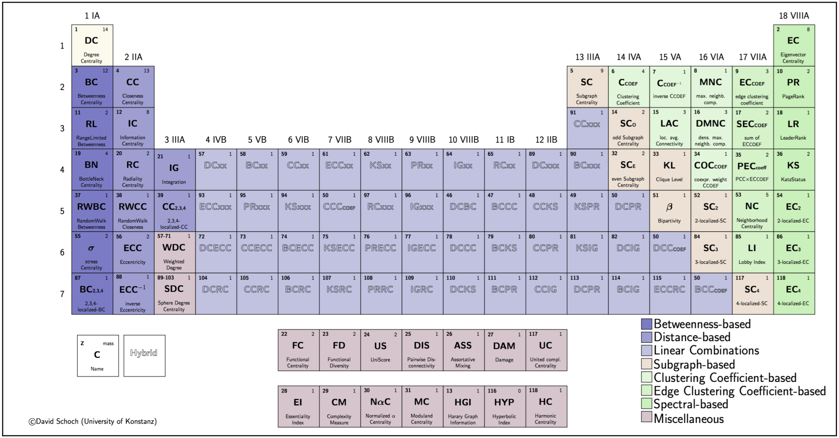
- *μ-corrected embeddedness* $\widetilde{\theta}$ in partial LPP network‖
- *highest* $\widetilde{\theta}_{ij} = 0.4$ links are $\{$*Pošta*, *Konzorcij*$\}$ *etc.*



‖ reduced to largest connected component of simple undirected graph

bridging *overview*

which *links* are most *important*?



bridging *references*

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