# Label propagation for community detection: A review

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#### December 2012

#### Abstract

Label propagation is an efficient approach for community detection initially proposed by Raghavan et al. (2007). We here give a comprehensive review various advances of label propagation that improve on its performance, stability, complexity and other.

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# 1 Introduction

Complex real-world networks commonly consist of communities, i.e., groups of nodes that are densely linked within and only sparsely linked between (also dense subgraphs of sparse graphs). Label propagation is an efficient approach for community detection initially proposed by Raghavan et al. (2007). Here labels are propagated among the nodes thus each node is assigned a label shared by most of its neighbors (i.e., majority label). When an equilibrium is reached, connected groups of nodes sharing unique label are classified into the same community. Note that the approach has near linear time complexity and is applicable to networks with millions of nodes and links on a standard computer.

In this work we give a comprehensive review of various advances of standard label propagation that improve on its performance, stability, complexity and other.

#### **Preliminaries**

For the purposes of this work, a network is represented as a simple undirected weighted graph. In the case of directed weighted multi-networks, directed links are first treated as undirected, whereas all loops are discarded. Multiple links between nodes are then replaced by a single link with weight equal to the sum of weights of the original links (or equal to the number of links for unweighted networks).<sup>1</sup>

#### Terminology

A brief description of used terms is given below.

- G network represented as a simple undirected weighted graph
- $\mathbf{C}$  set of (non-overlapping) communities of network G
- **N** list of nodes of network G, n = |N|
- **L** set of links of network G, m = |L|
- $\Theta$  coloring of network G,  $\chi = |\Theta|$
- $\mathbf{N}^{\theta}$  nodes with color  $\theta \in \Theta$ ,  $N^{\theta} \subseteq N$
- $\mathbf{N^c}$  nodes in community  $c \in C$ ,  $N^c \subseteq N$
- $\Gamma_{\mathbf{i}}$  neighbors of node  $i \in N$ ,  $\Gamma_i \subseteq N$
- $\Gamma_{\mathbf{i}}^{\mathbf{c}}$  neighbors of node  $i \in N$  in  $c \in C$ ,  $\Gamma_{i}^{c} \subseteq \Gamma_{i}$
- $\psi_{\mathbf{i}}$  normalized index of node  $i \in N, \psi_i \in [0, 1)$
- $\theta_{\mathbf{i}}$  color of node  $i \in N$ ,  $\theta_i \in \Theta$
- $\mathbf{c_i}$  label of node  $i \in N, c_i \in C$
- $\mathbf{p_i}$  preference of node  $i \in N, p_i \in \mathbb{R}$
- $\mathbf{k_i}$  degree of node  $i \in N$ ,  $k_i = |\Gamma_i|$
- $\mathbf{k_i^c}$  community degree of node  $i \in N$ ,  $k_i = |\Gamma_i^c|$
- $\Gamma_{ij}$  common neighbors of nodes  $i, j \in N$ ,  $\Gamma_{ij} = \Gamma_i \cap \Gamma_j$
- $\mathbf{w_{ij}}$  weight of link  $ij \in L$ ,  $w_{ij} \in \mathbb{R}$
- $\mathbf{k_{ij}}$  common degree of nodes  $i, j \in N, k_{ij} = |\Gamma_{ij}|$

<sup>&</sup>lt;sup>1</sup>Link weights must be taken into account when computing network statistics.

# 2 Label propagation

In the following we review different label propagation strategies.

#### 2.1 Synchronous propagation

```
References. (Raghavan et al., 2007; Leung et al., 2009)
```

Description. Synchronous label propagation, where nodes' labels are identified in parallel (Alg. 1).

Comments. Problems with convergence in, e.g., near-bipartite or star-like networks.

#### Algorithm 1 Synchronous propagation

```
Input: network G
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i {Unique label.}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       {Synchronous propagation.}
       for all i \in N do
 8:
         {Label identification.}
 9:
         c_i \leftarrow \operatorname{argmax}_c k_i^c \{ \text{Majority label.} \}
10:
       end for
11:
12: end while
```

#### 2.2 Asynchronous propagation

```
References. (Raghavan et al., 2007; Leung et al., 2009)
```

Description. Asynchronous label propagation, where nodes' labels are identified sequentially (Alg. 2).

Comments. Problems with performance and stability in, e.g., large networks.

# Algorithm 2 Asynchronous propagation

```
Input: network G
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i {Unique label.}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       {Asynchronous propagation.}
 7:
      \mathbf{shuffle}(N)
 8:
      for i \in N do
 9:
         {Label identification.}
10:
         c_i \leftarrow \operatorname{argmax}_c k_i^c \text{ {Majority label.}}
11:
       end for
12:
13: end while
```

# 2.3 Semi-synchronous propagation

```
References. (Cordasco & Gargano, 2010, 2011)
```

Description. Semi-synchronous label propagation, where independent nodes' labels are identified in parallel (Alg. 3). (Independent nodes are decided according to a coloring of the network.)

Comments. Network coloring can be computationally expensive.

# Algorithm 3 Semi-synchronous propagation

```
Input: network G, coloring \Theta
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i \text{ {Unique label.}}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
 7:
       {Semi-synchronous propagation.}
       \mathbf{shuffle}(\Theta)
 8:
       for \theta \in \Theta do
 9:
          for all i \in N^{\theta} do
10:
             {Label identification.}
11:
             c_i \leftarrow \operatorname{argmax}_c k_i^c \{ \text{Majority label.} \}
12:
          end for
13:
       end for
14:
15: end while
```

# 3 Label identification

In the following we review different majority label identification strategies.

#### 3.1 General strategies

In the following we review general strategies for majority label identification.

# 3.1.1 Standard propagation

```
References. (Raghavan et al., 2007)
```

Description. Majority label identification based solely on the neighbors' labels (Alg. 4).

### Algorithm 4 Standard propagation

```
Input: network G
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i \text{ {Unique label.}}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       for all i \in N do
 7:
          {Label identification.}
 8:
          c_i \leftarrow \operatorname{argmax}_c k_i^c \{ \text{Standard propagation.} \}
 9:
10:
       end for
11: end while
```

#### 3.1.2 Weighted propagation

```
References. (Šubelj & Bajec, 2011c)
```

Description. Majority label identification based on neighbors' labels and link weights (Alg. 5).

# Algorithm 5 Weighted propagation

```
Input: network G
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i {Unique label.}
 3:
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       for all i \in N do
 7:
         {Label identification.}
 8:
         c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_i^c} w_{ij} {Weighted propagation.}
 9:
       end for
11: end while
```

#### 3.1.3 Preferenced propagation

```
References. (Leung et al., 2009)
```

Description. Majority label identification based on neighbors' labels and preferences (Alg. 6).

# Algorithm 6 Preferenced propagation

```
Input: network G
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i {Unique label.}
       p_i \leftarrow \dots {Initial preference.}
 5: end for
 6: {Label propagation.}
 7: while not terminated do
       for all i \in N do
          {Label identification.}
 9:
          p_i \leftarrow \dots \{ \text{Preference update.} \}
10:
          c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_i^c} p_j {Preferenced propagation.}
11:
          p_i \leftarrow \dots \{ \text{Preference update.} \}
12:
        end for
13:
14: end while
```

#### 3.2 Performance-based strategies

In the following we review majority label identification strategies that improve performance.

#### 3.2.1 Strength propagation

```
References. (Xie & Szymanski, 2011)
```

Description. Majority label identification based on neighbors' preferences that are decided according to a strength propagation strategy (Alg. 7). (Parameter w is set to 1.)

Comments. Main rationale here is to improve performance by considering neighbors equivalence.

### Algorithm 7 Strength propagation

```
Input: network G, parameter w
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i {Unique label.}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       for all i \in N do
 7:
          {Label identification.}
 8:
          c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_c^c} w_{ij} + w k_{ij} \{ \text{Strength propagation.} \}
 9:
       end for
10:
11: end while
```

#### 3.2.2 Degree propagation

```
References. (Leung et al., 2009)
```

Description. Majority label identification based on neighbors' preferences that are decided according to a degree propagation strategy (Alg. 8). (Parameter  $\alpha$  is set to 0.1.)

Comments. Presumably, rationale here is to improve performance in scale-free real-world networks.

#### Algorithm 8 Degree propagation

```
Input: network G, parameter \alpha
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i {Unique label.}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       for all i \in N do
 7:
          {Label identification.}
 8:
          c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_i^c} k_j^{\alpha} w_{ij} \text{ [Degree propagation.]}
 9:
       end for
10:
11: end while
```

#### 3.2.3 Defensive propagation

```
References. (Subelj & Bajec, 2011c, 2010)
```

Description. Majority label identification based on neighbors' preferences that are decided according to a defensive propagation strategy (Alg. 9).

Comments. Main rationale here is to improve performance in (denser) real-world networks.

## Algorithm 9 Defensive propagation

```
Input: network G
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
        c_i \leftarrow i \text{ {Unique label.}}
       p_i \leftarrow 1/n {Initial preference.}
 5: end for
 6: {Label propagation.}
 7: while not terminated do
        for all i \in N do
           {Label identification.}
 9:
           c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_c^c} p_j w_{ij}  {Defensive propagation.}
10:
          p_i \leftarrow \sum_{j \in \Gamma_i^{c_i}} p_j / k_j^{c_i} {Defensive preference.}
11:
13: end while
```

#### 3.2.4 Offensive propagation

```
References. (Šubelj & Bajec, 2011c, 2010)
```

Description. Majority label identification based on neighbors' preferences that are decided according to a offensive propagation strategy (Alg. 10).

Comments. Main rationale here is to improve performance in sparser real-world networks.

#### Algorithm 10 Offensive propagation

```
Input: network G
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
        c_i \leftarrow i {Unique label.}
        p_i \leftarrow 1/n {Initial preference.}
 5: end for
 6: {Label propagation.}
 7: while not terminated do
        for all i \in N do
 8:
           {Label identification.}
 9:
           c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_i^c} (1 - p_j) w_{ij} {Offensive propagation.}
10:
           p_i \leftarrow \sum_{j \in \Gamma_i^{c_i}} p_j / \tilde{k}_j^{\tilde{c}_i} {Offensive preference.}
11:
        end for
12:
13: end while
```

#### 3.2.5 Modularity propagation

```
References. (Barber & Clark, 2009; Liu & Murata, 2009a)
```

Description. Majority label identification based on neighbors' preferences that are decided according to a modularity optimization strategy (Alg. 11).

Comments. Main rationale here is to improve performance by directly optimizing community quality function denoted modularity Q (Newman & Girvan, 2004).

#### Algorithm 11 Modularity propagation

```
Input: network G
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i {Unique label.}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
 7:
       for all i \in N do
          {Label identification.}
 8:
          c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_i^c} 2w_{ij} - k_i k_j / m \text{ {Modularity propagation.}}
 9:
       end for
10:
11: end while
```

#### 3.2.6 Potts model propagation

```
References. (Boldi et al., 2011a; Ronhovde & Nussinov, 2010; Tibély & Kertész, 2008)
```

Description. Majority label identification based on neighbors' preferences that are decided according to a Potts model strategy (Alg. 12). (Parameter  $\gamma$  should be set between 0 and 1.)

Comments. Main rationale here is to improve performance by directly optimizing Hamiltonian of the corresponding Potts model. (Density of each community is above  $\gamma/(\gamma+1)$ .)

#### Algorithm 12 Potts model propagation

```
Input: network G, parameter \gamma
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i \text{ {Unique label.}}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       for all i \in N do
 7:
          {Label identification.}
 8:
         c_i \leftarrow \operatorname{argmax}_c k_i^c - \gamma (N^c - k_i^c) {Potts model propagation.}
 9:
       end for
10:
11: end while
```

#### 3.3 Stability-based strategies

In the following we review majority label identification strategies that improve stability.

#### 3.3.1 Momentum propagation

```
References. (Soman & Narang, 2011)
```

Description. Majority label identification based on nodes' preferences that are decided according to a momentum propagation strategy (Alg. 13). (Parameter w should be set to 1.)

Comments. Main rationale here is to improve stability by suppressing the occurrence of an epidemic spread (i.e., major community). (Realized by adding a loop with weight w to each node.)

#### Algorithm 13 Momentum propagation

```
Input: network G, parameter w
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i \text{ {Unique label.}}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
 7:
       for all i \in N do
          {Label identification.}
 8:
          c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_c^c} w_{ij} + w\delta(c, c_i) {Momentum propagation.}
 9:
10:
       end for
11: end while
```

#### 3.3.2 Controlled propagation

```
References. (Soman & Narang, 2011)
```

Description. Majority label identification based on neighbors' preferences that are decided according to a controlled propagation strategy (Alg. 14).

Comments. Main rationale here is to improve stability by suppressing the occurrence of an epidemic spread (i.e., major community). (Realized by restricting the spread of larger communities.)

# Algorithm 14 Controlled propagation

```
Input: network G
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i \text{ {Unique label.}}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       for all i \in N do
 7:
          {Label identification.}
 8:
          c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma^c} (2 - \sum_{l \in N^c} k_l / m) w_{ij} {Controlled propagation.}
 9:
       end for
10:
11: end while
```

#### 3.3.3 Attenuated propagation

```
References. (Leung et al., 2009; Šubelj & Bajec, 2011c)
```

Description. Majority label identification based on neighbors' preferences that are decided according to a attenuated propagation strategy (Alg. 15). (Parameter  $\delta_{\text{max}}$  is fixed to 0.5, whereas  $\delta_{\text{step}}$  should be set around 0.05. Alternatively, one can set parameter  $\delta$  to, e.g., 0.15.)

Comments. Main rationale here is to improve stability by suppressing the occurrence of an epidemic spread (i.e., major community). (Realized by restricting how far a community can spread.)

#### Algorithm 15 Attenuated propagation

```
Input: network G, parameters \delta_{\text{max}}, \delta_{\text{step}}
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
        c_i \leftarrow i {Unique label.}
        p_i \leftarrow 0 {Initial preference.}
 5: end for
 6: {Parameter initialization.}
 7: \delta \leftarrow \delta_{\text{max}} or \delta \leftarrow 0 {Initial parameter.}
 8: {Label propagation.}
 9: while not terminated do
        for all i \in N do
10:
11:
            {Label identification.}
           c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_i^c} (1 - \delta p_j) w_{ij} {Attenuated propagation.}
12:
            if c_i changed then
13:
               p_i \leftarrow 1 + \min_{j \in \Gamma_i^{c_i}} p_j {Attenuated preference.}
14:
            end if
15:
        end for
16:
17:
        {Parameter estimation.}
        \delta \leftarrow \max\{0, \delta - \delta_{\text{step}}\}\ \text{or}\ \delta \leftarrow \min\{\delta_{\text{max}}, ratio\ of\ labels\ changed\}\ \{\text{Attenuated\ parameter.}\}
19: end while
```

#### 3.3.4 Balanced propagation

```
References. (Subelj & Bajec, 2011b)
```

Description. Majority label identification based on neighbors' preferences that are decided according to a balanced propagation strategy (Alg. 16). (Parameter  $\lambda$  is fixed to 0.5, whereas  $\mu$  should be set between 0 and 2.)

Comments. Main rationale here is to improve stability of the asynchronous label propagation. (Increasing parameter  $\mu$  improves stability, but it also increases complexity.)

# Algorithm 16 Balanced propagation

```
Input: network G, parameters \lambda, \mu
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
        c_i \leftarrow i \text{ {Unique label.}}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       \mathbf{shuffle}(N)
 8:
        for i \in N do
 9:
           {Label identification.}
           p_i \leftarrow \psi_i \text{ or } p_i \leftarrow 1/(1 + e^{-\mu(\psi_i - \lambda)}) \text{ {Balanced preference.}}
10:
           c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_i^c} p_j w_{ij} {Balanced propagation.}
11:
        end for
12:
13: end while
```

#### Algorithm 17 Selective propagation

```
Input: network G, parameter \omega
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i {Unique label.}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       for all i \in N do
 7:
 8:
          {Label identification.}
          if k_i^{c_i}/k_i \leq \omega then
 9:
             c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_i^c} w_{ij} {Selective propagation.}
10:
          end if
11:
       end for
12:
13: end while
```

# Algorithm 18 Passive propagation

```
Input: network G
Output: communities C
 1: {Label initialization.}
 2: for i \in N do
       c_i \leftarrow i {Unique label.}
 4: end for
 5: {Label propagation.}
 6: while not terminated do
       for all i \in N do
 7:
           {Label identification.}
 8:
          if \exists c \in C \setminus \{c_i\} : k_i^c \ge k_i^{c_i} then
 9:
             c_i \leftarrow \operatorname{argmax}_c \sum_{j \in \Gamma_i^c} w_{ij} \text{ {Passive propagation.}}
10:
          end if
11:
        end for
12:
13: end while
```

# 3.4 Complexity-based strategies

In the following we review majority label identification strategies that improve complexity.

## 3.4.1 Selective propagation

```
References. (Leung et al., 2009)
```

Description. Majority label identification based on neighbors' labels and link weights, and a selective propagation strategy (Alg. 17). (Parameter  $\omega$  is set to 0.5.)

Comments. Main rationale here is to improve complexity by propagating the labels selectively. (Realized by discarding the nodes whose labels are unlikely to change.)

#### 3.4.2 Passive propagation

```
References. (Xie & Szymanski, 2011)
```

Description. Majority label identification based on neighbors' labels and link weights, and a passive propagation strategy (Alg. 18).

Comments. Main rationale here is to improve complexity by propagating the labels passively. (Realized by discarding the nodes whose labels cannot change.)

# 4 Label ties

In the following we review different strategies for resolving majority label ties.

#### 4.1 Random label

```
References. (Raghavan et al., 2007)

Description. Ties are broken uniformly at random.
```

#### 4.2 Label retention

```
References. (Barber & Clark, 2009)
```

Description. Ties are broken uniformly at random, while a label is retained if among majority labels.

#### 4.3 Label inclusion

```
References. (Leung et al., 2009)
```

Description. Ties are broken uniformly at random, while a label is included in the majority label identification.

# 4.4 Label priority

```
References. (Cordasco & Gargano, 2010, 2011)
```

Description. Ties are broken due to label priority.

Comments. Label priority is an arbitrary number defined a priori.

# 5 Propagation criteria

In the following we review different propagation termination criteria.

#### 5.1 Label equilibrium

```
References. (Raghavan et al., 2007)
```

Description. Propagation is terminated, when each node's label equals the majority label.

## 5.2 Label convergence

```
References. (Barber & Clark, 2009)
```

Description. Propagation is terminated, when each node's label equals the label on the previous step.

# 5.3 Label semi-convergence

```
References. (Cordasco & Gargano, 2010, 2011)
```

Description. Propagation is terminated, when each node's label equals the label on the previous step, or the step before.

# 5.4 Threshold convergence

```
References. (Šubelj & Bajec, 2011c, 2010)
```

Description. Propagation is terminated, when the number of steps exceeds the defined threshold. (The threshold is set to, e.g., 100.)

# 6 Advanced propagation

In the following we give references to other advances of label propagation not reviewed in this work.

# 6.1 Hierarchical propagation

References. (Leung et al., 2009; Šubelj & Bajec, 2010, 2011c, 2012a; Xie & Szymanski, 2012)

# 6.2 Refining propagation

References. (Šubelj & Bajec, 2011c, 2010; Coscia et al., 2012)

# 6.3 Hybrid propagation

References. (Barber & Clark, 2009; Liu & Murata, 2009a; Šubelj & Bajec, 2010, 2011c,b; Gregory, 2010; Wu et al., 2012)

# 6.4 Divisive propagation

References. (Pang et al., 2009a)

## 6.5 Online propagation

References. (Pang et al., 2009b; Leung et al., 2009)

# 6.6 Parallel propagation

References. (Soman & Narang, 2011; Leung et al., 2009; Cordasco & Gargano, 2010, 2011; Boldi et al., 2011a; Rees & Gallagher, 2012a)

# 7 Other networks

In the following we give references to extensions of label propagation to other types of network.

#### Bipartite networks

```
References. (Liu & Murata, 2009b,c,a; Xie & Szymanski, 2012)
```

# Multi-partite networks

```
References. (Liu & Murata, 2011)
```

# 8 Other groups

In the following we give references to extensions of label propagation to other groups of network nodes.

# Overlapping community detection

```
References. (Gregory, 2010; Xie et al., 2011; Xie & Szymanski, 2012; Coscia et al., 2012; Wu et al., 2012; Rees & Gallagher, 2012b,a; Leung et al., 2009; Soman & Narang, 2011)
```

# Community and module detection

```
References. (Šubelj & Bajec, 2012b, 2011a, 2012a)
```

# 9 Applications

In the following we give references to different applications of label propagation.

#### Network robustness

```
References. (Boldi et al., 2011b)
```

# Network compression

```
References. (Boldi et al., 2011a)
```

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