node *mixing*

introduction to *network analysis* (*ina*)

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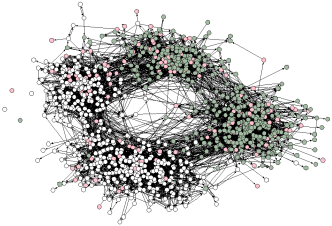
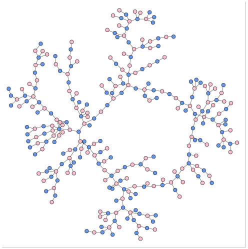
mixing *definition*

- *node mixing* = *correlations between linked* nodes
- in *assortative mixing* nodes are *linked to similar* others
- in *disassortative mixing* nodes *linked to dissimilar* others



assortative mixing by age/race



disassortative mixing by gender

mixing *degree*

- special case of *node mixing by degree* [New02]
- majority of *social networks degree assortative*
- most *other networks* are *degree disassortative*

$$p_{kk'} = k\frac{k'}{2m-1} = m\frac{kk'}{\binom{2m}{2}} \approx \frac{kk'}{2m}$$

celebrity hubs date hubs

but $10^3/10^8 = 0.00001$

protein hubs avoid hubs

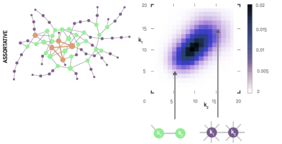
but $p_{56,13} = 0.16 \gg p_{1,2} = 0.0004$

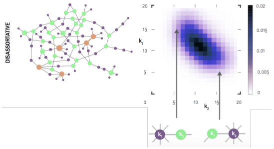
mixing *matrix*

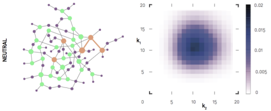
- *endpoints degree distribution* $e_{kk'}$ defined as
  - $e_{kk'}$ is *link probability* between *degree-k & -k'* nodes
  - q_k is *neighbor non-excess degree distribution* $\frac{kp_k}{\langle k \rangle}$

$$\sum_{kk'} e_{kk'} = 1 \qquad \sum_{k'} e_{kk'} = q_k = n_k \frac{k}{2m-1} \approx \frac{kp_k}{\langle k \rangle}$$

$e_{kk'} = q_k q_{k'}$ in *neutral networks* but impractical for (*dis*)*assortative networks*







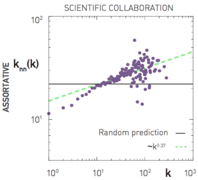
mixing *exponent*

— *neighbor degree function* k_{nn} [PSVV01] defined as

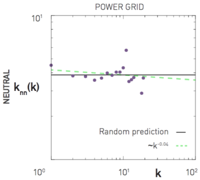
- $k_{nn}(k)$ is *average neighbor degree* of *degree-k* nodes
- $\mathrm{P}(k'|k)$ is *link probability* of *degree-k to -k'* node
- μ is *degree mixing power-law exponent* [VPSV02]

$$k_{nn}(k) = \sum_{k'} k'\mathrm{P}(k'|k) = \sum_{k'} k' \frac{e_{kk'}}{\sum_{k'} e_{kk'}}$$

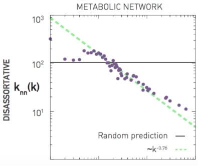
$k_{nn} = \frac{\langle k^2 \rangle}{\langle k \rangle}$ in *neutral networks* and $k_{nn}(k) \sim k^{\mu}$ in (*dis*)*assortative networks*



$\mu = 0.37 \pm 0.11$



$\mu = 0.04 \pm 0.05$



$\mu = -0.76 \pm 0.04$

mixing *coefficient*

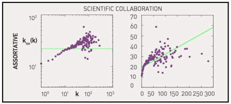
— *degree mixing coefficient r* [New02, Est11] defined as

  – r is *Pearson correlation* of *linked nodes' degrees* [New03]

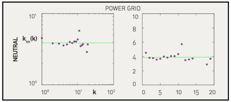
  – q_k is *neighbor excess degree distribution* $\frac{(k+1)p_{k+1}}{\langle k\rangle}$

$$r = \sum_{kk'} \frac{kk'(e_{kk'} - q_k q_{k'})}{\sum_k k^2 q_k - \left(\sum_k k q_k\right)^2}$$

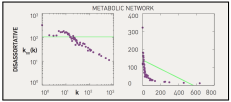
$r = 0$ in *neutral networks* and $k_{nn}(k) \sim rk$ in *(dis)assortative networks*



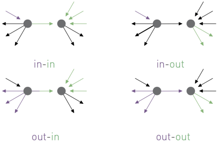
$r = 0.13$

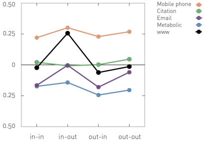


$r = 0$



$r = -0.04$



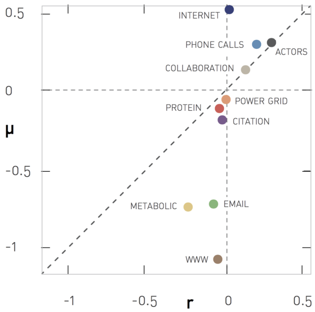


mixing *networks*

— *coefficient & exponent r & μ* in real networks [Bar16]

— *r & μ correlate* in *assortative* regime and $\text{sgn}(r) = \text{sgn}(\mu)$

NETWORK	N	r	μ
Internet	192,244	0.02	0.56
WWW	325,729	-0.05	-1.11
Power Grid	4,941	0.003	0.0
Mobile Phone Calls	36,595	0.21	0.33
Email	57,194	-0.08	-0.74
Science Collaboration	23,133	0.13	0.16
Actor Network	702,388	0.31	0.34
Citation Network	449,673	-0.02	-0.18
E. Coli Metabolism	1,039	-0.25	-0.76
Protein Interactions	2,018	0.04	-0.1



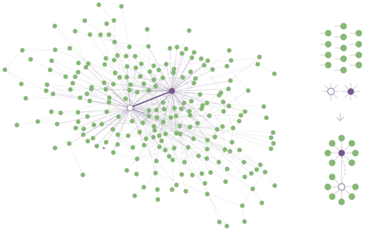
mixing *structural*

- *structural disassortativity* $\frac{E_{kk'}}{m_{kk'}} > 1$ [MSZ04] in real networks
  - $E_{kk'}$ is *number of links* between *degree-k & -k'* nodes
  - $m_{kk'}$ is *maximum* $E_{kk'}$ hence $\min(kn_k, k'n_{k'}, n_k n_{k'})$

$$E_{kk'} = 2me_{kk'} = \langle k \rangle ne_{kk'}$$

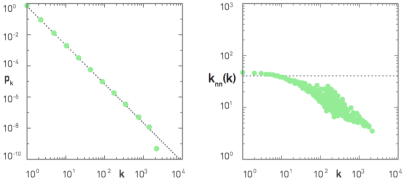
natural cutoff $k_{max} \sim n^{\frac{1}{\gamma-1}}$ and *structural cutoff* $k_s \sim \sqrt{\langle k \rangle n}$

- *structural disassortativity* in *scale-free* networks with $\gamma < 3$

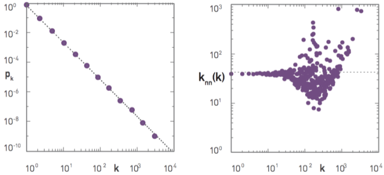


$k = 55$ and $k' = 46$ then $E_{kk'} = 2.81 > 1$

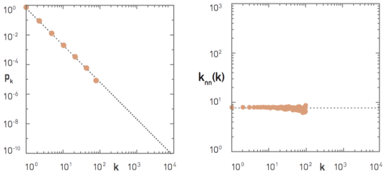
mixing *scale-free*



configuration *scale-free* network as *simple* *graph*

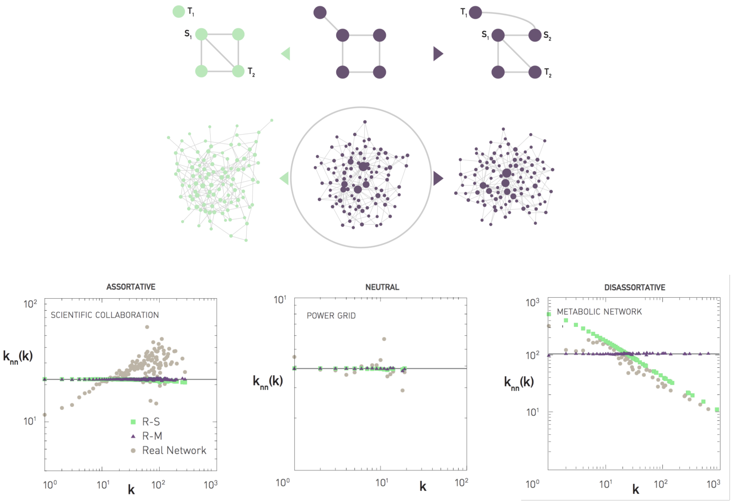


configuration *scale-free* network as *multigraph*



configuration *scale-free* network *without* ***hubs*** $k \geq k_s$

mixing *randomization*



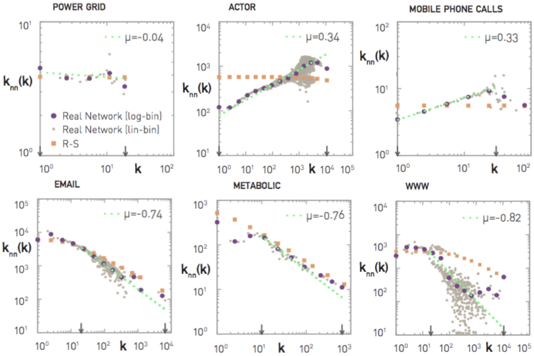
degree-preserving randomization with *simple/multi* links *retains/destroys structural disassortativity*

mixing *networks*

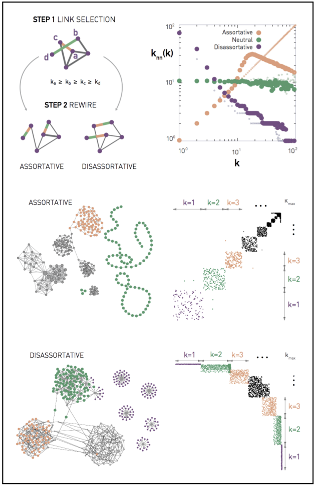
— *neighbor degree k_{nn}* in real networks [Bar16]

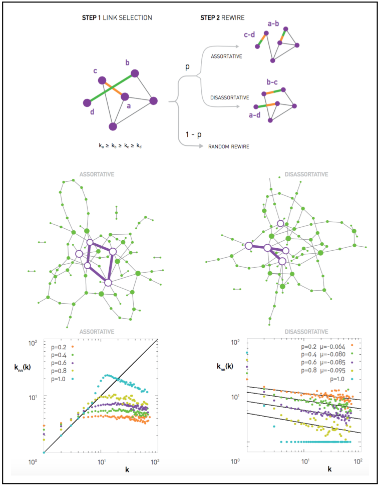
— *collaboration assortative* and *technological neutral*

— *biological/information (structurally) disassortative*



mixing *models*

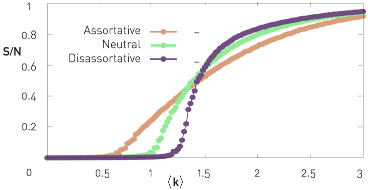


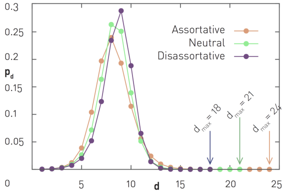


(dis)assortative degree-preserving randomization [XBS05]

mixing *impact*

- *degree mixing* impacts *connectivity* and *distances* [New02]
- *assortative mixing* coexists with *community structure* [NP03]
- *mixing* influences *resilience* [VM03] and *controllability* [LSB11]





mixing *references*

A.-L. Barabási.
Network Science.
Cambridge University Press, Cambridge, 2016.

Wouter de Nooy, Andrej Mrvar, and Vladimir Batagelj.
Exploratory Social Network Analysis with Pajek: Expanded and Revised Second Edition.
Cambridge University Press, Cambridge, 2011.

David Easley and Jon Kleinberg.
Networks, Crowds, and Markets: Reasoning About a Highly Connected World.
Cambridge University Press, Cambridge, 2010.

Ernesto Estrada and Philip A. Knight.
A First Course in Network Theory.
Oxford University Press, 2015.

Ernesto Estrada.
Combinatorial study of degree assortativity in networks.
Phys. Rev. E, 84(4):047101, 2011.

Yang-Yu Liu, Jean-Jacques Slotine, and Albert-Laszlo Barabasi.
Controllability of complex networks.
Nature, 473(7346):167–173, 2011.

Sergei Maslov, Kim Sneppen, and Alexei Zaliznyak.
Detection of topological patterns in complex networks: Correlation profile of the internet.
Physica A, 333:529–540, 2004.

M. E. J. Newman.
Assortative mixing in networks.
Phys. Rev. Lett., 89(20):208701, 2002.

mixing *references*

M. E. J. Newman.
Mixing patterns in networks.
Phys. Rev. E, 67(2):026126, 2003.

Mark E. J. Newman.
Networks.
Oxford University Press, Oxford, 2nd edition edition, 2018.

M. E. J. Newman and Juyong Park.
Why social networks are different from other types of networks.
Phys. Rev. E, 68(3):036122, 2003.

Romualdo Pastor-Satorras, Alexei Vázquez, and Alessandro Vespignani.
Dynamical and correlation properties of the Internet.
Phys. Rev. Lett., 87(25):258701, 2001.

Alexei Vázquez and Yamir Moreno.
Resilience to damage of graphs with degree correlations.
Phys. Rev. E, 67(1):015101, 2003.

Alexei Vázquez, Romualdo Pastor-Satorras, and Alessandro Vespignani.
Large-scale topological and dynamical properties of the Internet.
Phys. Rev. E, 65(6):066130, 2002.

R. Xulvi-Brunet and I. M. Sokolov.
Changing correlations in networks: Assortativity and dissortativity.
Acta Phys. Pol. B, 36:1431–1455, 2005.