

link analysis

introduction to *network analysis* (*ina*)

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link analysis

which *web pages* are most *important*?

- *node centrality measures* for (*un*)*directed* networks
- *link analysis algorithms* primarily for *directed web graphs*
 - Google *search ranking PageRank* [BP98, PBMW99]
 - hyperlink-induced *topic search HITS* [Kle99]



Sergey Brin



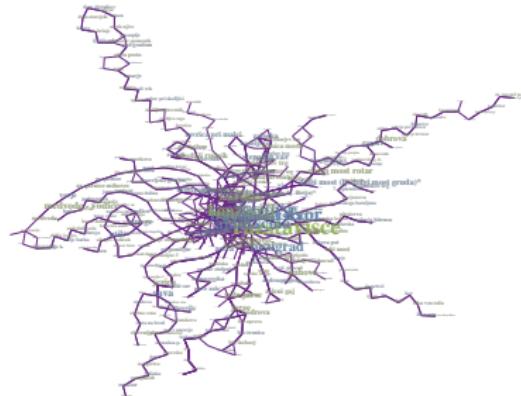
Lawrence Page



Jon Kleinberg

analysis LPP

- corrected *LPP public bus transport network**
- $n = 408$ bus stops with $\langle k \rangle = 5.73$ connections
- *giant component* 95.3% nodes (6 components)
- “*small-world*” with $\langle C \rangle = 0.10$ and $\langle d \rangle = 14.43$
- “*scale-free*” with $\gamma = 2.60$ for cutoff $k_{min} = 5$



* reduced to largest connected component

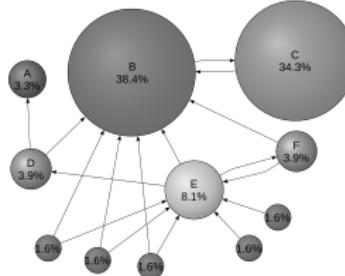
link *PageRank*

ranking algorithm for web page importance

- for *directed G* *PageRank rank p* [BP98] of *i* is
 - α is *positive constant* traditionally $\alpha = 0.85$

$$p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j^{out}} + \frac{1 - \alpha}{n}$$

- p_i probability that *random surfer with teleports* lands on *i*
- just *random walk oscillates in traps & leaks from dead ends*



analysis *PageRank*

- *PageRank ranks* p in corrected LPP network
- *highest* p nodes are *Razstavišče* and *Ajdotvčina*

#	bus stop	k_i	p_i
1	Razstavišče	43	0.010601
2	Ajdotvčina	36	0.007694
3	Bežigrad	23	0.007161
4	Bavarski dvor	30	0.007013
5	Konzorcij	30	0.006884
6	Gospovetska	30	0.006527
7	Stara cerkev	26	0.005485
8	Sava	12	0.005165
9	Tobačna	22	0.005136
10	Kino Šiška	18	0.004907
11	Medvode	4	0.004853
12	Tivoli	26	0.004838

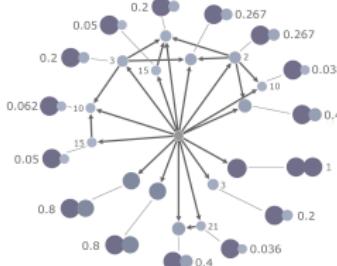
link *random walk with restarts*

ranking algorithm for *web page similarity*

- for *directed G random walk rank w* [TFP06] for *t* of *i* is
 - α is *positive constant* traditionally $\alpha = 0.85$

$$w_i = \alpha \sum_j A_{ij} \frac{w_j}{k_j^{\text{out}}} + (1 - \alpha) \delta_{it}$$

- w_i probability *random surfer with teleport t* lands on *i*
- *personalized PageRank* and *SimRank* [PBMW99, JW02]



analysis *random walk with restarts*

- *random walk ranks* w in corrected LPP network
- *highest* w nodes for *Razstavišče* and *Hajdrihova*

#	bus stop	k_i	w_i	#	bus stop	k_i	w_i
1	Razstavišče	43	0.236115	1	Hajdrihova	14	0.201318
2	Bavarski dvor	30	0.065124	2	Tobačna	22	0.091186
3	Bezigrad	23	0.057260	3	Ilirija	12	0.051714
4	Astra	16	0.047765	4	Stara cerkev	26	0.046825
5	Ajdovščina	36	0.040099	5	Tabor	10	0.038395
6	Kozolec	10	0.038384	6	Vič	16	0.034478
7	Gospovshtska	30	0.030981	7	Avtomontaža	6	0.030372
8	Konzorcij	30	0.020278	8	Stan in dom	4	0.030296
9	Bavarski dvor	8	0.019262	9	Kino Šiška	18	0.028569
10	Polje	10	0.014254	10	Tivoli	26	0.028180
11	Stadion	8	0.013294	11	Glince	8	0.027528
12	Topniška	8	0.013235	12	Na klancu	10	0.023836

link HITS

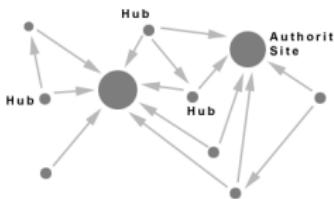
ranking algorithm for web hubs & authorities

- for directed G hub & authority ranks h & a [Kle99] of i

- h is eigenvector of $A^T A$ with eigenvalue $(\alpha\beta)^{-1}$
 - a is eigenvector of AA^T with eigenvalue $(\alpha\beta)^{-1}$
 - α and β are some positive constants

$$h_i = \alpha \sum_j A_{ji} a_j \quad a_i = \beta \sum_j A_{ij} h_j$$

- a measures content and h measures table of content
- $a = 0$ for $k^{in} = 0$ nodes and $h = 0$ for $k^{out} = 0$ nodes



analysis *HITS*

- hub & authority ranks h & a in corrected LPP network
- highest h node is *Ajdovščina* and highest a node is *Konzorcij*

#	bus stop	k_i	h_i
1	Ajdovščina	36	0.715370
2	Razstavišče	43	0.455771
3	Tivoli	26	0.286178
4	Drama	23	0.256027
5	Gospovshtska	30	0.175142
6	Bavarski dvor	30	0.129155
7	Pošta	9	0.111497
8	Kolodvor	4	0.090644
9	Konzorcij	30	0.083028
10	Tavčarjeva	7	0.069477
11	Kozolec	10	0.068749
12	Stara cerkev	26	0.064760

#	bus stop	k_i	a_i
1	Konzorcij	30	0.656745
2	Bavarski dvor	30	0.512119
3	Gospovshtska	30	0.235790
4	Kozolec	10	0.224651
5	Bežigrad	23	0.176839
6	Astra	16	0.172509
7	Stara cerkev	26	0.172482
8	Ajdovščina	36	0.161840
9	Razstavišče	43	0.110391
10	Tivoli	26	0.106024
11	Bavarski dvor	8	0.096486
12	Kolizej	4	0.088636

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