

link *bridging*

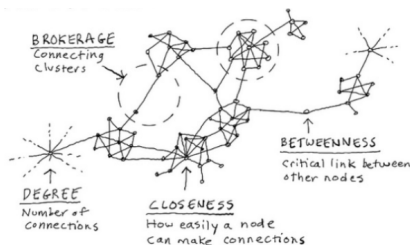
introduction to *network analysis* (*ina*)

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bridging *measures*

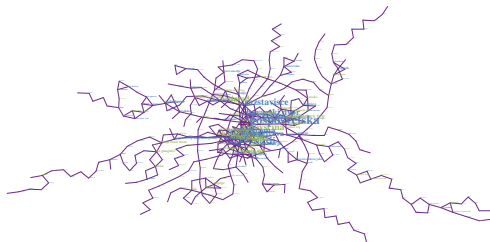
which *links* are most *important*?

- *link bridging measures* for (un)directed networks
 - *betweenness-based* centrality [Fre77, FBW91, New05]
- *link embeddedness measures* for (un)directed networks
 - *topological overlap* measures [RSM⁺02, OSH⁺07, dNMB11]



networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 2.72$ connections
- *giant component* 95.4% nodes (6 components)
- “small-world” with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- “scale-free” with $\gamma = 2.43$ for cutoff $k_{min} = 2$



* reduced to largest connected component of simple undirected graph

bridging *betweenness*

important *links* are *between other nodes*

- for (un)directed G link betweenness σ [Fre77] of $\{i, j\}$ is
 - g_{st} is number of *geodesic paths between s and t*
 - g_{st}^{ij} is number of *such geodesic paths through $\{i, j\}$*

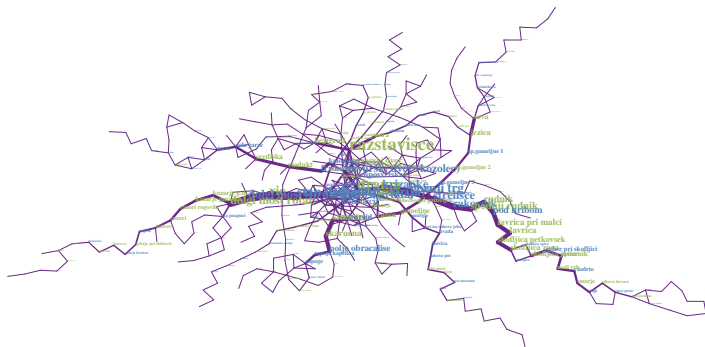
$$\sigma_{ij} = \sum_{st \notin \{i, j\}} \frac{g_{st}^{ij}}{g_{st}}$$

- σ considers *only geodesic paths* [FBW91, New05]



networkology *betweenness*

- *link betweenness* σ in partial LPP network[†]
- *highest* $\sigma_{ij} = 0.176n^2$ link is {*Vič, Stan in dom*}



[†] reduced to largest connected component of simple undirected graph

important *links* are *bridges between nodes*

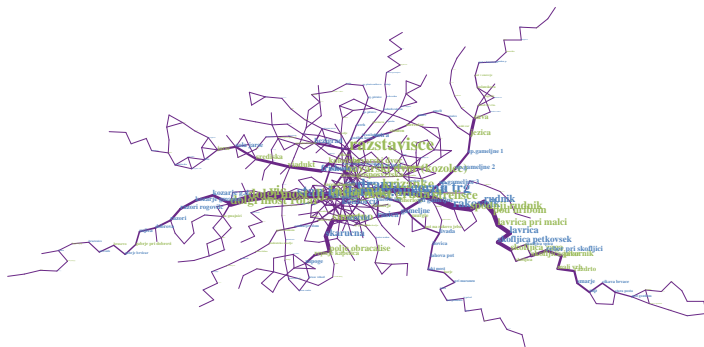
- for (un)directed G *link bridgeness* $\tilde{\sigma}$ [JMK⁺16] of $\{i, j\}$ is
 - g_{st} is number of *geodesic paths between s and t*
 - g_{st}^{ij} is number of *such geodesic paths through $\{i, j\}$*

$$\tilde{\sigma}_{ij} = \sigma_{ij} - \sum_{st \in \Gamma_i \cup \Gamma_j} \frac{g_{st}^{ij}}{g_{st}} = \sum_{st \notin \Gamma_i \cup \Gamma_j} \frac{g_{st}^{ij}}{g_{st}}$$

- σ mixes *local centers* with *global bridges* [JMK⁺16]



- *link bridgeness* $\tilde{\sigma}$ in partial LPP network[‡]
- *highest* $\tilde{\sigma}_{ij} = 0.169n^2$ link is {*Vič, Stan in dom*}



[‡] reduced to largest connected component of simple undirected graph

bridging *embeddedness*

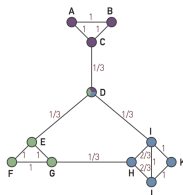
important *links* are *embedded between nodes*

- for *undirected G link embeddedness*[§] θ [OSH⁺07] of $\{i, j\}$ is
 - Γ_i is set of *neighbors* or *neighborhood* of i

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \quad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$

- μ -corrected *link embeddedness* $\tilde{\theta}$ [dNMB11] of $\{i, j\}$ is
 - μ is *maximum* number of *triangles* over *links*

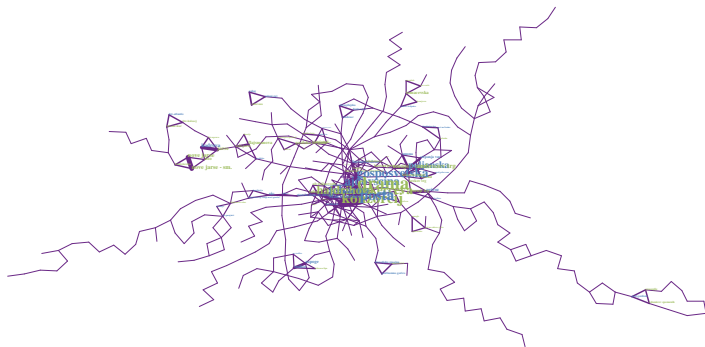
$$\tilde{\theta}_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{\mu + \max(k_i, k_j) - 1 - |\Gamma_i \cap \Gamma_j|}$$



[§] θ & $\tilde{\theta}$ better known as topological overlap indices/weights

networkology μ -embeddedness

- μ -corrected embeddedness $\tilde{\theta}$ in partial LPP network^{||}
- highest $\tilde{\theta}_{ij} = 0.4$ links are $\{Pošta, Konzorcij\}$ etc.



^{||} reduced to largest connected component of simple undirected graph

bridging *overview*

which *links* are most *important*?

| | | | | | | | | | | | | | | | | | | | | | | | |
|--------|--------------------------|------------------------|--------------------------|---------------------------|--------------------------|------------------|-------------------------|------------------|------------------|------------------|------------------|------------------|------------------|-------------------------|-------------------------|---------------------------------|-------------------------|-----------------------------|-----------------|--|--|--|--|
| 1 IA | | | | | | | | | | | | 18 VIIIA | | | | | | | | | | | |
| 1 | 34 | | | | | | | | | | | 2 | | | | | | | | | | | |
| 1 | DC | | | | | | | | | | | | | | | | | | | | | | |
| | Degree Centrality | | | | | | | | | | | | | | | | | | | | | | |
| 2 | BC | CC | | | | | | | | | | | 5 | SC | C _{COEF} | C _{COEF} ⁻¹ | MNC | EC _{COEF} | PR | | | | |
| | Betweenness Centrality | Closeness Centrality | | | | | | | | | | | | Subgraph Centrality | Clustering Coefficient | Inverse COEF | max. neigh. comp. | edge clustering coefficient | PageRank | | | | |
| 3 | RL | IC | | | | | | | | | | | 91 | CC _{COEF} | SC _{Co} | LAC | DMNC | SEC _{COEF} | LR | | | | |
| | Range-Linked Betweenness | Information Centrality | | | | | | | | | | | | odd Subgraph Centrality | old Subgraph Centrality | loc. avg. Connectivity | dens. max. length comp. | sum of EC _{COEF} | LeaderRank | | | | |
| 4 | BN | RC | IG | DC _{Co} | BC _{Co} | CC _{Co} | EC _{Co} | KS _{Co} | PR _{Co} | IG _{Co} | RC _{Co} | DC _{Co} | BC _{Co} | SC _{Co} | KL | COC _{Co} | DMNC | SEC _{Co} | KS | | | | |
| | Betweenness Centrality | Radiality Centrality | Integration | | | | | | | | | | | new Subgraph Centrality | Clique Level | cooper. weight COEF | dens. max. length comp. | sum of EC _{Co} | KatzRank | | | | |
| 5 | RWBC | RWCC | CC _{2,3,4} | EC _{Co} | PR _{Co} | KS _{Co} | CC _{Co} | RC _{Co} | IG _{Co} | DC _{Co} | BC _{Co} | CKS | KSPR | DCPR | β | SC ₂ | SC ₃ | SC ₄ | EC ₂ | | | | |
| | RandomWalk Betweenness | RandomWalk Closeness | 2,3,4-localized CC | | | | | | | | | | | | Bipartivity | 2-localized SC | Neighborhood Centrality | 2-localized EC | | | | | |
| 6 | σ | ECC | WDC | DC _{Co} | BC _{Co} | CC _{Co} | EC _{Co} | KS _{Co} | PR _{Co} | IG _{Co} | DC _{Co} | BC _{Co} | CCPR | KBG | DCG | DC _{Co} | SC ₂ | LI | EC ₂ | | | | |
| | sigma Centrality | Eccentricity | Weighted Degree | | | | | | | | | | | | | | 3-localized SC | Lobby Index | 3-localized EC | | | | |
| 7 | BC _{2,3,4} | ECC ⁻¹ | SDC | DC _{Co} | BC _{Co} | CC _{Co} | EC _{Co} | KS _{Co} | PR _{Co} | IG _{Co} | DC _{Co} | BC _{Co} | CCIG | DCPR | BCIG | EC _{Co} | SC ₂ | EC ₂ | | | | | |
| | 2,3,4-localized-BC | Inverse Eccentricity | Sphere Degree Centrality | | | | | | | | | | | | | | 4-localized SC | 4-localized EC | | | | | |
| Z | | | | | | | | | | | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | | | | | | | | | | | |
| Name | | | | | | | | | | | | | | | | | | | | | | | |
| Hybrid | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | |
| 22 | FC | FD | US | DIS | ASS | DAM | UC | | | | | | | | | | | | | | | | |
| | Functional Centrality | Functional Diversity | UniScore | Pairwise Dis-connectivity | Assortative Mixing | Damage | United comp. Centrality | | | | | | | | | | | | | | | | |
| 28 | EI | CM | NαC | MC | HGI | HYP | HC | | | | | | | | | | | | | | | | |
| | Essentiality Index | Complexity Measure | Normalized α Centrality | Modular Centrality | Hungry Graph Information | Hypertonic Index | Harmonic Centrality | | | | | | | | | | | | | | | | |

- Betweenness-based
- Distance-based
- Linear Combinations
- Subgraph-based
- Clustering Coefficient-based
- Edge Clustering Coefficient-based
- Spectral-based
- Miscellaneous

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bridging *references*



A.-L. Barabási.

Network Science.

Cambridge University Press, Cambridge, 2016.



Wouter de Nooy, Andrej Mrvar, and Vladimir Batagelj.

Exploratory Social Network Analysis with Pajek: Expanded and Revised Second Edition.

Cambridge University Press, Cambridge, 2011.



David Easley and Jon Kleinberg.

Networks, Crowds, and Markets: Reasoning About a Highly Connected World.

Cambridge University Press, Cambridge, 2010.



Ernesto Estrada and Philip A. Knight.

A First Course in Network Theory.

Oxford University Press, 2015.



Linton C. Freeman, Stephen P. Borgatti, and Douglas R. White.

Centrality in valued graphs: A measure of betweenness based on network flow.

Soc. Networks, 13(2):141–154, 1991.



L. Freeman.

A set of measures of centrality based on betweenness.

Sociometry, 40(1):35–41, 1977.



Pablo Jensen, Matteo Morini, Marton Karsai, Tommaso Venturini, Alessandro Vespignani, Mathieu Jacomy, Jean-Philippe Cointet, Pierre Merckle, and Eric Fleury.

Detecting global bridges in networks.

J. Complex Netw., 4(3):319–329, 2016.

bridging *references*



M. E. J. Newman.

A measure of betweenness centrality based on random walks.
Soc. Networks, 27(1):39–54, 2005.



Mark E. J. Newman.

Networks.
Oxford University Press, Oxford, 2nd edition edition, 2018.



J.-P. Onnela, J. Saramäki, J. Hyvönen, G. Szabó, D. Lazer, K. Kaski, J. Kertész, and A.-L. Barabási.
Structure and tie strengths in mobile communication networks.
P. Natl. Acad. Sci. USA, 104(18):7332–7336, 2007.



E. Ravasz, A. L. Somera, D. A. Mongru, Z. N. Oltvai, and Albert László Barabási.
Hierarchical organization of modularity in metabolic networks.
Science, 297(5586):1551–1555, 2002.