network *representations*

introduction to *network analysis* (*ina*)

Lovro Šubelj
University of Ljubljana
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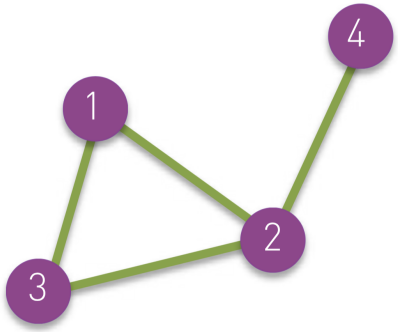
network *representations*



undirected graph

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

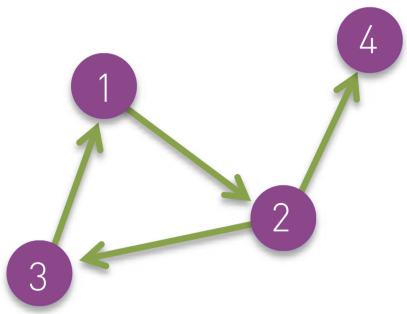
adjacency matrix

1: [2, 3]
2: [1, 3, 4]
3: [1, 2]
4: [2]

adjacency list

{1, 2}
{1, 3}
{2, 3}
{2, 4}

edge list



directed graph

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

adjacency matrix

[3] :1: [2]
[1] :2: [3, 4]
[2] :3: [1]
[2] :4: [ ]

adjacency list

(1, 2)
(2, 3)
(2, 4)
(3, 1)

edge list

---

*adjacency list can also be implemented with maps or trees & edge list does not represent isolated nodes

network *representations*

— *adjacency matrix* for elegant *analytical derivations*

most derivations based on matrix representation†

— *adjacency list* for efficient *algorithms implementation*

ideal complexity since most algorithms only require incidence†

— *edge list* for efficient *network storing/manipulation*

easy editing since each is edge stored only once

† some derivations can also be based on adjacency list & some algorithms require edge list

network *structures*

— *edge list edges data structure* complexity

data structure	link manipulation	random node	random link
array	none	$\mathcal{O}(m)$	$\mathcal{O}(1)$
array list	addition	$\mathcal{O}(m)$	$\mathcal{O}(1)$
hash map	any	$\mathcal{O}(m)$	$\mathcal{O}(m)$

— *adjacency list nodes data structure* complexity

data structure	node manipulation	random node	random link
array	none	$\mathcal{O}(1)$	$\mathcal{O}(m)$
array list	addition	$\mathcal{O}(1)$	$\mathcal{O}(m)$
hash map	any	$\mathcal{O}(n)$	$\mathcal{O}(m)$

— *adjacency list neighbors data structure* complexity

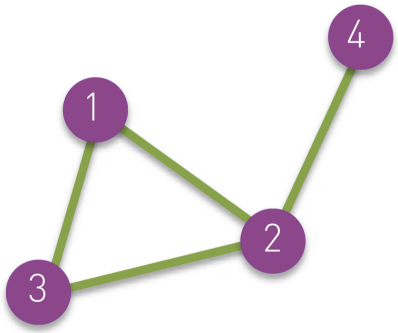
data structure	link manipulation	node incidence	random neighbor
array	none	$\mathcal{O}(k)$	$\mathcal{O}(1)$
array list	addition	$\mathcal{O}(k)$	$\mathcal{O}(1)$
hash map	any	$\approx \mathcal{O}(1)$	$\mathcal{O}(k)$
tree map	any	$\mathcal{O}(\log k)$	$\mathcal{O}(k)$

— *hash maps* for *construction* and *arrays* for *analysis*

— use *directed adjacency list* with *undirected flag*

‡ random link selection equivalent to random node selection by degree

network *formats*



undirected graph

```
# undirect
1 2
1 3
2 3
2 4
```

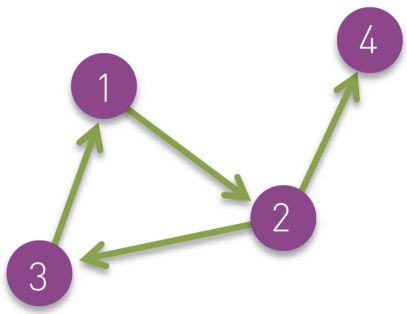
edge list

```
*vertices 4
1 "1"
2 "2"
3 "3"
4 "4"
*edges
1 2
1 3
2 3
2 4
```

Pajek format

```
# undirect
# 0 "1"
# 1 "2"
# 2 "3"
# 3 "4"
#
0 1
0 2
1 2
1 3
```

another format



directed graph

```
# directed
1 2
2 3
2 4
3 1
```

edge list

```
*vertices 4
1 "1"
2 "2"
3 "3"
4 "4"
*arcs
1 2
2 3
2 4
3 1
```

```
# directed
# 0 "1"
# 1 "2"
# 2 "3"
# 3 "4"
#
0 1
1 2
1 3
2 0
```

§ ad-hoc edge list and Pajek format most popular & other formats include GML, GraphML and JSON

network *data*

- easily obtained from *online sources*
- already present in many *standard datasets*
- *personal web pages* of network researchers
- popular *network repositories/collections*
  - **Network Catalogue and Repository** [Netzschleuder]
  - Colorado Index of Complex Networks [ICON]
  - Stanford Network Analysis Project [SNAP]
  - Koblenz Network Collection [KONECT]
  - Open Graph Benchmark [OGB]
  - Network Repository [NetRepo]
  - Pajek datasets [Pajek]

network *software*

- most popular *Python libraries*
  - igraph [https://igraph.org]
  - **NetworkX** [https://networkx.org]
  - graph-tool [https://graph-tool.skewed.de]
  - Snap.py [https://snap.stanford.edu/snappy]
  - Pajek [http://mrvar.fdv.uni-lj.si/pajek]

- most popular *network software*
  - **Gephi** [https://gephi.org]
  - visone [https://visone.ethz.ch]
  - Pajek [http://mrvar.fdv.uni-lj.si/pajek]