

node *centrality*

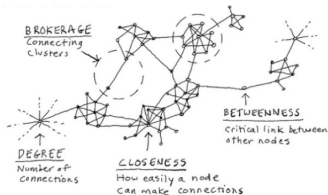
introduction to *network analysis* (*ina*)

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centrality *measures*

which *nodes* are most *important*?

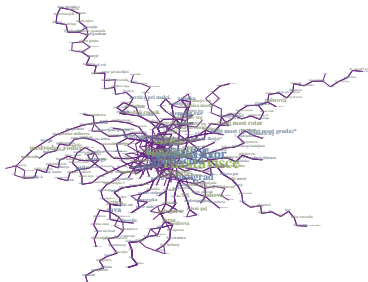
- *node centrality measures* for (*un*)*directed* networks
 - *clustering coefficients* [WS98, SV05, dNMB05]
 - *distance-based* measures [Fre77, FBW91, New05]
 - *spectral analysis* measures [Kat53, Bon87, BP98]
 - *fragment-based* measures [MSOI⁺02, Prž07, EK15]



- *link analysis algorithms* primarily for *directed* networks

networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 5.62$ connections
- *giant component* 95.4% nodes (6 components)
- “small-world” with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- “scale-free” with $\gamma = 2.62$ for cutoff $k_{min} = 5$



* reduced to largest connected component

centrality *clustering*

important *nodes* are *strongly embedded*

- for *undirected* G *clustering coefficient* C [WS98] of i is
 - t_i is number of *linked neighbors* or *triangles* of i

$$C_i = \frac{2t_i}{k_i(k_i-1)} \quad C_i = 0 \text{ for } k_i \leq 1$$

- ω -*corrected clustering coefficient* C^ω [SV05] of i is
 - ω_i is *maximum possible* t_i with *respect to* $\{k\}$

$$C_i^\omega = \frac{t_i}{\omega_i} \quad C_i^\omega = 0 \text{ for } \omega_i = 0$$

- μ -*corrected clustering coefficient* C^μ [dNMB05] of i is
 - μ is *maximum* number of *triangles* over *links*

$$C_i^\mu = \frac{2t_i}{k_i\mu} \quad C_i^\mu = 0 \text{ for } k_i = 0$$

centrality *closeness*

important *nodes* are *close to other* nodes

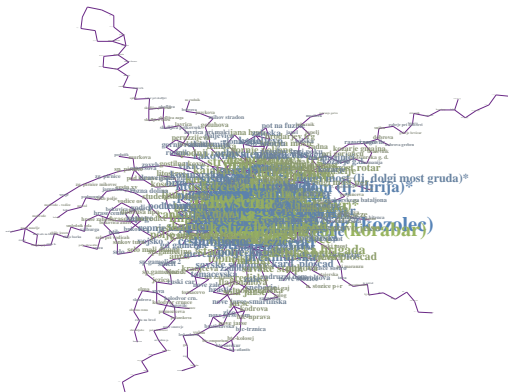
- for (*un*)*directed* G *closeness centrality* ℓ^{-1} [New10] of i is
 - d_{ij} is (*un*)*directed distance* between i and j
 - $d_{ij} = \infty$ for nodes in *different components*

$$\ell_i^{-1} = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

- ℓ^{-1} spans *small range* in *small-world* networks

networkology *closeness*

- *closeness centrality* ℓ^{-1} in partial LPP network[§]
- *highest* $\ell_i^{-1} = 0.208$ node is *Gospodsvetska* with $k_i = 14$



[§] reduced to simple undirected graph

centrality *betweenness*

important *nodes* are *bridges between other* nodes

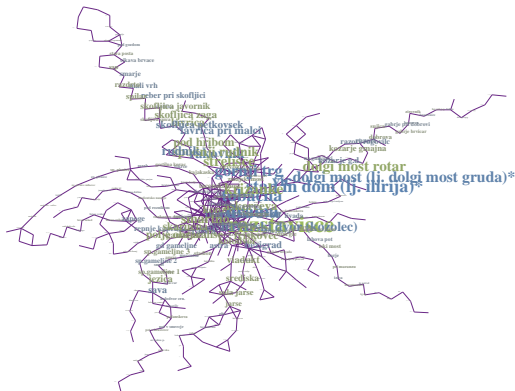
- for (*un*)*directed* G *betweenness centrality* σ [Fre77] of i is
 - g_{st} is number of *shortest paths between* s and t
 - g_{st}^i is number of *such shortest paths through* i

$$\sigma_i = \frac{1}{n^2} \sum_{st} \frac{g_{st}^i}{g_{st}}$$

- σ considers *only shortest paths* [FBW91, New05]
- σ mixes *local centers* with *global bridges* [JMK⁺16]

networkology *betweenness*

- *betweenness centrality* σ in partial LPP network
- *highest* $\sigma_i = 0.235$ node is *Razstavišče* with $k_i = 11$



reduced to simple undirected graph

centrality *degrees*

important *nodes* are *linked by many* nodes

- for *undirected* G *degree centrality* d of i is

$$d_i = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i}{n-1}$$

- in *directed* G *in-degree centrality* d^{in} of i is

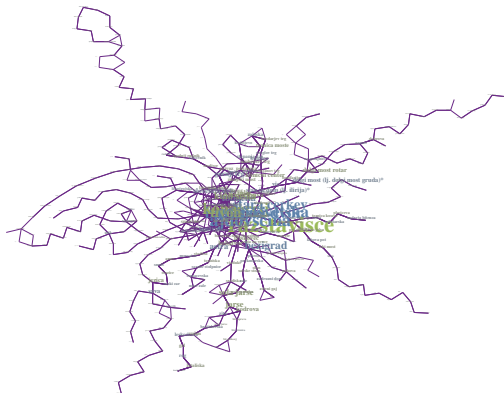
$$d_i^{in} = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i^{in}}{n-1}$$

- in *directed* G *out-degree centrality* d^{out} of i is

$$d_i^{out} = \frac{1}{n-1} \sum_{j \neq i} A_{ji} = \frac{k_i^{out}}{n-1}$$

networkology *degrees*

- *degree centrality* d in partial LPP network
- *highest* $d_i = 0.099$ node is *Razstavišče* with $k_i = 41$
- *highest* d_i^{in} node is *Razstavišče* with $k_i^{in} = 20$ and $k_i^{out} = 21$



centrality *eigenvector*

important *nodes* are *linked by important* nodes

- for (*un*)*directed* G *eigenvector centrality* e [Bon87] of i is
 - v and λ are *eigenvectors* and *eigenvalues* of A
 - e is *proportional* to *leading eigenvector* v_1

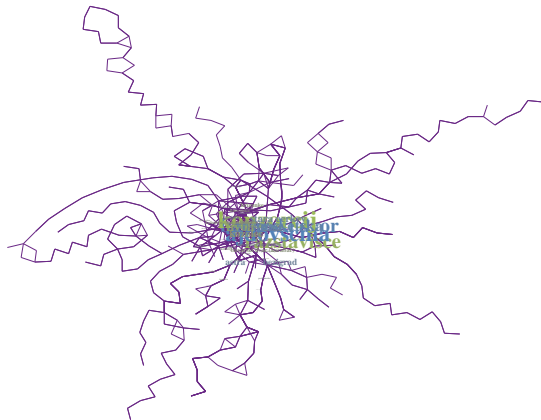
$$e(t) = A^t e(0) = A^t \sum_i C_i v_i = \sum_i C_i \lambda_i^t v_i = \lambda_1^t \sum_i C_i \left[\frac{\lambda_i}{\lambda_1} \right]^t v_i \rightarrow C_1 \lambda_1^t v_1$$

$$e_i = \lambda_1^{-1} \sum_j A_{ij} e_j$$

- in *directed* G $e = 0$ for $k^{in} = 0$ *nodes etc.*

networkology *eigenvector*

- *eigenvector centrality* e in partial LPP network
- *highest* $e_i = 0.082$ node is *Konzorcij* with $k_i = 30$



centrality *Katz*

nodes get small amount of *importance* for free

- for (un)directed G *Katz centrality* z [Kat53] of i is
 - α and β_i are some *positive constants*

$$z_i = \alpha \sum_j A_{ij} z_j + \beta_i$$

- for *convenience* $\beta_i = 1$ whereas $\alpha < \lambda_1^{-1}$
 - λ_1 is *leading eigenvalue* of A

centrality *PageRank*

nodes distribute equal amount of importance

- for (un)directed G *PageRank centrality* p [BP98] of i is
 - α and β_i are some *positive constants*

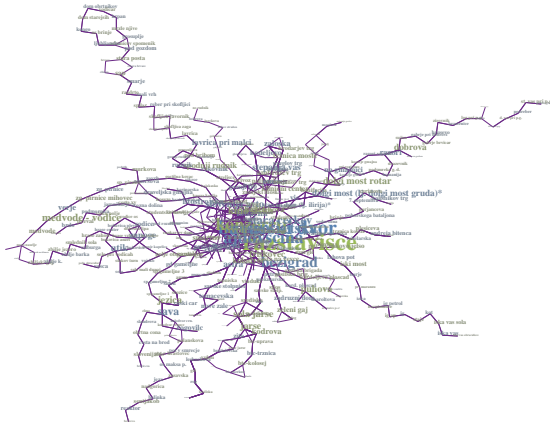
$$p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j^{\text{out}}} + \beta_i$$

- for *convenience* $\beta_i = \frac{1-\alpha}{n}$ whereas $\alpha = 0.85$

see PageRank algorithm *NetLogo* demo

networkology *PageRank*

- *PageRank centrality* p in partial LPP network
- *highest* $p_i = 0.011$ node is *Razstavišče* with $k_i = 41$



centrality *overview*

which *nodes* are most *important*?

1 IA												2 IA																	
1	DC	34											2	EC	8														
	Degree Centrality													Eigenvector Centrality															
2	BC	12	4	2 IIA	13											5	SC	0	14 IVA	7	15 VA	16 VIA	17 VIIA	18 VIIIA	19	20			
	Betweenness Centrality			Closeness Centrality													Subgraph Centrality			Clustering Coefficient		max. neigh. comp.		edge clustering coefficient		PageRank			
3	RL	2	12	8												6	CCoef	14	2	35	5	16	3	17	2	18	1		
	Range-Linked Betweenness			Information Centrality													old Subgraph Centrality		loc. avg. Connectivity		dens. max. length comp.		sum of ECCoef		LeaderRank				
4	BN	39	1	2		21	3 IIIA	4 IVB	5 VB	6 VIB	7 VIIB	8 VIIIB	9 VIIIB	10 VIIIB	11 IB	12 IIIB	13	SC ₀	32	2	33	1	34	1	35	2	36	2	
	BrandNet Centrality			Radiality Centrality			Integration											new Subgraph Centrality		Clique Level		COCoef	PECoef	PCC+ECoef		Kendatlas			
5	RWBC	57	1	38	1	93	1	93	1	94	1	95	1	96	1	97	1	98	1	99	1	100	1	101	1	102	1	103	1
	RandomWalk Betweenness			2,3,4-localized CC																									
6	σ	56	2	50	2	57-71	1	72	1	73	1	74	1	75	1	76	1	77	1	78	1	79	1	80	1	81	1	82	1
	sigma Centrality			Eccentricity			Weighted Degree																						
7	BC _{2,3,4}	87	1	80	1	80-103	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1
	2,3,4-localized-BC			Inverse Eccentricity			Sphere Degree Centrality																						
Z mass												1																	
C												2																	
Name												1																	
Hybrid												1																	
												</																	

centrality *references*



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centrality *references*



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