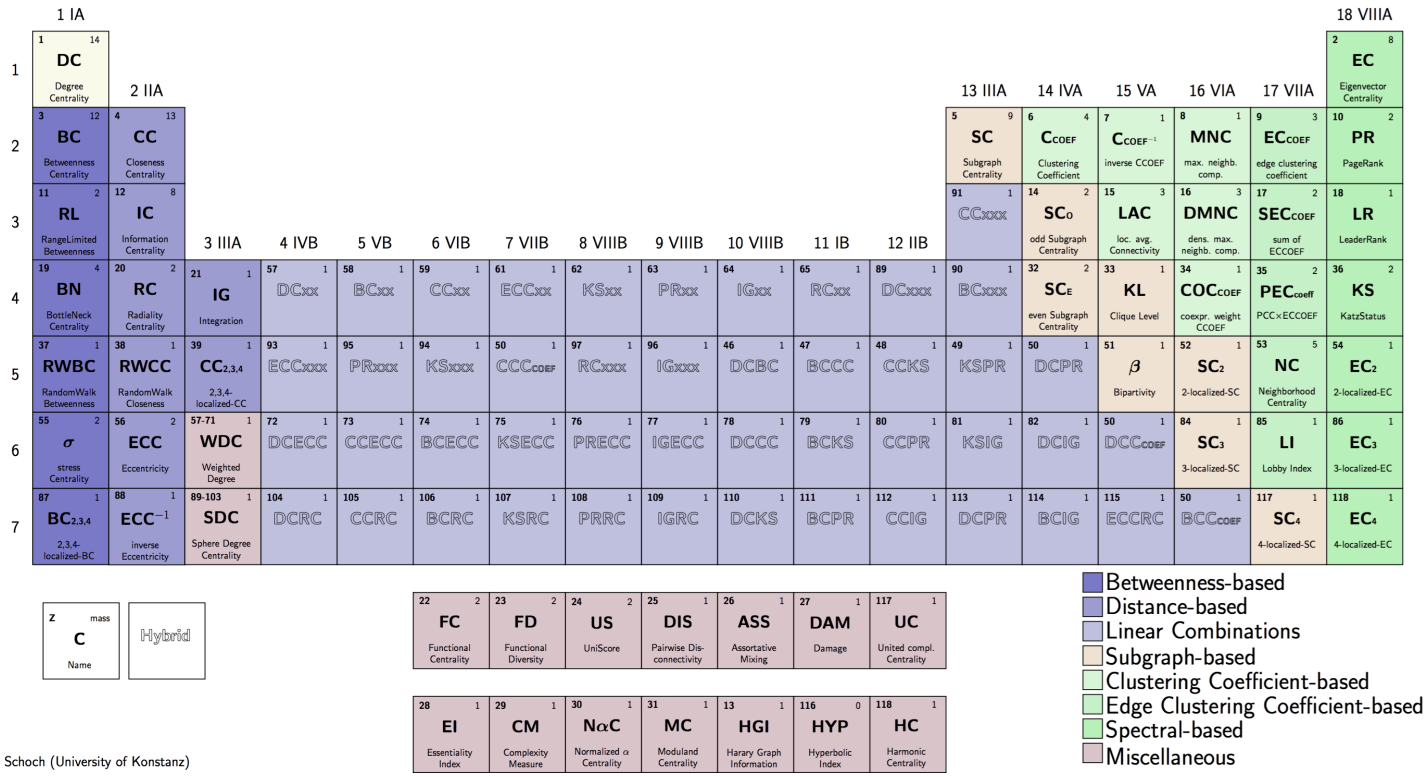
Measures of centrality, PageRank algorithm

You are given iMDB actors collaboration network in Pajek format. Your task is to find the most important actors according to different measures of centrality.



I. Degree centrality and clustering coefficients

1. **(code)** Find the most important actors according to degree centrality $d_i = \frac{k_i}{n-1}$, where n is the number of network nodes and k_i is the degree of node i. What kind of actors have the highest d_i (e.g. Hollywood, international, unknown)?

2. **(code)** Find the most important actors according to clustering coefficient $C_i = \frac{2t_i}{k_i(k_i-1)}$, where k_i is the degree of node i and t_i is the number of triads including node i. You should use the link triad counting algorithm from previous labs. What kind of actors have the highest C_i (e.g. Hollywood, international, unknown)?

3. **(answer)** Find the most important actors according to μ-corrected clustering coefficient $C_i^{\mu} = \frac{2t_i}{k_i \mu}$, where k_i is the degree of node i, t_i is the number of triads including node i and μ is the maximum number of triads over a link. You should use the link triad counting algorithm from previous labs. What

kind of actors have the highest C_i^{μ} (e.g. Hollywood, international, unknown)?

II. Closeness and betweenness centrality

1. **(code)** Find the most important actors according to closeness centrality $\ell_i^{-1} = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$, where n is the number of network nodes and d_{ij} is the distance between nodes i and j. You should use the breadth-first search algorithm from previous labs. What kind of actors have the highest ℓ_i^{-1} (e.g. Hollywood, international, unknown)?

2. **(answer)** Find the most important actors according to betweenness centrality $\sigma_i = \frac{1}{n^2} \sum_{st} \frac{g_{st}^i}{g_{st}}$, where n is the number of network nodes, g_{st} is the number of geodesic paths between nodes s and t, and g_{st}^i is the number of such paths through node i. You should ask the course instructor to do these computations for you. What kind of actors have the highest σ_i (e.g. Hollywood, international, unknown)?

III. Eigenvector centrality and PageRank algorithm

1. **(code)** Find the most important actors according to eigenvector centrality $e_i = \lambda_1^{-1} \sum_j A_{ij} e_j$, where A is the network adjacency matrix and λ_1 is a normalizing constant. You should use the power iteration algorithm shown below. What kind of actors have the highest e_i (e.g. Hollywood, international, unknown)?

2. **(code)** Find the most important actors according to PageRank algorithm $p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j} + \frac{1-\alpha}{n}$, where A is the network adjacency matrix, n is the number of network nodes, k_i is the degree of node i and α is the damping factor set to 0.85. You should use the PageRank algorithm shown below. What kind of actors have the highest p_i (e.g. Hollywood, international, unknown)?

```
input  graph G, precision ϵ
output eigenvector centrality E
E ← array of ones
do
    U ← array of zeros
    for nodes i ∈ N do
        for neighbors j ∈ Γ_i do
            U[i] ← U[i] + E[j]
    u ← ||U||
    for nodes i ∈ N do
        U[i] ← U[i] · n/u
    Δ ← ||E − U||
    E ← U
while Δ > ϵ
return E
```

```
input  graph G, damping α, precision ϵ
output PageRank ranks P
P ← array of n^−1-s
do
    U ← array of zeros
    for nodes i ∈ N do
        for predecessors j ∈ Γ_i^in do
            U[i] ← U[i] + P[j] · α/k_j^out
    u ← ||U||
    for nodes i ∈ N do
        U[i] ← U[i] + (1 − u)/n
    Δ ← ||P − U||
    P ← U
while Δ > ϵ
return P
```