

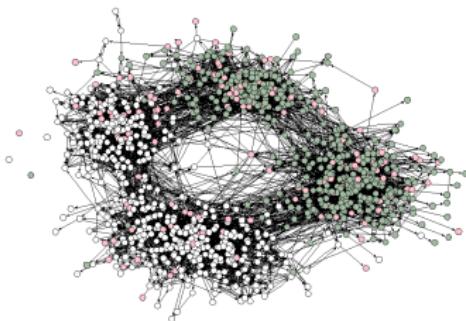
node *mixing*

introduction to *network analysis* (*ina*)

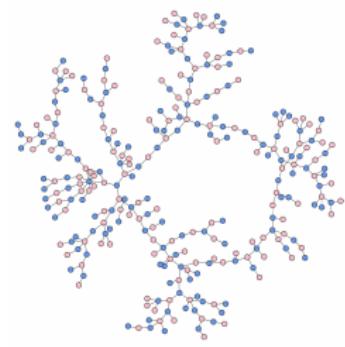
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mixing *definition*

- *node mixing* = *correlations between linked nodes*
- in *assortative mixing* nodes are *linked to similar others*
- in *disassortative mixing* nodes *linked to dissimilar others*



assortative mixing by age/race



disassortative mixing by gender

mixing *degree*

- special case of *node mixing by degree* [New02]
- majority of *social networks* *degree assortative*
- most *other networks* are *degree disassortative*

$$p_{kk'} = k \frac{k'}{2m-1} = m \frac{kk'}{\binom{2m}{2}} \approx \frac{kk'}{2m}$$



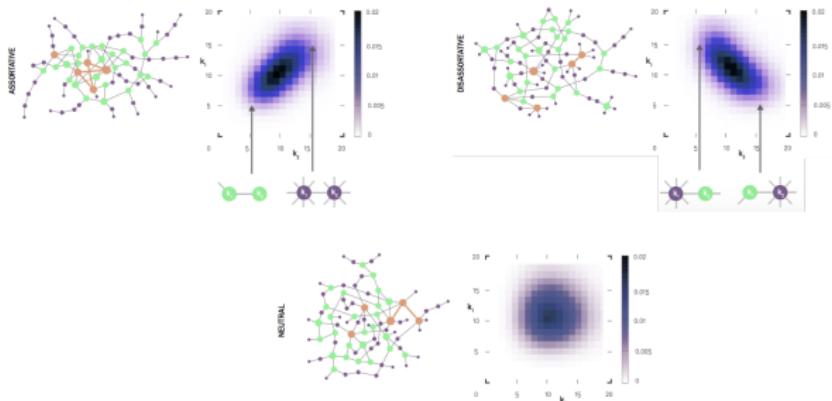
celebrity hubs date hubs
but $10^3/10^8 = 0.00001$



protein hubs avoid hubs
but $p_{56,13} = 0.16 \gg p_{1,2} = 0.0004$

mixing matrix

- endpoints degree distribution $e_{kk'}$ defined as
 - $e_{kk'}$ is link probability between degree- k & - k' nodes
 - q_k is neighbor non-excess degree distribution $\frac{kp_k}{\langle k \rangle}$
$$\sum_{kk'} e_{kk'} = 1 \quad \sum_{k'} e_{kk'} = q_k = n_k \frac{k}{2m-1} \approx \frac{kp_k}{\langle k \rangle}$$
$$e_{kk'} = q_k q_{k'} \text{ in neutral networks but impractical for (dis)assortative networks}$$

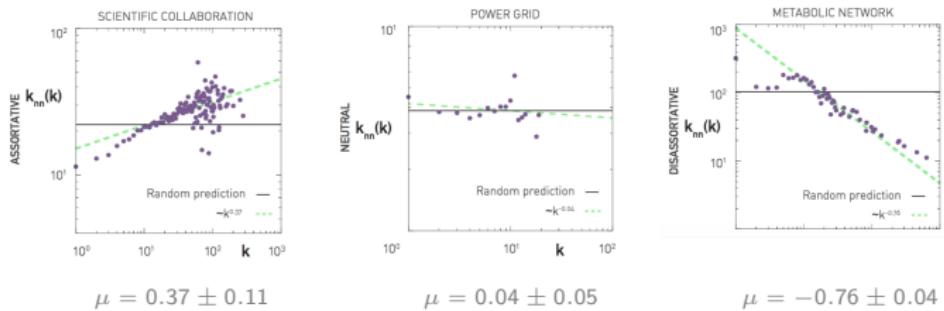


mixing exponent

- neighbor degree function k_{nn} [PSVV01] defined as
 - $k_{nn}(k)$ is average neighbor degree of degree- k nodes
 - $P(k'|k)$ is link probability of degree- k to - k' node
 - μ is degree mixing power-law exponent [VPSV02]

$$k_{nn}(k) = \sum_{k'} k' P(k'|k) = \sum_{k'} k' \frac{e_{kk'}}{\sum_{k'} e_{kk'}}$$

$k_{nn} = \frac{\langle k^2 \rangle}{\langle k \rangle}$ in neutral networks and $k_{nn}(k) \sim k^\mu$ in (dis)assortative networks

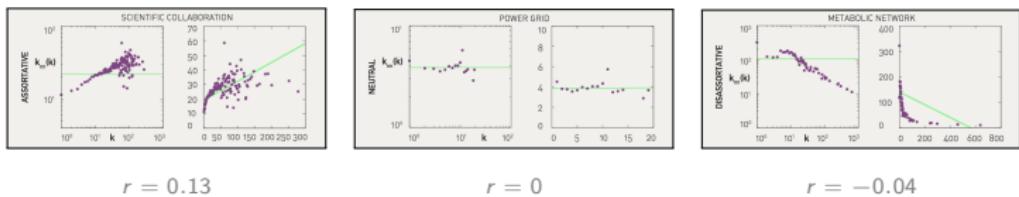


mixing coefficient

- *degree mixing coefficient* r [New02, Est11] defined as
 - r is Pearson correlation of *linked nodes' degrees* [New03]
 - q_k is *neighbor excess degree distribution* $\frac{(k+1)p_{k+1}}{\langle k \rangle}$

$$r = \sum_{kk'} \frac{kk'(e_{kk'} - q_k q_{k'})}{\sum_k k^2 q_k - (\sum_k k q_k)^2}$$

$r = 0$ in *neutral networks* and $k_{nn}(k) \sim rk$ in (*dis*)*assortative networks*



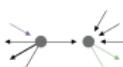
$$r = 0.13$$

$$r = 0$$

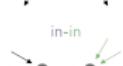
$$r = -0.04$$



in-in



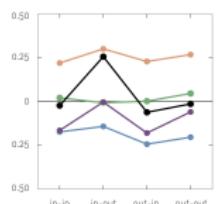
in-out



out-in



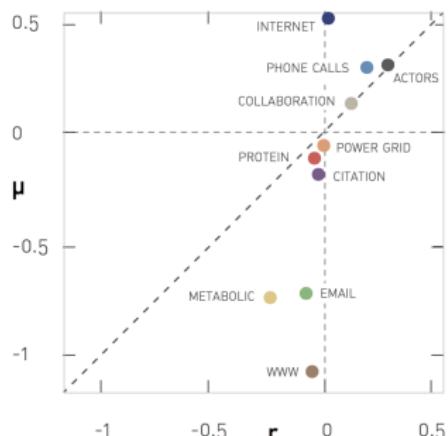
out-out



mixing *networks*

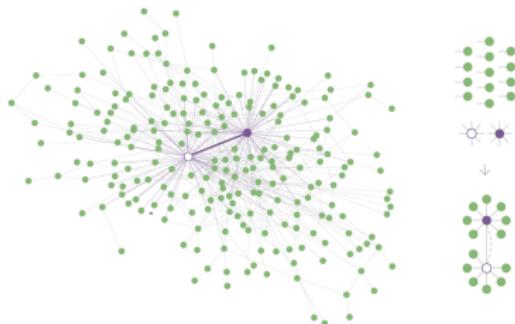
- coefficient & exponent r & μ in real networks [Bar16]
- r & μ correlate in assortative regime and $\text{sgn}(r) = \text{sgn}(\mu)$

NETWORK	N	r	μ
Internet	192,244	0.02	0.56
WWW	325,729	-0.05	-1.11
Power Grid	4,941	0.003	0.0
Mobile Phone Calls	36,595	0.21	0.33
Email	57,194	-0.08	-0.74
Science Collaboration	23,133	0.13	0.16
Actor Network	702,388	0.31	0.34
Citation Network	449,673	-0.02	-0.18
E. Coli Metabolism	1,039	-0.25	-0.76
Protein Interactions	2,018	0.04	-0.1



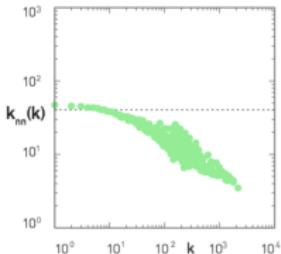
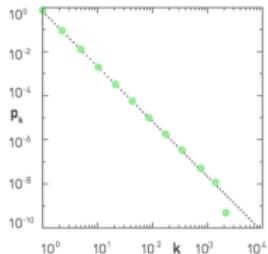
mixing *structural*

- *structural disassortativity* $\frac{E_{kk'}}{m_{kk'}} > 1$ [MSZ04] in real networks
 - $E_{kk'}$ is *number of links* between degree- k & $-k'$ nodes
 - $m_{kk'}$ is *maximum $E_{kk'}$* hence $\min(kn_k, k'n_{k'}, n_k n_{k'})$
- $E_{kk'} = 2me_{kk'} = \langle k \rangle n e_{kk'}$
natural cutoff $k_{max} \sim n^{\frac{1}{\gamma-1}}$ and *structural cutoff* $k_s \sim \sqrt{\langle k \rangle n}$
- *structural disassortativity* in *scale-free* networks with $\gamma < 3$

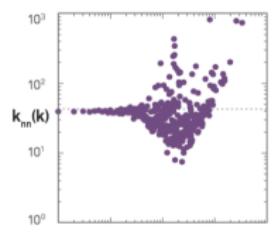
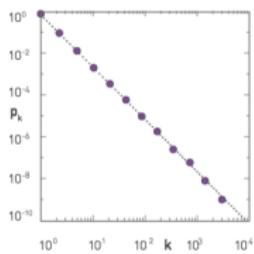


$$k = 55 \text{ and } k' = 46 \text{ then } E_{kk'} = 2.81 > 1$$

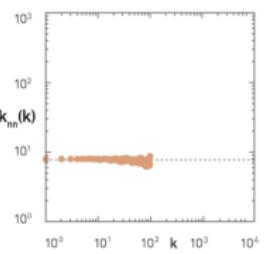
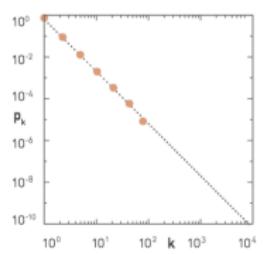
mixing *scale-free*



configuration *scale-free* network as *simple graph*

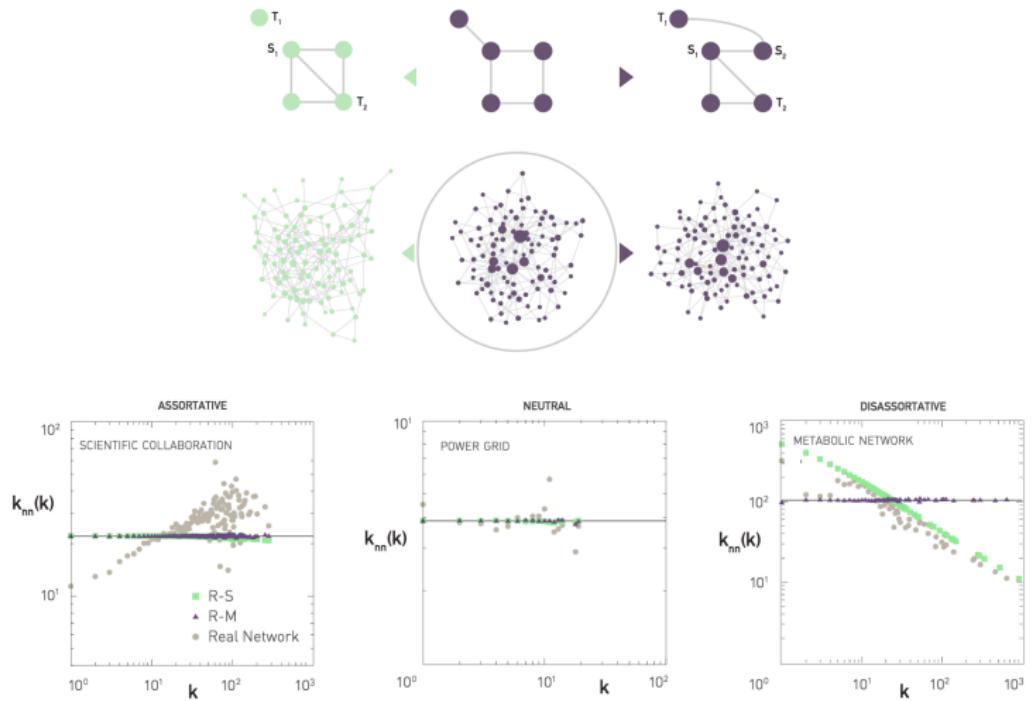


configuration *scale-free* network as *multigraph*



configuration *scale-free* network *without hubs* $k \geq k_s$

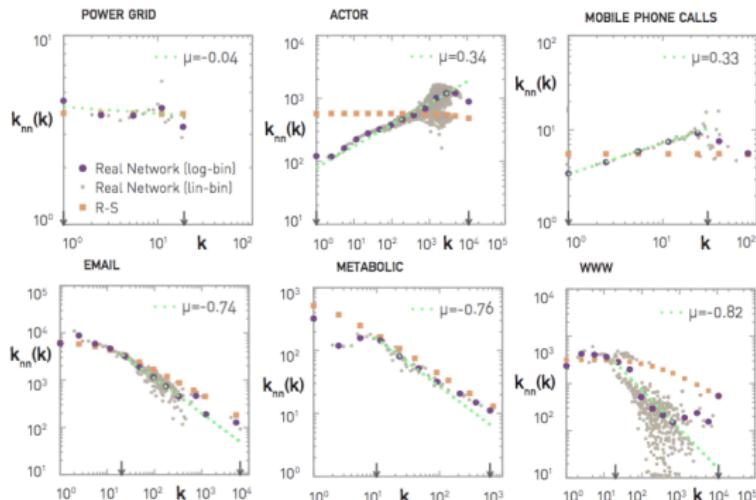
mixing *randomization*



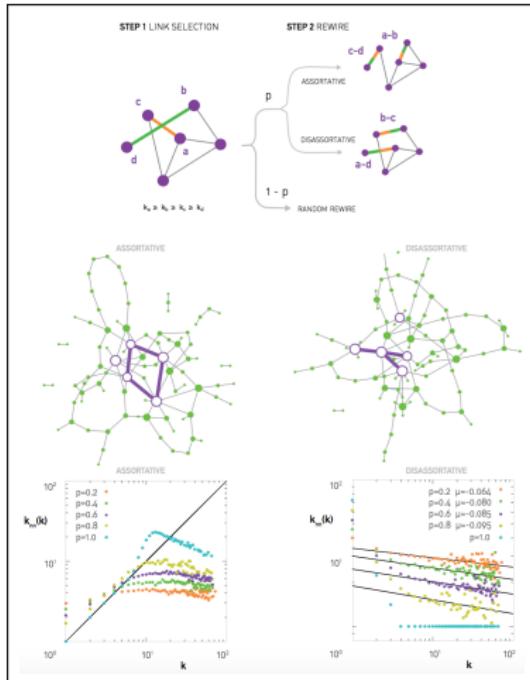
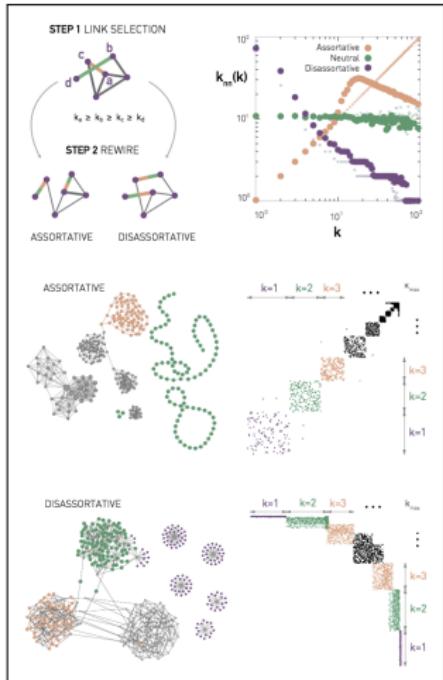
degree-preserving randomization with *simple/multi* links *retains/destroys* structural *disassortativity*

mixing *networks*

- neighbor degree k_{nn} in real networks [Bar16]
- collaboration assortative and technological neutral
- biological/information (structurally) disassortative



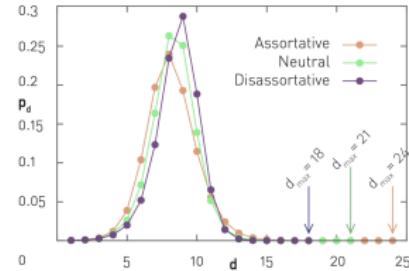
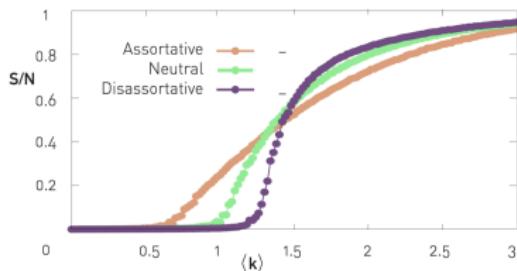
mixing *models*



(dis)assortative degree-preserving randomization [XBS05]

mixing *impact*

- *degree mixing* impacts *connectivity* and *distances* [New02]
- *assortative mixing* coexists with *community structure* [NP03]
- *mixing* influences *resilience* [VM03] and *controllability* [LSB11]



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