

link *bridging*

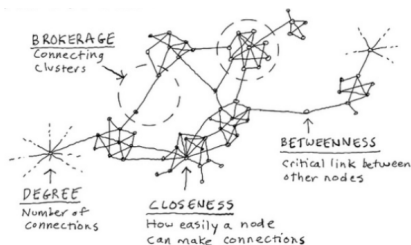
introduction to *network analysis* (*ina*)

Lovro Šubelj
University of Ljubljana
spring 2022/23

bridging *measures*

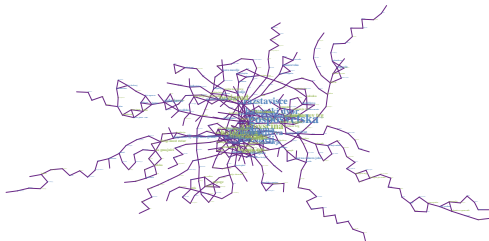
which *links* are most *important*?

- *link bridging measures* for (un)directed networks
 - *betweenness-based* centrality [Fre77, FBW91, New05]
- *link embeddedness measures* for (un)directed networks
 - *topological overlap* measures [RSM⁺02, OSH⁺07, dNMB11]



networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 2.72$ connections
- *giant component* 95.4% nodes (6 components)
- “small-world” with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- “scale-free” with $\gamma = 2.43$ for cutoff $k_{min} = 2$



* reduced to largest connected component of simple undirected graph

bridging *betweenness*

important *links* are *between other nodes*

- for (*un*)*directed* G *link betweenness* σ [Fre77] of $\{i, j\}$ is
 - g_{st} is number of *geodesic paths between* s and t
 - g_{st}^{ij} is number of *such geodesic paths through* $\{i, j\}$

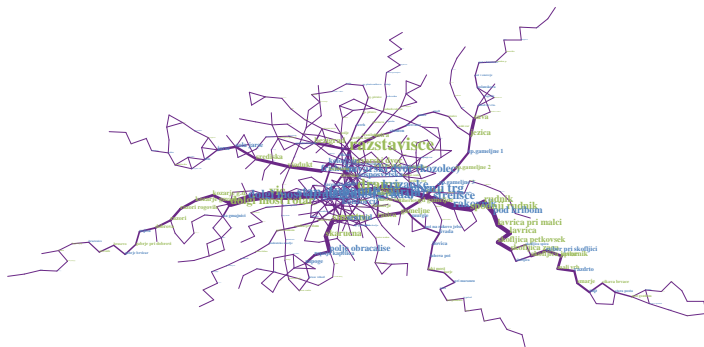
$$\sigma_{ij} = \sum_{st \notin \{i, j\}} \frac{g_{st}^{ij}}{g_{st}}$$

- σ considers *only geodesic paths* [FBW91, New05]



networkology *betweenness*

- *link betweenness* σ in partial LPP network[†]
- *highest* $\sigma_{ij} = 0.176n^2$ link is {*Vič, Stan in dom*}



[†] reduced to largest connected component of simple undirected graph

bridging *bridgeness*

important *links* are *bridges between nodes*

- for (un)directed G *link bridgeness* $\tilde{\sigma}$ [JMK⁺16] of $\{i, j\}$ is
 - g_{st} is number of *geodesic paths between s and t*
 - g_{st}^{ij} is number of *such geodesic paths through $\{i, j\}$*

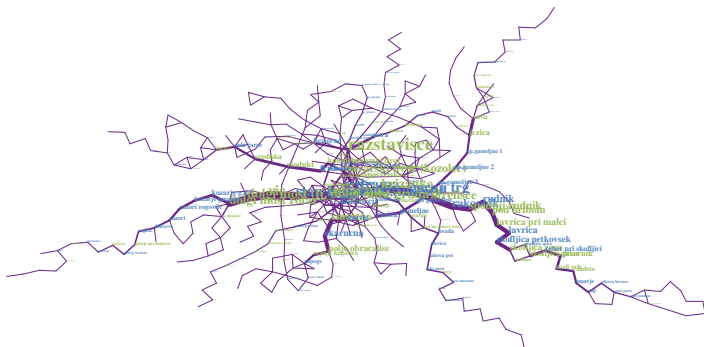
$$\tilde{\sigma}_{ij} = \sigma_{ij} - \sum_{st \in \Gamma_i \cup \Gamma_j} \frac{g_{st}^{ij}}{g_{st}} = \sum_{st \notin \Gamma_i \cup \Gamma_j} \frac{g_{st}^{ij}}{g_{st}}$$

- σ mixes *local centers* with *global bridges* [JMK⁺16]



networkology *bridgeness*

- *link bridgeness* $\tilde{\sigma}$ in partial LPP network[‡]
- *highest* $\tilde{\sigma}_{ij} = 0.169n^2$ link is {*Vič, Stan in dom*}



[‡] reduced to largest connected component of simple undirected graph

bridging *embeddedness*

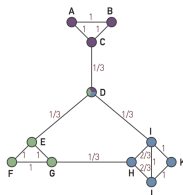
important *links* are *embedded between nodes*

- for *undirected* G *link embeddedness*[§] θ [OSH⁺07] of $\{i, j\}$ is
 - Γ_i is set of *neighbors* or *neighborhood* of i

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \quad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$

- μ -*corrected link embeddedness* $\tilde{\theta}$ [dNMB11] of $\{i, j\}$ is
 - μ is *maximum* number of *triangles* over *links*

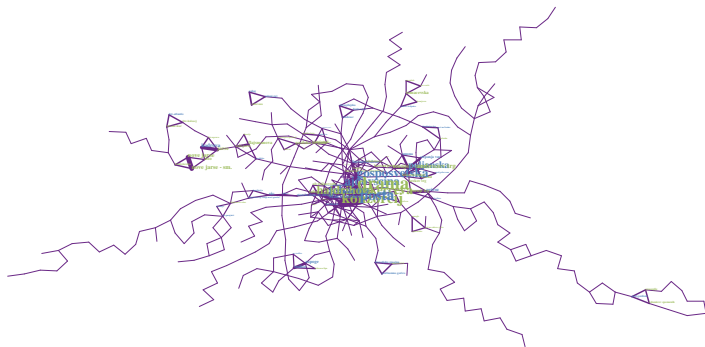
$$\tilde{\theta}_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{\mu + \max(k_i, k_j) - 1 - |\Gamma_i \cap \Gamma_j|}$$



[§] θ & $\tilde{\theta}$ better known as topological overlap indices/weights

networkology μ -embeddedness

- μ -corrected embeddedness $\tilde{\theta}$ in partial LPP network^{||}
- highest $\tilde{\theta}_{ij} = 0.4$ links are {*Pošta*, *Konzorcij*} etc.



^{||} reduced to largest connected component of simple undirected graph

bridging *overview*

which *links* are most *important*?

1 IA												18 VIIIA												
1	DC																						2	EC
	Degree Centrality																							Eigenvector Centrality
2	BC	CC											13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA						3	PR
	Betweenness Centrality	Closeness Centrality											SC	C _{coef}	C _{coef} ⁻¹	MNC	EC _{coef}							PageRank
3	RL	IC											91	SC _o	LAC	DMNC	SEC _{coef}						17	LR
	Range-Linked Betweenness	Information Centrality											C _{coef}	old Subgraph Centrality	loc. avg. Connectivity	dens. max. length comp.	sum of EC _{coef}						28	LeaderRank
4	BN	RC	IG	DC _{ox}	BC _{ox}	CC _{ox}	EC _{ox}	KS _{ox}	PR _{ox}	IG _{ox}	RC _{ox}	DC _{ox}	BC _{ox}	SC _i	KL	COC _{coef}	PEC _{coef}					35	KS	
	BetweenRank Centrality	Radiality Centrality	Integration											new Subgraph Centrality	Clique Level	cooper. weight CO _{coef}	PCC + EC _{coef}					36	KatzRank	
5	RWBC	RWCC	CC _{2,3,4}	EC _{coef}	PR _{ox}	KS _{ox}	CC _{coef}	RC _{ox}	IG _{ox}	DC _{ox}	BC _{ox}	CC _{ox}	KS _{ox}	SC _i	β	SC ₂	NC					53	EC ₃	
	RandomWalk Betweenness	RandomWalk Closeness	2,3,4-localized CC											new Subgraph Centrality	Bipartivity	2-localized SC	Neighborhood Centrality					54	EC ₄	
6	σ	ECC	WDC	DC _{ox}	CC _{ox}	BC _{ox}	KS _{ox}	PR _{ox}	IG _{ox}	DC _{ox}	BC _{ox}	CC _{ox}	KS _{ox}	SC _i	β	SC ₂	LI					85	EC ₅	
	sigma Centrality	Eccentricity	Weighted Degree											new Subgraph Centrality	Bipartivity	3-localized SC	Lobby Index					117	EC ₆	
7	BC _{2,3,4}	ECC ⁻¹	SDC	DC _{ox}	CC _{ox}	BC _{ox}	KS _{ox}	PR _{ox}	IG _{ox}	DC _{ox}	BC _{ox}	CC _{ox}	KS _{ox}	SC _i	β	SC ₂	EC ₄					118	EC ₇	
	2,3,4-localized-BC	Inverse Eccentricity	Sphere Degree Centrality											new Subgraph Centrality	Bipartivity	4-localized SC	4-localized EC							

Z	max	
C		Hybrid
Name		

22	2	23	2	24	2	25	1	26	1	27	1	117	1
FC		FD		US		DIS		ASS		DAM		UC	
Functional Centrality		Functional Diversity		UniScore		Pairwise Dis-connectivity		Assortative Mixing		Damage		United comd. Centrality	

28	1	29	1	30	1	31	1	13	1	116	0	118	1
EI		CM		NαC		MC		HGI		HYP		HC	
Essentiality Index		Complexity Measure		Normalized α Centrality		Modular Centrality		Hungry Graph Information		Hyperbolic Index		Harmonic Centrality	

Betweenness-based

Distance-based

Linear Combinations

Subgraph-based

Clustering Coefficient-based

Edge Clustering Coefficient-based

Spectral-based

Miscellaneous

©David Schoch (University of Konstanz)

bridging *references*



A.-L. Barabási.

Network Science.

Cambridge University Press, Cambridge, 2016.



Wouter de Nooy, Andrej Mrvar, and Vladimir Batagelj.

Exploratory Social Network Analysis with Pajek: Expanded and Revised Second Edition.

Cambridge University Press, Cambridge, 2011.



David Easley and Jon Kleinberg.

Networks, Crowds, and Markets: Reasoning About a Highly Connected World.

Cambridge University Press, Cambridge, 2010.



Ernesto Estrada and Philip A. Knight.

A First Course in Network Theory.

Oxford University Press, 2015.



Linton C. Freeman, Stephen P. Borgatti, and Douglas R. White.

Centrality in valued graphs: A measure of betweenness based on network flow.

Soc. Networks, 13(2):141–154, 1991.



L. Freeman.

A set of measures of centrality based on betweenness.

Sociometry, 40(1):35–41, 1977.



Pablo Jensen, Matteo Morini, Marton Karsai, Tommaso Venturini, Alessandro Vespignani, Mathieu Jacomy, Jean-Philippe Cointet, Pierre Merckle, and Eric Fleury.

Detecting global bridges in networks.

J. Complex Netw., 4(3):319–329, 2016.

bridging *references*



M. E. J. Newman.

A measure of betweenness centrality based on random walks.

Soc. Networks, 27(1):39–54, 2005.



Mark E. J. Newman.

Networks.

Oxford University Press, Oxford, 2nd edition, 2018.



J.-P. Onnela, J. Saramäki, J. Hyvönen, G. Szabó, D. Lazer, K. Kaski, J. Kertész, and A.-L. Barabási.

Structure and tie strengths in mobile communication networks.

P. Natl. Acad. Sci. USA, 104(18):7332–7336, 2007.



E. Ravasz, A. L. Somera, D. A. Mongru, Z. N. Oltvai, and Albert László Barabási.

Hierarchical organization of modularity in metabolic networks.

Science, 297(5586):1551–1555, 2002.