network *inference*

introduction to *network analysis* (*ina*)

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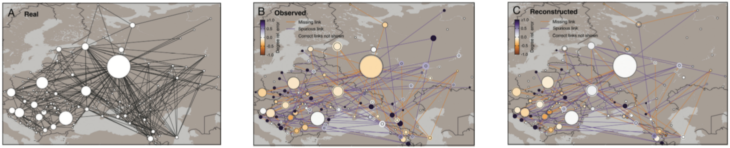
inference *overview*

- *inferring missing/spurious/hidden nodes/links*
  - due to *sampling*, *errors*, *noise* or *other* [GSP09, MBN15]
  - from *network structure*, *dynamics* or *other* [GRLK12]
- popular *predicting future links* that are likely to occur
  - recommendation of *friendship ties* on *Facebook* [BL11]
  - prediction of *product ratings* on *Amazon* [GLGMSP16]
  - prediction for costly *protein interaction* networks etc.



real, observed & reconstructed air transportation network [GSP09]

link *prediction*

introduction to *network analysis* (*ina*)

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prediction *overview*

which *links* are most *likely to occur*?

- link prediction by *local structure/dynamics*
  - *structural equivalence* [LW71] and *topological overlap* [RSM+02]
  - *node similarity* [LHN06] and *local dynamics* indices [ZLZ09]
- link prediction by *global structure/dynamics*
  - *regular equivalence* [WR83] and *link analysis* algorithms [JW02]
  - *community detection* [GN02] and *blockmodeling* [DBF05, Pei15]
- link prediction by *maximum likelihood* methods
  - *hierarchical* [CMN08] and *stochastic block* models [GSP09]
- link prediction by *probabilistic inference* methods
  - *probabilistic relational* models [FGKP99, SPH06]

prediction *equivalence*

links predicted by *structural equivalence*

— *common neighbors index* [LW71] for i and j is

$$s_{ij} = \sum_x A_{ix}A_{xj} = |\Gamma_i \cap \Gamma_j|$$

— *Jaccard neighbors index* [Jac01] for i and j is

$$s_{ij} = \frac{\sum_x A_{ix}A_{xj}}{\sum_x A_{ix} + \sum_x A_{xj} - \sum_x A_{ix}A_{xj}} = \frac{|\Gamma_i \cap \Gamma_j|}{|\Gamma_i \cup \Gamma_j|}$$

— *Salton cosine similarity* [SM83] for i and j is

$$s_{ij} = \cos\theta_{ij} = \frac{\sum_x A_{ix}A_{xj}}{\sqrt{\sum_x A_{ix}^2}\sqrt{\sum_x A_{jx}^2}} = \frac{|\Gamma_i \cap \Gamma_j|}{\sqrt{k_i k_j}}$$

— *Leicht similarity index* [LHN06] for i and j is

$$s_{ij} = \frac{n\sum_x A_{ix}A_{xj}}{\sum_x A_{ix}\sum_x A_{jx}} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i k_j / n} \approx \frac{|\Gamma_i \cap \Gamma_j|}{k_i k_j}$$

prediction *overlap*

links predicted by *topological overlap*

— *Sørensen neighbors index* [Sør48] for i and j is

$$s_{ij} = \frac{\sum_x A_{ix}A_{xj}}{\frac{1}{2}\left(\sum_x A_{ix} + \sum_x A_{xj}\right)} = \frac{|\Gamma_i \cap \Gamma_j|}{\frac{1}{2}(k_i + k_j)} \approx \frac{|\Gamma_i \cap \Gamma_j|}{k_i + k_j}$$

— *hub promoted index* [RSM+02] for i and j is

$$s_{ij} = \frac{\sum_x A_{ix}A_{xj}}{\min\left(\sum_x A_{ix}, \sum_x A_{xj}\right)} = \frac{|\Gamma_i \cap \Gamma_j|}{\min(k_i, k_j)}$$

— *hub depressed index* [LZ10] for i and j is

$$s_{ij} = \frac{\sum_x A_{ix}A_{xj}}{\max\left(\sum_x A_{ix}, \sum_x A_{xj}\right)} = \frac{|\Gamma_i \cap \Gamma_j|}{\max(k_i, k_j)}$$

prediction *models*

links predicted by *graph/network models*

— *configuration model index* [LHN06] for i and j is

$$s_{ij} = \frac{n\sum_x A_{ix}A_{xj}}{\sum_x A_{ix}\sum_x A_{jx}} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i k_j / n} \approx \frac{|\Gamma_i \cap \Gamma_j|}{k_i k_j}$$

— *preferential attachment index* [BA99] for i and j is

$$s_{ij} = \sum_x A_{ix}\sum_x A_{xj} = k_i k_j$$

— *random graph index* [ER59] for i and j is

$$s_{ij} = \frac{\langle k \rangle}{n-1} \approx const.$$

prediction *dynamics*

links predicted by *local* *dynamics*

- *resource allocation index* [ZLZ09] for i and j is

$$s_{ij} = \sum_x \frac{A_{ix} A_{xj}}{\sum_y A_{xy}} = \sum_{x \in \Gamma_i \cap \Gamma_j} \frac{1}{k_x}$$

- *Adamic-Adar similarity index* [AA03] for i and j is

$$s_{ij} = \sum_x \frac{A_{ix} A_{xj}}{\log \sum_y A_{xy}} = \sum_{x \in \Gamma_i \cap \Gamma_j} \frac{1}{\log k_x}$$

- *WalkRank similarity index* [TFP06] for i and j is

$$p_i^t = \alpha \sum_{j \in \Gamma_i} p_j^t / k_j + (1 - \alpha) \delta_{it} \qquad s_{ij} = p_i^j + p_j^i$$

prediction *clusters*

links predicted by *node clusters*

— *community structure index* [YG11] for i and j is

- $\{C\}$ *communities* by *Louvain* [BGLL08] or *Infomap* [RB08]
- n_i and m_{c_i} *number of nodes* and *links* within C_i

$$s_{ij} = \begin{cases} \frac{m_{c_i}}{\binom{n_i}{2}} & \text{if } c_i = c_j \\ -\infty & \text{otherwise} \end{cases}$$

— *block model index* [HLL83, DBF05] for i and j is

- $\{C\}$ *clusters* by *stochastic block models* [Pei15]
- $m_{c_i c_j}$ *number of links* between C_i and C_j

$$s_{ij} = \begin{cases} \frac{m_{c_i}}{\binom{n_i}{2}} & \text{if } c_i = c_j \\ \frac{m_{c_i c_j}}{n_i n_j} & \text{otherwise} \end{cases}$$

prediction *framework*

link *prediction* as *ranking problem*

— *standard link prediction* setting

1. $L_N \leftarrow$ randomly *sample* $m/10$ *unlinked nodes* $\{i,j\} \notin L$
2. $L_P \leftarrow$ *remove* random $m/10$ *node links* $\{i,j\} \in L$
3. *compute* s_{ij} for $\{i,j\} \in L_N \cup L_P$ on *resulting* L

— *temporal link prediction* setting

1. $L_N \leftarrow$ randomly *sample* $|L_P|$ *unlinked nodes* $\{i,j\} \notin L$
2. $L_P \leftarrow$ *remove node links* $\{i,j\} \in L$ *after time* t
3. *compute* s_{ij} for $\{i,j\} \in L_N \cup L_P$ on L *at time* t

— *Pearson/Spearman correlation* or *AUC measure*

$$[\overbrace{0\ 0\ 0\ \ldots\ 0\ 0\ 0}^{\text{ideal } s_{ij} \text{ for } L_N}\ \overbrace{1\ 1\ 1\ \ldots\ 1\ 1\ 1}^{\text{ideal } s_{ij} \text{ for } L_P}]$$

prediction *scale-free*

— *link prediction* in synthetic *scale-free graph* [BA99]

1st *highest AUC* by *stochastic block models* [Pei15]

2nd *highest AUC* by *preferential attachment* [BA99]

class	index	Pearson	Spearman	AUC
models	*preferential*	0.128	0.347	0.701
equivalence	neighbors	0.105	0.135	0.530
	Jaccard	0.019	0.131	0.529
	Salton	0.043	0.131	0.529
	Leicht	−0.008	0.131	0.529
dynamics	allocation	0.091	0.135	0.530
	Adamic-Adar	0.104	0.135	0.530
clusters	modularity	0.002	0.005	0.502
	map equation	0.009	0.034	0.503
	block model	0.168	0.370	0.711
baseline	random	−0.001	−0.001	0.499

prediction *small-world*

— *link prediction* in synthetic *small-world graph* [WS98]

1st *highest AUC* by *stochastic block models* [Pei15]

2nd *highest AUC* by *common neighbors index* [LW71]

class	**index**	**Pearson**	**Spearman**	**AUC**
models	preferential	−0.563	−0.547	0.187
equivalence	*neighbors*	0.721	0.786	0.903
	Jaccard	0.686	0.785	0.903
	Salton	0.729	0.785	0.902
	Leicht	0.730	0.785	0.903
dynamics	*allocation*	0.719	0.785	0.902
	Adamic-Adar	0.720	0.785	0.902
clusters	modularity	0.743	0.754	0.885
	map equation	0.643	0.649	0.807
	block model	0.737	0.754	0.931
baseline	random	−0.003	−0.002	0.499

prediction *human*

— *link prediction* in *human protein interaction* map

1st *highest AUC* by *stochastic block models* [Pei15]

2nd *highest AUC* by *preferential attachment* [BA99]

class	**index**	**Pearson**	**Spearman**	**AUC**
models	*preferential*	0.231	0.719	0.915
equivalence	neighbors	0.342	0.676	0.845
	Jaccard	0.301	0.648	0.830
	Salton	0.391	0.646	0.830
	Leicht	−0.005	0.625	0.819
dynamics	allocation	0.291	0.681	0.847
	Adamic-Adar	0.343	0.680	0.847
clusters	modularity	0.284	0.381	0.672
	map equation	0.220	0.408	0.660
	block model	0.344	0.746	0.929
baseline	random	0.000	0.000	0.500

prediction *P2P*

— *link prediction* in *P2P file transfer* network [LKF07]

1st *highest AUC* by *stochastic block models* [Pei15]

2nd *highest AUC* by *preferential attachment* [BA99]

class	**index**	**Pearson**	**Spearman**	**AUC**
models	*preferential*	0.379	0.378	0.717
equivalence	neighbors	0.113	0.120	0.515
	Jaccard	0.093	0.120	0.515
	Salton	0.098	0.120	0.515
	Leicht	0.055	0.120	0.515
dynamics	allocation	0.087	0.120	0.515
	Adamic-Adar	0.102	0.120	0.515
clusters	modularity	0.081	0.121	0.531
	map equation	0.096	0.113	0.513
	block model	0.487	0.621	0.837
baseline	random	−0.002	−0.002	0.499

prediction *iMDB*

— *link prediction* in *iMDB collaboration* network [BA99]

1st *highest AUC* by *stochastic block models* [Pei15]

2nd *highest AUC* by *resource allocation index* [ZLZ09]

class	**index**	**Pearson**	**Spearman**	**AUC**
models	preferential	0.359	0.589	0.840
equivalence	*neighbors*	0.491	0.875	0.970
	Jaccard	0.609	0.876	0.970
	Salton	0.724	0.877	0.970
	Leicht	0.355	0.869	0.967
dynamics	*allocation*	0.627	0.878	0.971
	Adamic-Adar	0.520	0.876	0.970
clusters	modularity	0.345	0.826	0.948
	map equation	0.421	0.785	0.909
	block model	0.544	0.856	0.986
baseline	random	−0.003	−0.003	0.498

prediction *nd.edu*

— *link prediction* in *nd.edu* *web* graph [BA99]

1st *highest AUC* by *modularity optimization* [BGLL08]

2nd *highest AUC* by *resource allocation index* [ZLZ09]

class	index	Pearson	Spearman	AUC
models	preferential	0.094	0.548	0.816
equivalence	*neighbors*	0.346	0.717	0.855
	Jaccard	0.453	0.716	0.854
	Salton	0.526	0.716	0.854
	Leicht	0.257	0.715	0.854
dynamics	*allocation*	0.181	0.718	0.855
	Adamic-Adar	0.334	0.718	0.855
clusters	*modularity*	0.197	0.767	0.893
	map equation	0.391	0.703	0.844
	block model	-	-	-
baseline	random	−0.001	−0.001	0.499

prediction *WoS*

— *link prediction* in *WoS citation* network [ŠF17]

1st *highest AUC* by *modularity optimization* [BGLL08]

2nd *highest AUC* by *common neighbors index* [LW71]

class	index	Pearson	Spearman	AUC
models	preferential	0.082	0.509	0.794
equivalence	*neighbors*	0.434	0.754	0.880
	Jaccard	0.499	0.753	0.880
	Salton	0.574	0.753	0.880
	Leicht	0.258	0.753	0.880
dynamics	*allocation*	0.449	0.754	0.880
	Adamic-Adar	0.454	0.754	0.880
clusters	*modularity*	0.082	0.779	0.908
	map equation	0.392	0.546	0.734
	block model	-	-	-
baseline	random	0.000	0.000	0.500

prediction *Texas*

— *link prediction* in *Texas road* map [LLDM09]

1st *highest AUC* by *modularity optimization* [BGLL08]

2nd *highest AUC* by *map equation method* [RB08]

class	index	Pearson	Spearman	AUC
models	*preferential*	−0.353	−0.311	0.322
equivalence	neighbors	0.230	0.233	0.551
	Jaccard	0.217	0.232	0.551
	Salton	0.225	0.232	0.551
	Leicht	0.202	0.232	0.551
dynamics	allocation	0.225	0.232	0.551
	Adamic-Adar	0.225	0.232	0.551
clusters	*modularity*	0.060	0.736	0.868
	map equation	0.335	0.362	0.616
	block model	-	-	-
baseline	random	0.000	0.000	0.500

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