graph theory $\rightarrow$ *network science*

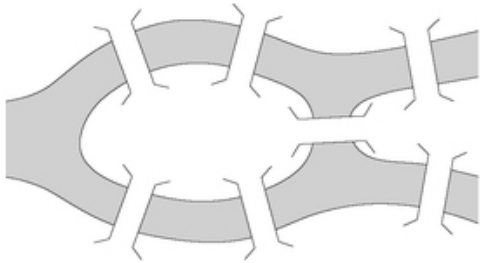
introduction to *network analysis* (*ina*)

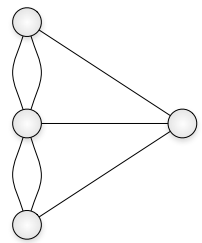
Lovro Šubelj
University of Ljubljana
spring 2023/24

history *graph theory*

1736 seven *bridges of Königsberg* [Eul36] (Leonhard Euler)

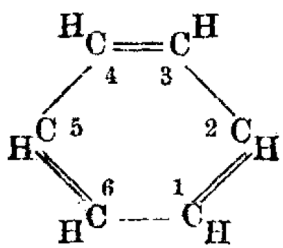
1800s *travelling salesman* problem (William Hamilton)

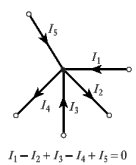


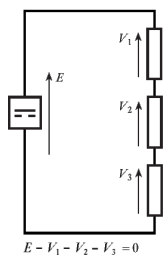


1845 *electrical circuit* laws (Gustav Kirchhoff)

1857 *chemical structure* theory (August Kekulé)







history *operations research*

1956 *shortest paths* (Edsger Dijkstra)

1956 minimum *spanning tree* (Joseph Kruskal)

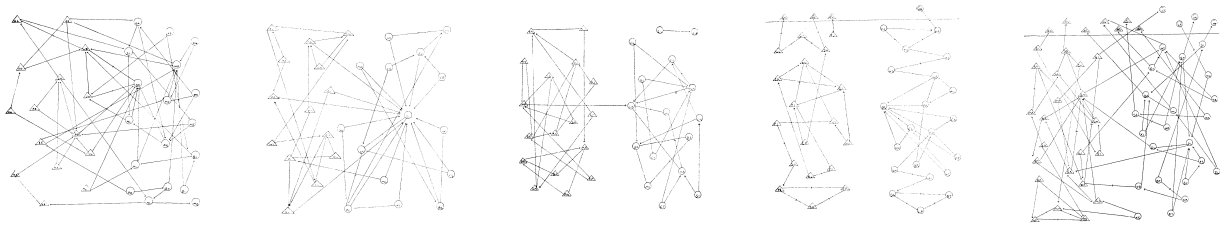
1956 maximum *flow*/minimum *cut* (Ford & Fulkerson)

1956 *signed graph* theory [CH56] (Cartwright & Harary)

1959 *random graph* theory [ER59] (Erdős & Rényi)

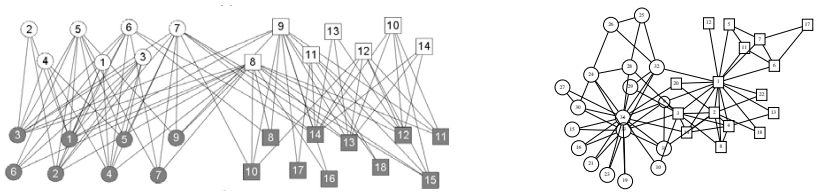
history *sociometry*

1934 children *sociograms* [Mor34] (Jacob Moreno)



1941 *Southern women* [DGG41] (Allison Davis)

1970 university *karate club* [Zac77] (Wayne Zachary)



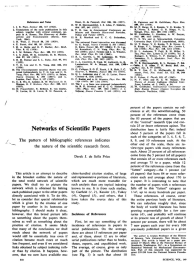
1967 *small-world* experiment [Mil67] (Stanley Milgram)

1973 strength of *weak ties* [Gra73] (Mark Granovetter)

1977 measures of *centrality* [Fre77] (Linton Freeman)

history *bibliometrics*

1965 scientific *paper citations* [Pri65] (Derek de Solla Price)

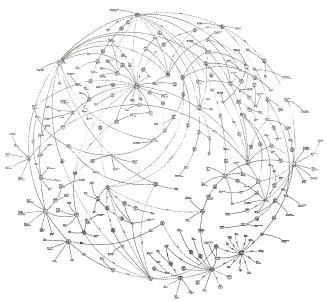

Networks of Scientific Papers

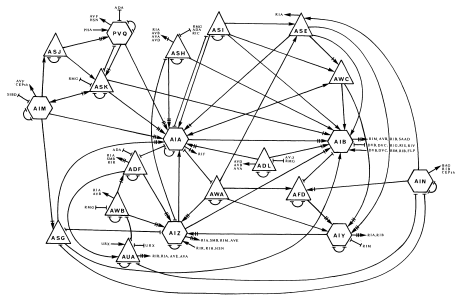
SCIENCE CITATION INDEX

1980s *political scandals* [HL03] (Mark Lombardi)

1986 *neural wirings* [WSTB86] (White et al.)

1999 transportation [Pel99] (Jon Pelletier)





revolution *data*

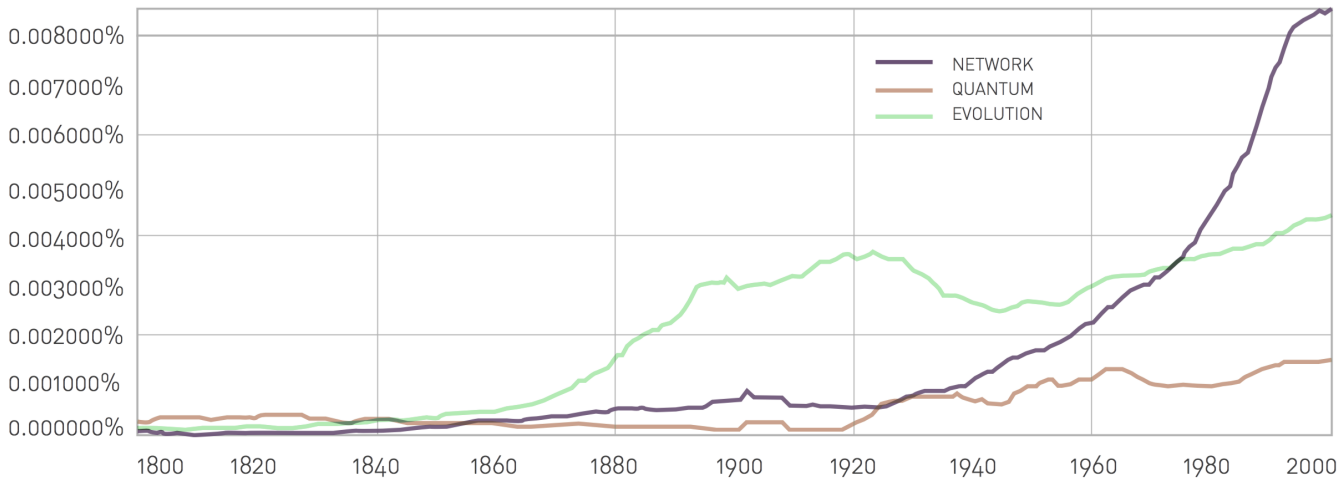
< 2000 *small graphs* 10^2-10^3 nodes

≈ 2000 *communication networks* 10^5-10^8 nodes

≈ 2005 *online social networks* 10^8 nodes

today *Facebook graph* $> 10^9$ users

today *Web graph* $> 10^{12}$ pages



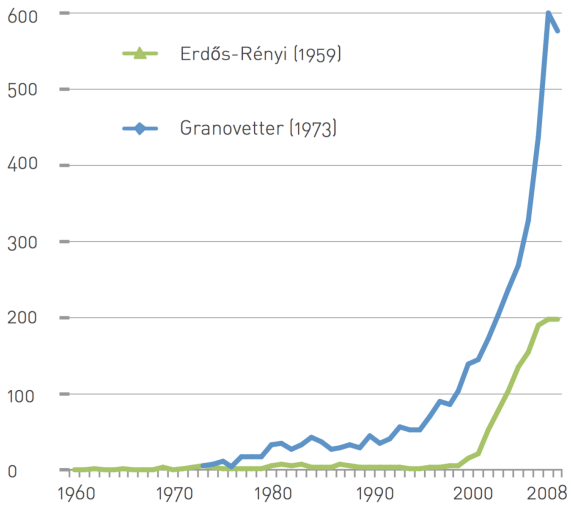
revolution *models*

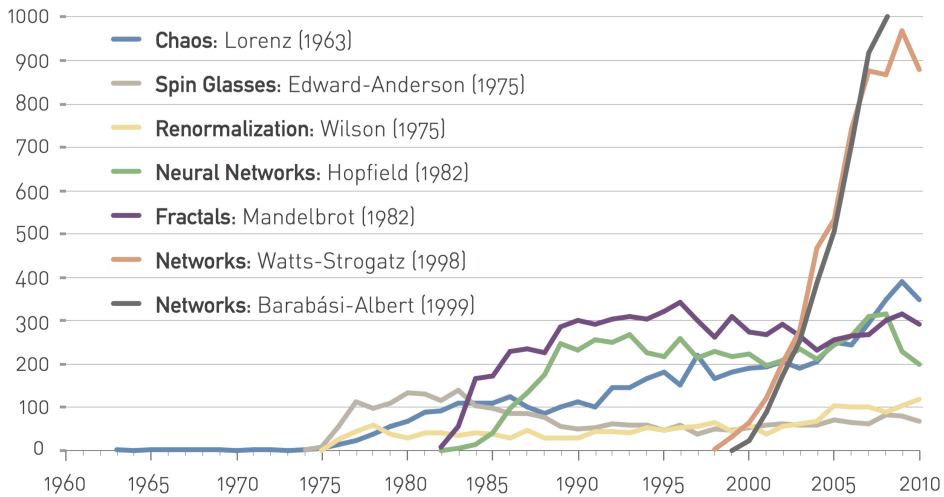
1959 *random graph* models [ER59]

1973 *valued graphs* models [Gra73]

1998 *small-world network* structure [WS98]

1999 *scale-free network* structure [BA99]





revolution *language*

"A key discovery of network science is that the architecture of networks emerging in various domains of science, nature, and technology are similar to each other, a consequence of being governed by the same organizing principles. Consequently we can use a common set of tools to explore these systems."

Albert-László Barabási

"Networks are ideal structures to describe problems of organized complexity."

César A. Hidalgo

"I think the 21st century will be the century of complexity."

Stephen Hawking

network *impact*

- *management:* internal structure of organization
- *economic:* from web search to social networking
- *epidemics:* from forecasting to halting deadly viruses
- *health:* from drug design to metabolic engineering
- *security:* fraud detection and fighting terrorism
- *neuroscience:* mapping human brain
- *many other:* ...

network *science*

problem

understanding *real networks*

means

study of *network properties*

design of *mathematical models*

implementation of *efficient algorithms*

goals

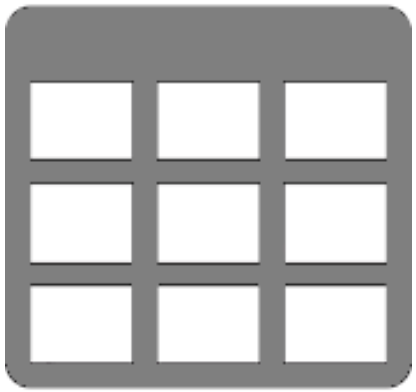
network *structure* and *evolution*

nodes, links, fragments, clusters, layers, networks

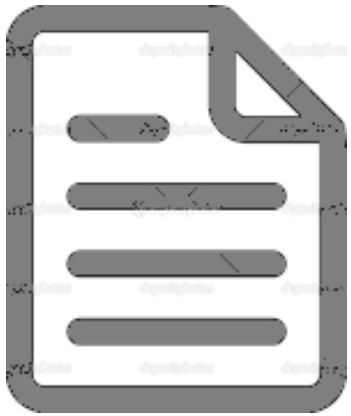
network *dynamics* and *processes*

spreading, diffusion, epidemics

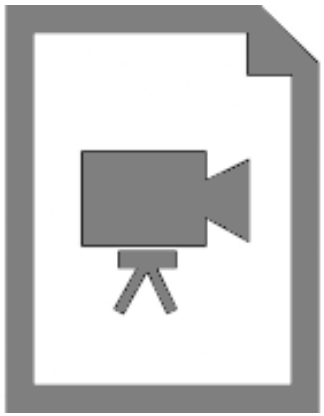
network *analysis*



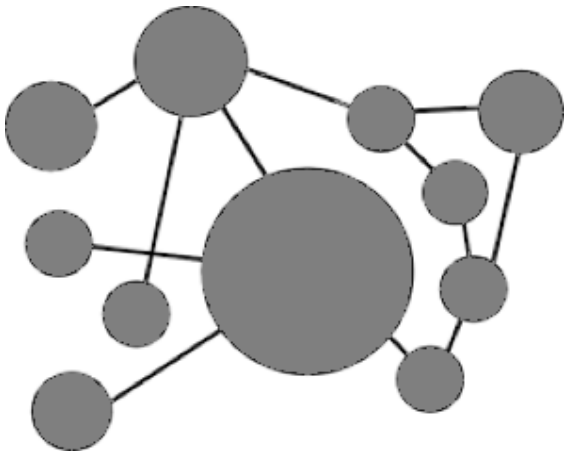
data mining



text mining



computer vision



network analysis

history *references*

A.-L. Barabási and R. Albert.

Emergence of scaling in random networks.
Science, 286(5439):509–512, 1999.

A.-L. Barabási.

Network Science.
Cambridge University Press, Cambridge, 2016.

Dorwin Cartwright and Frank Harary.

Structural balance: A generalization of Heider's theory.
Psychological Review, 63(5):277–293, 1956.

A. Davis, B. B. Gardner, and M. R. Gardner.

Deep South.
Chicago University Press, Chicago, 1941.

Wouter de Nooy, Andrej Mrvar, and Vladimir Batagelj.

Exploratory Social Network Analysis with Pajek: Expanded and Revised Second Edition.
Cambridge University Press, Cambridge, 2011.

David Easley and Jon Kleinberg.

Networks, Crowds, and Markets: Reasoning About a Highly Connected World.
Cambridge University Press, Cambridge, 2010.

Ernesto Estrada and Philip A. Knight.

A First Course in Network Theory.
Oxford University Press, 2015.

P. Erdős and A. Rényi.

On random graphs I.
Publ. Math. Debrecen, 6:290–297, 1959.

history *references*

Leonhard Euler.

Solutio problematis ad geometriam situs pertinentis.
Comment. Academiae Sci. I. Petropolitanae, 8:128–140, 1736.

L. Freeman.

A set of measures of centrality based on betweenness.
Sociometry, 40(1):35–41, 1977.

Mark S. Granovetter.

The strength of weak ties.
Am. J. Sociol., 78(6):1360–1380, 1973.

Robert Hobbs and Mark Lombardi.

Mark Lombardi: Global Networks.
Independent Curators International, New York, 2003.

Stanley Milgram.

The small world problem.
Psychol. Today, 1(1):60–67, 1967.

J. L. Moreno.

Who Shall Survive?
Beacon House, Beacon, 1934.

Mark E. J. Newman.

Networks.
Oxford University Press, Oxford, 2nd edition, 2018.

Jon D. Pelletier.

Self-organization and scaling relationships of evolving river networks.
Journal of Geophysical Research, 104(4):7359–7375, 1999.

history *references*

D. J. de Solla Price.

Networks of scientific papers.
Science, 149:510–515, 1965.

D. J. Watts and S. H. Strogatz.

Collective dynamics of 'small-world' networks.
Nature, 393(6684):440–442, 1998.

J. G. White, E. Southgate, J. N. Thomson, and S. Brenner.

The structure of the nervous system of the nematode Caenorhabditis elegans.
Phil. Trans. R. Soc. Lond. B, 314(1165):1–340, 1986.

Wayne W. Zachary.

An information flow model for conflict and fission in small groups.
J. Anthropol. Res., 33(4):452–473, 1977.