

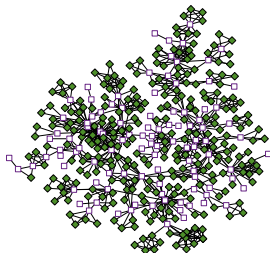
backbones & *skeletons*

introduction to *network analysis* (*ina*)

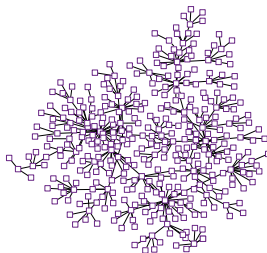
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spring 2024/25

skeletons *overview*

- *network backbone* retains *strongest links/information flow*
sparsification technique that removes as many links as possible
- *network skeleton* retains *micro/meso/macro structure* [CN17]
simplification technique that retains as many links as possible



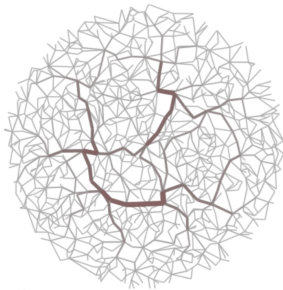
coauthorship network



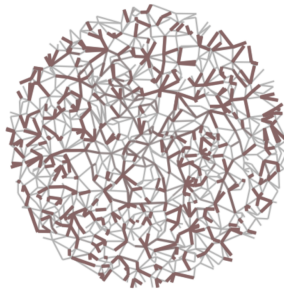
network spanning tree

skeletons *geodesics*

- *betweenness/salience skeletons* of *synthetic graph* [GTB12]
betweenness/salience \sim number of geodesics/spanning trees including edge



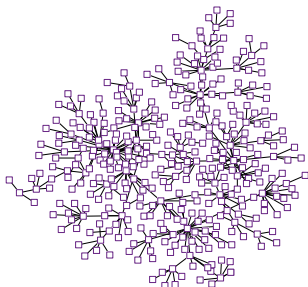
high-betweenness backbone



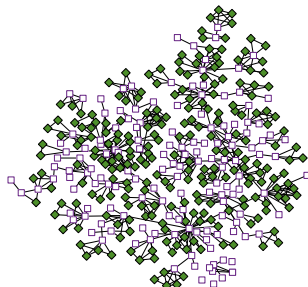
high-salience skeleton

skeletons *convexity*

- *convex skeleton* of famous *network scientists* [MŠ18, Šub18]
largest skeleton where every subgraph includes most geodesics



network spanning tree



convex skeleton \sim tree with cliques

skeletons *statistics*

— *large-scale statistics* of *skeletons of networks* [Šub18]

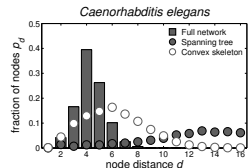
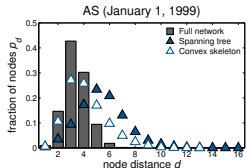
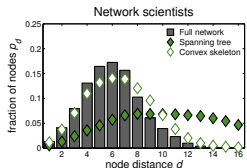
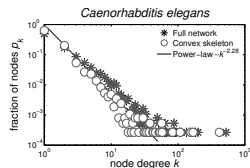
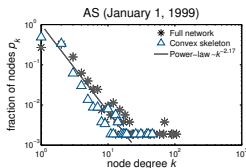
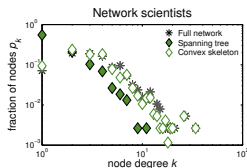
convex skeletons retain connectivity, clustering and geodesics

| | clustering $\langle C \rangle$ | | | geodesics $\langle \sigma \rangle$ | | | convexity X_s | | |
|---------------------------------|--------------------------------|------|------|------------------------------------|------|------|-----------------|------|------|
| | N | CS | ST | N | CS | ST | N | CS | ST |
| Jazz musicians | 0.62 | 0.81 | 0.00 | 9.71 | 1.97 | 1.00 | 0.12 | 0.84 | 1.00 |
| Network scientists | 0.74 | 0.75 | 0.00 | 2.66 | 1.47 | 1.00 | 0.85 | 0.95 | 1.00 |
| Computer scientists | 0.48 | 0.54 | 0.00 | 4.08 | 1.42 | 1.00 | 0.64 | 0.95 | 1.00 |
| <i>Plasmodium falciparum</i> | 0.02 | 0.07 | 0.00 | 3.71 | 1.77 | 1.00 | 0.43 | 0.95 | 1.00 |
| <i>Saccharomyces cerevisiae</i> | 0.07 | 0.10 | 0.00 | 2.58 | 1.19 | 1.00 | 0.68 | 0.88 | 1.00 |
| <i>Caenorhabditis elegans</i> | 0.06 | 0.12 | 0.00 | 6.79 | 3.03 | 1.00 | 0.56 | 0.85 | 1.00 |
| AS (January 1, 1998) | 0.18 | 0.21 | 0.00 | 3.87 | 2.32 | 1.00 | 0.66 | 0.91 | 1.00 |
| AS (January 1, 1999) | 0.18 | 0.27 | 0.00 | 3.54 | 2.05 | 1.00 | 0.49 | 0.95 | 1.00 |
| AS (January 1, 2000) | 0.20 | 0.25 | 0.00 | 4.81 | 3.07 | 1.00 | 0.59 | 0.90 | 1.00 |
| Little Rock Lake | 0.32 | 0.69 | 0.00 | 22.13 | 4.32 | 1.00 | 0.02 | 0.82 | 1.00 |
| Florida Bay (wet) | 0.33 | 0.79 | 0.00 | 9.17 | 1.37 | 1.00 | 0.03 | 0.92 | 1.00 |
| Florida Bay (dry) | 0.33 | 0.82 | 0.00 | 9.37 | 1.65 | 1.00 | 0.03 | 0.93 | 1.00 |

skeletons *distributions*

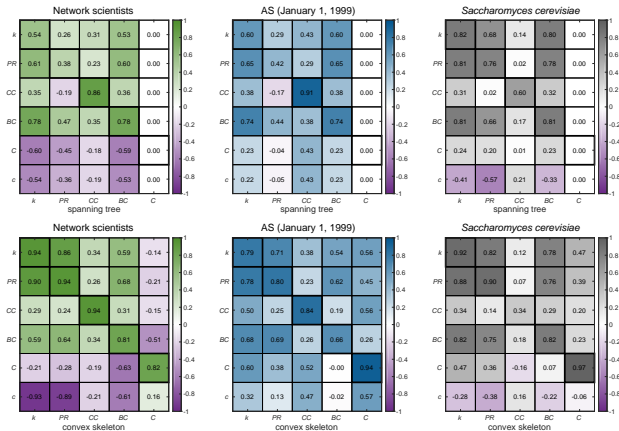
— *node distributions* of *skeletons of networks* [Šub18]

convex skeletons retain node degrees and distances



skeletons *centralities*

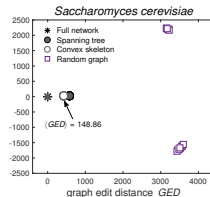
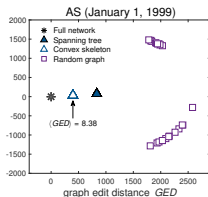
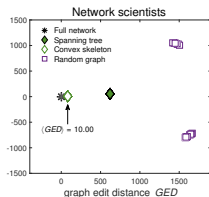
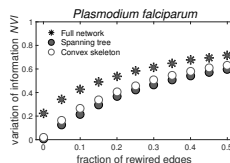
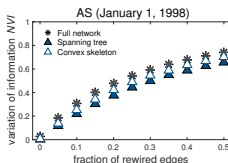
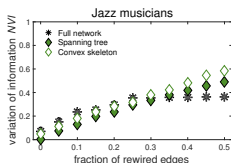
- *node centralities* in *skeletons of networks* [Šub18]
convex skeletons retain node centralities and clustering



skeletons *communities*

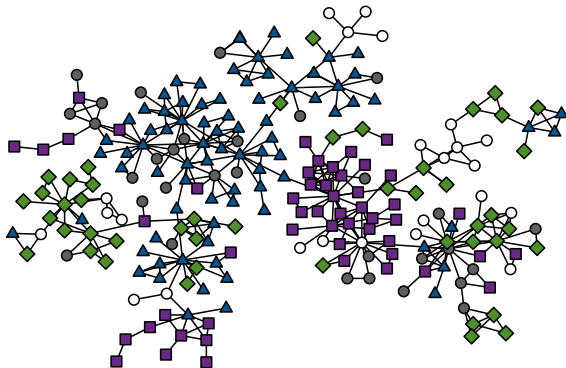
— *communities & robustness* of *skeletons of networks* [Šub18]

convex skeletons highlight communities with short edit distance



skeletons *coauthorships*

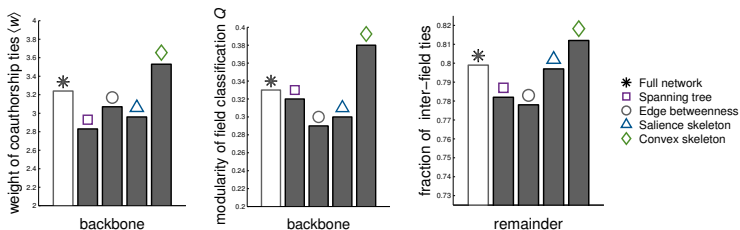
— *convex skeleton* of SI *computer scientists* [Šub18, ŠFCK19]
computer theory (◆), information syst. (■), intelligent syst. (▲), programming (○) etc.



skeletons *properties*

— *properties* of *skeletons of coauthorships* [Šub18, ŠFCK19]

convex skeletons retain strong intra-field coauthorships



skeletons *references*



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skeletons *references*



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