

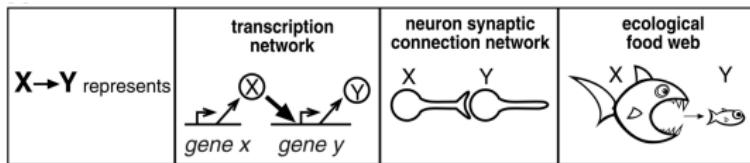
subgraphs or *fragments*

introduction to *network analysis* (*ina*)

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spring 2022/23

fragments *definition*

- small *subgraphs* are *building blocks* of networks
- *subgraphs* characterize *local network structure*



- *fragments* = *connected subgraphs* of networks [EK15]
- *motifs* = *frequent non-induced* subgraphs [MSOI⁺02]
- *graphlets* = *specific induced* subgraphs [PCJ04]

see **mfinder** and **orca** for implementations

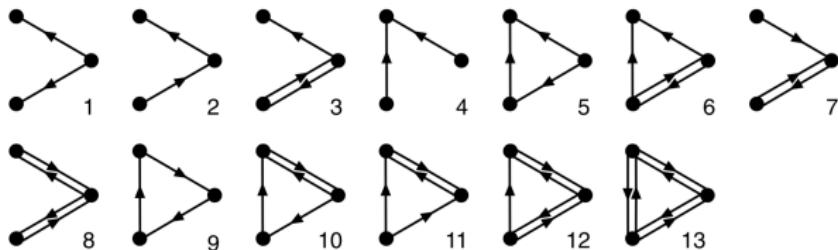
network *motifs*

introduction to *network analysis* (*ina*)

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motifs *definition*

- *fragments* characterize *network-wise local structure*
- *motifs* are *frequent non-induced fragments* [MSOI⁺02]
 - probability of *motif appearing in random graph*
equal or greater number of times is < 0.01
- (*un*)*directed motifs* consisting of *three to five/six/seven nodes*



all 13 directed three-node motifs

motifs *significance*

— motif significance Z with normal distribution $N(0, 1)$

- \tilde{n}_i is number of motifs i in random graph with variance $\tilde{\sigma}_i^2$
- n_i is number of motifs i in real network

$$Z_i = \frac{n_i - \langle \tilde{n}_i \rangle}{\tilde{\sigma}_i} \quad n_i - \langle \tilde{n}_i \rangle > 0.1 \langle \tilde{n}_i \rangle$$

— $\tilde{n}/\tilde{\sigma}$ estimated by motif preserving randomization [MSOI⁺02]

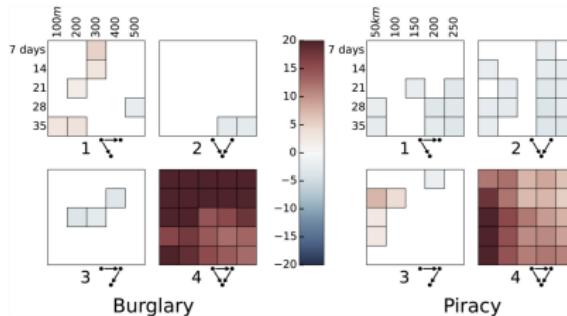
Network	Nodes	Edges	N_{real}	$N_{\text{rand}} \pm \text{SD}$	Z score	N_{real}	$N_{\text{rand}} \pm \text{SD}$	Z score	N_{real}	$N_{\text{rand}} \pm \text{SD}$	Z score
Neurons											
<i>C. elegans</i>	252	509	125	90 ± 10	3.7	127	55 ± 13	5.3	227	35 ± 10	20
Food webs											
Little Rock	92	984	3219	3120 ± 50	2.1	7295	2220 ± 210	25			
Ythan	83	391	1182	1020 ± 20	7.2	1357	230 ± 50	23			
St. Martin	42	205	469	450 ± 10	NS	382	130 ± 20	12			
Electronic circuits (forward logic chips)											
s15850	10,383	14,240	424	2 ± 2	285	1040	1 ± 1	1200	480	2 ± 1	335
s38584	20,717	34,204	413	10 ± 3	120	1739	6 ± 2	800	711	9 ± 2	320
s38417	23,843	33,661	612	3 ± 2	400	2404	1 ± 1	2550	531	2 ± 2	340
World Wide Web											
nd.edu\$	325,729	1.46e6	1.1e5	$2e3 \pm 1e2$	800	6.8e6	$5e4 \pm 4e2$	15,000	1.2e6	$1e4 \pm 2e2$	5000

motifs examples

motif Z-scores of class *software networks* [VS05]

Network	Nodes	Edges	N _{ref}	N _{test}	Z _{test}	N _{ref}	N _{test}	Z _{test}	N _{ref}	N _{test}	Z _{test}	N _{ref}	N _{test}	Z _{test}	N _{ref}	N _{test}	Z _{test}
Software Networks (medium)																	
Faure	186	180	41	11.6±3.3	8.94	18	7.9±3.5	3.01	33	9.5±5.5	4.25						
Aime	143	319	68	29.4±6.1	7.86	30	10.2±4.5	4.38	55	31.8±9.3	2.49						
Filezilla221a	183	331	77	29.4±6.1	7.86	25	10.6±4.6	3.15	68	14±5.9	9.08						
Artec	255	391	68	26±5.4	7.82	86	10.2±4.8	6.75	182	80.2±19.6	5.18						
Exalt	261	504	107	36.3±8.4	6.01												
Software Networks (large)																	
blender26	495	834	486	138±30.3	11.4	33	16±5.2	3.2	123	7.8±5.8	20	22	3.7±3.6	5.04	18	4.2±3.2	4.37
glk221	748	1347	175	13.7±3.7	7.77	119	25±5.2	12.5	175	26±6.7	3.5	21	2.8±3.6	10.9	19	4.2±3.2	3.41
vk	1362	512	262±39.9	6.24		159	16±4.3	5.58	41	13±3.2	10.7	93	17.7±14.7	0.7	122	12.6±5.8	18.7
jars2	1364	1947	816	180±35.5	17.7	173	48±10.8	11.5	345	14.8±14.4	23.3	22	2.2±2.1	0.5	17	3.8±2.2	5.99
prevaly	1993	4987	22750	1840±171	12.2	3848	32±2.42	103.1	1080	144±50.6	18.4	210	28.7±9.2	19.8	1318	55.5±14.7	85.9
Software Networks (large)																	
blender26	126	33.8±6.1	15	N/A		1976	766±162	7.43	436	196±65	3.7	94	26.2±48.6	7.88			
glk221	126	47.7±9.6	7.31	15	3.1±2.3	5.06	4177	194±139.6	5.6	1462	748±261	2.73	188	68.1±13.7	8.73		
vk	229	81.6±10.6	13.9	30	13±6.7	2.53	707	38±8±1.6	5.17	333	217±44	2.62	718	212.1±49.1	10.3		
java2	176	46.2±9.9	4.8	8	1.8±1.1	4.57	10212	634±611.0	1.5	2494	1397±522	2.1	257	52.5±17.6	11.6		
prevaly	1169	272821	42.6	282	30.8±10.4	24.2	25997	1548±21790.5	5.5	5742	4183±752	2.1	2699	736±101.4	21.4		

motif Z-scores of spatio-temporal *crime networks* [DM15]



motifs *profiles*

- motif *significance profile* SP [MSOI⁺02] defined as

- Z_i is *significance of motif i in real network*

$$SP_i = \frac{Z_i}{\sqrt{\sum_i Z_i^2}} \quad Z_i = \frac{n_i - \langle \tilde{n}_i \rangle}{\tilde{\sigma}_i} \quad n_i \geq 4$$

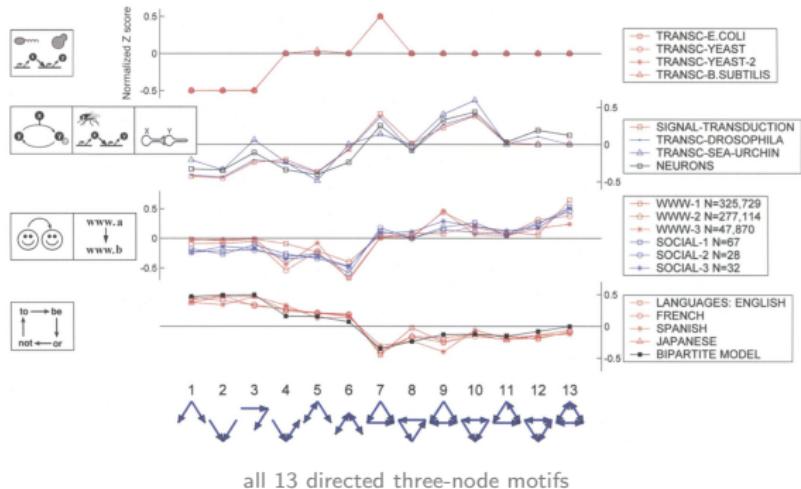
- motif *abundance/ratio profile* RP [MIK⁺04] defined as

- A_i is *abundance of motif i in real network*

$$RP_i = \frac{A_i}{\sqrt{\sum_i A_i^2}} \quad A_i = \frac{n_i - \langle \tilde{n}_i \rangle}{n_i + \langle \tilde{n}_i \rangle + \epsilon_i} \quad \epsilon_i = 4$$

motifs *families*

- directed *motif significance profiles* [MSOI⁺02]
- profiles reveal (*super*)families of real networks



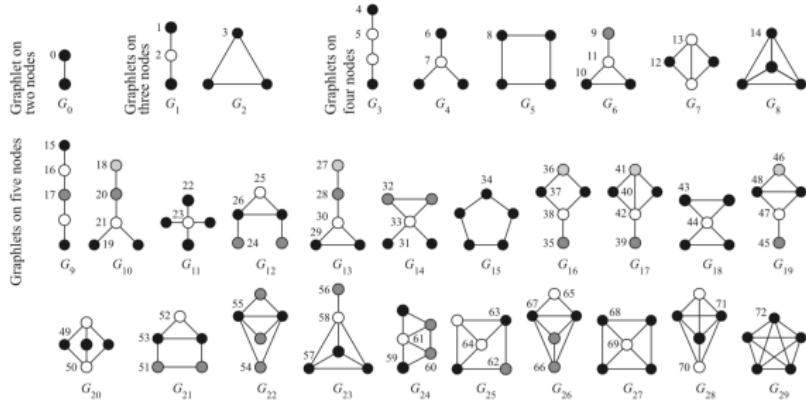
network *graphlets*

introduction to *network analysis* (*ina*)

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graphlets *definition*

- *fragments* characterize *node-wise local structure*
- *graphlets* are *specific induced fragments* [PCJ04]
- *graphlet orbits* are *automorphisms of graphlets* [Prž07]
- (*un*)*directed graphlets* consisting of *three to five/... nodes*

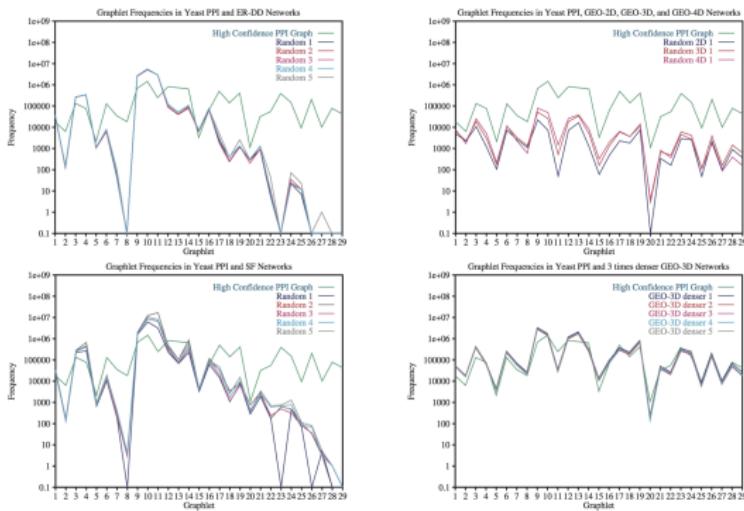


all 30 undirected two- to five-node graphlets with 73 orbits

graphlets *frequency*

- *relative graphlet frequency* F [PCJ04] defined as
 - n_i is *number of graphlets i in real network*

$$F_i = \frac{n_i}{\sum_i n_i}$$



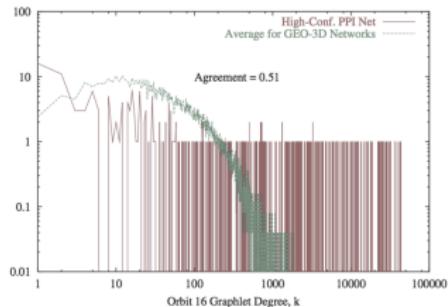
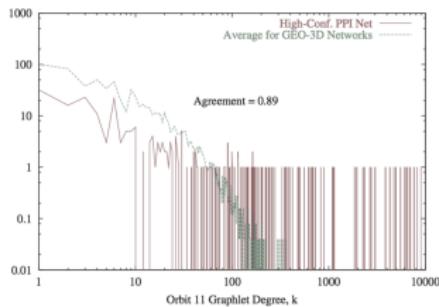
graphlet frequency in protein network and random graphs

graphlets *distribution*

- *i-th orbit graphlet degree distribution* p_k^i [Prž07] defined as
 - p_k^0 is *degree distribution* p_k of *real network*
 - p_k^i is *graphlet degree distribution* for *i-th orbit*
 - \tilde{p}_k^i is *scaled graphlet degree distribution* for *i-th orbit*

$$\tilde{p}_k^i \sim p_k^i/k$$

$$\tilde{p}_k^0 = p_k^0 = p_k$$



11th and 16th orbit graphlet degree distributions of protein network and random graph

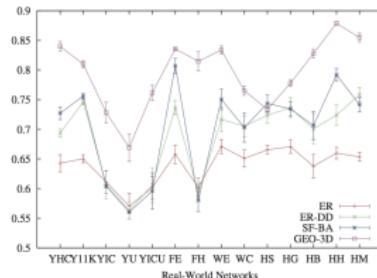
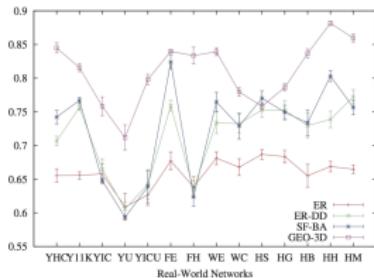
graphlets *agreement*

- *i-th orbit graphlet agreement* A_i [Prž07] defined as
 - \tilde{p}_k^i is *i-th orbit graphlet degree distribution* of *first network*
 - \tilde{q}_k^i is *i-th orbit graphlet degree distribution* of *second network*

$$A_i = 1 - \sqrt{\frac{1}{2} \sum_k (\log \tilde{q}_k^i - \log \tilde{p}_k^i)^2}$$

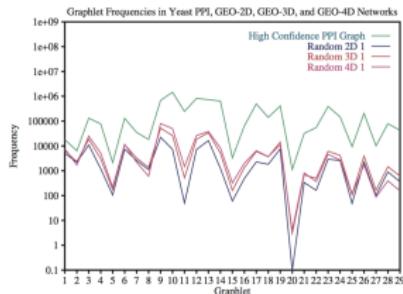
- *arithmetic/geometric graphlet agreement* A defined as

$$A = \frac{1}{73} \sum_i A_i \quad A = (\prod_i A_i)^{\frac{1}{73}}$$

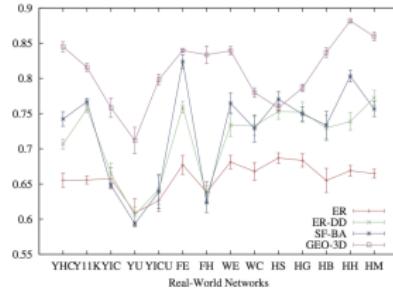


arithmetic/geometric graphlet agreement of protein networks and random graphs

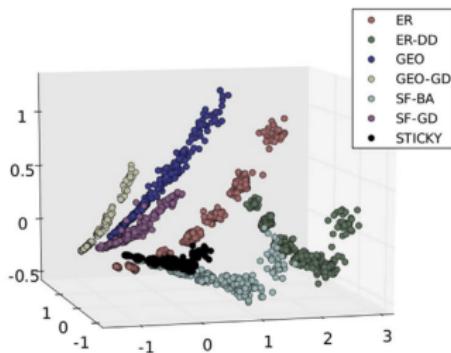
graphlets *measures*



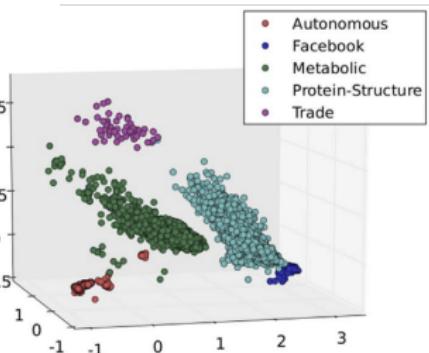
relative graphlet frequency [PCJ04]



graphlet distribution agreement [Prž07]



graphlet correlation matrix and distance [YMDD⁺14]



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