

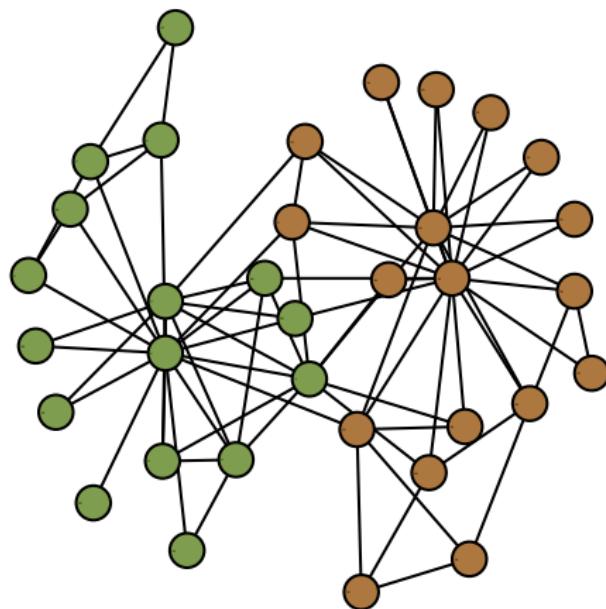
community structure

introduction to *network analysis* (*ina*)

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spring 2024/25

community *structure*

karate club *network split* [Zac77]

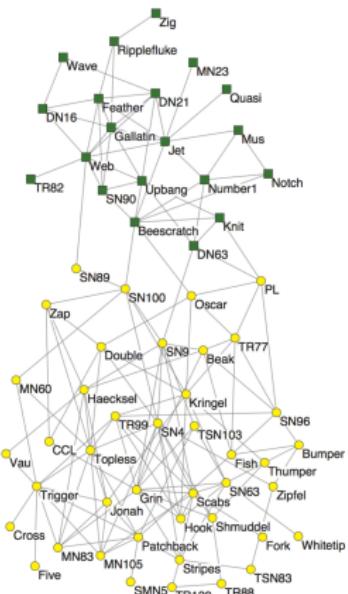


community *detection*

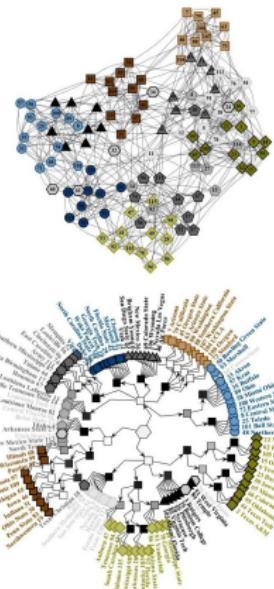
karate club *split detection* [RAK07]

community examples

most social networks contain *communities* [GN02]



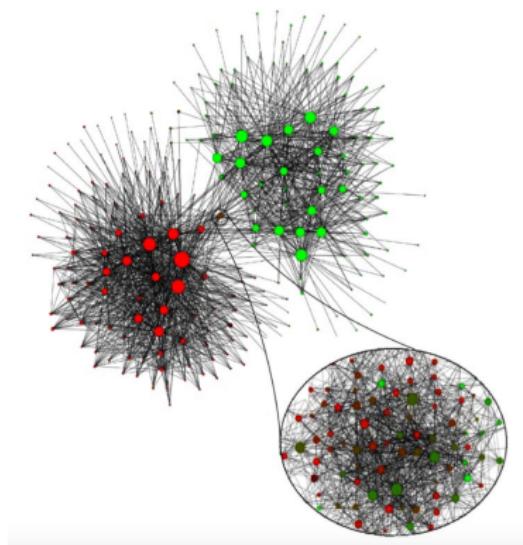
bottlenose dolphins [LSB⁺⁰³]



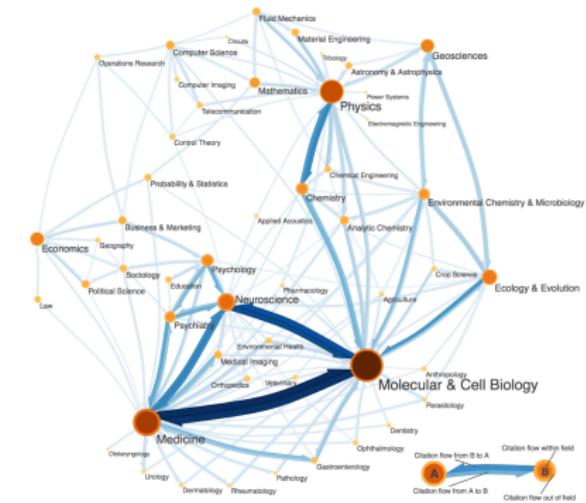
american football [GN02]

community *examples*

many *information networks* contain *communities* [FLG00]



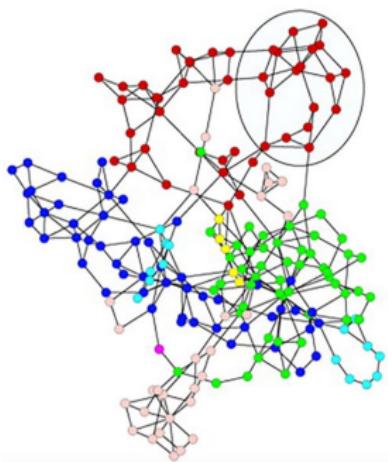
mobile communications [BGLL08]



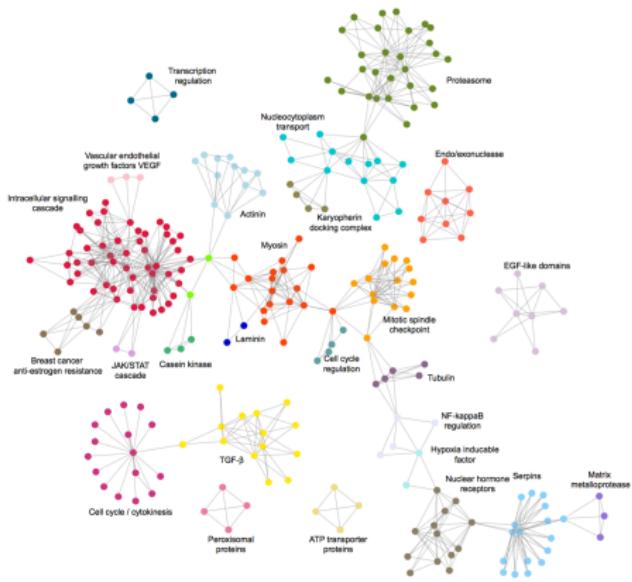
journal citations [RB08]

community *examples*

many biological networks contain *communities* [RSM⁺02]



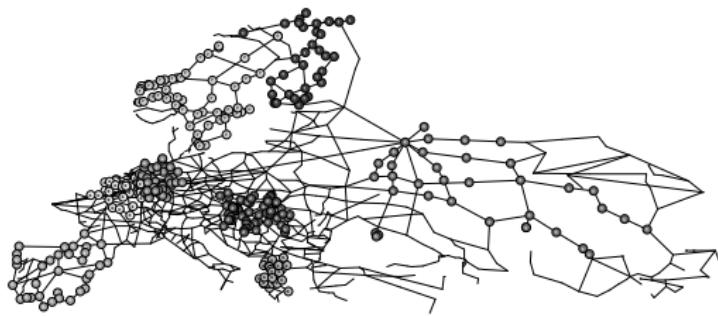
E. coli metabolism [RSM⁺02]



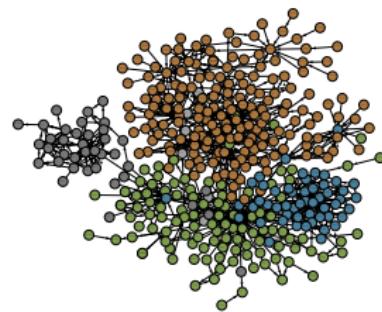
protein interactions [JCZB06]

community *examples*

technological networks rarely contain *communities* [ŠB11a]



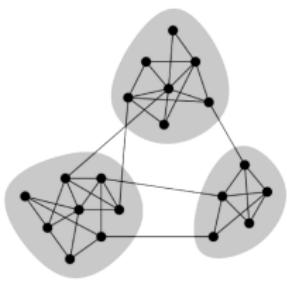
European highways [ŠB11b]



JUNG dependencies [ŠB11a]

community *explanation*

- *weak & strong ties* according to *information flow*
- *bridges & embedded ties* according to *network span*
 - removal of *local bridge* $\{i,j\}$ causes $d_{ij} > 2$
 - removal of *bridge* $\{i,j\}$ causes $d_{ij} = \infty$
 - *embedded tie* $\{i,j\}$ has $C_{ij} > 0$

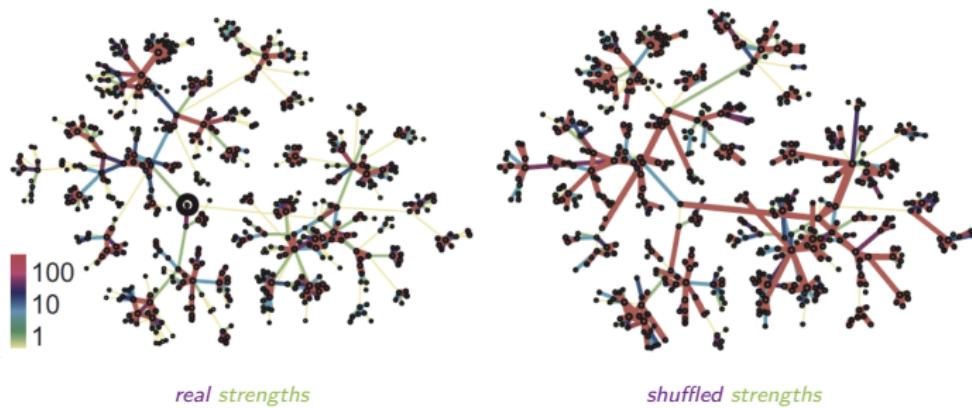


strength of weak ties or weakness of strong ties

- *weak ties are (local) bridges under triadic closure* [Gra73]
- *assortative mixing* and *homophily* in (social) networks [NG03]

community *experiment*

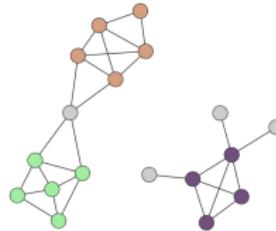
- *tie strength* in mobile communications [OSH⁺07]
- *weak ties are (local) bridges* in real networks



strength of weak ties or weakness of strong ties

community *definition*

- *clique* is *complete subgraph of some graph*
 - also *k-plexes*, *k-cores*, *k-cliques*, *k-clubs*, *k-clans*
- *community* is *dense subgraph of sparse network* [GN02]
- *strong* and *weak community* C [FLG00, RCC⁺04] defined as
 - k_i^{int} and k_i^{ext} are *internal* and *external degree* of i
$$\forall i \in C : k_i^{\text{int}} > k_i^{\text{ext}} \quad \sum_{i \in C} k_i^{\text{int}} > \sum_{i \in C} k_i^{\text{ext}}$$
- *community detection* is \gg *graph partitioning* [For10]



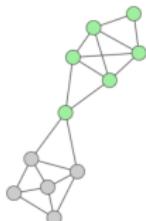
connected communities



maximum clique



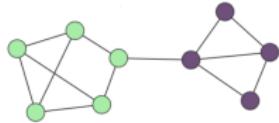
strong community



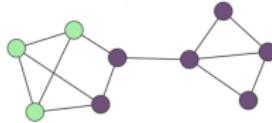
weak community

community *modularity*

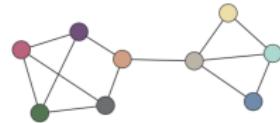
- random graphs should lack community structure
- modularity Q [GN02] of communities $\{C\}$ defined as
 - $k_c = \sum_{i \in C} k_i$ is total degree and m_c is number of links in C
$$\frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta_{c_i c_j} = \frac{1}{2m} \sum_C \sum_{ij \in C} \left(A_{ij} - \frac{k_i k_j}{2m} \right) = \sum_C \frac{m_c}{m} - \left(\frac{k_c}{2m} \right)^2$$
$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta_{c_i c_j} = \sum_C \frac{m_c}{m} - \left(\frac{k_c}{2m} \right)^2$$
- modularity Q popular quality/optimization function [For10]



optimal $Q = 0.41$



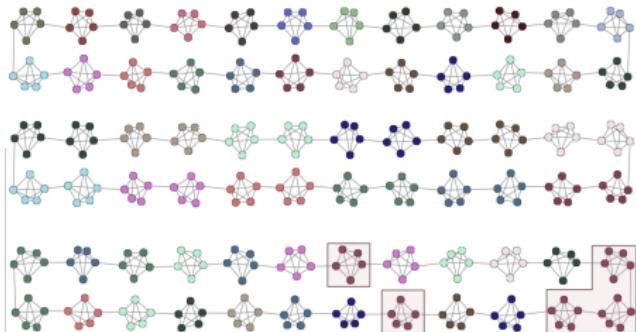
suboptimal $Q = 0.22$



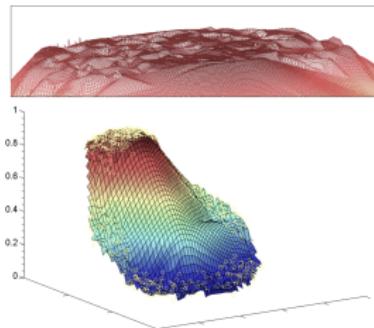
isolates $Q = -0.12$

community \neg modularity

- modularity $Q \gg 0$ also in random graphs [GSPA04]
- modularity Q has resolution limit at $k_c \leq \sqrt{2m}$ [FB07]
- modularity Q lacks clear optimum in real networks [GdMC10]

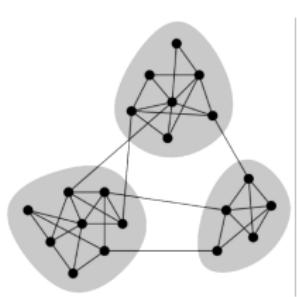


intuitive $Q = 0.867$, optimal $Q = 0.871$ and random $Q = 0.8$

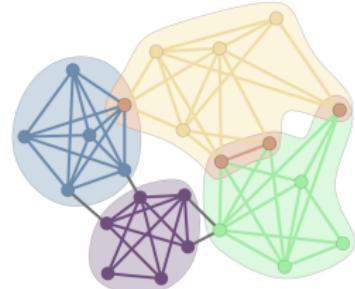


Q plateau and maxima

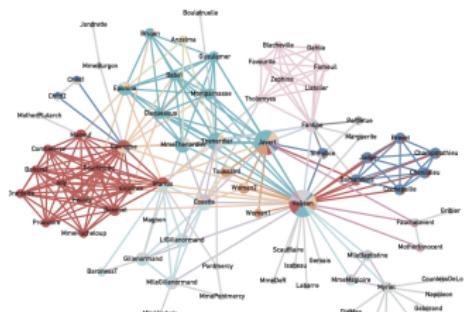
community *overview*



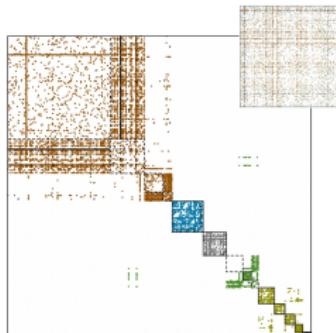
communities [GN02]



overlapping communities [PDFV05]



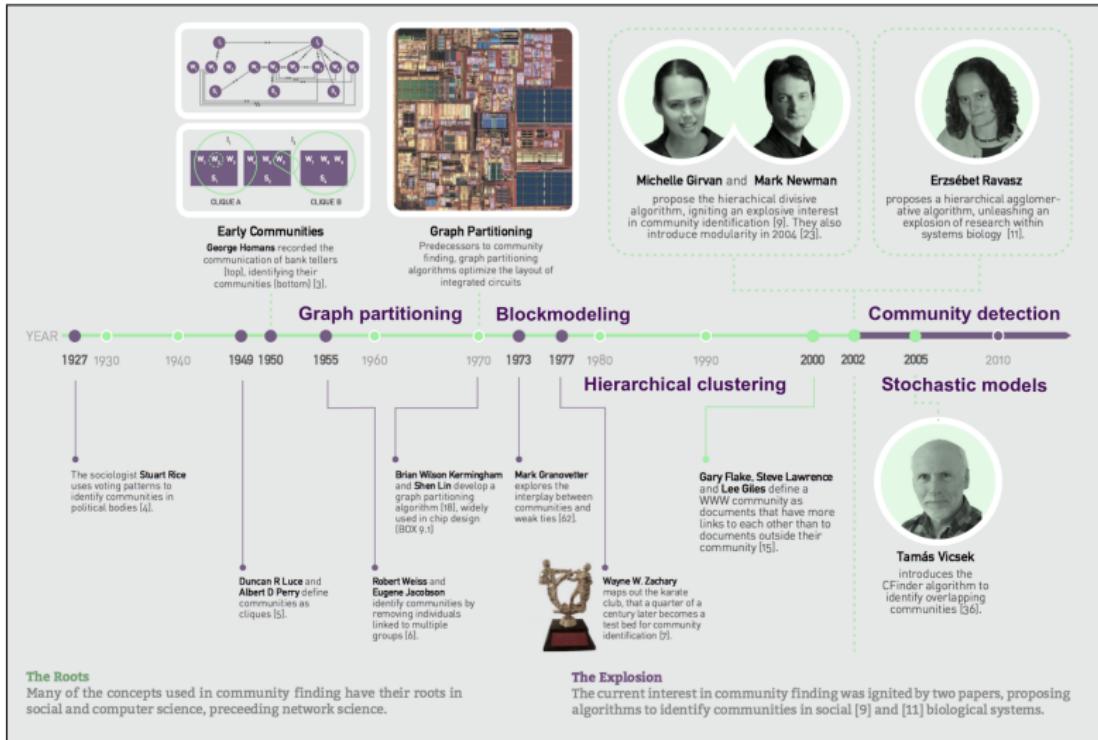
link communities [EL09, ABL10]



block models, blockmodeling etc.

`javax.swing, javax.management, javax.xml, javax.print, javax.naming, javax.lang`

community *history*



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