

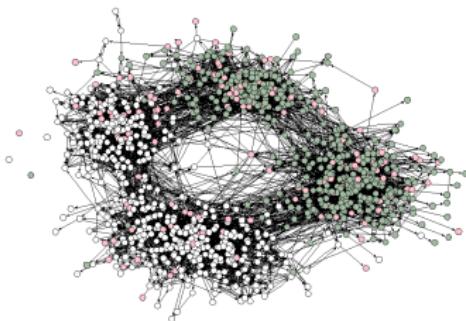
node *mixing*

introduction to *network analysis* (*ina*)

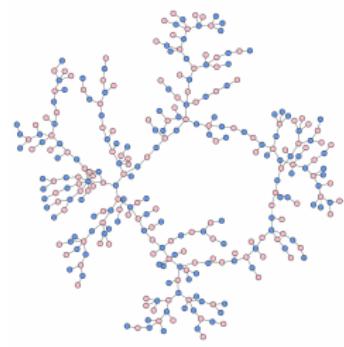
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## mixing *definition*

- *node mixing* = *correlations between linked nodes*
- in *assortative mixing* nodes are *linked to similar others*
- in *disassortative mixing* nodes *linked to dissimilar others*



assortative mixing by age/race



disassortative mixing by gender

# mixing *degree*

- special case of *node mixing by degree* [New02]
- majority of *social networks* *degree assortative*
- most *other networks* are *degree disassortative*

$$p_{kk'} = k \frac{k'}{2m-1} = m \frac{kk'}{\binom{2m}{2}} \approx \frac{kk'}{2m}$$



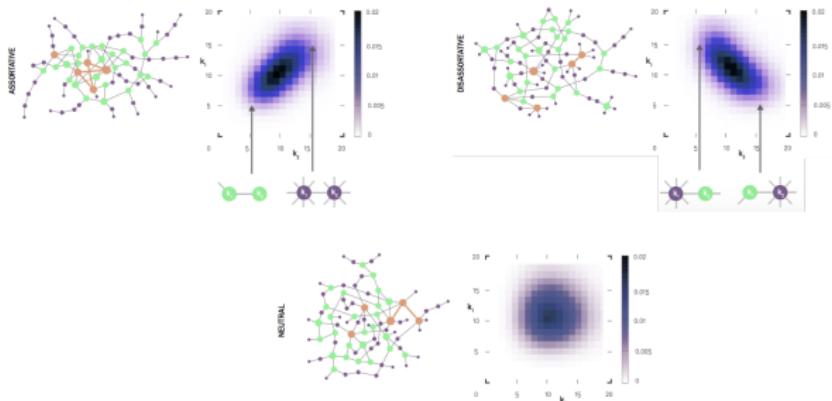
celebrity hubs date hubs  
but  $10^3/10^8 = 0.00001$



protein hubs avoid hubs  
but  $p_{56,13} = 0.16 \gg p_{1,2} = 0.0004$

# mixing matrix

- endpoints degree distribution  $e_{kk'}$  defined as
  - $e_{kk'}$  is link probability between degree- $k$  & - $k'$  nodes
  - $q_k$  is neighbor non-excess degree distribution  $\frac{kp_k}{\langle k \rangle}$
$$\sum_{kk'} e_{kk'} = 1 \quad \sum_{k'} e_{kk'} = q_k = n_k \frac{k}{2m-1} \approx \frac{kp_k}{\langle k \rangle}$$
$$e_{kk'} = q_k q_{k'} \text{ in neutral networks but impractical for (dis)assortative networks}$$

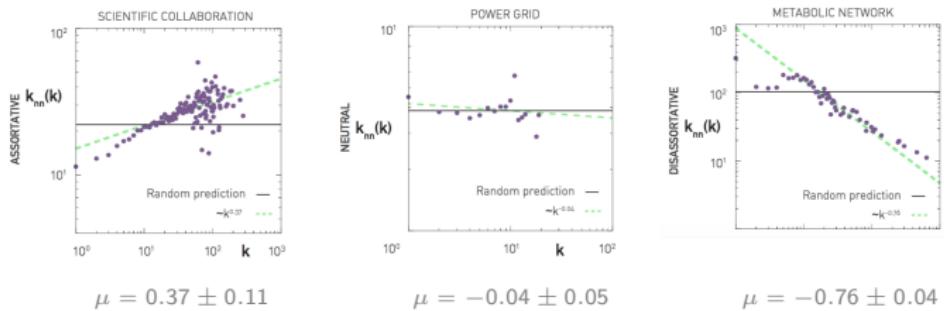


# mixing exponent

- neighbor degree function  $k_{nn}$  [PSVV01] defined as
  - $k_{nn}(k)$  is average neighbor degree of degree- $k$  nodes
  - $P(k'|k)$  is link probability of degree- $k$  to - $k'$  node
  - $\mu$  is degree mixing power-law exponent [VPSV02]

$$k_{nn}(k) = \sum_{k'} k' P(k'|k) = \sum_{k'} k' \frac{e_{kk'}}{\sum_{k'} e_{kk'}}$$

$k_{nn} = \frac{\langle k^2 \rangle}{\langle k \rangle}$  in neutral networks and  $k_{nn}(k) \sim k^\mu$  in (dis)assortative networks

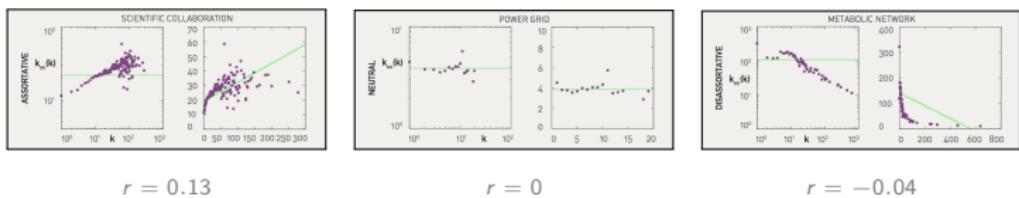


# mixing coefficient

- *degree mixing coefficient*  $r$  [New02, Est11] defined as
  - $r$  is Pearson correlation of linked nodes' degrees [New03]
  - $q_k$  is neighbor excess degree distribution  $\frac{(k+1)p_{k+1}}{\langle k \rangle}$

$$r = \sum_{kk'} \frac{kk'(\epsilon_{kk'} - q_k q_{k'})}{\sum_k k^2 q_k - (\sum_k k q_k)^2}$$

$r = 0$  in *neutral networks* and  $k_{nn}(k) \sim rk$  in (*dis*)*assortative networks*



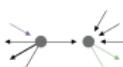
$$r = 0.13$$

$$r = 0$$

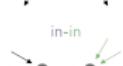
$$r = -0.04$$



in-in



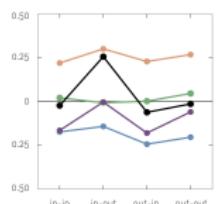
in-out



out-in



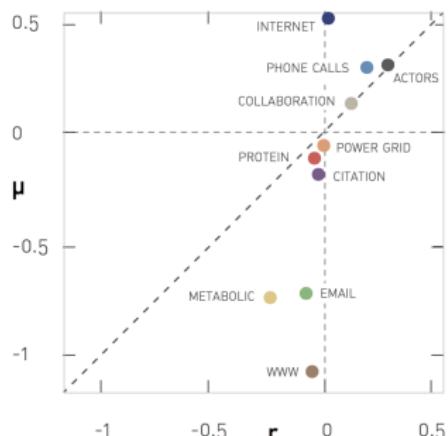
out-out



# mixing *networks*

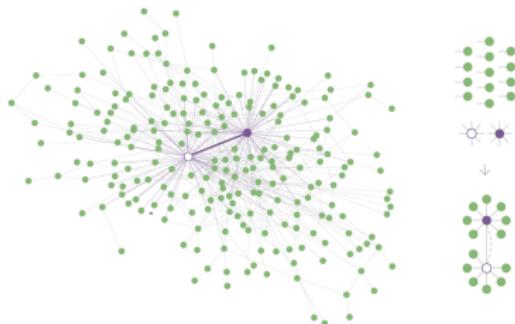
- coefficient & exponent  $r$  &  $\mu$  in real networks [Bar16]
- $r$  &  $\mu$  correlate in assortative regime and  $\text{sgn}(r) = \text{sgn}(\mu)$

NETWORK	N	r	$\mu$
Internet	192,244	0.02	0.56
WWW	325,729	-0.05	-1.11
Power Grid	4,941	0.003	0.0
Mobile Phone Calls	36,595	0.21	0.33
Email	57,194	-0.08	-0.74
Science Collaboration	23,133	0.13	0.16
Actor Network	702,388	0.31	0.34
Citation Network	449,673	-0.02	-0.18
E. Coli Metabolism	1,039	-0.25	-0.76
Protein Interactions	2,018	0.04	-0.1



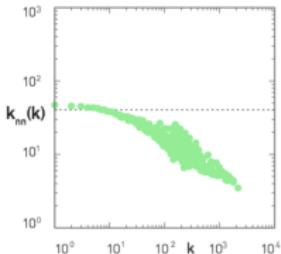
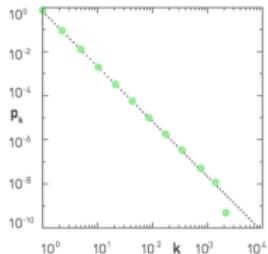
## mixing *structural*

- *structural disassortativity*  $\frac{E_{kk'}}{m_{kk'}} > 1$  [MSZ04] in real networks
  - $E_{kk'}$  is *number of links* between degree- $k$  &  $-k'$  nodes
  - $m_{kk'}$  is *maximum  $E_{kk'}$*  hence  $\min(kn_k, k'n_{k'}, n_k n_{k'})$
- $E_{kk'} = 2me_{kk'} = \langle k \rangle n e_{kk'}$   
*natural cutoff*  $k_{max} \sim n^{\frac{1}{\gamma-1}}$  and *structural cutoff*  $k_s \sim \sqrt{\langle k \rangle n}$
- *structural disassortativity* in *scale-free* networks with  $\gamma < 3$

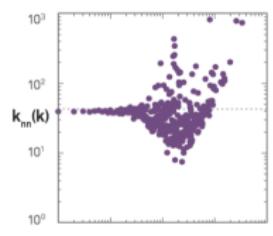
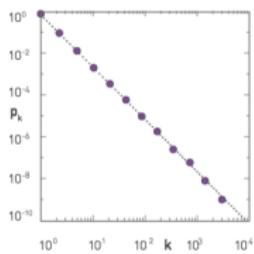


$$k = 55 \text{ and } k' = 46 \text{ then } E_{kk'} = 2.81 > 1$$

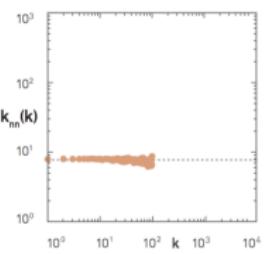
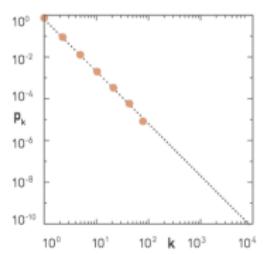
# mixing *scale-free*



configuration *scale-free* network as *simple graph*

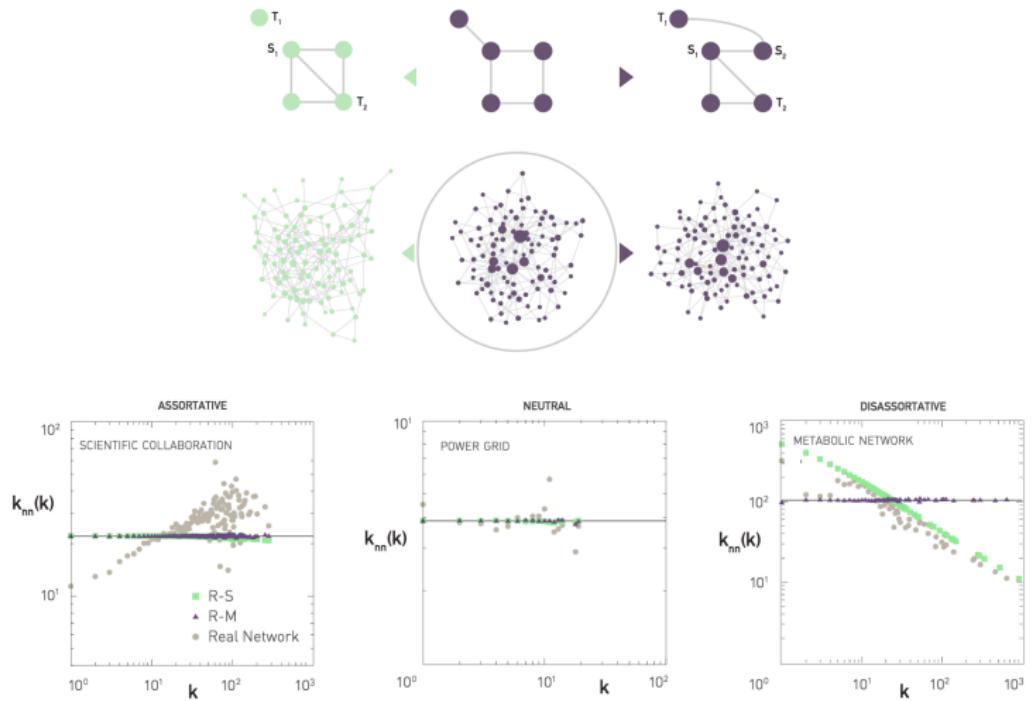


configuration *scale-free* network as *multigraph*



configuration *scale-free* network *without hubs*  $k \geq k_s$

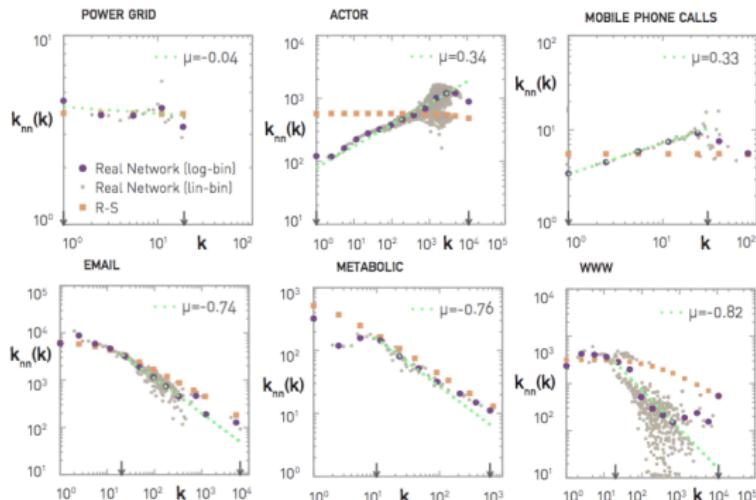
# mixing *randomization*



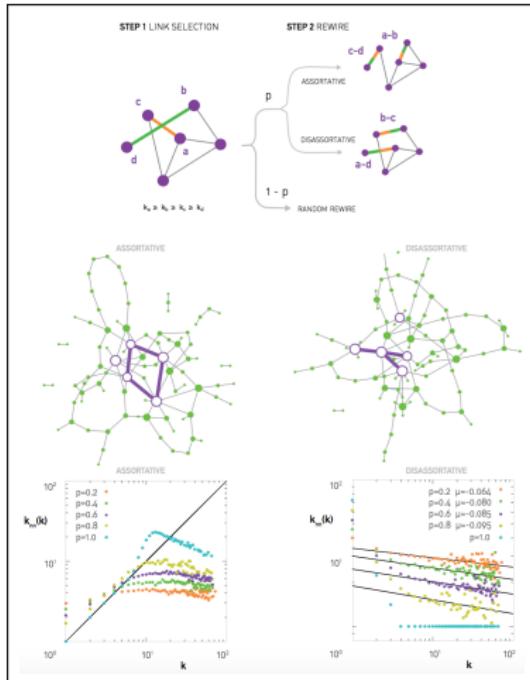
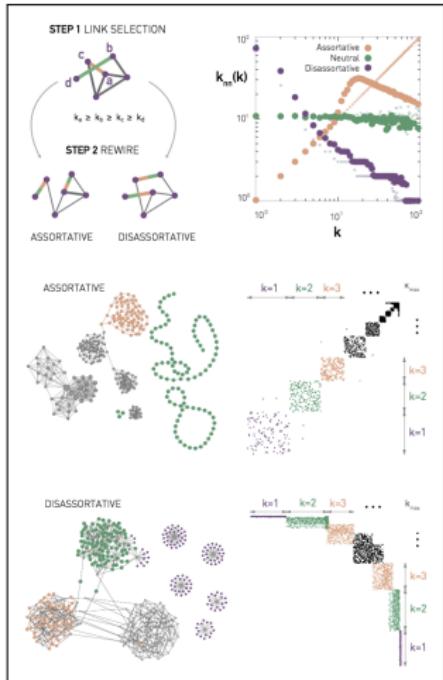
*degree-preserving randomization* with *simple/multi* links *retains/destroys* structural *disassortativity*

# mixing *networks*

- neighbor degree  $k_{nn}$  in real networks [Bar16]
- collaboration assortative and technological neutral
- biological/information (structurally) disassortative



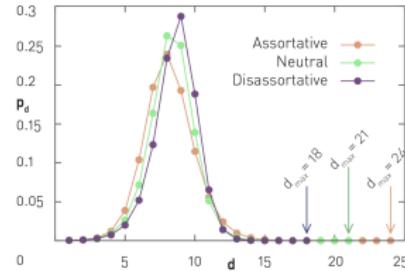
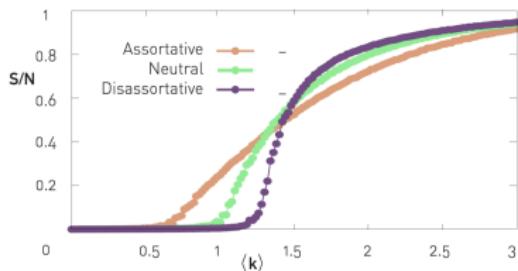
# mixing *models*



(dis)assortative degree-preserving randomization [XBS05]

# mixing *impact*

- *degree mixing* impacts *connectivity* and *distances* [New02]
- *assortative mixing* coexists with *community structure* [NP03]
- *mixing* influences *resilience* [VM03] and *controllability* [LSB11]



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