advanced topics in *network science* (*ants*)

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spring 2019/20

announcements *F3 week*

- *low complexity* out *today*
- *low complexity* due *next week*
- *PhD presentations* later *today*

- *project & paper* details *today*
- *project presentations* in *two weeks*
- *project proposals* in *three weeks*

- think about *course project*
- keep your *reading list*!

challenge *F3 week*

low complexity challenge

node *centrality*

advanced topics in *network science* (*ants*)

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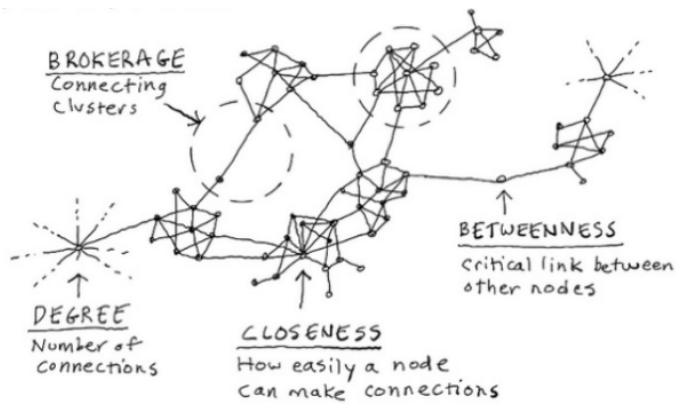
centrality *measures*

which *nodes* are most *important*?

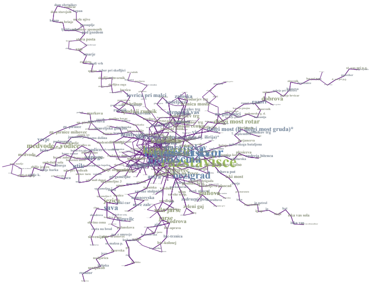
— *node centrality measures* for (*un*)*directed* networks
  - *clustering coefficients* [WS98, SV05, dNMB05]
  - *distance-based* centrality [Fre77, FBW91, New05]
  - *spectral analysis* centrality [Kat53, Bon87, BP98]
  - *fragment-based* centrality [MSOI+02, Pržˇ07, EK15]



— *link analysis algorithms* for *directed* networks

networkology *LPP*

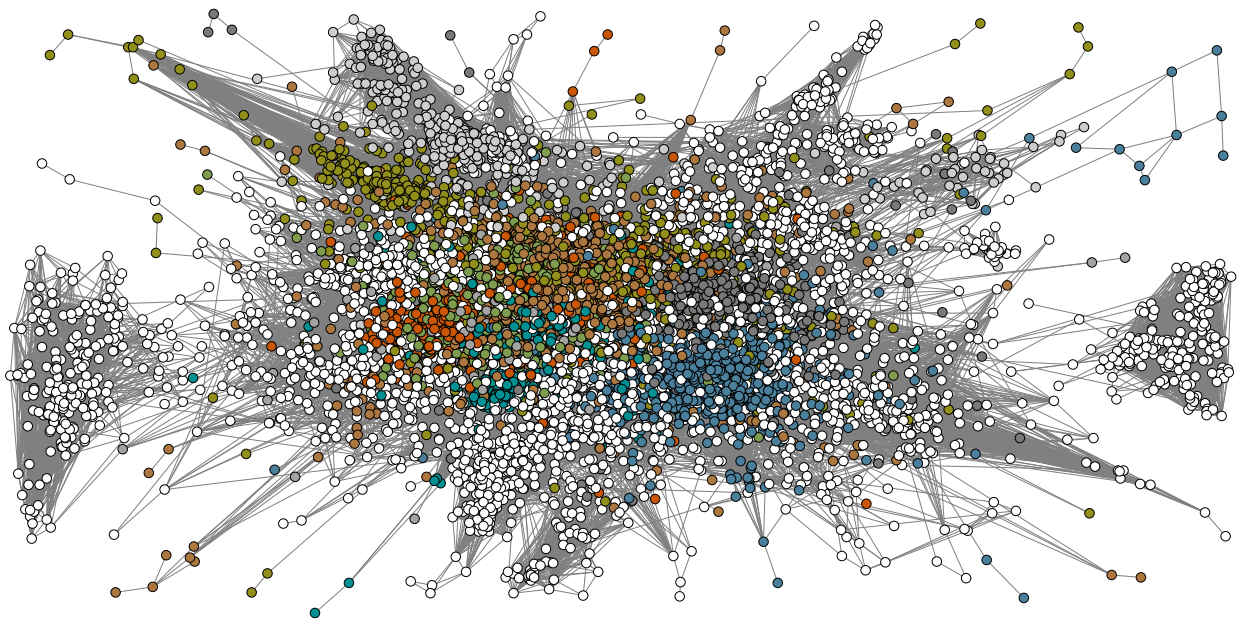
- partial *LPP* *public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 5.62$ connections
- *giant component* 95.4% nodes (6 components)
- *"small-world"* with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- *"scale-free"* with $\gamma = 2.62$ for cutoff $k_{min} = 5$



*reduced to largest connected component

networkology *iMDB*

- *iMDB movie actors collaboration network*
- $n = 17\,577$ actors with $\langle k \rangle = 32.7$ collaborations
- *giant component* 99.3% nodes (19 components)
- *small-world* with $\langle C \rangle = 0.34$ and $\langle d \rangle = 4.82$
- *scale-free* with $\gamma = 2.21$ for cutoff $k_{min} = 25$



centrality *clustering*

important *nodes* are *strongly embedded*

— for *undirected* G *clustering coefficient* C [WS98] of i is
  - t_i is number of *linked neighbors* or *triangles* of i

$$C_i = \frac{2t_i}{k_i(k_i-1)} \qquad C_i = 0 \text{ for } k_i \leq 1$$

— ω-*corrected clustering coefficient* C^ω [SV05] of i is
  - ω_i is *maximum possible* t_i with *respect to* $\{k\}$

$$C_i^\omega = \frac{t_i}{\omega_i} \qquad C_i^\omega = 0 \text{ for } \omega_i = 0$$

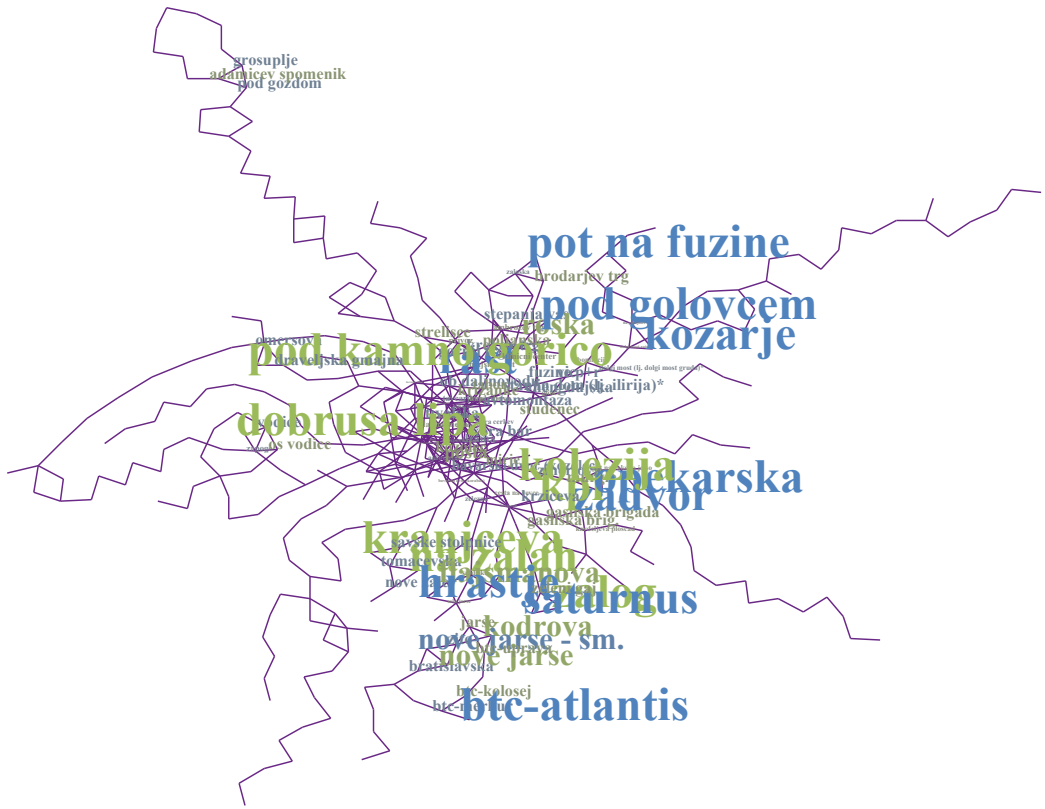
— μ-*corrected clustering coefficient* C^μ [dNMB05] of i is
  - μ is *maximum* number of *triangles* over *links*

$$C_i^\mu = \frac{2t_i}{k_i\mu} \qquad C_i^\mu = 0 \text{ for } k_i = 0$$

networkology *LPP*

— *clustering coefficient* C in partial LPP network[†]

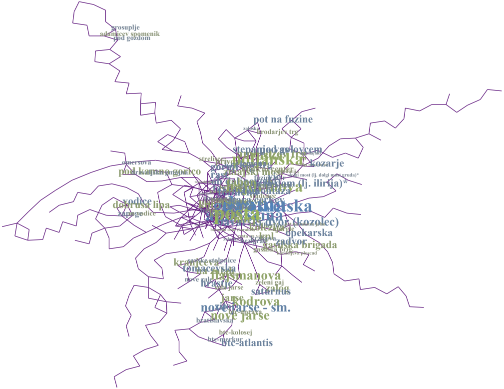
— *highest* $C_i = 1.0$ nodes are *Na Žalah etc.* with $k_i = 2$



[†] reduced to simple undirected graph

networkology *LPP*

- μ-*corrected clustering* C^{μ} in partial LPP network‡
- *highest* $C_i^{\mu} = 0.44$ node is *Drama* with $k_i = 10$



‡reduced to simple undirected graph

networkology *iMDB*

— *clustering coefficient* C in iMDB network

— *highest* C nodes are *less known actors*

#	**actor**	k	C
1	Rachner, Jonathan	46	1.0000
2	Doucette, Jeff	23	1.0000
3	Willis, Susan	20	1.0000
4	Andersson, Kris	20	1.0000
5	Kurtis, Bill	19	1.0000
6	Cantillana, Nestor	16	1.0000
7	Tolkien, J.R.R.	15	1.0000
8	Faris, Anna	15	1.0000
9	Kurata, Tetsuo	13	1.0000
10	Margera, Jess	12	1.0000
11	Rakeyohn	12	1.0000
12	Raab, Chris	12	1.0000

networkology *iMDB*

— *μ-corrected clustering C^{μ}* in iMDB network

— *highest C^{μ}* nodes are *wrestling actors/models*

#	actor	k	C^{μ}
1	Helms, Shane	216	0.3689
2	Wilson, Torrie	217	0.3688
3	Matthews, Darren	212	0.3688
4	Greenwald, Nora	215	0.3687
5	Keibler, Stacy	215	0.3687
6	Jindrak, Mark	214	0.3683
7	Wight, Paul	221	0.3677
8	Guerrero Jr., Chavo	215	0.3676
9	Bischoff, Eric	218	0.3675
10	Hugger, John	212	0.3675
11	Flair, Ric	219	0.3672
12	Layfield, John	192	0.3667

centrality *closeness*

important *nodes* are *close to other* nodes

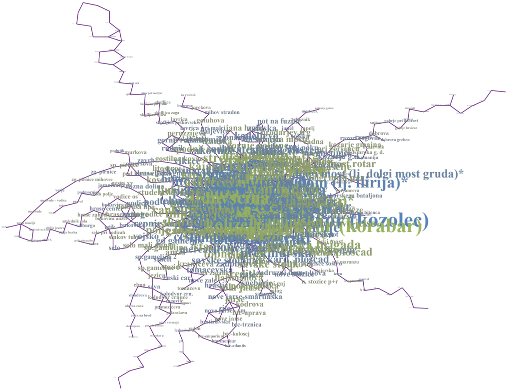
— for (*un*)*directed* G *closeness centrality* ℓ^{-1} [New10] of i is
  - d_{ij} is (*un*)*directed distance* between i and j
  - $d_{ij} = \infty$ for nodes in *different components*

$$\ell_i^{-1} = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

— ℓ^{-1} spans *small range* in *small-world* networks

networkology *LPP*

- *closeness centrality* ℓ^{-1} in partial LPP network§
- *highest* $\ell_i^{-1} = 0.208$ node is *Gosposvetska* with $k_i = 14$



§ reduced to simple undirected graph

— *closeness centrality* ℓ^{-1} in iMDB network[¶]

— *highest* ℓ^{-1} nodes are *Hollywood actors*

#	actor	k	ℓ^{-1}
1	Goldberg, Whoopi	398	0.3506
2	Hanks, Tom	457	0.3500
3	Jackson, Samuel L.	427	0.3497
4	Berry, Halle	376	0.3468
5	Diaz, Cameron	361	0.3459
6	Stiller, Ben	382	0.3452
7	Lopez, Jennifer	410	0.3427
8	Myers, Mike	345	0.3409
9	Douglas, Michael	263	0.3403
10	Cruise, Tom	336	0.3401
11	Travolta, John	335	0.3388
12	Schwarzenegger, Arnold	333	0.3385

[¶] reduced to largest connected component

centrality *betweenness*

important *nodes* are *bridges for other* nodes

— for (*un*)*directed* G *betweenness centrality* σ [Fre77] of i is
  - g_{st} is number of *geodesic paths between* s and t
  - g_{st}^{i} is number of *such geodesic paths through* i

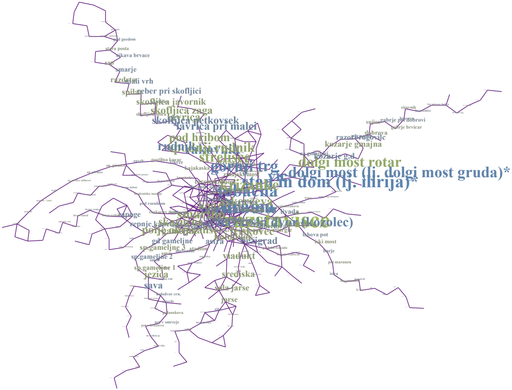
$$\sigma_i = \frac{1}{n^2} \sum_{st} \frac{g_{st}^{i}}{g_{st}}$$

— σ considers *only geodesic paths* [FBW91, New05]

— σ mixes *local centers* with *global bridges* [JMK+16]

networkology *LPP*

- *betweenness centrality* σ in partial LPP network[‖]
- *highest* $\sigma_i = 0.235$ node is *Razstavišče* with $k_i = 11$



‖ reduced to simple undirected graph

— *betweenness centrality* σ in iMDB network**

— *highest* σ nodes are *international actors*

#	actor	k	σ
1	Jeremy, Ron	471	0.0640
2	Chan, Jackie	135	0.0310
3	Cruz, Penelope	182	0.0284
4	Shahlavi, Darren	8	0.0282
5	Del Rosario, Monsour	6	0.0280
6	Depardieu, Gerard	159	0.0265
7	Bachchan, Amitabh	66	0.0169
8	Jackson, Samuel L.	427	0.0167
9	Soualem, Zinedine	121	0.0155
10	Del Rio, Olivia	168	0.0152
11	Jaenicke, Hannes	73	0.0140
12	Hayek, Salma	185	0.0139

** reduced to largest connected component

centrality *degrees*

important *nodes* are *linked by many* nodes

— for *undirected* G *degree centrality* d of i is

$$d_i = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i}{n-1}$$

— in *directed* G *in-degree centrality* d^{in} of i is

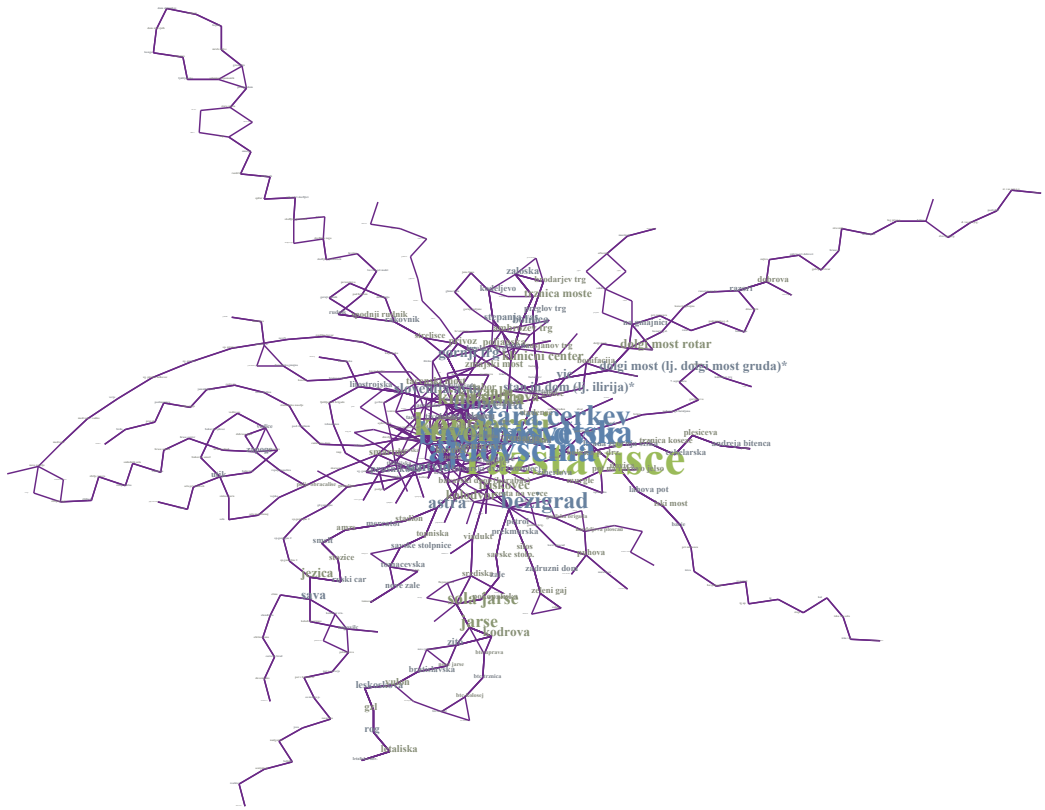
$$d_i^{in} = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i^{in}}{n-1}$$

— in *directed* G *out-degree centrality* d^{out} of i is

$$d_i^{out} = \frac{1}{n-1} \sum_{j \neq i} A_{ji} = \frac{k_i^{out}}{n-1}$$

networkology *LPP*

- *degree centrality* d in partial LPP network
- *highest* $d_i = 0.099$ node is *Razstavišče* with $k_i = 41$
- *highest* $d_i^{\cdot}$ node is *Razstavišče* with $k_i^{in} = 20$ and $k_i^{out} = 21$



— *degree centrality d* in iMDB network

— *highest d* nodes are *pornographic actors*

#	**actor**	*k*	*d*
1	Davis, Mark	784	0.0446
2	Sanders, Alex	610	0.0347
3	North, Peter	599	0.0341
4	Marcus, Mr.	584	0.0332
5	Tedeschi, Tony	561	0.0319
6	Dough, Jon	555	0.0316
7	Stone, Lee	545	0.0310
8	Voyeur, Vince	533	0.0303
9	Lawrence, Joel	500	0.0284
10	Steele, Lexington	493	0.0280
11	Ashley, Jay	490	0.0279
12	Boy, T.T.	475	0.0270

centrality *eigenvector*

important *nodes* are *linked by important* nodes

— for (*un*)*directed* G *eigenvector centrality* e [Bon87] of i is
  - v and λ are *eigenvectors* and *eigenvalues* of A
  - e is *proportional* to *leading eigenvector* v_1

$$e(t) = A^t e(0) = A^t \sum_i C_i v_i = \sum_i C_i \lambda_i^t v_i = \lambda_1^t \sum_i C_i \left[\frac{\lambda_i}{\lambda_1}\right]^t v_i \to C_1 \lambda_1^t v_1$$

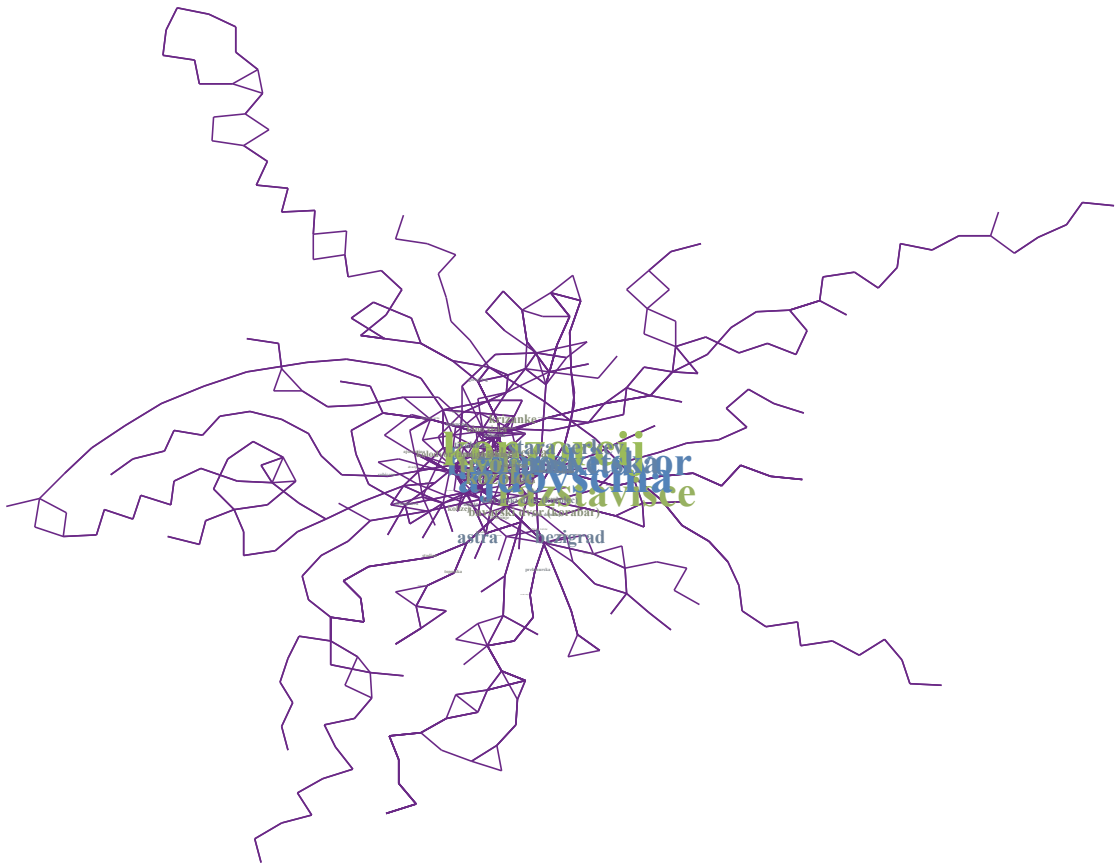
$$e_i = \lambda_1^{-1} \sum_j A_{ij} e_j$$

— in *directed* G $e = 0$ for $k^{in} = 0$ *nodes etc.*

networkology *LPP*

— *eigenvector centrality* e in partial LPP network

— *highest* $e_i = 0.082$ node is *Konzorcij* with $k_i = 30$



— *eigenvector centrality* e in iMDB network

— *highest* e nodes are *wrestling actors*

#	**actor**	k	e
1	Benoit, Chris	225	0.005261
2	Guerrero, Eddie	225	0.005261
3	Storm, Lance	225	0.005236
4	Wight, Paul	221	0.005231
5	Jericho, Chris	253	0.005230
6	Runnels, Dustin	220	0.005206
7	Flair, Ric	219	0.005203
8	Huffman, Booker	219	0.005199
9	Bischoff, Eric	218	0.005192
10	Wilson, Torrie	217	0.005192
11	Gruner, Peter	218	0.005183
12	Levy, Scott	226	0.005181

centrality *Katz*

nodes get small amount of *importance for free*

— for (*un*)*directed* G *Katz centrality* z [Kat53] of i is
  - α and β are some *positive constants*

$$z_i = \alpha \sum_j A_{ij} z_j + \beta_i$$

— for *convenience* $\beta = 1$ whereas $\alpha < \lambda_1^{-1}$
  - λ_1 is *leading eigenvalue* of A

centrality *PageRank*

nodes distribute equal amount of *importance*

— for (*un*)*directed* G *PageRank centrality* p [BP98] of i is
  - α and β are some *positive constants*

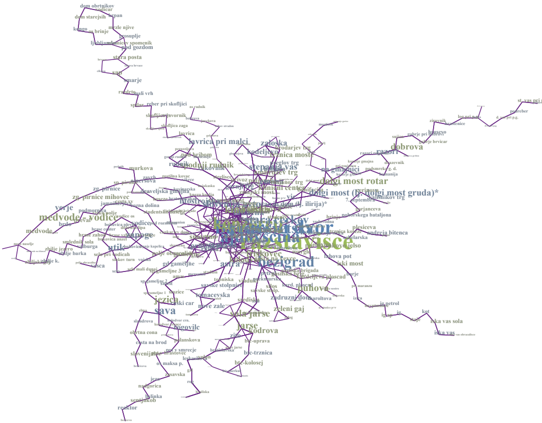
$$p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j^{out}} + \beta_j$$

— for *convenience* $\beta = \frac{1-\alpha}{n}$ whereas $\alpha = 0.85$

see PageRank algorithm NetLogo demo

networkology *LPP*

- *PageRank centrality* p in partial LPP network
- *highest* $p_i = 0.011$ node is *Razstavišče* with $k_i = 41$



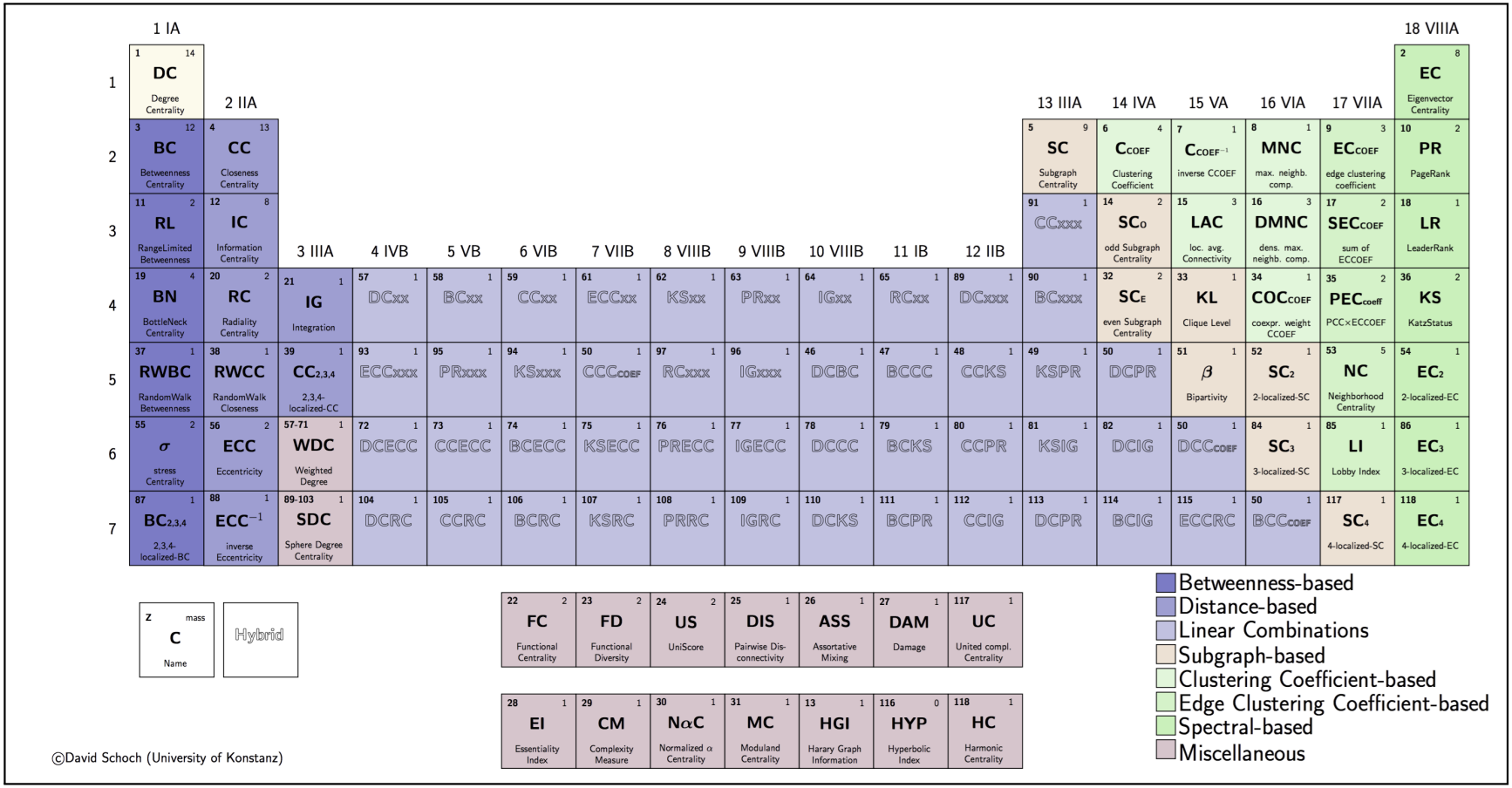
— *PageRank centrality p* in iMDB network

— *highest p* nodes are *Hollywood actors*

#	**actor**	*k*	*p*
1	Hanks, Tom	457	0.000660
2	Jackson, Samuel L.	427	0.000642
3	Goldberg, Whoopi	398	0.000607
4	Stiller, Ben	382	0.000561
5	Davis, Mark	784	0.000547
6	Lopez, Jennifer	410	0.000542
7	Berland, Francois	194	0.000540
8	Berry, Halle	376	0.000536
9	Diaz, Cameron	361	0.000514
10	Travolta, John	335	0.000486
11	Jeremy, Ron	471	0.000476
12	Myers, Mike	345	0.000468

centrality *overview*

which *nodes* are most *important*?



centrality *references*

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link *analysis*

advanced topics in *network science* (*ants*)

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spring 2019/20

link *analysis*

which *web pages* are most *important*?

— *node centrality measures* for (*un*)*directed* networks

— *link analysis algorithms* primarily for *directed web graphs*

- Google *search ranking PageRank* [BP98, PBMW99]
- hyperlink-induced *topic search HITS* [Kle99]

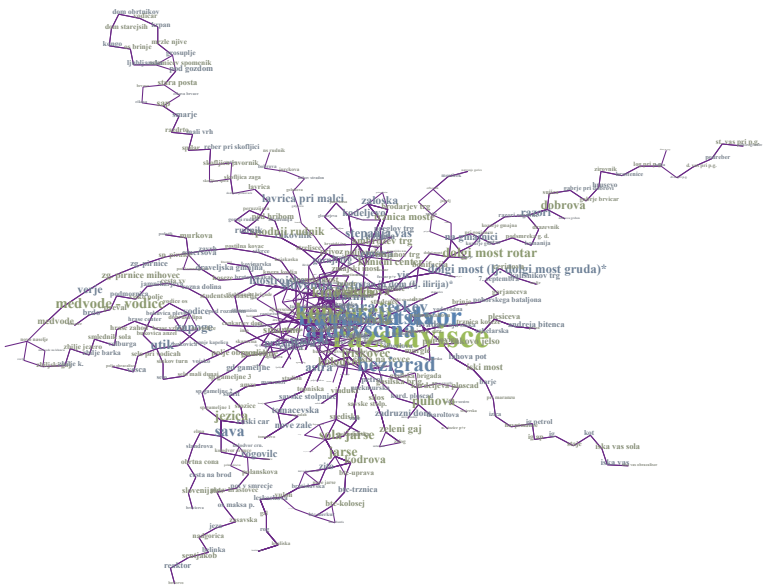
Sergey Brin

Lawrence Page

Jon Kleinberg

networkology *LPP*

- corrected *LPP public bus transport network**
- $n = 408$ bus stops with $\langle k \rangle = 5.73$ connections
- *giant component* 95.3% nodes (6 components)
- *"small-world"* with $\langle C \rangle = 0.10$ and $\langle d \rangle = 14.43$
- *"scale-free"* with $\gamma = 2.60$ for cutoff $k_{min} = 5$



* reduced to largest connected component

analysis *PageRank*

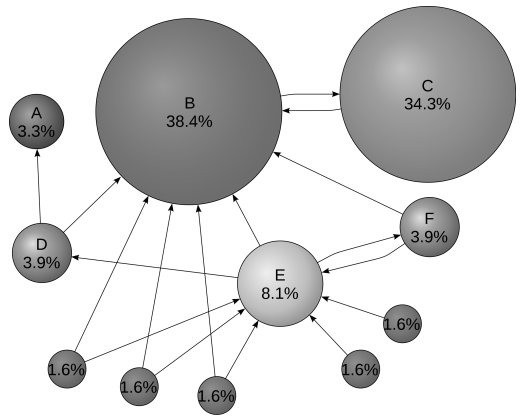
ranking algorithm for *web page importance*

— for *directed* G *PageRank rank* p [BP98] of i is
  – α is *positive constant* traditionally $\alpha = 0.85$

$$p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j^{out}} + \frac{1-\alpha}{n}$$

— p *oscillates* in *spider traps* and *leaks out* of *dead ends*

— p_i probability *random surfer with teleports* lands on i



networkology *PageRank*

— *PageRank ranks* p in corrected LPP network

— *highest* p nodes are *Razstavišče* and *Ajdovščina*

#	**bus stop**	k_i	p_i
1	Razstavišče	43	0.010601
2	Ajdovščina	36	0.007694
3	Bežigrad	23	0.007161
4	Bavarski dvor	30	0.007013
5	Konzorcij	30	0.006884
6	Gosposvetska	30	0.006527
7	Stara cerkev	26	0.005485
8	Sava	12	0.005165
9	Tobačna	22	0.005136
10	Kino Šiška	18	0.004907
11	Medvode	4	0.004853
12	Tivoli	26	0.004838

analysis *WalkRank*

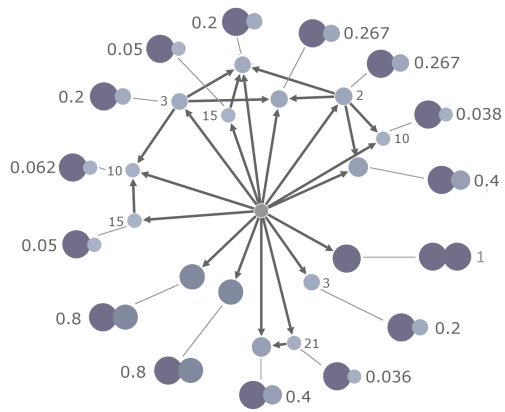
ranking algorithm for *web page similarity*

— for *directed* G *WalkRank rank* w [TFP06] for t of i is
  – α is *positive constant* traditionally $\alpha = 0.85$

$$w_i^t = \alpha \sum_j A_{ij} \frac{w_j^t}{k_j^{out}} + (1-\alpha)\delta_{it}$$

— w_i^t probability *random surfer with teleport* t lands on i

— *personalized PageRank* and *SimRank* [PBMW99, JW02]



— *WalkRank ranks w* in corrected LPP network

— *highest w* nodes for *Razstavišče* and *Hajdrihova*

#	bus stop	k_i	w_i
1	Razstavišče	43	0.236115
2	Bavarski dvor	30	0.065124
3	Bezigrad	23	0.057260
4	Astra	16	0.047765
5	Ajdovščina	36	0.040099
6	Kozolec	10	0.038384
7	Gosposvetska	30	0.030981
8	Konzorcij	30	0.020278
9	Bavarski dvor	8	0.019262
10	Polje	10	0.014254
11	Stadion	8	0.013294
12	Topniška	8	0.013235

#	bus stop	k_i	w_i
1	Hajdrihova	14	0.201318
2	Tobačna	22	0.091186
3	Ilirija	12	0.051714
4	Stara cerkev	26	0.046825
5	Tabor	10	0.038395
6	Vič	16	0.034478
7	Avtomontaža	6	0.030372
8	Stan in dom	4	0.030296
9	Kino Šiška	18	0.028569
10	Tivoli	26	0.028180
11	Glince	8	0.027528
12	Na klancu	10	0.023836

analysis *HITS*

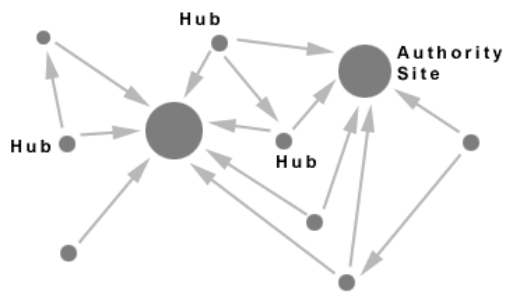
ranking algorithm for *web hubs & authorities*

— for *directed* G *hub & authority ranks* h & a [Kle99] of i
  - h is *eigenvector* of $A^T A$ with *eigenvalue* $(\alpha\beta)^{-1}$
  - a is *eigenvector* of AA^T with *eigenvalue* $(\alpha\beta)^{-1}$
  - α and β are some *positive constants*

$$h_i = \alpha \sum_j A_{ji} a_j \qquad a_i = \beta \sum_j A_{ij} h_j$$

— a measures *content* and h measures *table of content*

— $a = 0$ for $k^{in} = 0$ *nodes* and $h = 0$ for $k^{out} = 0$ *nodes*



— *hub & authority ranks h & a* in corrected LPP network

— *highest h* node is *Ajdovščina* and *highest a* node is *Konzorcij*

#	bus stop	k_i	h_i
1	Ajdovščina	36	0.715370
2	Razstavišče	43	0.455771
3	Tivoli	26	0.286178
4	Drama	23	0.256027
5	Gosposvetska	30	0.175142
6	Bavarski dvor	30	0.129155
7	Pošta	9	0.111497
8	Kolodvor	4	0.090644
9	Konzorcij	30	0.083028
10	Tavčarjeva	7	0.069477
11	Kozolec	10	0.068749
12	Stara cerkev	26	0.064760

#	bus stop	k_i	a_i
1	Konzorcij	30	0.656745
2	Bavarski dvor	30	0.512119
3	Gosposvetska	30	0.235790
4	Kozolec	10	0.224651
5	Bežigrad	23	0.176839
6	Astra	16	0.172509
7	Stara cerkev	26	0.172482
8	Ajdovščina	36	0.161840
9	Razstavišče	43	0.110391
10	Tivoli	26	0.106024
11	Bavarski dvor	8	0.096486
12	Kolizej	4	0.088636

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community structure

advanced topics in *network science* (*ants*)

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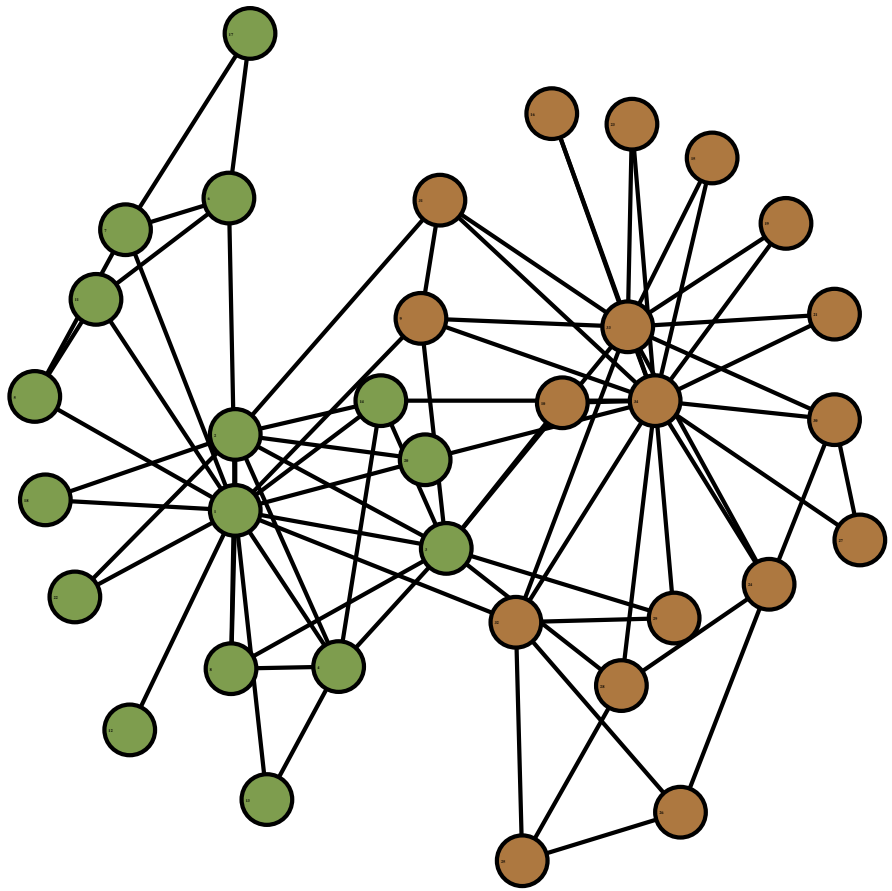
University of Ljubljana
spring 2019/20

community *structure*

karate club *network split* [Zac77]

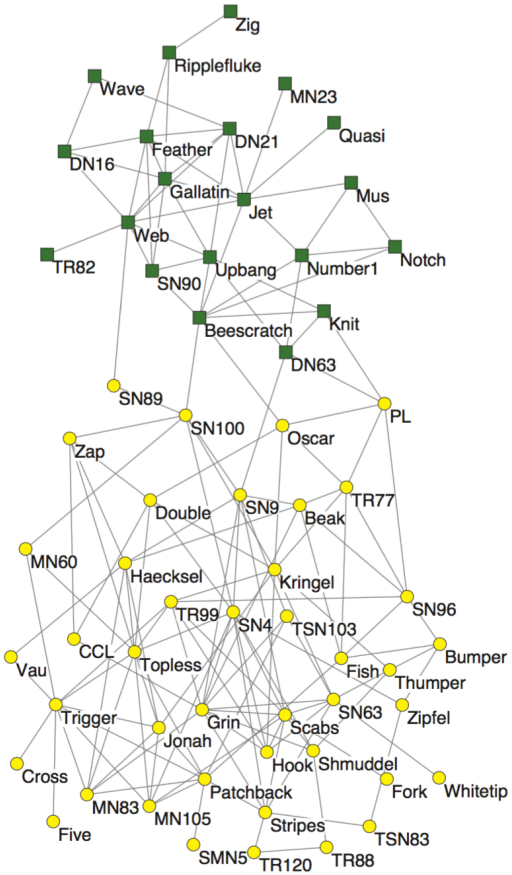


community *detection*

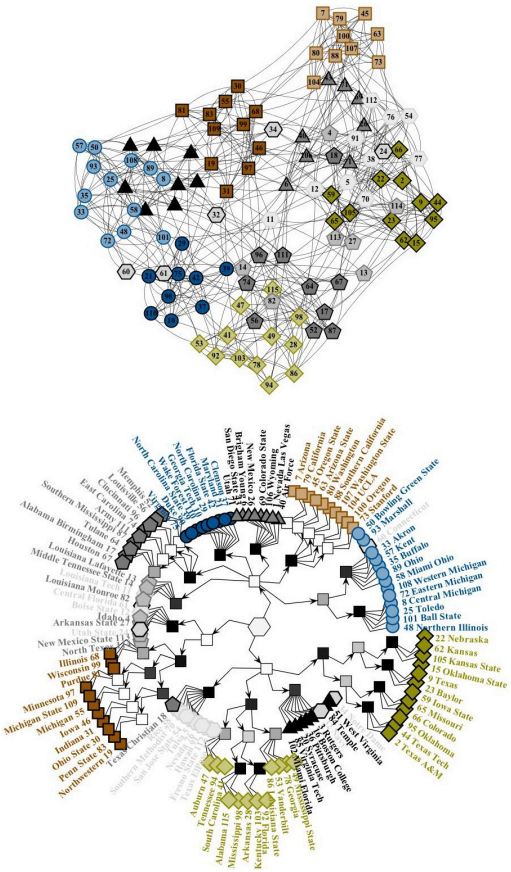
karate club *split detection* [RAK07]

community *examples*

most social networks contain *communities* [GN02]



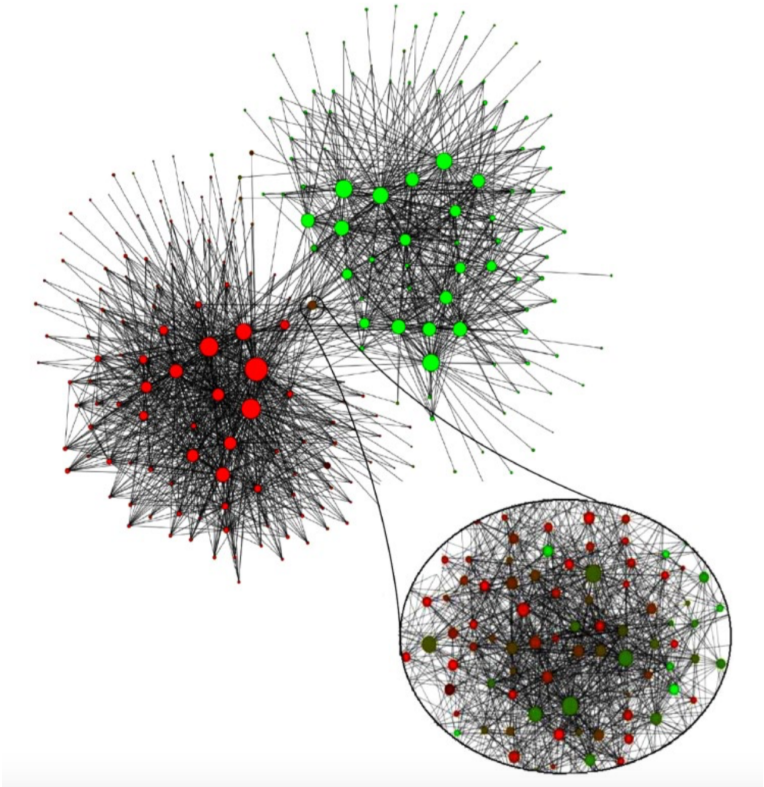
bottlenose dolphins [LSB+03]



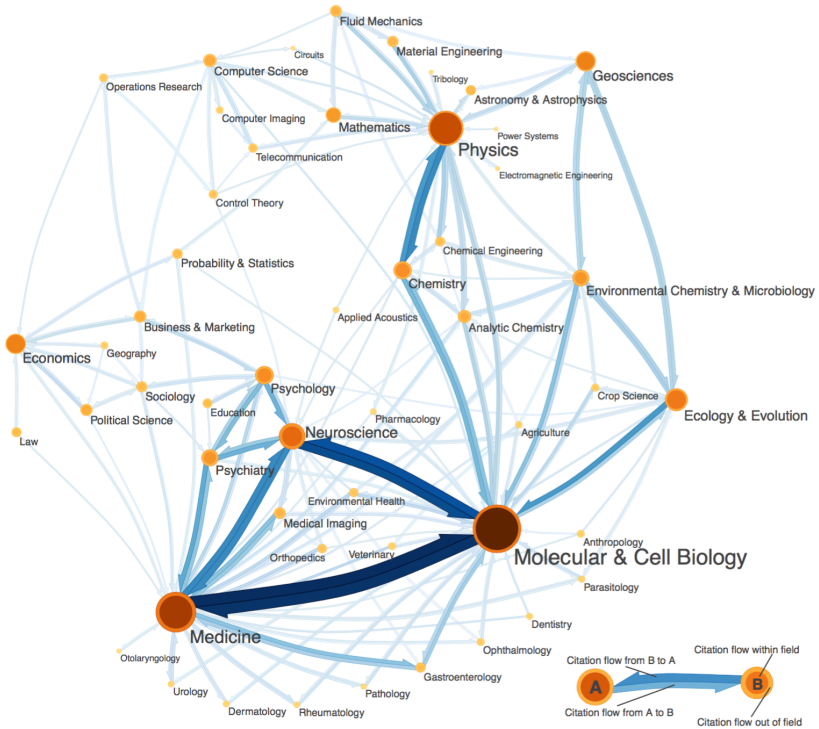
college football [GN02]

community *examples*

many information networks contain *communities* [FLG00]



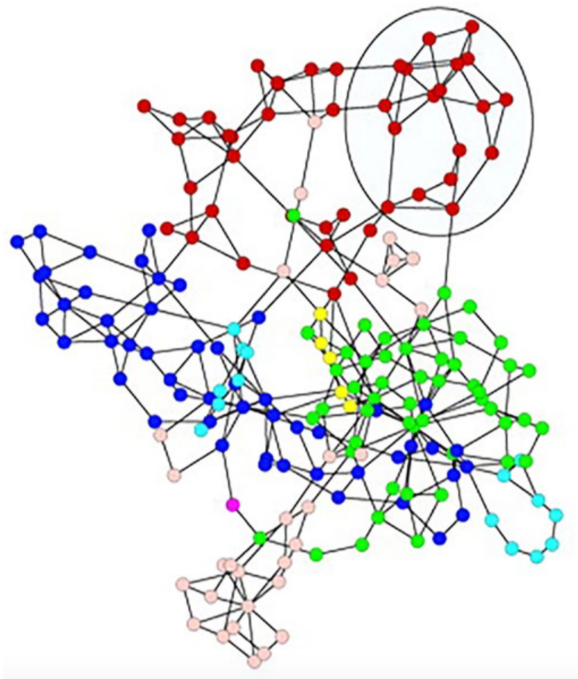
mobile communications [BGLL08]



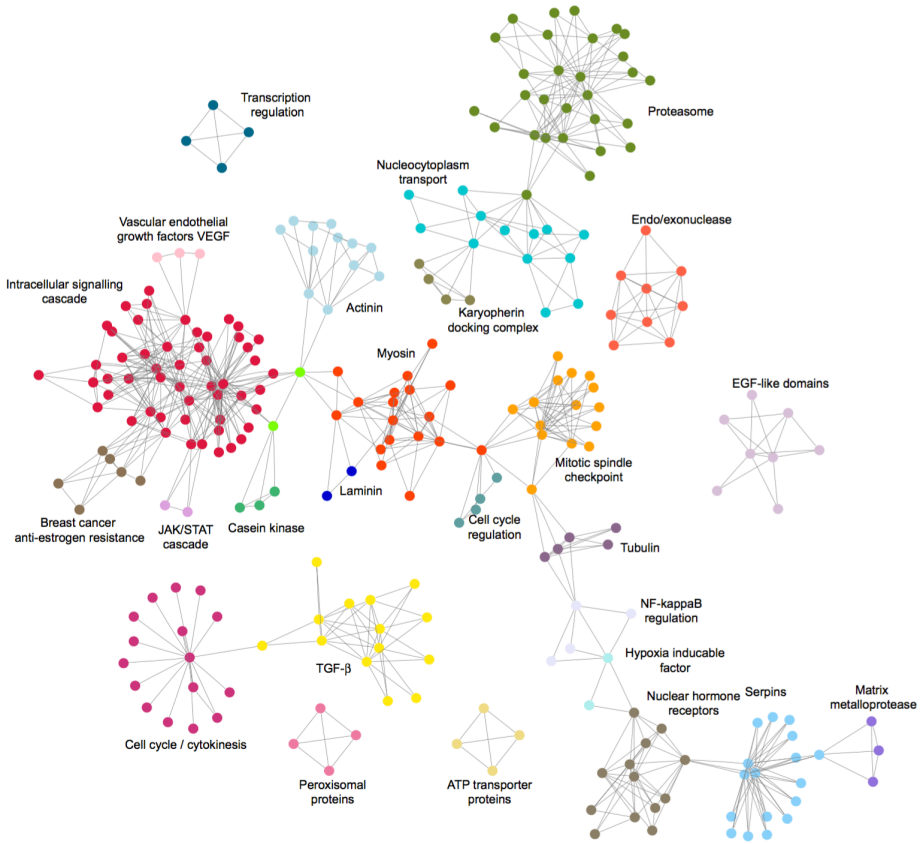
journal citations [RB08]

community *examples*

many biological networks contain *communities* [RSM+02]



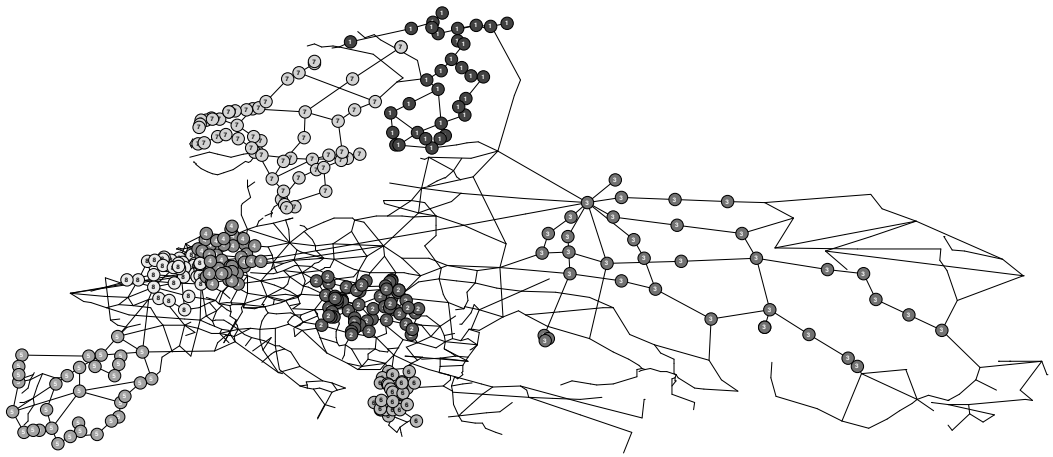
E. coli metabolism [RSM+02]



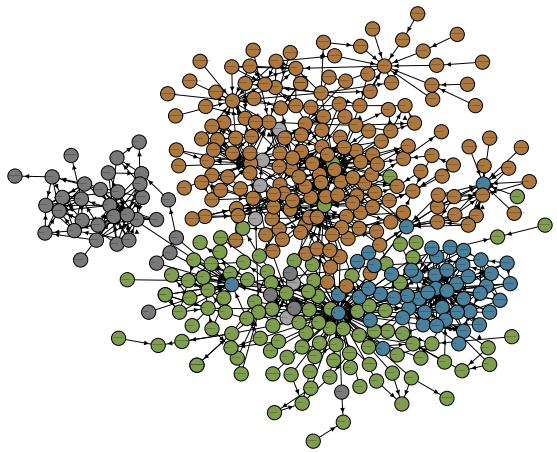
protein interactions [JCZB06]

community *examples*

some technological networks contain *communities* [ŠB11a]



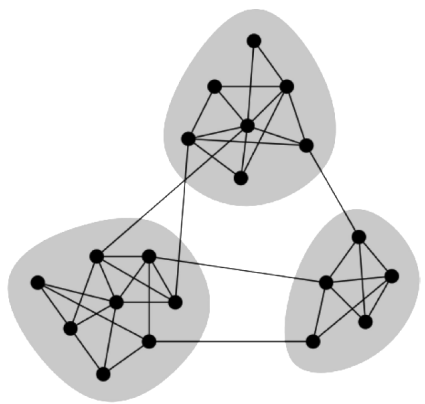
European highways [ŠB11b]



JUNG dependencies [ŠB11a]

community *explanation*

- *weak* & *strong ties* according to *information flow*
- *bridges* & *embedded ties* according to *network span*
  - removal of *local bridge* $\{i, j\}$ causes $d_{ij} > 2$
  - removal of *bridge* $\{i, j\}$ causes $d_{ij} = \infty$
  - *embedded tie* $\{i, j\}$ has $C_{ij} > 0$



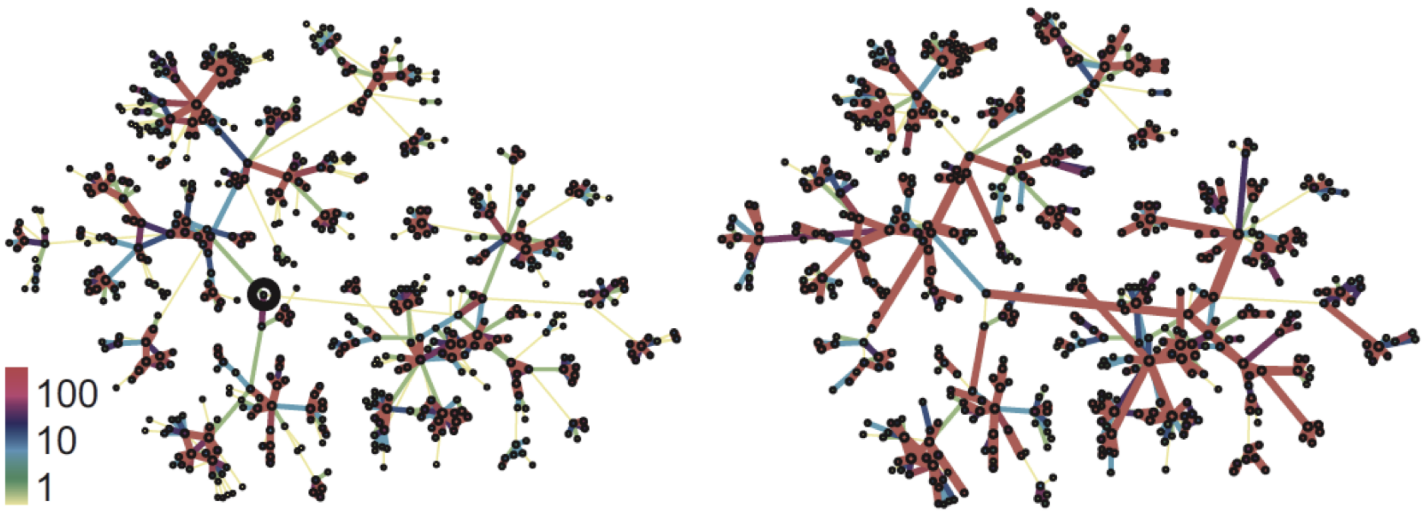
strength of weak ties or weakness of strong ties

- *weak ties are* (*local*) *bridges* under *triadic closure* [Gra73]
- *assortative mixing* and *homophily* in (social) networks [NG03]

community *experiment*

— *tie strength* in mobile communications [OSH+07]

— *weak ties are* (*local*) *bridges* in real networks



real strengths *shuffled strengths*

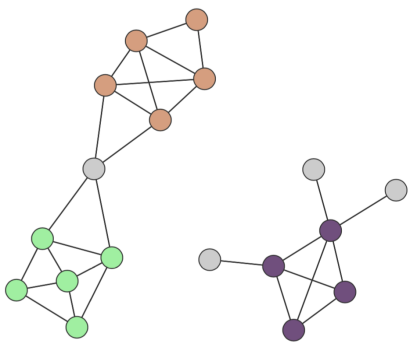
strength of weak ties or weakness of strong ties

community *definition*

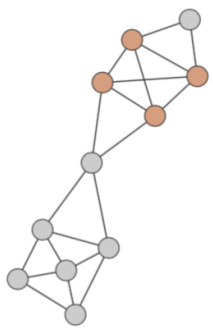
- *clique* is *complete subgraph of* some *graph*
  - also *k-plexes*, *k-cores*, *k-cliques*, *k-clubs*, *k-clans*
- *community* is *dense subgraph of sparse network* [GN02]
- *strong* and *weak community* C [FLG00, RCC+04] defined as
  - k_i^{int} and k_i^{ext} are *internal* and *external degree* of i

$$\forall i \in C : k_i^{int} > k_i^{ext} \qquad \sum_{i \in C} k_i^{int} > \sum_{i \in C} k_i^{ext}$$

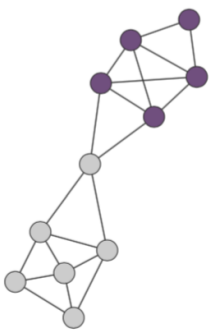
- *community detection* is *not graph partitioning* [For10]



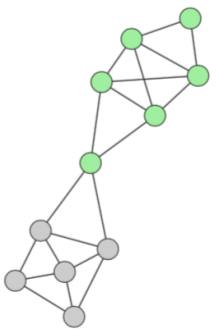
connected communities



maximum clique



strong community



weak community

community *modularity*

— *random graph lacks community structure*

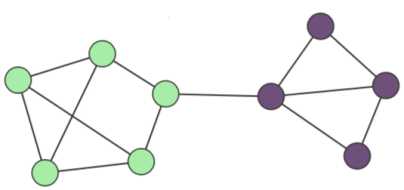
— *modularity* Q [GN02] of *communities* $\{C\}$ defined as

– $k_c = \sum_{i \in C} k_i$ is *total degree* and m_c is *number of links* in C

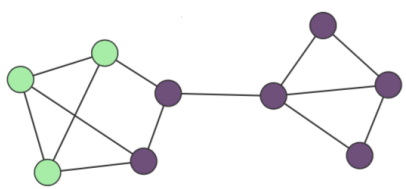
$$\frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta_{c_i c_j} = \frac{1}{2m} \sum_{C} \sum_{ij \in C} \left(A_{ij} - \frac{k_i k_j}{2m} \right) = \sum_{C} \frac{m_c}{m} - \left(\frac{k_c}{2m} \right)^2$$

$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta_{c_i c_j} = \sum_{C} \frac{m_c}{m} - \left(\frac{k_c}{2m} \right)^2$$

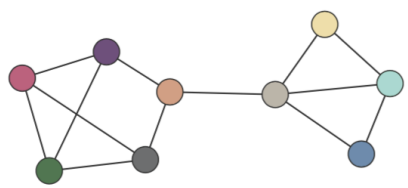
— *modularity* Q popular *quality*/*optimization function* [For10]



optimal $Q = 0.41$



suboptimal $Q = 0.22$



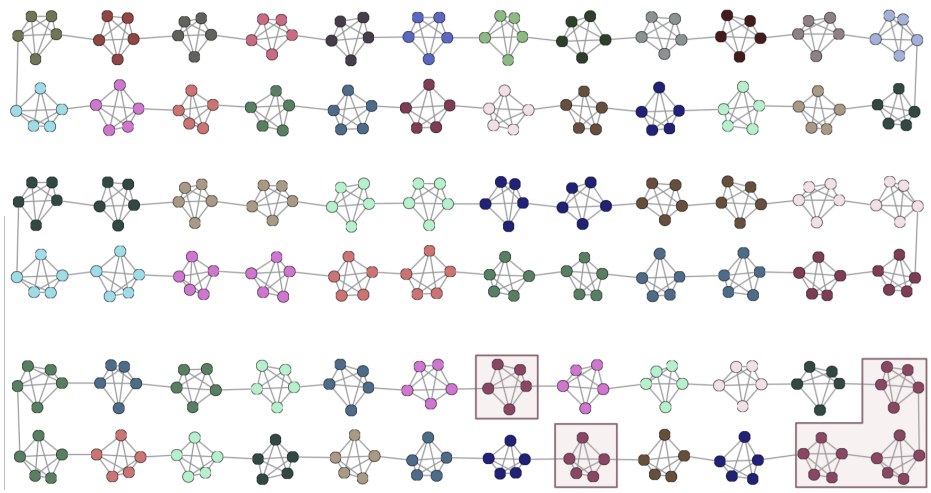
isolates $Q = -0.12$

community ¬*modularity*

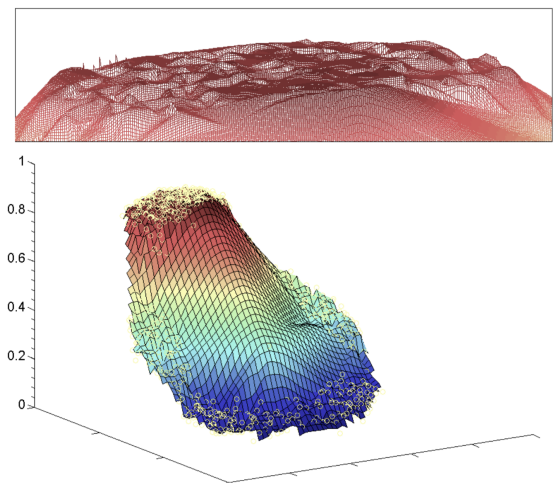
— *modularity* $Q \gg 0$ also in *random graphs* [GSPA04]

— *modularity* Q has *resolution limit* at $k_c \leq \sqrt{2m}$ [FB07]

— *modularity* Q *lacks clear optimum* in real networks [GdMC10]

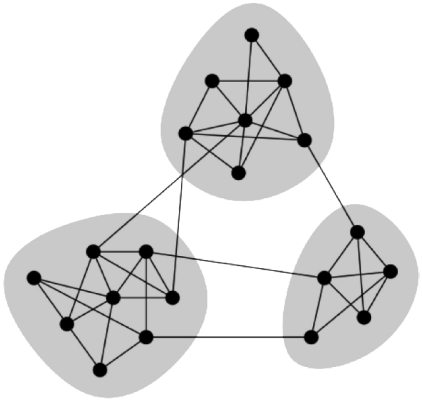


intuitive $Q = 0.867$, *optimal* $Q = 0.871$ and random $Q = 0.8$

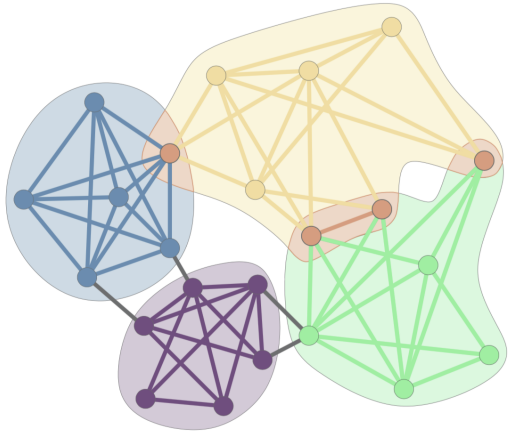


Q *plateau* and maxima

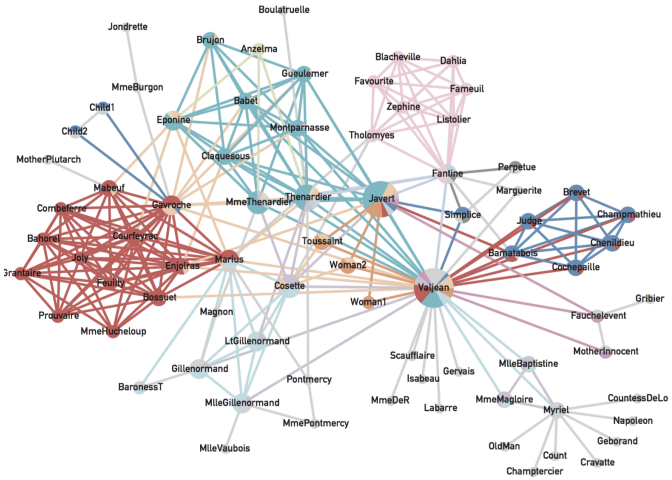
community *overview*



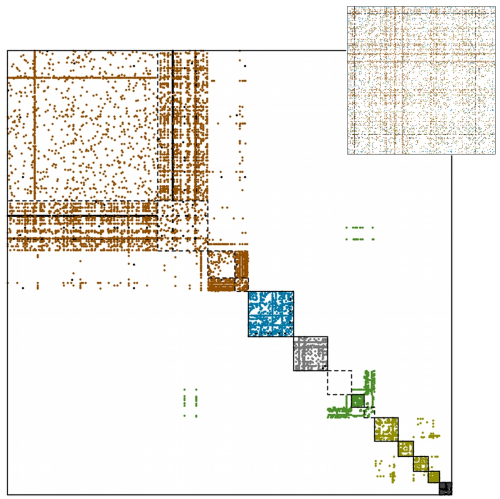
communities [GN02]



overlapping communities [PDFV05]



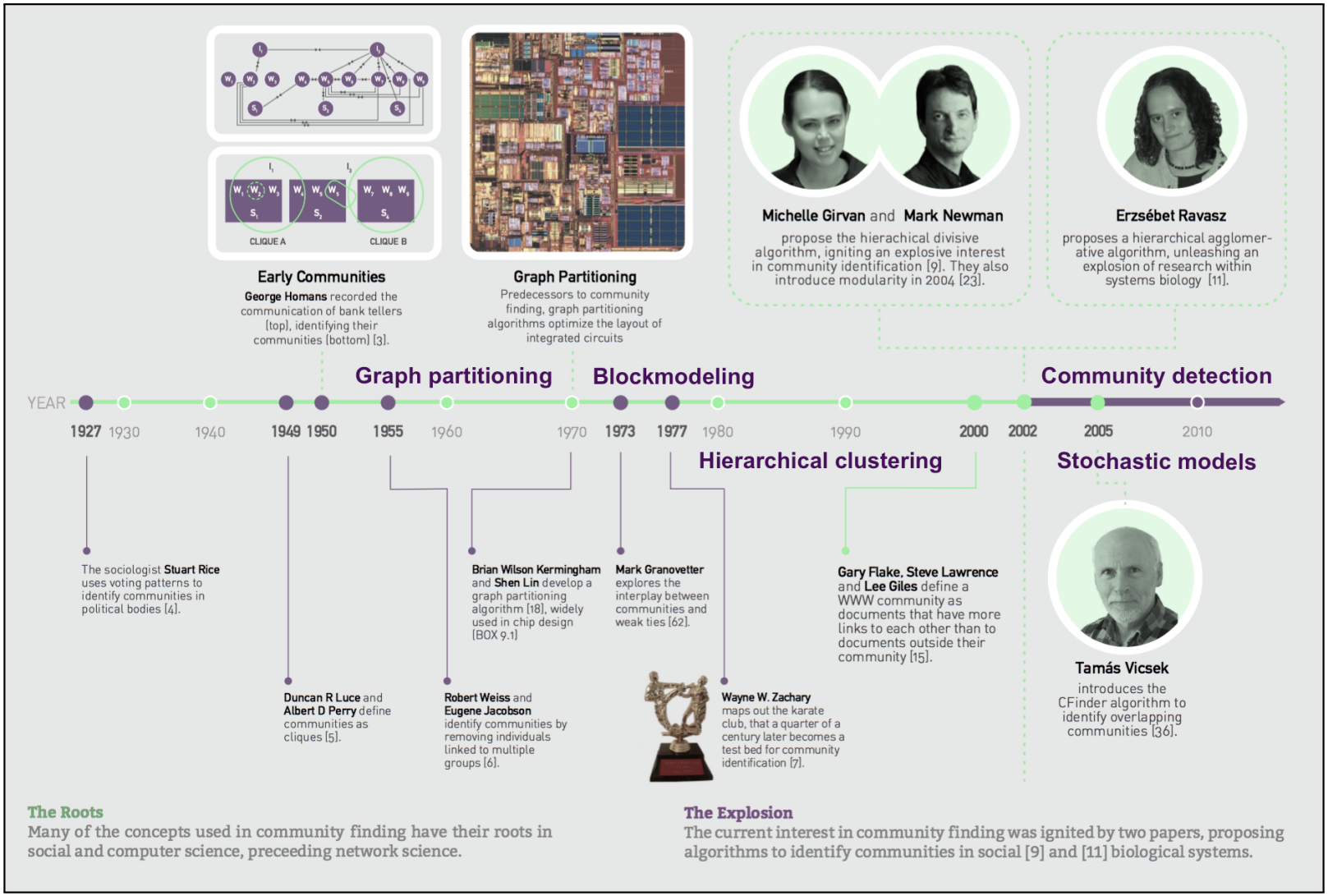
link communities [EL09, ABL10]



block models, *blockmodeling* etc.

`javax.swing`, `javax.management`, `javax.xml`, `javax.print`, `javax.naming`, `javax.lang`

community *history*



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network *clustering*

advanced topics in *network science* (*ants*)

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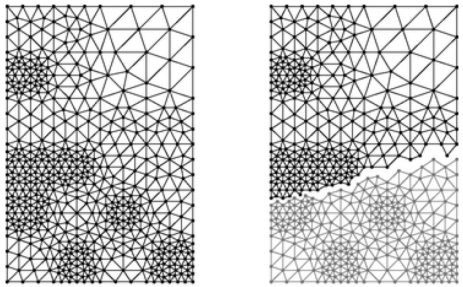
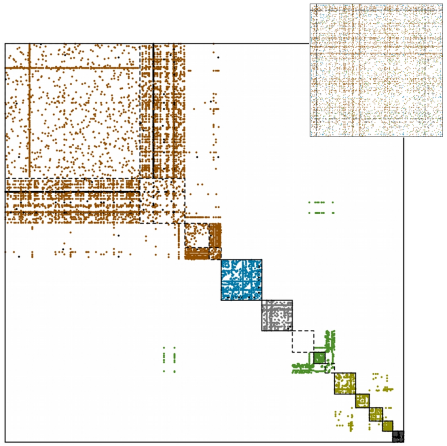
spring 2019/20

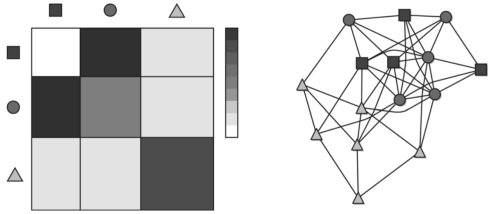
clustering *overview*



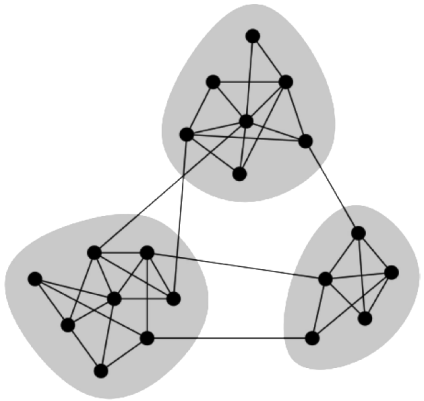
graph partitioning [KL70, Fie73]



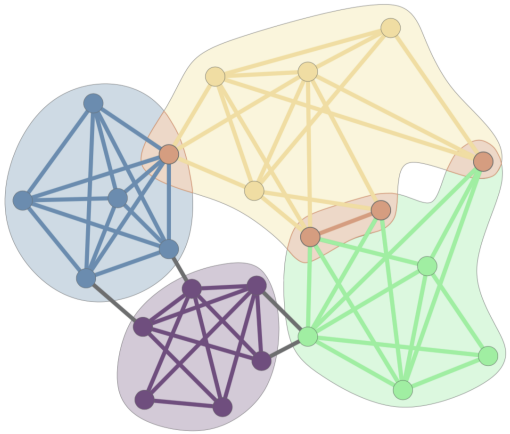
blockmodeling [LW71, WR83]



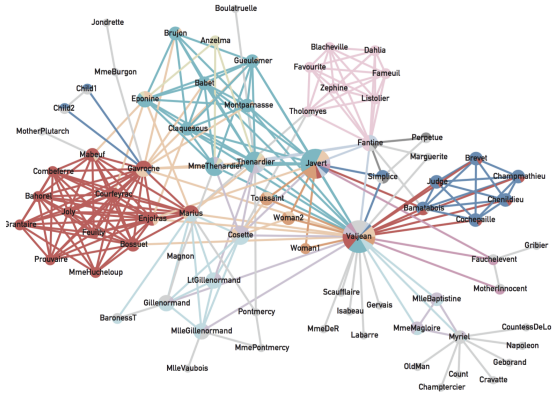
stochastic block models [Pei15]



communities [GN02]

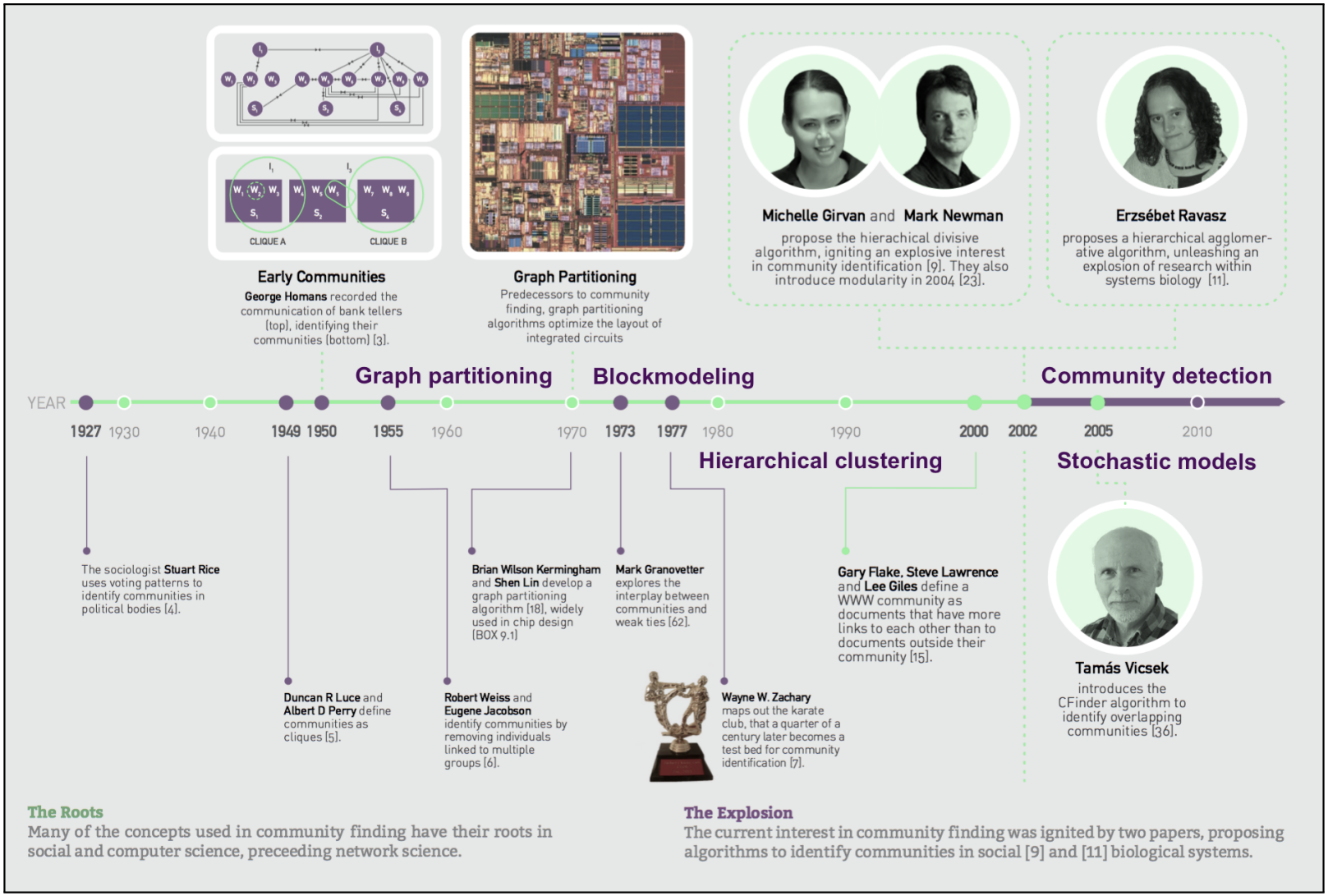


overlapping communities [PDFV05]



link communities [EL09, ABL10]

clustering *history*



graph *partitioning*

advanced topics in *network science* (*ants*)

Lovro Šubelj & Jure Leskovec
University of Ljubljana
spring 2019/20

partitioning *bisection*

— Kernighan-Lin *graph bisection* [KL70]

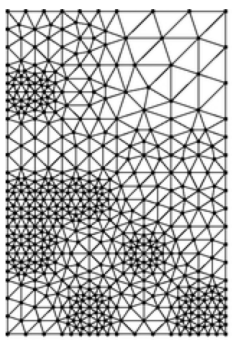
– define *bisection quality* as *cut size*

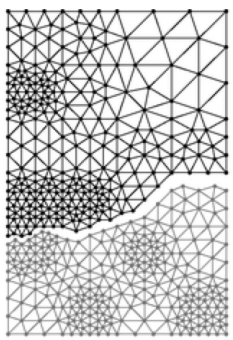
$$R = \frac{1}{2}\sum_{ij} A_{ij}(1 - \delta_{c_i c_j}) \qquad \forall i : c_i = \pm 1$$

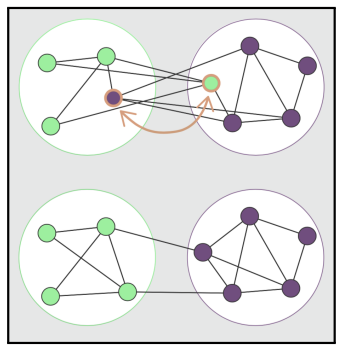
1. *swap nodes* by *minimizing cut size* $\mathcal{O}(cn^2m)$

$$\Delta R_{ij} = k_i^{ext} - k_i^{in} + k_j^{ext} - k_j^{in} - 2A_{ij}$$

2. repeat 1. *until* $\min(n_1, n_2)$ *nodes swapped*
3. return *bisection minimizing cut size*







*example mesh bisection with cut size equal to 40

partitioning *spectral*

— Fiedler *graph bisection* [Fie73]

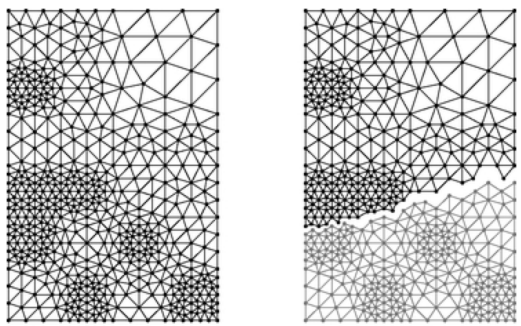
– define *bisection quality* as *cut size*

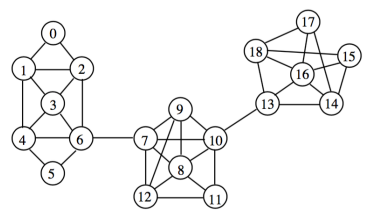
$$R = \frac{1}{4}\sum_{ij} A_{ij}(1 - s_i s_j) \qquad \forall i : s_i = \delta_{c_i c_1} - \delta_{c_i c_2}$$

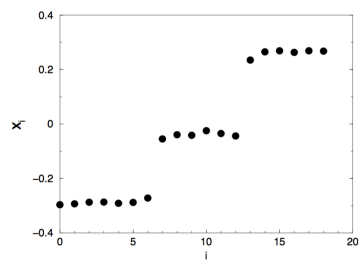
– formulate *eigenvector problem* of *graph Laplacian*

$$R = \frac{1}{4}\sum_i k_i s_i^2 - \frac{1}{4}\sum_{ij} A_{ij} s_i s_j = \frac{1}{4}\sum_{ij}(k_i\delta_{ij} - A_{ij})s_i s_j = \frac{1}{4}s^T L s \simeq \frac{1}{4}v^T L v = \frac{n_1 n_2}{n}\lambda$$

1. find *eigenvector* v_2 with *algebraic connectivity* λ_2 $\mathcal{O}(nm)$
2. assign n_1 *nodes* with *largest*/*smallest* v_2 to C_1
3. return *bisection minimizing cut size*







see graclus and metis implementations

† example mesh bisection with cut size equal to 46

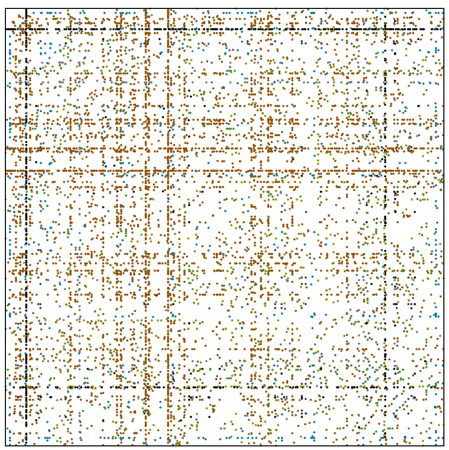
partitioning *blockmodeling*

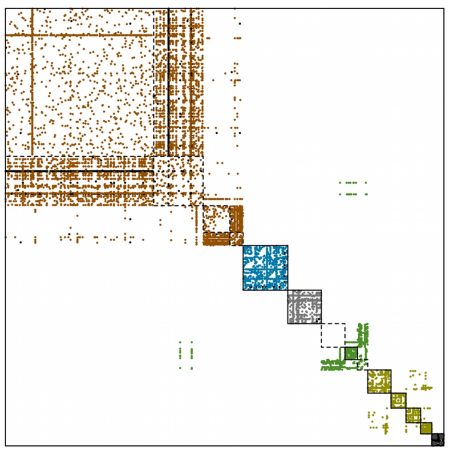
— standard *equivalence blockmodeling* [DBF05]

– define *node similarity* as *(structural) equivalence*

$$s_{ij} \sim |\Gamma_i \cap \Gamma_j|$$

1. *blockmodeling* by *(hierarchical) clustering* $\mathcal{O}(n^2)$
2. return *block model* at desired *clustering resolution*





see catrege implementation

‡ `javax.swing`, `javax.management`, `javax.naming`, `javax.print`, `javax.xml`, `javax.lang` etc.

community detection

advanced topics in *network science* (*ants*)

Lovro Šubelj & Jure Leskovec
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spring 2019/20

community *agglomerative*

— Ravasz *hierarchical clustering* [RSM+02]

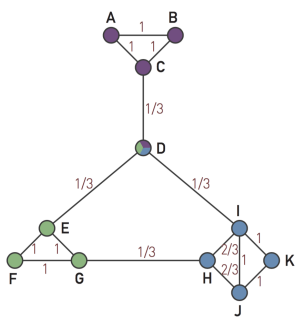
- define *node similarity* as *topological overlap*

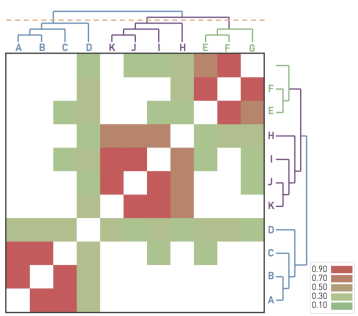
$$s_{ij} = \frac{|\Gamma_i \cap \Gamma_j| + A_{ij}}{\min(k_i, k_j)}$$

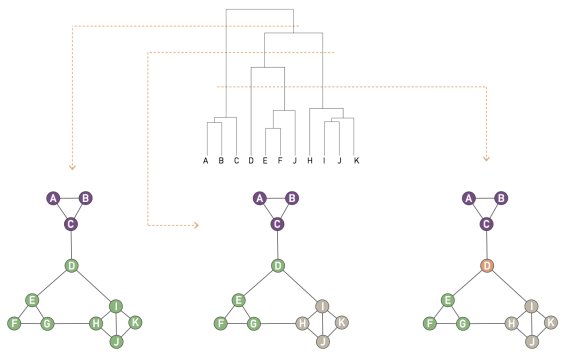
- define *cluster similarity* as *average linkage*

$$S_{ij} = \frac{1}{n_i n_j} \sum_{xy} s_{xy} \delta_{c_x c_i} \delta_{c_y c_j}$$

1. bottom-up *agglomerative hierarchical clustering* $\mathcal{O}(n^2)$
2. cut *cluster dendrogram* at desired *clustering resolution*







community *divisive*

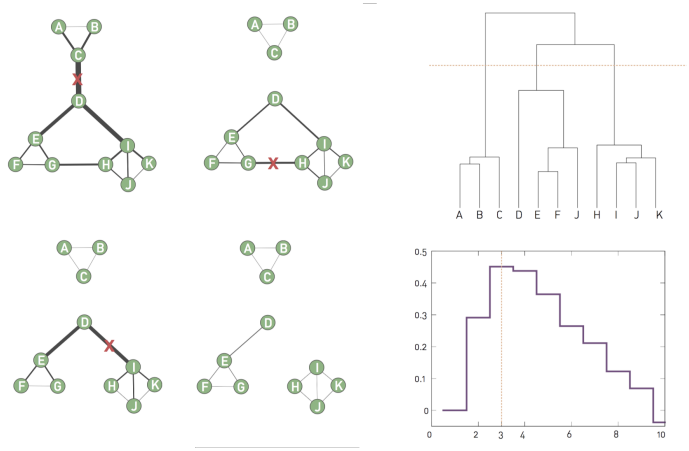
— Girvan-Newman *hierarchical clustering* [GN02]

– define *node dissimilarity* as *link betweenness*

$$\sigma_{ij} = \sum_{st \notin \{i,j\}} \frac{g_{st}^{ij}}{g_{st}}$$

1. top-down *divisive hierarchical clustering* $\mathcal{O}(nm^2)$
2. cut *cluster dendrogram* at *maximum modularity*

$$Q = \frac{1}{2m} \sum_{ij} (A_{ij} - \frac{k_i k_j}{2m}) \delta_{c_i c_j}$$

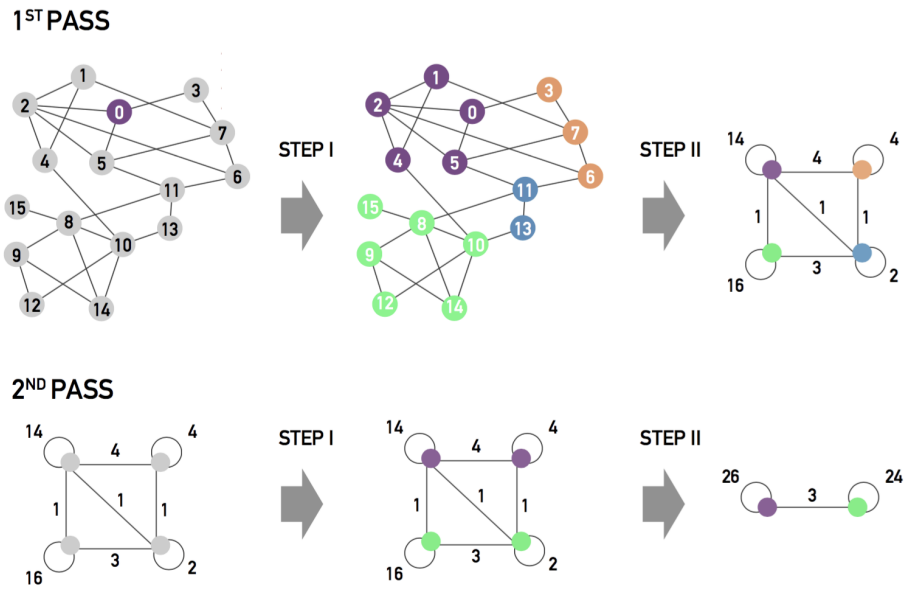


community *modularity*

— Louvain *modularity optimization* [BGLL08]

1. set *node community* by *modularity optimization* $\mathcal{O}(cm)$
2. *aggregate community nodes into supernodes* and repeat 1.
3. return *community structure maximizing modularity*

$$Q = \frac{1}{2m} \sum_{ij} (A_{ij} - \frac{k_i k_j}{2m}) \delta_{c_i c_j}$$



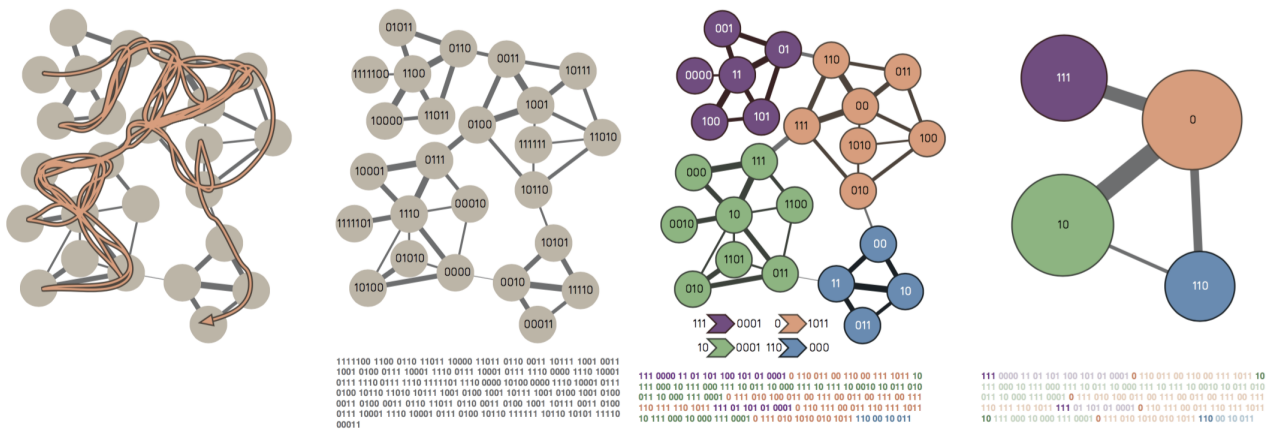
see findcommunities implementation

community *map equation*

— Infomap *map equation compression* [RB08]

1. set *node community* by *optimal coding* $\mathcal{O}(m \log m)$
2. *compress community nodes into supernodes* and repeat 1.
3. return *community structure maximizing map equation*

$$\mathcal{L} = \sum_i p_{i\rightsquigarrow} H(\widetilde{\mathcal{C}}) + \sum_i p_{i\hookleftarrow} H(\mathcal{C}_i)$$



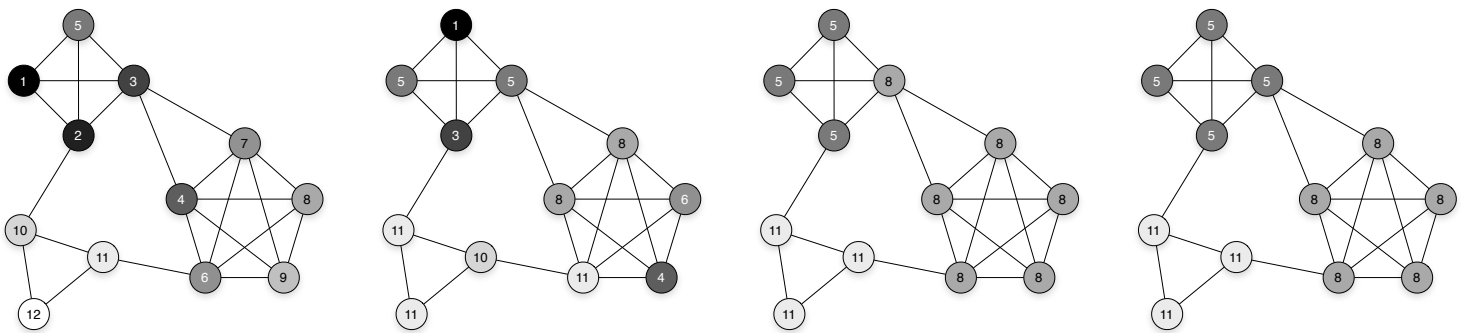
see mapequation implementation

community *propagation*

— Raghavan *label propagation* [RAK07, ŠB11]

1. set *node community* by *neighbors frequency* $\mathcal{O}(cm)$
2. *randomly shuffle nodes* and repeat 1. *until convergence*
3. return *community structure connected components*

$$\forall i : c_i = \arg\max_c \sum_j A_{ij}\delta_{c_j c}$$



see balanced implementation

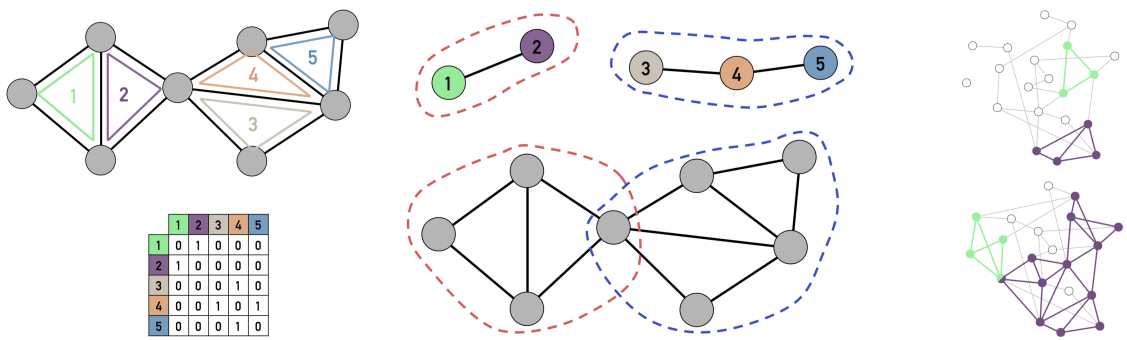
community *percolation*

— Palla *clique percolation* [PDFV05, KKKS08]

1. find *k-node cliques* by *sequential enumeration* $\mathcal{O}(n_k)$
2. *merge clique nodes into supernodes* and *link adjacent*

   adjacent k-node cliques share $k-1$ nodes
3. return *clique structure connected components*

   clique percolation at $(kn-n)^{\frac{1}{1-k}}$



see kclique implementation

community *links*

— Ahn *link clustering* [EL09, ABL10]

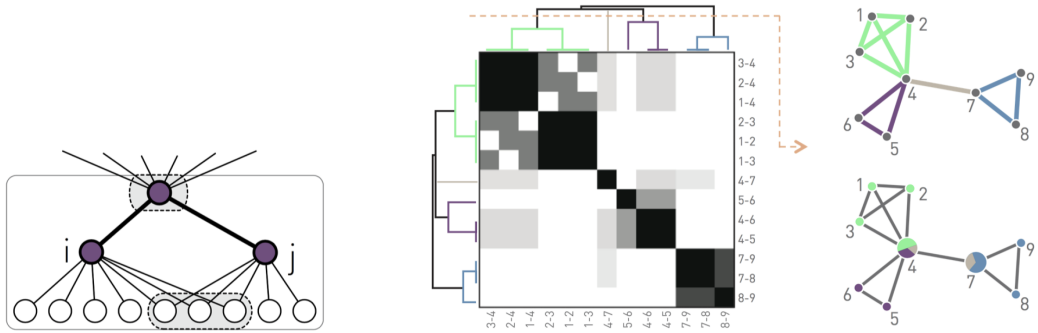
- define *link similarity* as *neighbors index*

$$\forall ij \in \Gamma_x : s_{ij}^x = \frac{|\Gamma_i^+ \cap \Gamma_j^+|}{|\Gamma_i^+ \cup \Gamma_j^+|}$$

- define *cluster similarity* as *single linkage*

$$S_{ij} = \max_{xy \in \Gamma_z} (s_{xy}^z \delta_{c_{xz} c_i} \delta_{c_{yz} c_j})$$

1. bottom-up *agglomerative hierarchical clustering* $\mathcal{O}(m^2)$
2. cut *cluster dendrogram* at desired *clustering resolution*



see linkcomm implementation

community *measures*

— *degree* K, *expansion* E and *Flake* F [FLG00, RCC+04] of $\{C\}$

$$K = \frac{1}{n}\sum_{ij} A_{ij}\delta_{c_i c_j} = \langle k \rangle - E \qquad F = \frac{|\{i{:}\sum_j A_{ij}\delta_{c_i c_j} < k_i/2\}|}{n}$$

— *normalized mutual information* NMI [DDGDA05] of $\{C\}$,$\{D\}$

- p_c & p_{cd} are *standard* & *joint distributions* of $\{C\}$,$\{D\}$
- $H(\mathcal{C})$ & $H(\mathcal{C}|\mathcal{D})$ are *standard* & *conditional entropies*
- MI & VI are *mutual* & *variation* of *information*

$$NMI = \frac{2MI(\mathcal{C},\mathcal{D})}{H(\mathcal{C})+H(\mathcal{D})} = \frac{2H(\mathcal{C})-2H(\mathcal{C}|\mathcal{D})}{H(\mathcal{C})+H(\mathcal{D})} = \frac{2H(\mathcal{C})+2\sum_{CD} p_{cd}\log\frac{p_{cd}}{p_d}}{-\sum_C p_c \log p_c + H(\mathcal{D})}$$

— *normalized variation of information* NVI [Mei07, KLN08]

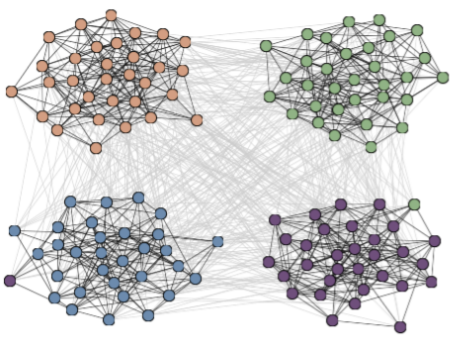
$$NVI = \frac{VI(\mathcal{C},\mathcal{D})}{\log n} = \frac{H(\mathcal{C}|\mathcal{D})+H(\mathcal{D}|\mathcal{C})}{\log n}$$

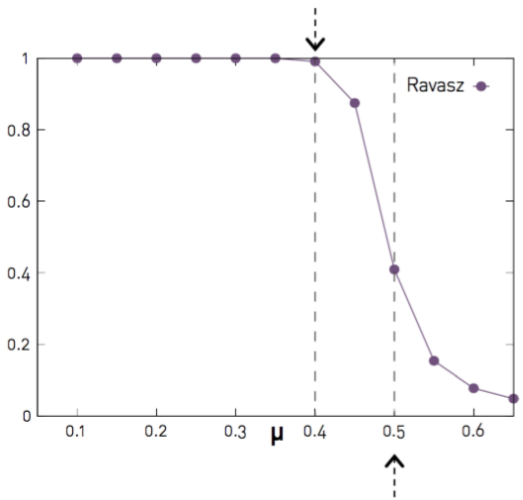
community *benchmarks*

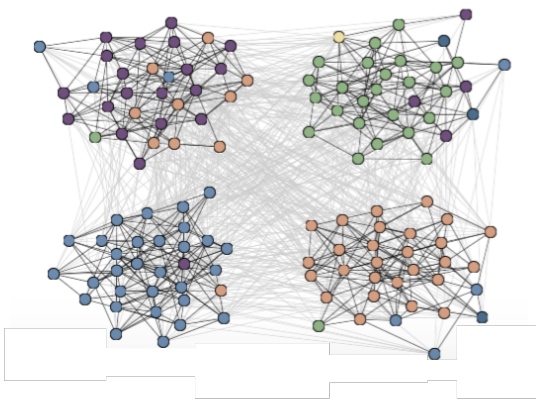
— Girvan-Newman *synthetic graphs* [GN02]

— *planted partition* controlled by *mixing parameter* μ

$n = 128$ $\quad \langle k \rangle = \langle k^{int} \rangle + \langle k^{ext} \rangle = 16$ $\quad \mu = \frac{\langle k^{ext} \rangle}{\langle k \rangle}$







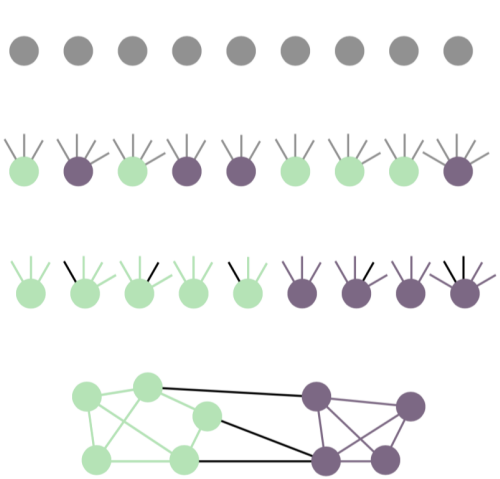
community *benchmarks*

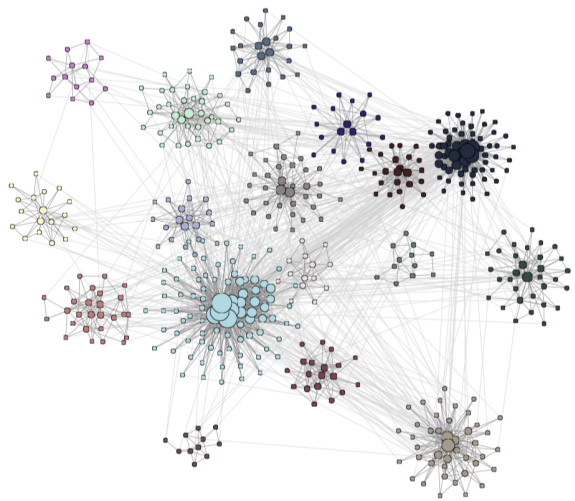
— Lanchichinetti *synthetic graphs* [LFR08]

— *power-law distributions* $p_k \sim k^{-\gamma_k}$ & $p_s \sim s^{-\gamma_s}$

— *planted communities* controlled by *mixing parameter* μ

$n = 1000,\ n_c \in [10, 50]$ $\gamma_k \in [2, 3],\ \gamma_s \in [1, 2]$ $\mu = \frac{\langle k^{ext} \rangle}{\langle k \rangle}$





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Lovro Šubelj and Marko Bajec.

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course *project*

advanced topics in *network science* (*ants*)

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University of Ljubljana
spring 2019/20

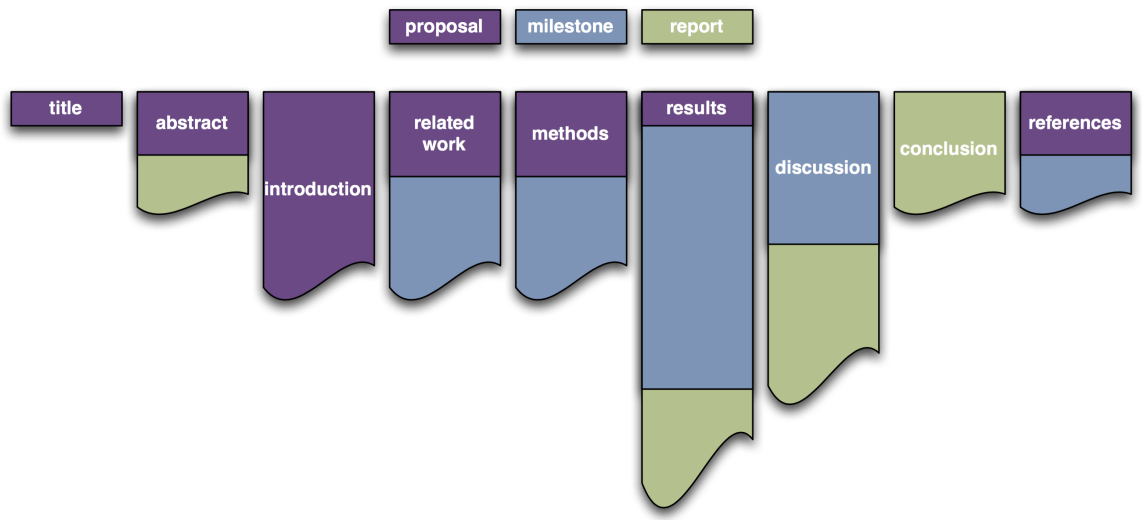
project *overview*

- substantial *network science project*
- networks *combined with PhD research*
- *scientific paper submitted* (to arXiv.org)
- project should go *beyond this course*

- *analytical derivation* of *theoretical results*
- *empirical evaluation* of *methods* or *models*
- *design of novel methods*, *models* or *algorithms*
- *scalable implementation* of existing *algorithms*

project *delivery*

- course *project delivery* breakdown
  - *reaction paper* and *project proposal*
  - *midterm project milestone report*
  - final *eight-page scientific paper*
- informal *project presentations* to get *feedback*
  - *proposal/milestone presentations* in front of *class*
  - *final presentation* in front of *INA students* (*seminar*)



project *proposal*

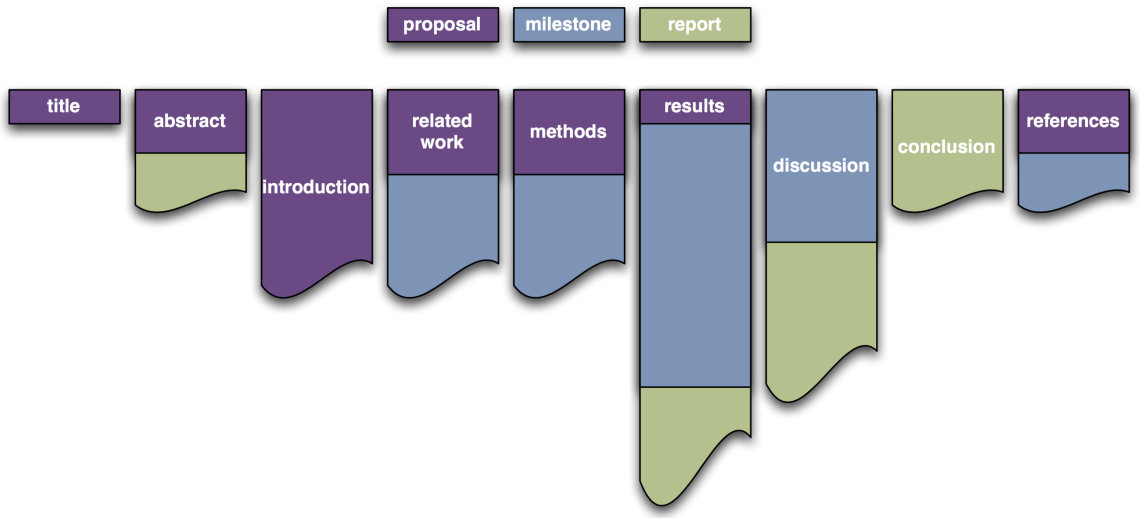
- *cover* component ($\leq$ *paragraph*)
  - project *title* and short *abstract*
- *introductory* component ($\leq$ *page*)
  - project *motivation* and *background*
  - *detailed* project *problem description*
- *reaction paper* component ($>$ *page*)
  - survey of existing *related work*
  - *critical evaluation* of existing work
  - highlight *strengths* and *weaknesses*
- *project proposal* component ($\leq$ *page*)
  - *detailed* project *proposal description*
  - proposed *methods* and expected *results*
  - highlight *value*, *novelty* and *feasibility*

project *milestone*

— *midterm project milestone*

— *at least* 50% of *research done*

— *at least* 50% of final *paper written*



project *deadlines*

- *hard-copy* in *submission box* (grading)
- *electronic version* to eUcilnica (archive)
- *cover sheet* with signed *honor code*

week	lectures	presentations	assignments
. . .	. . .	. . .	. . .
7		*proposal* (Apr 2nd)	
8			*proposal* (Apr 9th)
9			
10			
11	*advanced topics in network science*	*milestone* (Apr 30th)	
12			*milestone* (May 7th)
13			
14			
15	*applications of network science*	*project* (May 28th)	
16			*paper* (Jun 4th)
. . .			

— *assignments* due on *Thursdays at 3:00pm*

— *late days* expire on *Tuesdays at 3:00pm*

— *presentations* on *Thursdays at 6:00pm*