### graph theory o *network science*

advanced topics in *network science* (ants)

Lovro Šubelj & Jure Leskovec University of Ljubljana spring 2019/20

## history graph theory

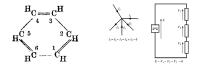
1736 seven *bridges of Königsberg* [Eul36] (Leonhard Euler)

1800s travelling salesman problem (William Hamilton)



1845 electrical circuit laws (Gustav Kirchhoff)

1857 chemical structure theory (August Kekulé)



## history operations research

```
1956 shortest paths (Edsger Dijkstra)
1956 minimum spanning tree (Joseph Kruskal)
1956 maximum flow/minimum cut (Ford & Fulkerson)
1956 signed graph theory [CH56] (Cartwright & Harary)
1959 random graph theory [ER59] (Erdős & Rényi)
```

## history sociometry

1934 children *sociograms* [Mor34] (Jacob Moreno)



1941 Southern women [DGG41] (Allison Davis)

1970 university karate club [Zac77] (Wayne Zachary)





1967 small-world experiment [Mil67] (Stanley Milgram)

1973 strength of weak ties [Gra73] (Mark Granovetter)

1977 measures of *centrality* [Fre77] (Linton Freeman)

## history bibliometrics

1965 scientific paper citations [Pri65] (Derek de Solla Price)



#### SCIENCE CITATION INDEX

1980s political scandals [HL03] (Mark Lombardi)

1986 neural wirings [WSTB86] (White et al.)

1999 transportation [Pel99] (Jon Pelletier)







#### networks boom

- < 2000 small graphs  $10^2$ - $10^3$  nodes
- pprox 2000 communication networks  $10^5\text{-}10^8$  nodes
- pprox 2005 online social networks  $10^8$  nodes

today Bitcoin blockchain 108-109 addresses

today  $Facebook\ graph > 10^9\ users$ 

today  $Web\ graph > 10^{12}\ pages$ 

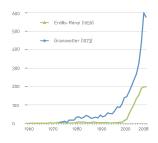


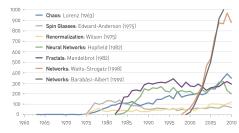
#### network *models*

1959 random graph theory [ER59] 1973 valued graphs theory [Gra73]

1998 *small-world network* structure [WS98]

1999 scale-free network structure [BA99]





## networks language

"A key discovery of network science is that the architecture of networks emerging in various domains of science, nature, and technology are similar to each other, a consequence of being governed by the same organizing principles. Consequently we can use a common set of tools to explore these systems."

Albert-László Barabási

"Networks are ideal structures to describe problems of organized complexity."

César A. Hidalgo

"I think the next century will be the century of complexity."

Stephen Hawking

## networks impact

- management: internal structure of organization
- economic: from web search to social networking
- epidemics: from forecasting to halting deadly viruses
- health: from drug design to metabolic engineering
- security: fraud detection and fighting terrorism
- neuroscience: mapping human brain
- many other: your course project

#### network *science*

# problem understanding real networks

means

study of network properties design of mathematical models implementation of efficient algorithms

goals

network structure and evolution nodes, fragments, clusters, layers, network network dynamics and processes spreading, diffusion, epidemics

## network *analysis*

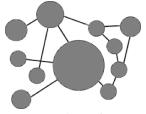




text mining



computer vision



network analysis

#### history references



A.-L. Barabási and R. Albert.

Emergence of scaling in random networks. Science, 286(5439):509–512, 1999.



A.-L. Barabási.

Network Science.

Cambridge University Press, Cambridge, 2016.



Dorwin Cartwright and Frank Harary.

Structural balance: A generalization of Heider's theory.

Psychological Review, 63(5):277-293, 1956.



A. Davis, B. B. Gardner, and M. R. Gardner.

Deep South.

Chicago University Press, Chicago, 1941.



David Easley and Jon Kleinberg.

Networks, Crowds, and Markets: Reasoning About a Highly Connected World.
Cambridge University Press. Cambridge. 2010.



P. Erdős and A. Rénvi.

On random graphs I.

Publ. Math. Debrecen, 6:290-297, 1959.



Leonhard Euler.

Solutio problematis ad geometriam situs pertinentis. Comment. Academiae Sci. I. Petropolitanae, 8:128–140, 1736.



L. Freeman

A set of measures of centrality based on betweenness. *Sociometry*, 40(1):35–41, 1977.

500,0,10(1),50 11, 1511

#### history references



Mark S. Granovetter.

The strength of weak ties.

Am. J. Sociol., 78(6):1360-1380, 1973.



César A. Hidalgo.

Disconnected, fragmented, or united? A trans-disciplinary review of network science. Appl. Netw. Sci., 1:6, 2016.



Robert Hobbs and Mark Lombardi.

Mark Lombardi: Global Networks.

Independent Curators International, New York, 2003.



Stanley Milgram.

The small world problem. Psychol. Today, 1(1):60–67, 1967.



J. L. Moreno.

Who Shall Survive?

Beacon House, Beacon, 1934.



Mark E. J. Newman.

Networks: An Introduction.

Oxford University Press, Oxford, 2010.



Jon D. Pelletier.

Self-organization and scaling relationships of evolving river networks. Journal of Geophysical Research, 104(4):7359–7375, 1999.



D. J. de Solla Price.

Networks of scientific papers.

Science, 149:510-515, 1965.

### history references



D. J. Watts and S. H. Strogatz.

Collective dynamics of 'small-world' networks. *Nature*, 393(6684):440–442, 1998.



J. G. White, E. Southgate, J. N. Thomson, and S. Brenner.

The structure of the nervous system of the nematode Caenorhabditis elegans. *Phil. Trans. R. Soc. Lond. B*, 314(1165):1–340, 1986.



Wayne W. Zachary.

An information flow model for conflict and fission in small groups.

J. Anthropol. Res., 33(4):452-473, 1977.