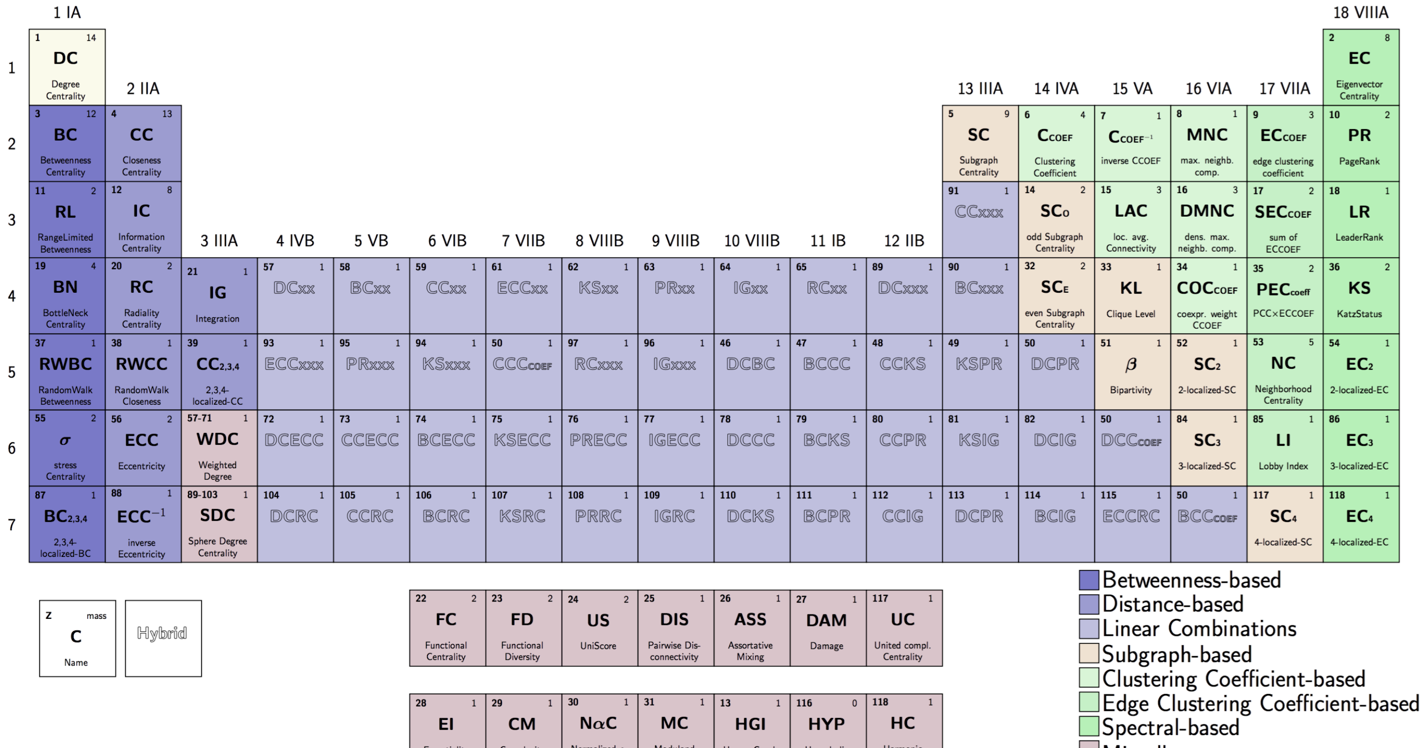
Measures of centrality, PageRank algorithm

You are given [iMDB actors collaboration network]() in Pajek format (edge list and LNA formats are also available). Your task is to find the most important actors according to different measures of centrality.



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I. Degree centrality and clustering coefficients

1. **(code)** Find the most important actors according to degree centrality $d_i = \frac{k_i}{n-1}$, where n is the number of network nodes and k_i is the degree of node i. What kind of actors have the highest d_i (e.g. Hollywood, Bollywood, international, unknown)?

2. **(code)** Find the most important actors according to clustering coefficient $C_i = \frac{2t_i}{k_i(k_i-1)}$, where k_i is the degree of node i and t_i is the number of triads including node i. You should use the link triad counting algorithm from previous labs. What kind of actors have the highest C_i (e.g. Hollywood, Bollywood, international, unknown)?

3. **(answer)** Find the most important actors according to μ-corrected clustering coefficient $C_i^\mu = \frac{2t_i}{k_i \mu}$, where k_i is the degree of node i, t_i is the number of triads including node i and μ is the maximum number of triads over a link. You should use the link triad counting algorithm from previous labs. What

kind of actors have the highest C_i^{μ} (e.g. Hollywood, Bollywood, international, unknown)?

II. Closeness and betweenness centrality

1. **(code)** Find the most important actors according to closeness centrality $\ell_i^{-1} = \frac{1}{n-1}\sum_{j\neq i}\frac{1}{d_{ij}}$, where n is the number of network nodes and d_{ij} is the distance between nodes i and j. You should use the breadth-first search algorithm from previous labs. What kind of actors have the highest ℓ_i^{-1} (e.g. Hollywood, Bollywood, international, unknown)?

2. **(answer)** Find the most important actors according to betweenness centrality $\sigma_i = \frac{1}{n^2}\sum_{st}\frac{g_{st}^i}{g_{st}}$, where n is the number of network nodes, g_{st} is the number of geodesic paths between nodes s and t, and g_{st}^i is the number of such paths through node i. You should ask the course instructor to do these computations for you. What kind of actors have the highest σ_i (e.g. Hollywood, Bollywood, international, unknown)?

III. Eigenvector centrality and PageRank algorithm

1. **(code)** Find the most important actors according to eigenvector centrality $e_i = \lambda_1^{-1}\sum_j A_{ij}e_j$, where A is the network adjacency matrix and λ_1 is a normalizing constant. You should use the power iteration algorithm shown below. What kind of actors have the highest e_i (e.g. Hollywood, Bollywood, international, unknown)?

2. **(code)** Find the most important actors according to PageRank algorithm $p_i = \alpha\sum_j A_{ij}\frac{p_j}{k_j} + \frac{1-\alpha}{n}$, where A is the network adjacency matrix, n is the number of network nodes, k_i is the degree of node i and α is the damping factor set to 0.85. You should use the PageRank algorithm shown below. What kind of actors have the highest p_i (e.g. Hollywood, Bollywood, international, unknown)?

```
input   graph G, precision ε
output  eigenvector centrality E
E ← array of ones
do
    U ← array of zeros
    for nodes i ∈ N do
        for neighbors j ∈ Γ_i do
            U[i] ← U[i] + E[j]
    u ← ||U||
    for nodes i ∈ N do
        U[i] ← U[i] · n/u
    Δ ← ||E − U||
    E ← U
while Δ > ε
return E
```

```
input   graph G, damping α, precision ε
output  PageRank ranks P
P ← array of n^-1-s
do
    U ← array of zeros
    for nodes i ∈ N do
        for predecessors j ∈ Γ_i^in do
            U[i] ← U[i] + P[j] · α/k_j^out
    u ← ||U||
    for nodes i ∈ N do
        U[i] ← U[i] + (1 − u)/n
    Δ ← ||P − U||
    P ← U
while Δ > ε
return P
```