community structure

introduction to *network analysis* (*ina*)

Lovro Šubelj
University of Ljubljana
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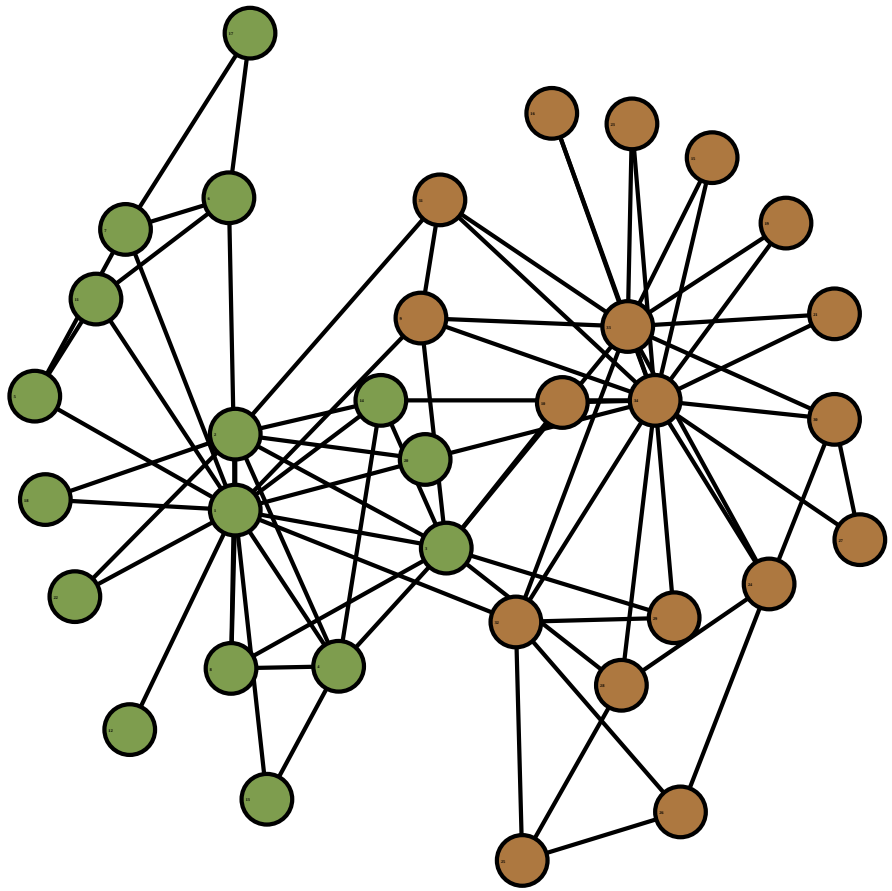
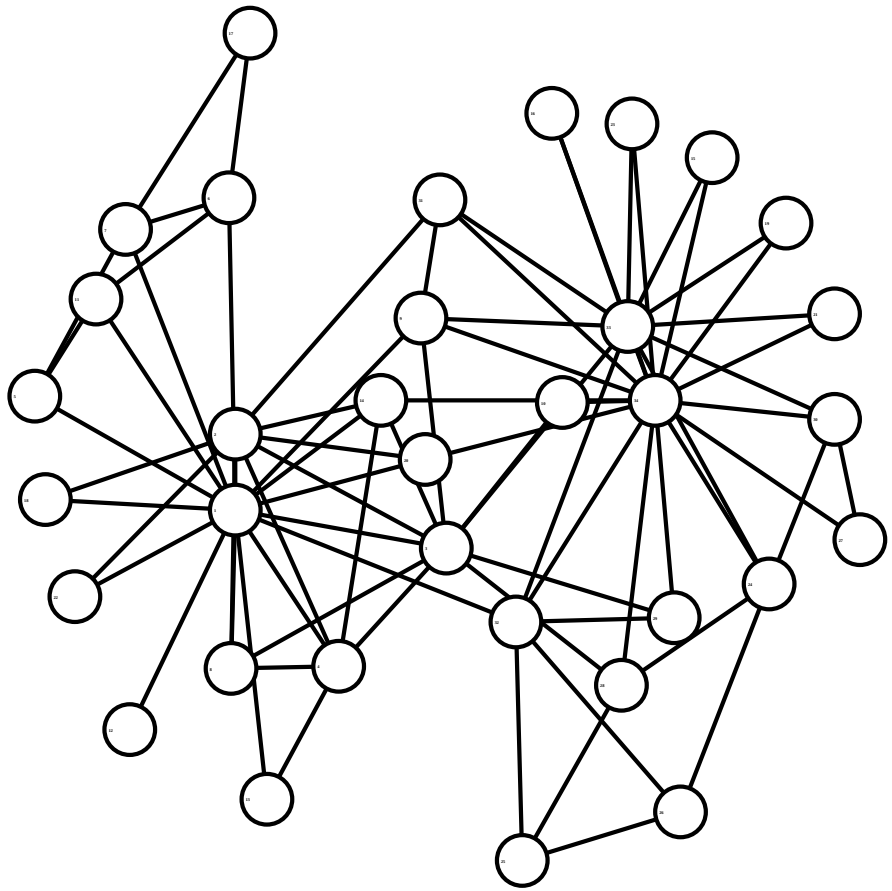
community *structure*

karate club *network split* [Zac77]



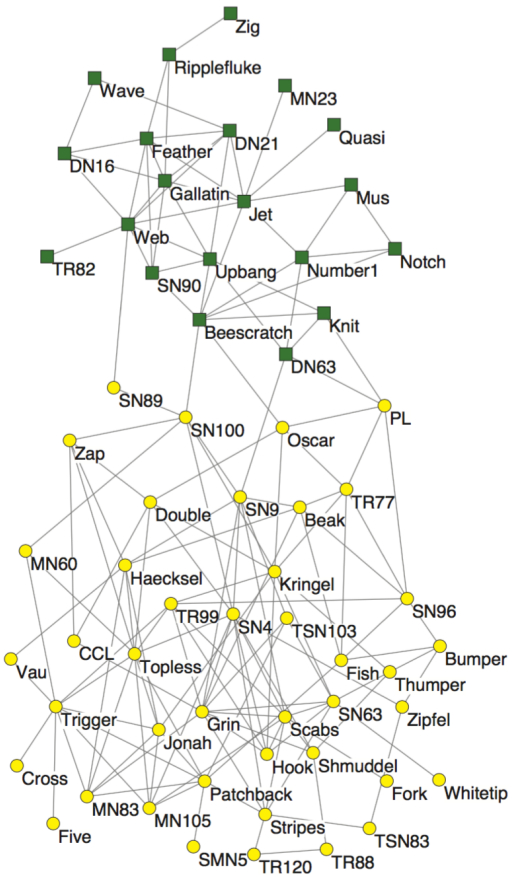
community *detection*

karate club *split detection* [RAK07]

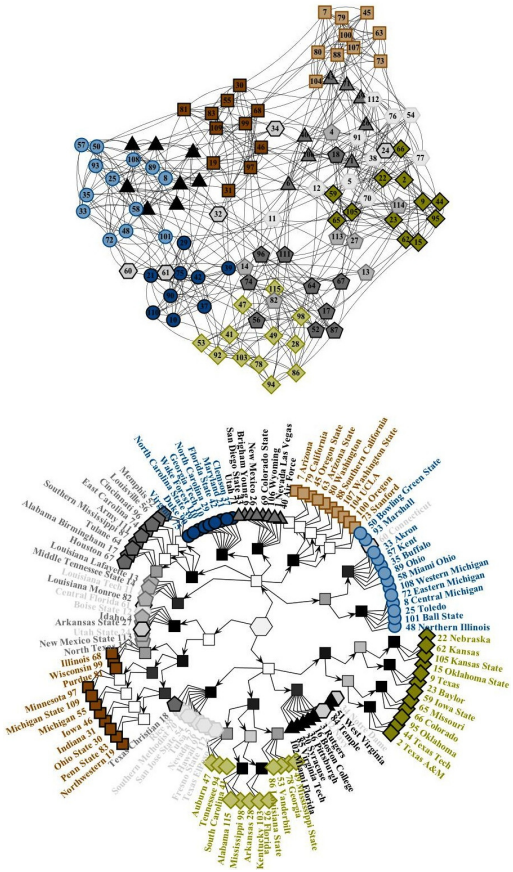


community *examples*

most social networks contain *communities* [GN02]



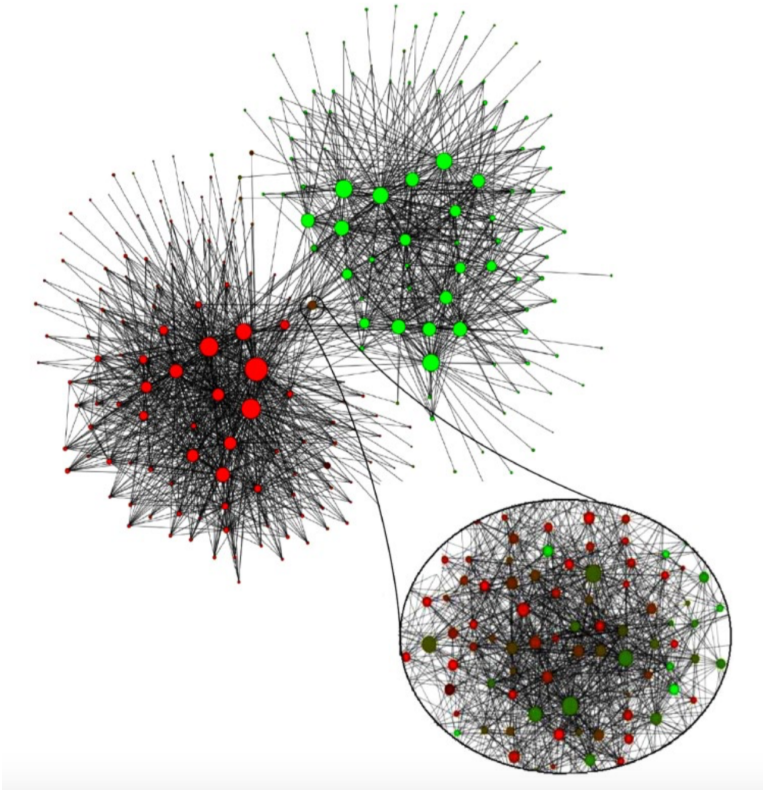
bottlenose dolphins [LSB+03]



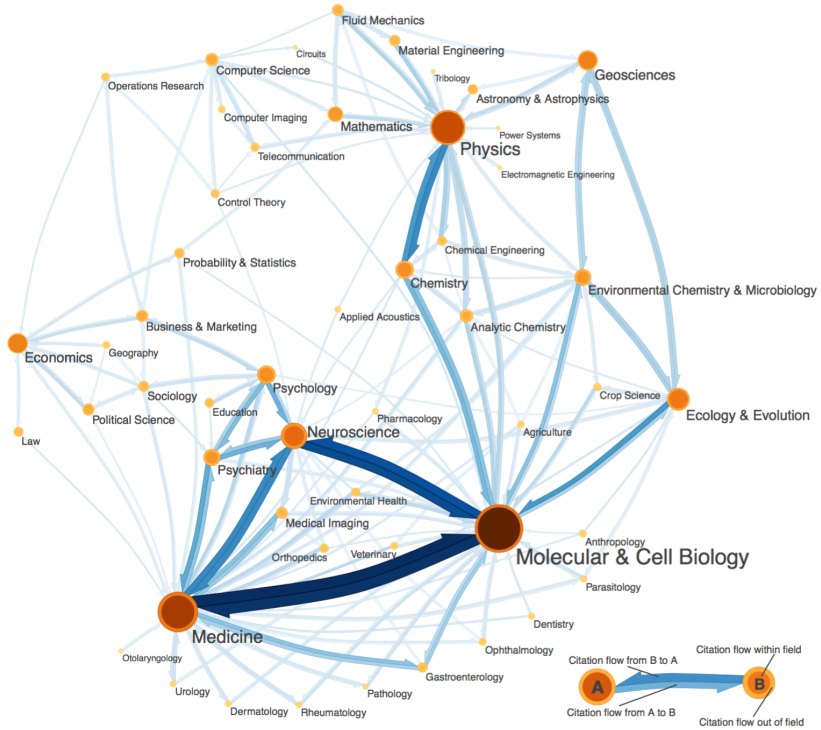
college football [GN02]

community *examples*

many information networks contain *communities* [FLG00]



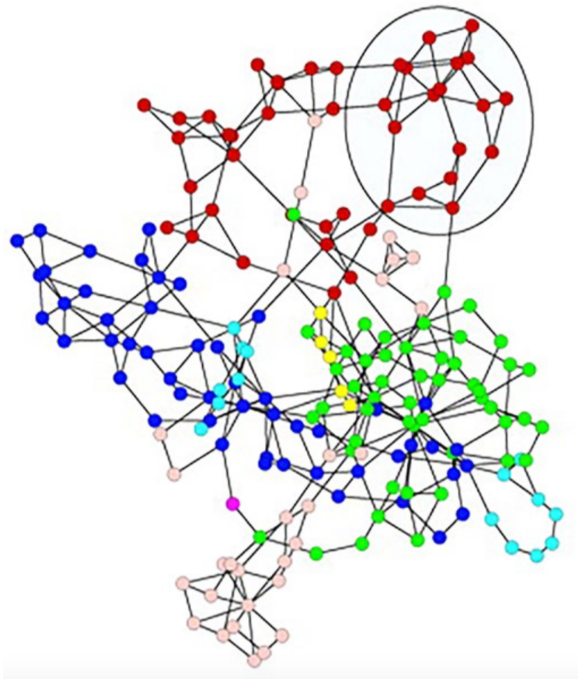
mobile communications [BGLL08]



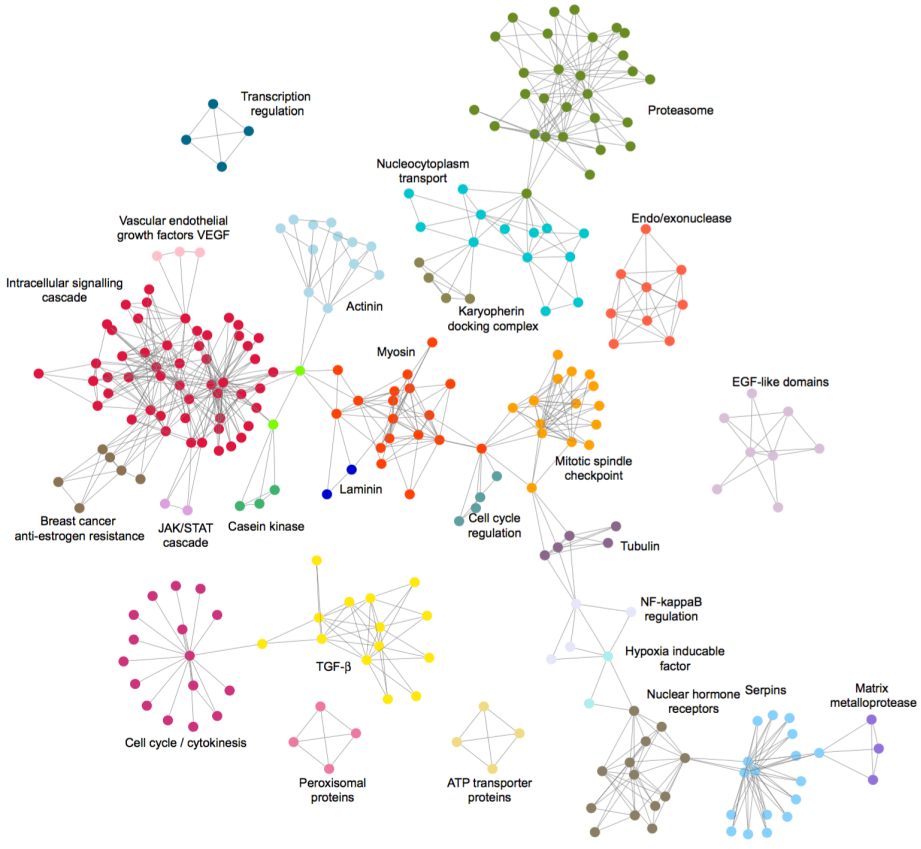
journal citations [RB08]

community *examples*

many biological networks contain *communities* [RSM+02]



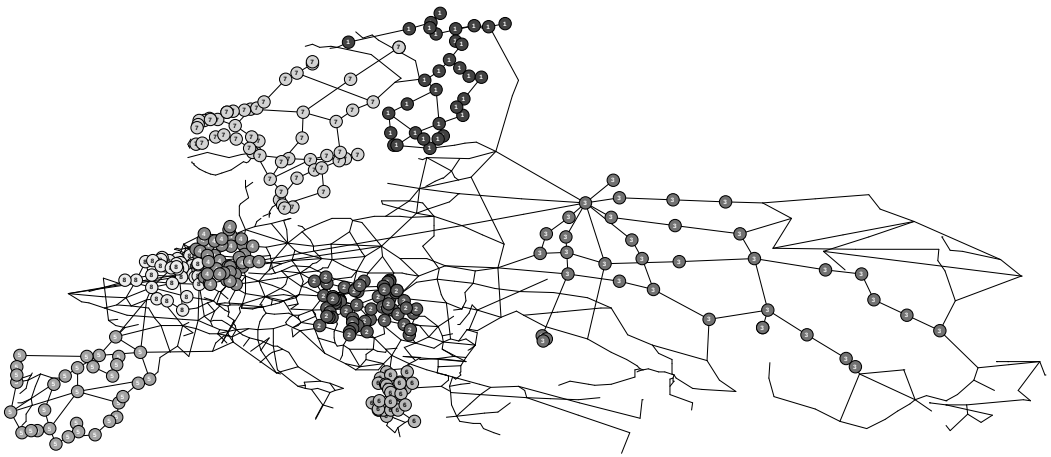
E. coli metabolism [RSM+02]



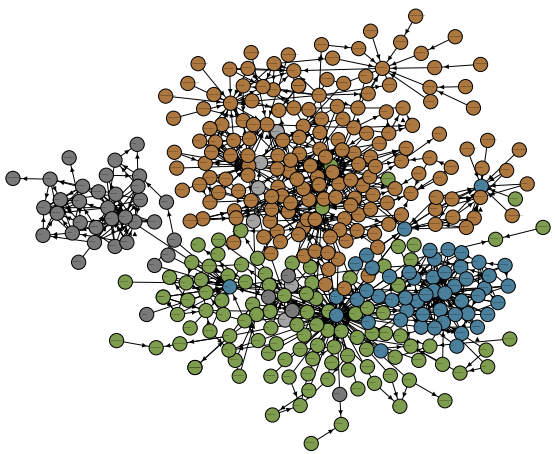
protein interactions [JCZB06]

community *examples*

technological networks *rarely* contain *communities* [ŠB11a]



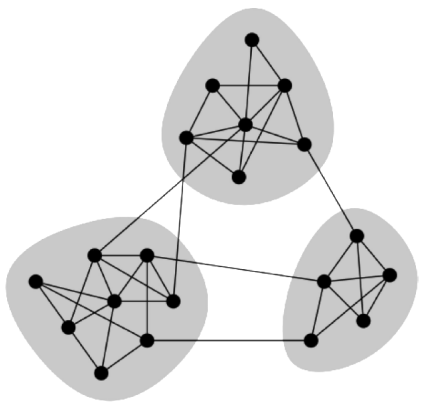
European highways [ŠB11b]



JUNG dependencies [ŠB11a]

community *explanation*

- *weak* & *strong ties* according to *information flow*
- *bridges* & *embedded ties* according to *network span*
  - removal of *local bridge* $\{i, j\}$ causes $d_{ij} > 2$
  - removal of *bridge* $\{i, j\}$ causes $d_{ij} = \infty$
  - *embedded tie* $\{i, j\}$ has $C_{ij} > 0$



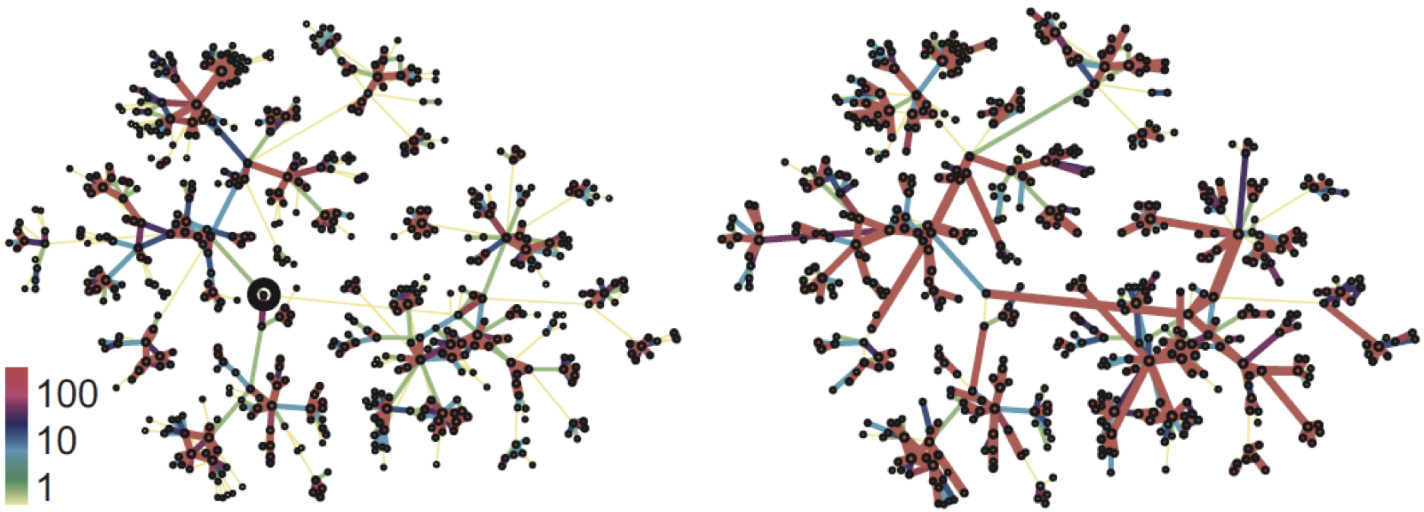
strength of weak ties or weakness of strong ties

- *weak ties are* (*local*) *bridges* under *triadic closure* [Gra73]
- *assortative mixing* and *homophily* in (social) networks [NG03]

community *experiment*

— *tie strength* in mobile communications [OSH+07]

— *weak ties are* (*local*) *bridges* in real networks



real strengths *shuffled strengths*

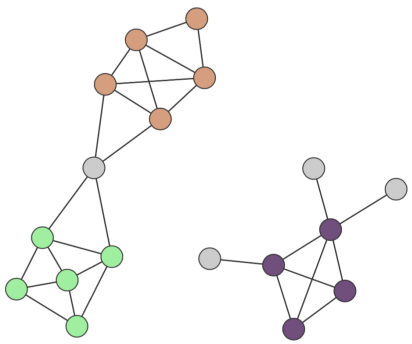
strength of weak ties or weakness of strong ties

community *definition*

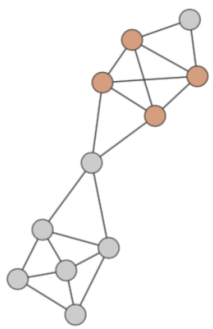
- *clique* is *complete subgraph of* some *graph*
  - also *k-plexes*, *k-cores*, *k-cliques*, *k-clubs*, *k-clans*
- *community* is *dense subgraph of sparse network* [GN02]
- *strong* and *weak community* C [FLG00, RCC+04] defined as
  - k_i^{int} and k_i^{ext} are *internal* and *external degree* of i

$$\forall i \in C : k_i^{int} > k_i^{ext} \qquad \sum_{i \in C} k_i^{int} > \sum_{i \in C} k_i^{ext}$$

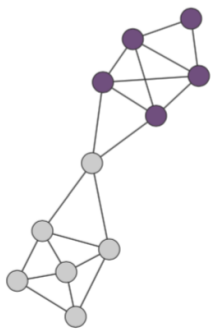
- *community detection* is $\gg$ *graph partitioning* [For10]



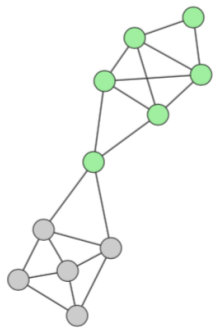
connected communities



maximum clique



strong community



weak community

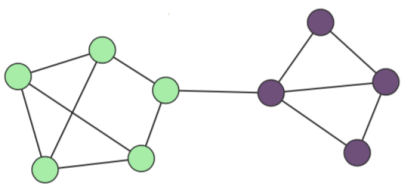
community *modularity*

- *random graphs* should *lack community structure*
- *modularity* Q [GN02] of *communities* $\{C\}$ defined as
  - $k_c = \sum_{i \in C} k_i$ is *total degree* and m_c is *number of links* in C

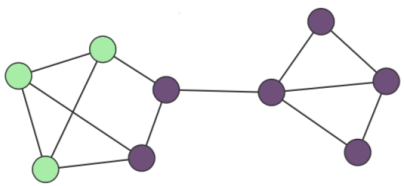
$$\frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta_{c_i c_j} = \frac{1}{2m} \sum_{C} \sum_{ij \in C} \left(A_{ij} - \frac{k_i k_j}{2m} \right) = \sum_{C} \frac{m_c}{m} - \left(\frac{k_c}{2m} \right)^2$$

$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta_{c_i c_j} = \sum_{C} \frac{m_c}{m} - \left(\frac{k_c}{2m} \right)^2$$

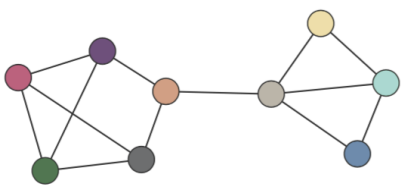
- *modularity* Q popular *quality*/*optimization function* [For10]



optimal $Q = 0.41$



suboptimal $Q = 0.22$



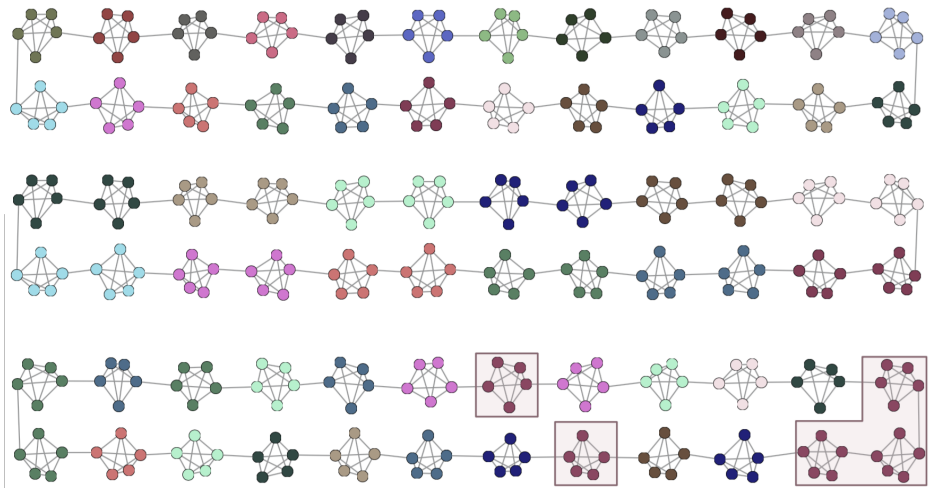
isolates $Q = -0.12$

community ¬*modularity*

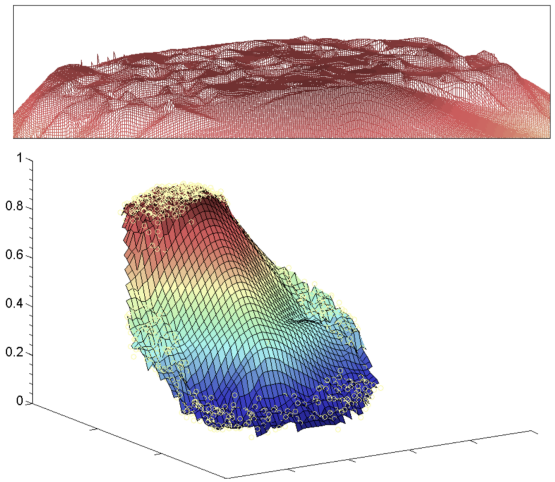
— *modularity* $Q \gg 0$ also in *random graphs* [GSPA04]

— *modularity* Q has *resolution limit* at $k_c \leq \sqrt{2m}$ [FB07]

— *modularity* Q *lacks clear optimum* in real networks [GdMC10]

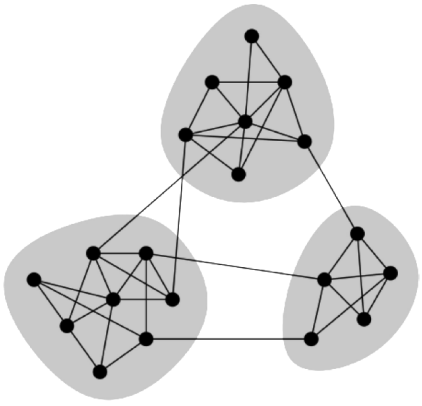


intuitive $Q = 0.867$, *optimal* $Q = 0.871$ and random $Q = 0.8$

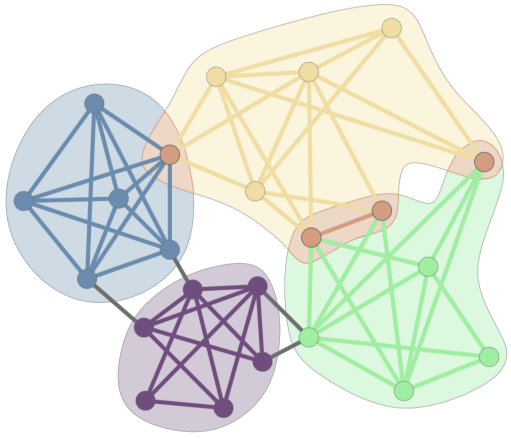


Q *plateau* and maxima

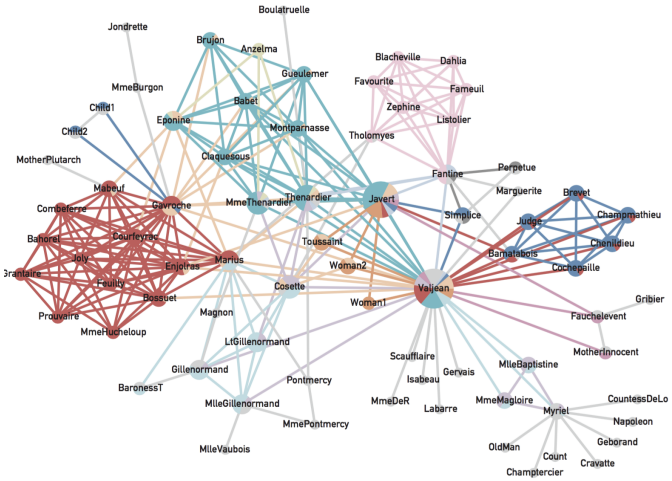
community *overview*



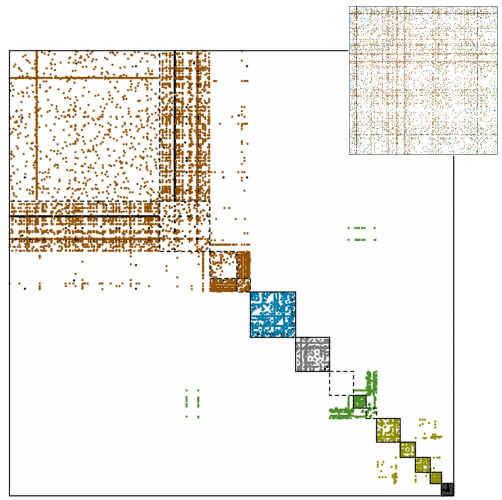
communities [GN02]



overlapping communities [PDFV05]



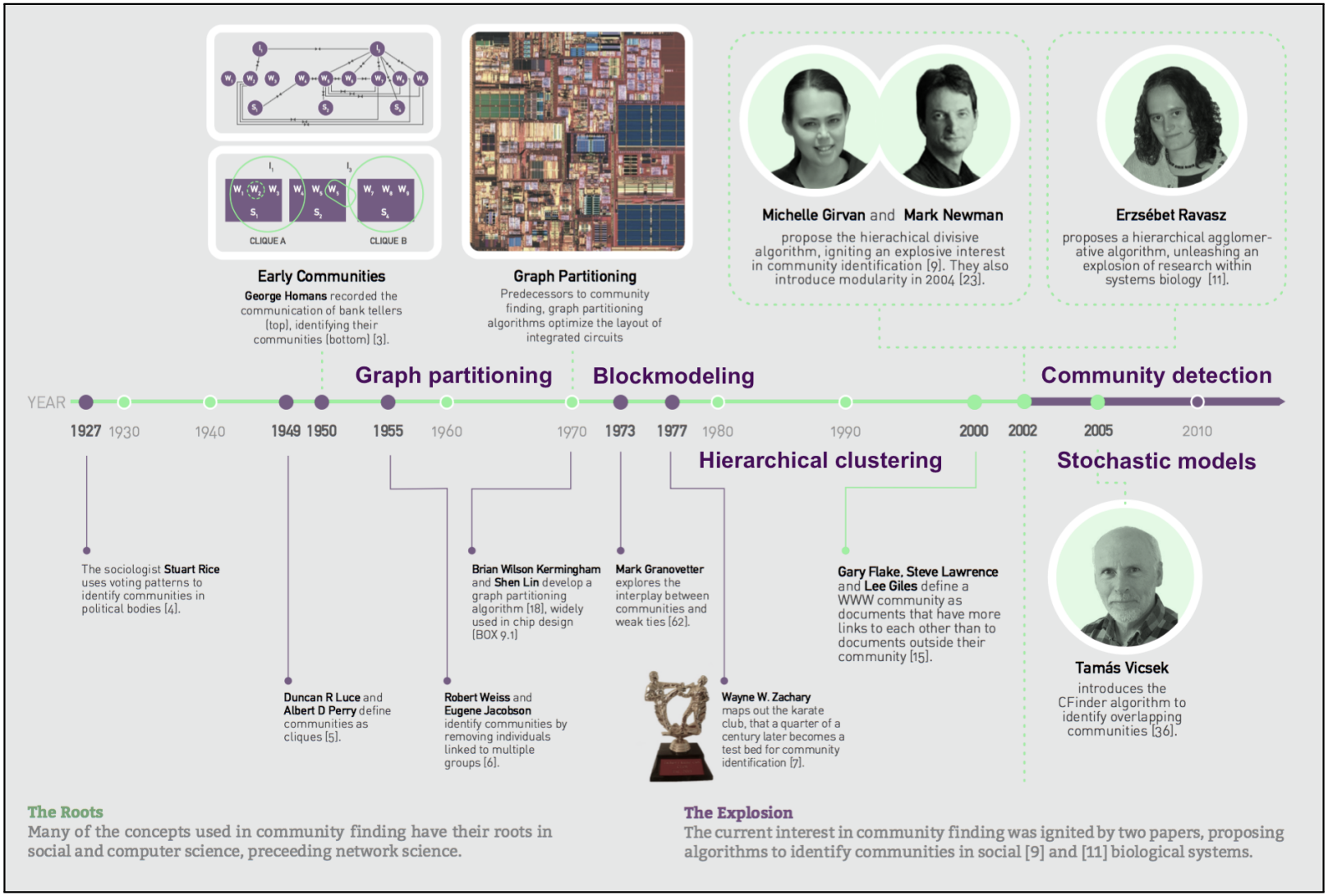
link communities [EL09, ABL10]



block models, *blockmodeling* etc.

`javax.swing`, `javax.management`, `javax.xml`, `javax.print`, `javax.naming`, `javax.lang`

community *history*



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