

link *bridging*

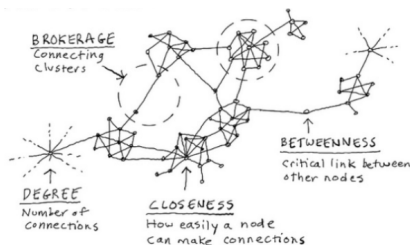
introduction to *network analysis* (*ina*)

Lovro Šubelj
University of Ljubljana
spring 2022/23

bridging *measures*

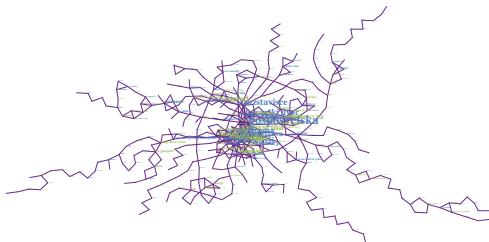
which *links* are most *important*?

- *link bridging measures* for (un)directed networks
 - *betweenness-based* centrality [Fre77, FBW91, New05]
- *link embeddedness measures* for (un)directed networks
 - *topological overlap* measures [RSM⁺02, OSH⁺07, dNMB11]



networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 2.72$ connections
- *giant component* 95.4% nodes (6 components)
- “small-world” with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- “scale-free” with $\gamma = 2.43$ for cutoff $k_{min} = 2$



* reduced to largest connected component of simple undirected graph

bridging *betweenness*

important *links* are *between other nodes*

- for (*un*)*directed* G *link betweenness* σ [Fre77] of $\{i, j\}$ is
 - g_{st} is number of *geodesic paths between* s and t
 - g_{st}^{ij} is number of *such geodesic paths through* $\{i, j\}$

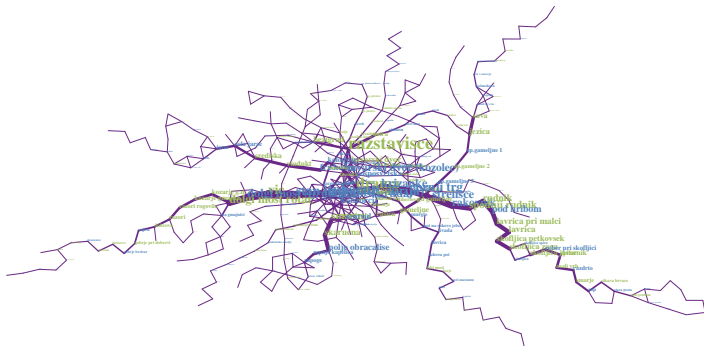
$$\sigma_{ij} = \sum_{st \notin \{i, j\}} \frac{g_{st}^{ij}}{g_{st}}$$

- σ considers *only geodesic paths* [FBW91, New05]



networkology *betweenness*

- *link betweenness* σ in partial LPP network[†]
- *highest* $\sigma_{ij} = 0.176n^2$ link is {*Vič, Stan in dom*}



[†] reduced to largest connected component of simple undirected graph

bridging *bridgeness*

important *links* are *bridges between nodes*

- for (un)directed G *link bridgeness* $\tilde{\sigma}$ [JMK⁺16] of $\{i, j\}$ is
 - g_{st} is number of *geodesic paths between s and t*
 - g_{st}^{ij} is number of *such geodesic paths through $\{i, j\}$*

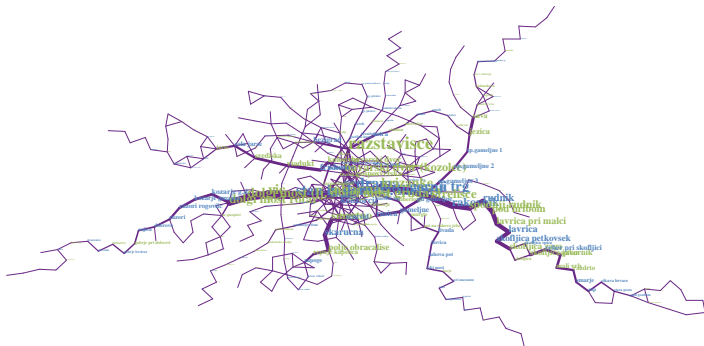
$$\tilde{\sigma}_{ij} = \sigma_{ij} - \sum_{st \in \Gamma_i \cup \Gamma_j} \frac{g_{st}^{ij}}{g_{st}} = \sum_{st \notin \Gamma_i \cup \Gamma_j} \frac{g_{st}^{ij}}{g_{st}}$$

- σ mixes *local centers* with *global bridges* [JMK⁺16]



networkology *bridgeness*

- *link bridgeness* $\tilde{\sigma}$ in partial LPP network[‡]
- *highest* $\tilde{\sigma}_{ij} = 0.169n^2$ link is {*Vič, Stan in dom*}



[‡] reduced to largest connected component of simple undirected graph

bridging *embeddedness*

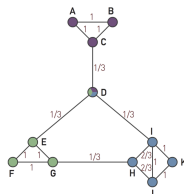
important *links* are *embedded between nodes*

- for *undirected G link embeddedness*[§] θ [OSH⁺07] of $\{i, j\}$ is
 - Γ_i is set of *neighbors* or *neighborhood* of i

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \quad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$

- μ -*corrected link embeddedness* $\tilde{\theta}$ [Bat19] of $\{i, j\}$ is
 - μ is *maximum* number of *triangles* over *links*

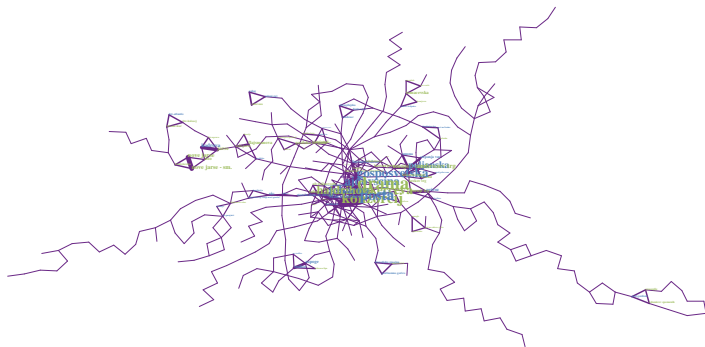
$$\tilde{\theta}_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{\mu + \max(k_i, k_j) - 1 - |\Gamma_i \cap \Gamma_j|}$$



[§] θ & $\tilde{\theta}$ better known as topological overlap indices/weights

networkology μ -embeddedness

- μ -corrected embeddedness $\tilde{\theta}$ in partial LPP network^{||}
- highest $\tilde{\theta}_{ij} = 0.4$ links are $\{Pošta, Konzorcij\}$ etc.



^{||} reduced to largest connected component of simple undirected graph

bridging *overview*

which *links* are most *important*?

1 IA												18 VIIIA												
1	DC																						2	EC
	Degree Centrality																							Eigenvector Centrality
2	BC	CC											13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA						3	PR
	Betweenness Centrality	Closeness Centrality											SC	C _{COEF}	C _{COEF} ⁻¹	MNC	EC _{COEF}							PageRank
3	RL	IC											CC _{COEF}	SC ₀	LAC	DMNC	SEC _{COEF}						4	LR
	Range-Linked Betweenness	Information Centrality											old Subgraph Centrality	old Subgraph Centrality	loc. avg. Connectivity	dens. max. length comp.	sum of EC _{COEF}							LeaderRank
4	BN	RC	IG	DC _{0x}	BC _{0x}	CC _{0x}	EC _{0x}	KS _{0x}	PR _{0x}	IG _{0x}	RC _{0x}	DC _{0x}	BC _{0x}	SC ₁	KL	COC _{COEF}	PEC _{COEF}					5	KS	
	Betweenness Centrality	Radiality Centrality	Integration											new Subgraph Centrality	Clique Level	cooper. weight CO _{COEF}	PCC + EC _{COEF}						KatzRank	
5	RWBC	RWCC	CC _{2,3,4}	EC _{0x0x}	PR _{0x0x}	KS _{0x0x}	CC _{0x0x}	RC _{0x0x}	IG _{0x0x}	DC _{0x0x}	BC _{0x0x}	CC _{0x0x}	KS _{0x0x}	SC ₂	β	SC ₂	NC					6	EC ₃	
	RandomWalk Betweenness	RandomWalk Closeness	2,3,4-localized CC											Bipartivity	2-localized SC	Neighborhood Centrality	2-localized EC							
6	σ	ECC	WDC	DC _{0x0x}	CC _{0x0x}	BC _{0x0x}	KS _{0x0x}	PR _{0x0x}	IG _{0x0x}	DC _{0x0x}	BC _{0x0x}	CC _{0x0x}	KS _{0x0x}	SC ₃	LI	LI	EC ₃					7	EC ₄	
	sigma Centrality	Eccentricity	Weighted Degree											3-localized SC	Lobby Index	3-localized EC								
7	BC _{2,3,4}	ECC ⁻¹	SDC	DC _{0x0x}	CC _{0x0x}	BC _{0x0x}	KS _{0x0x}	PR _{0x0x}	IG _{0x0x}	DC _{0x0x}	BC _{0x0x}	CC _{0x0x}	KS _{0x0x}	SC ₄	EC ₄									
	2,3,4-localized-BC	Inverse Eccentricity	Sphere Degree Centrality											4-localized SC	4-localized EC									

Z

mass

C

Name

Hybrid

22	FC	23	FD	24	US	25	DIS	26	ASS	27	DAM	117	UC
	Functional Centrality		Functional Closeness		UniScore		Pairwise Disconnectedness		Assortative Mixing		Damage		United coml. Centrality

28	EI	29	CM	30	N α C	31	MC	13	HGI	116	HYP	118	HC
	Essentiality Index		Complexity Measure		Normalized α Centrality		Modular Centrality		Hungry Graph Information		Hyperbolic Index		Harmonic Centrality

Betweenness-based

Distance-based

Linear Combinations

Subgraph-based

Clustering Coefficient-based

Edge Clustering Coefficient-based

Spectral-based

Miscellaneous

©David Schoch (University of Konstanz)

bridging *references*



A.-L. Barabási.

Network Science.

Cambridge University Press, Cambridge, 2016.



Vladimir Batagelj.

Corrected overlap weight and clustering coefficient.

e-print *arXiv:190604581v1*, 2019.



Wouter de Nooy, Andrej Mrvar, and Vladimir Batagelj.

Exploratory Social Network Analysis with Pajek: Expanded and Revised Second Edition.

Cambridge University Press, Cambridge, 2011.



David Easley and Jon Kleinberg.

Networks, Crowds, and Markets: Reasoning About a Highly Connected World.

Cambridge University Press, Cambridge, 2010.



Ernesto Estrada and Philip A. Knight.

A First Course in Network Theory.

Oxford University Press, 2015.



Linton C. Freeman, Stephen P. Borgatti, and Douglas R. White.

Centrality in valued graphs: A measure of betweenness based on network flow.

Soc. Networks, 13(2):141–154, 1991.



L. Freeman.

A set of measures of centrality based on betweenness.

Sociometry, 40(1):35–41, 1977.

bridging *references*



Pablo Jensen, Matteo Morini, Marton Karsai, Tommaso Venturini, Alessandro Vespignani, Mathieu Jacomy, Jean-Philippe Cointet, Pierre Merckle, and Eric Fleury.
Detecting global bridges in networks.
J. Complex Netw., 4(3):319–329, 2016.



M. E. J. Newman.
A measure of betweenness centrality based on random walks.
Soc. Networks, 27(1):39–54, 2005.



Mark E. J. Newman.
Networks.
Oxford University Press, Oxford, 2nd edition, 2018.



J.-P. Onnela, J. Saramäki, J. Hyvönen, G. Szabó, D. Lazer, K. Kaski, J. Kertész, and A.-L. Barabási.
Structure and tie strengths in mobile communication networks.
P. Natl. Acad. Sci. USA, 104(18):7332–7336, 2007.



E. Ravasz, A. L. Somera, D. A. Mongru, Z. N. Oltvai, and Albert-László Barabási.
Hierarchical organization of modularity in metabolic networks.
Science, 297(5586):1551–1555, 2002.