Modeling citation network topology: A natural graph model and applications to citation theory

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Abstract

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Keywords: graph model, citation networks, citation theory, infometrics, bibliometrics, scientometrics.

1. Introduction

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2. Related work

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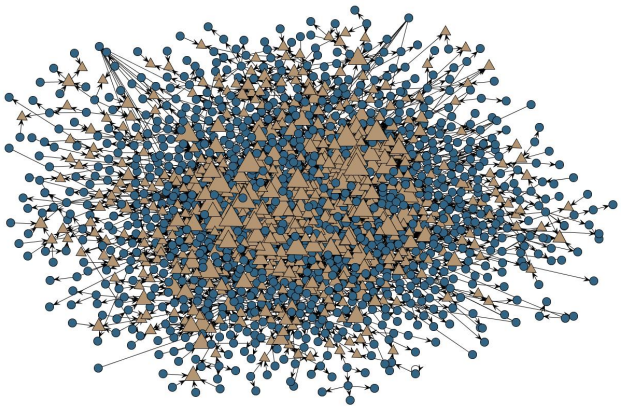

Figure 1: Largest connected component of citation network of WoS category *Information Science & Library Science* reduced to papers published before 1980 (see also text). Papers with more than one citation are shown as triangles, while node sizes are proportional to in-degrees. (Network layout was generated with Large Graph Layout [1].)

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3. Modeling citation networks

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3.1. Study of citation dynamics

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3.2. Graph model of citation networks

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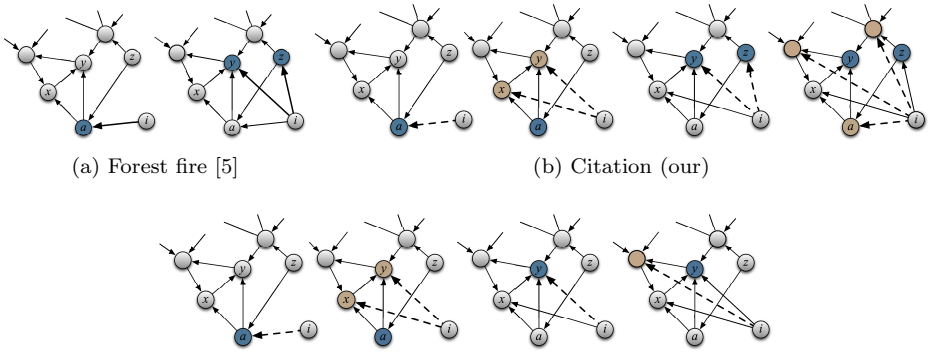

(a) Forest fire [5] (b) Citation (our)

(c) Random walk [9]

Figure 2: Linking dynamics of random graph models of citation networks. (a) In Forest fire model [5], newly added node i chooses an ambassador a uniformly at random and links to it (solid lines). Next, some of its neighbors are taken as the new ambassadors by following the in-links and out-links (e.g., y and z). (b) In the proposed Citation model, i links to a only with a certain probability (dashed lines), while also to some of its neighbors by following the out-links (e.g., x and y). (c) In Random walk model [9, 6], i traverses the network by following the out-links, taking a single new ambassador on each step (e.g., y).

Initially, the graph consists of a single link, while the model repeats the following procedure $n-2$ times.

(1) Newly added node i chooses an ambassador node a uniformly at random (we say that i burns a).
(2) i links to a with probability q_a and randomly selects $x_l \sim G(q_l)$ neighbors of a that were not yet linked by following the out-links, and links to them.
(3) i randomly selects $x_f \sim G(p_f)$ neighbors of a that were not yet burned $a_1, \ldots, a_{x_f}$ by following the out-links and $x_b \sim G(p_b)$ neighbors that were not yet burned $a_{x_f+1}, \ldots, a_{x_f+x_b}$ by following the in-links.
(4) $a_1, \ldots, a_{x_f+x_b}$ are taken as the new ambassadors of i (step (2)).

When i cannot select enough neighbors of a, the remaining are selected uniformly at random from all nodes.

$$\langle s \rangle = \sum_{x=0}^{\infty} \left(\frac{p_f}{1-p_f} + \frac{p_b}{1-p_b} \right)^x = \left(1 - \frac{p_f}{1-p_f} - \frac{p_b}{1-p_b} \right)^{-1} \tag{1}$$

for $\frac{p_f}{1-p_f} + \frac{p_b}{1-p_b} < 1$. Hence,

$$m = n \langle s \rangle \left(q_a + \frac{q_l}{1-q_l} \right) \tag{2}$$

for $q_l < 1$. Thus,

$$q_a = \frac{m}{n \langle s \rangle} - \frac{q_l}{1-q_l} \tag{3}$$

and

$$q_l = \left(1 + \left(\frac{m}{n \langle s \rangle} - q_a \right)^{-1} \right)^{-1}. \tag{4}$$

Also,

$$n_+ = \frac{nq - nq\left(p - pq\right)^{\langle s \rangle}}{1 - p\left(1 - q\right)} \approx \frac{nq}{1 - p\left(1 - q\right)} \tag{5}$$

for $p\left(1 - q\right) < 1$, where $p = p_f + p_b - p_f p_b$ and $q = q_a + q_l - q_a q_l$. Thus,

$$q_l \approx 1 - \frac{2n_+ \left\langle s \right\rangle \left(1 - p\right)}{2m - n_+ \left\langle k \right\rangle p}. \tag{6}$$

3.3. Alternative citation models

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3.3.1. Poisson citation distribution

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Hence,

$$m = n \left\langle s \right\rangle \left(q_a + q_l\right). \tag{7}$$

Thus,

$$q_a = \frac{m}{n \left\langle s \right\rangle} - q_l \tag{8}$$

and

$$q_l = \frac{m}{n \left\langle s \right\rangle} - q_a. \tag{9}$$

Also,

$$n_+ = \frac{n\left(1 - q\right)\left(1 - \left(pq\right)^{\langle s \rangle}\right)}{1 - pq} \approx \frac{n\left(1 - q\right)}{1 - pq} \tag{10}$$

for $pq < 1$, where $p = p_f + p_b - p_f p_b$ and $q = \left(1 - q_a\right) e^{-q_l}$. Thus,

$$q_l \approx \frac{m}{n \left\langle s \right\rangle} - W\left(\exp\left(\frac{m}{n \left\langle s \right\rangle}\right)\left(n - n_+ p\right)^{-1}\right) - 1, \tag{11}$$

where W is the Lambert function.

3.3.2. Preferential attachment citation

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3.3.3. Random walk citation traversal

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4. Citation data and methodology

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4.1. Web of Science data

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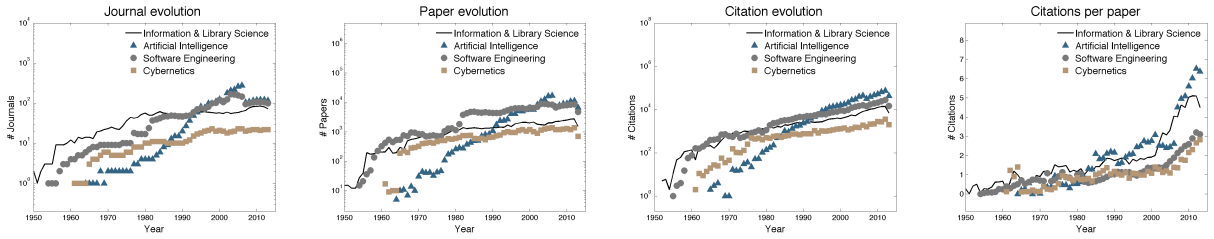
Table 1: Basic statistics of *Web of Science* citation data analyzed in the study.

Data	WoS category	Period	# Journals	# Papers	# Citations
ILS	*Information Science & Library Science*[1]	1945–2013	177	71253	176240
AI	*Computer Science, Artificial Intelligence*	1964–2013	877	179510	639126
SE	*Computer Science, Software Engineering*	1954–2013	560	215745	323444
CY	*Computer Science, Cybernetics*	1961–2013	66	38097	45755



(a) (b) (c) (d)

Figure 3: Journal, paper and citation evolution of *Web of Science* data analyzed in the study (mind semilogarithmic scales).

4.2. Statistics of citation networks

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$$c_i = \frac{t_i}{\binom{k_i}{2}} \text{ and } d_i = \frac{t_i}{\omega_i}. \tag{12}$$

α is the scale-free exponent of the power-law degree distribution $P(k) \sim k^{-\alpha}$ (similarly for α_{in} and α_{out}). $r_{(\alpha,\beta)}$ is the Pearson correlation coefficient of α-degrees and β-degrees at links' source and target nodes, respectively, $\alpha, \beta \in \{in, out\}$.

$$\mathcal{F} = \exp\left(-C\left(\frac{|m-m^*|}{m} + \frac{|\langle c\rangle - \langle c\rangle^*|}{\langle c\rangle} + \frac{|\langle d\rangle - \langle d\rangle^*|}{\langle d\rangle} + \frac{|\alpha-\alpha^*|}{\alpha} + \frac{|\alpha_{in}-\alpha^*_{in}|}{\alpha_{in}} + \frac{|\alpha_{out}-\alpha^*_{out}|}{\alpha_{out}} + \right.\right. \tag{13}$$
$$\left.\left. + |r_{(in,in)} - r^*_{(in,in)}| + |r_{(in,out)} - r^*_{(in,out)}| + |r_{(out,in)} - r^*_{(out,in)}| + |r_{(out,out)} - r^*_{(out,out)}|\right)\right),$$

where C is set to 0.4.

[1] Reduced to papers within WoS category *Computer Science* or its subcategories.

4.3. Parameters of citation models

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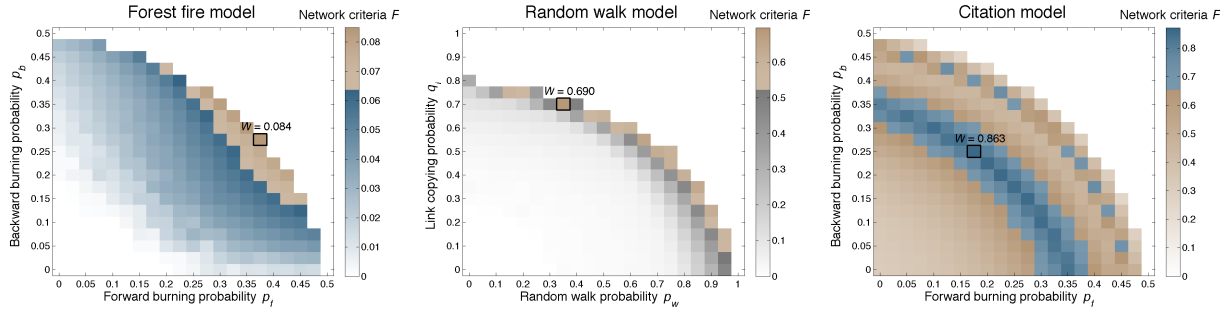

(a) (b) (c)

Figure 4: Random graph parameter estimation for citation network of WoS category *Information Science & Library Science* (note different color scales). (Results are estimates of the mean over 25 graph realizations with the global maximum highlighted.)

5. Results and applications

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5.1. Validation of citation model

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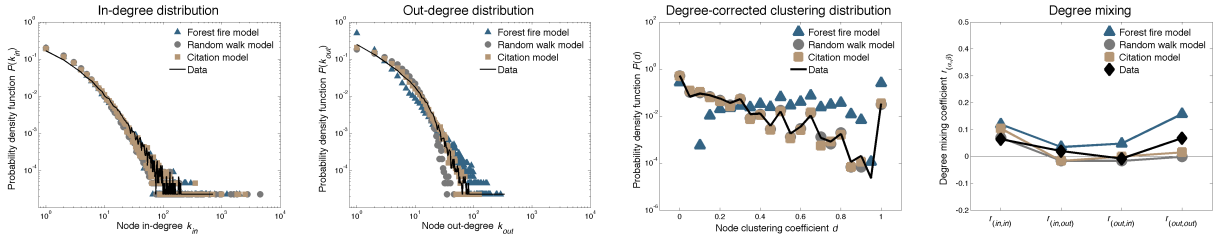

(a) (b) (c) (d)

Figure 5: Degree and clustering distributions and degree mixing in citation network of WoS category *Information Science & Library Science* and graphs generated with random models (see also Figs. B.9 and B.10).

Table 2: Degree and clustering distributions and degree mixing in citation network of WoS category *Information Science & Library Science* and graphs generated with random models (see also Tables B.8 and B.9). (Results are estimates of the mean over 25 graph realizations.)

Model	$\langle k \rangle$	$\langle c \rangle$	$\langle d \rangle$	α_{in}	α_{out}	$r_{(in,in)}$	$r_{(in,out)}$	$r_{(out,in)}$	$r_{(out,out)}$	$\mathcal{F}$
Data	8.05	0.11	0.14	2.47	3.35	0.06	0.02	−0.01	0.07	*1.000*
Citation (our)	7.97	0.11	0.13	2.58	3.31	0.12	−0.01	0.01	0.01	*0.863*
Random walk [9]	7.91	0.11	0.13	2.61	4.68	0.07	−0.01	−0.02	0.00	*0.695*
Random-citing [7]	7.48	0.21	0.26	2.36	3.26	0.16	−0.02	0.08	0.15	*0.400*
Forest fire [5]	7.88	0.40	0.52	2.51	2.45	0.13	0.04	0.06	0.17	*0.087*
Poisson citation	8.03	0.11	0.14	2.63	2.17	0.12	−0.02	0.00	0.01	*0.749*
Yule dynamics	8.06	0.11	0.14	2.14	2.16	0.12	−0.04	0.01	0.03	*0.697*
Yule burning	8.02	0.11	0.13	2.53	3.28	0.10	−0.01	0.01	0.02	*0.882*
Yule citing	8.01	0.11	0.14	2.59	2.17	0.10	−0.02	−0.01	0.02	*0.759*

5.2. Assessment of citation dynamics

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Recall that $\langle s \rangle$ in Eq. (1) is the number of papers actually read by an author, while q_a is the probability of citing such paper. The total number of cited papers is $\langle k \rangle /2$, while $\langle s \rangle q_l/(1-q_l)$ is the number of copied citations. The ratio between the papers discovered by citations and services is $p_f/(1-p_f) : p_b/(1-p_b)$.

Table 3: Paper citation, study and discovery dynamics of WoS category *Information Science & Library Science* as predicted by random graph models (see also Table C.10). (Results are estimates of the mean over 25 graph realizations.)

	Paper citation		Paper study		Paper discovery by		
Model	# Cite	% Copy	# Read	% Cite	% Citation	% Service	% Other
Citation (our)	3.98	86.1%	2.14	27.9%	29.2%	41.0%	29.8%
Random walk [9]	3.95	90.4%	1.54	29.2%	83.7%	-	16.3%
Random-citing [7]	3.73	65.4%	1.29	100.0%	-	-	-
Forest fire [5]	3.95	-	3.95	100.0%	62.3%	37.7%	-

5.3. Applications to citation theory

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Table 4: Paper citation, study and discovery dynamics of WoS categories *Information Science & Library Science* (ILS), *Computer Science, Artificial Intelligence* (AI), *Computer Science, Software Engineering* (SE) and *Computer Science, Cybernetics* (CY) as predicted by citation model (see also Table C.10). (Results are estimates of the mean over 25 graph realizations.)

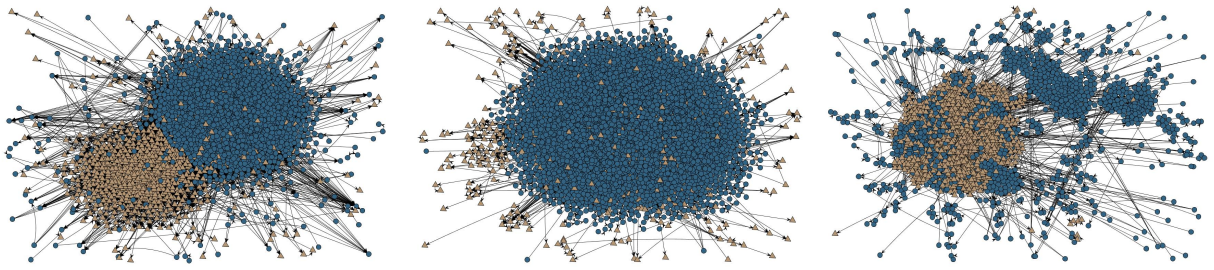
	Paper citation		Paper study		Paper discovery by		
Data	# Cite	% Copy	# Read	% Cite	% Citation	% Service	% Other
ILS	3.98	86.1%	2.14	27.9%	29.2%	41.0%	29.8%
AI	4.52	87.3%	1.47	40.9%	25.8%	47.6%	26.6%
SE	2.78	81.5%	1.58	36.4%	68.8%	2.0%	29.2%
CY	2.18	69.6%	1.59	43.2%	24.5%	37.8%	37.6%

Table 5: Paper citation, study and discovery evolution of WoS category *Information Science & Library Science* as predicted by citation model (see also Table D.11). (Results are estimates of the mean over 25 graph realizations.)

	Paper citation		Paper study		Paper discovery by		
Period	# Cite	% Copy	# Read	% Cite	% Citation	% Service	% Other
1945–2013	3.98	86.1%	2.14	27.9%	29.2%	41.0%	29.8%
1970–1980	2.23	52.1%	3.39	33.5%	41.4%	0.0%	58.5%
1980–1990	2.62	65.1%	2.96	33.0%	48.3%	1.1%	50.6%
1990–2000	3.42	81.6%	2.38	29.0%	40.3%	23.2%	36.5%
2000–2010	5.06	83.6%	2.90	32.2%	40.7%	27.5%	31.7%

Table 6: Paper citation, study and discovery evolution of WoS journals *Scientometrics* (SCI), *Expert Systems with Applications* (ESWA), *Communications of the ACM* (CACM) and *IEEE Transactions on Systems, Man, and Cybernetics* (TSMC) as predicted by citation model (see also text). (Results are estimates of the mean over 25 graph realizations.)

	Paper citation		Paper study		Paper discovery by		
Journal	# Cite	% Copy	# Read	% Cite	% Citation	% Service	% Other
SCI	3.83	87.6%	1.40	38.5%	54.3%	23.4%	22.3%
ESWA	2.12	81.0%	1.67	25.9%	34.7%	27.8%	37.6%
CACM	1.32	46.5%	1.70	42.0%	8.7%	38.5%	52.8%
TSMC	1.47	37.1%	1.66	56.8%	13.6%	39.9%	46.4%



(a) (b) (c)

Figure 6: Largest connected components of citation networks of WoS journals (a) *Scientometrics*, (b) *Expert Systems with Applications* and (c) *Communications of the ACM* (see also text). Papers published in the first or second half of the journal's lifespan are shown as triangles and circles, respectively. (Network layouts were generated with Large Graph Layout [1].)

6. Final remarks

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Acknowledgments

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Appendix A. Parameters of graph models

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Table A.7: Random graph model parameters for citation networks of WoS categories *Information Science & Library Science* (ILS), *Computer Science, Artificial Intelligence* (AI), *Computer Science, Software Engineering* (SE) and *Computer Science, Cybernetics* (CY).

		Burning probability		Linking probability	
Data	Model	p_f (p_w)	p_b	q_a	q_l
ILS	Citation (our)	0.181	0.237	0.279	0.615
	Random walk [9]	0.349	-	0.292	0.699
	Forest fire [5]	0.384	0.274	-	-
AI	Citation (our)	0.102	0.173	0.409	0.728
	Random walk [9]	0.261	-	0.417	0.747
	Forest fire [5]	0.423	0.209	-	-
SE	Citation (our)	0.264	0.010	0.364	0.588
	Random walk [9]	0.351	-	0.333	0.602
	Forest fire [5]	0.444	0.128	-	-
CY	Citation (our)	0.128	0.184	0.432	0.488
	Random walk [9]	0.255	-	0.427	0.549
	Forest fire [5]	0.343	0.191	-	-

		Kronecker graph initiator $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$			
Data	Model	a	b	c	d
ILS	Kronecker [4]	0.503	0.398	0.429	0.898
AI		0.450	0.506	0.505	0.701
SE		0.859	0.509	0.434	0.337
CY		0.536	0.356	0.371	0.908

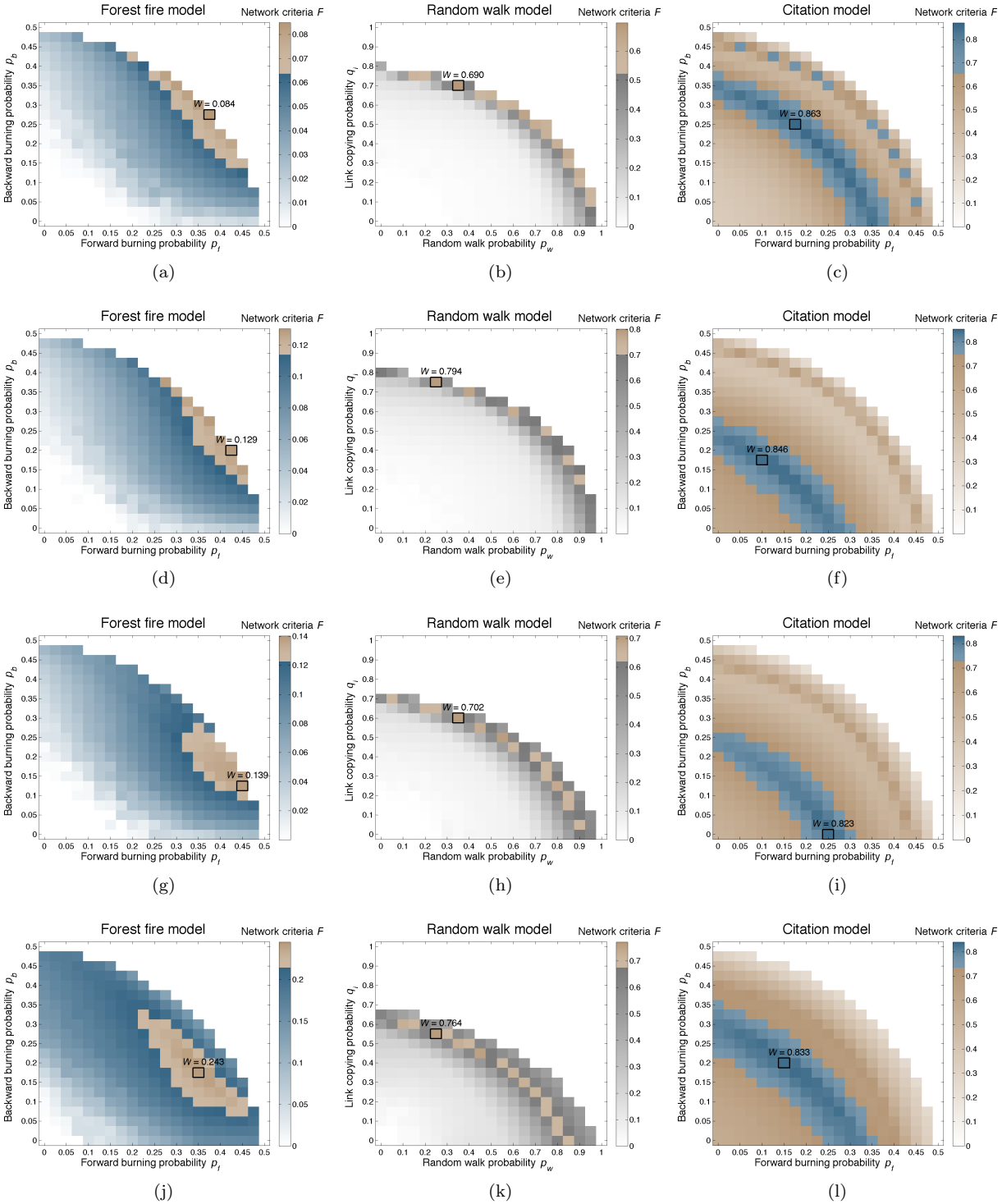

Figure A.7: Random graph parameter estimation for citation networks of WoS categories (a)–(c) *Information Science & Library Science*, (d)–(f) *Computer Science, Artificial Intelligence*, (g)–(i) *Computer Science, Software Engineering* and (j)–(l) *Computer Science, Cybernetics* (note different color scales). (Results are estimates of the mean over 25 graph realizations with the global maximum highlighted.)

Appendix B. Comparison of graph models

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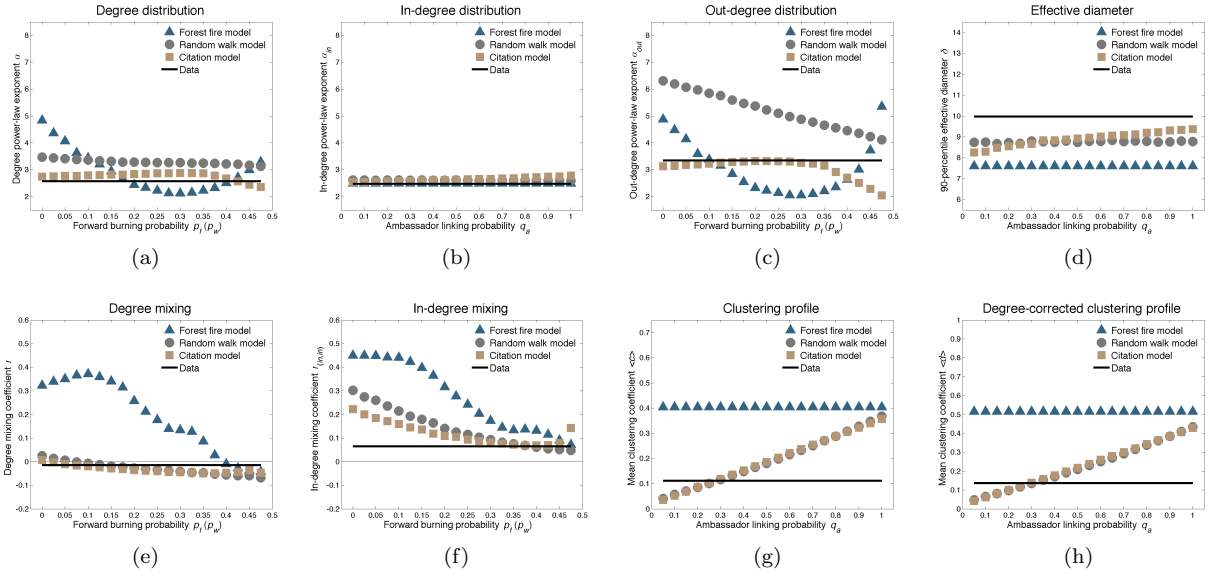

Figure B.8: Degree and clustering distributions and degree mixing in citation network of WoS category *Information Science & Library Science* and graphs generated by varying parameters of random models.

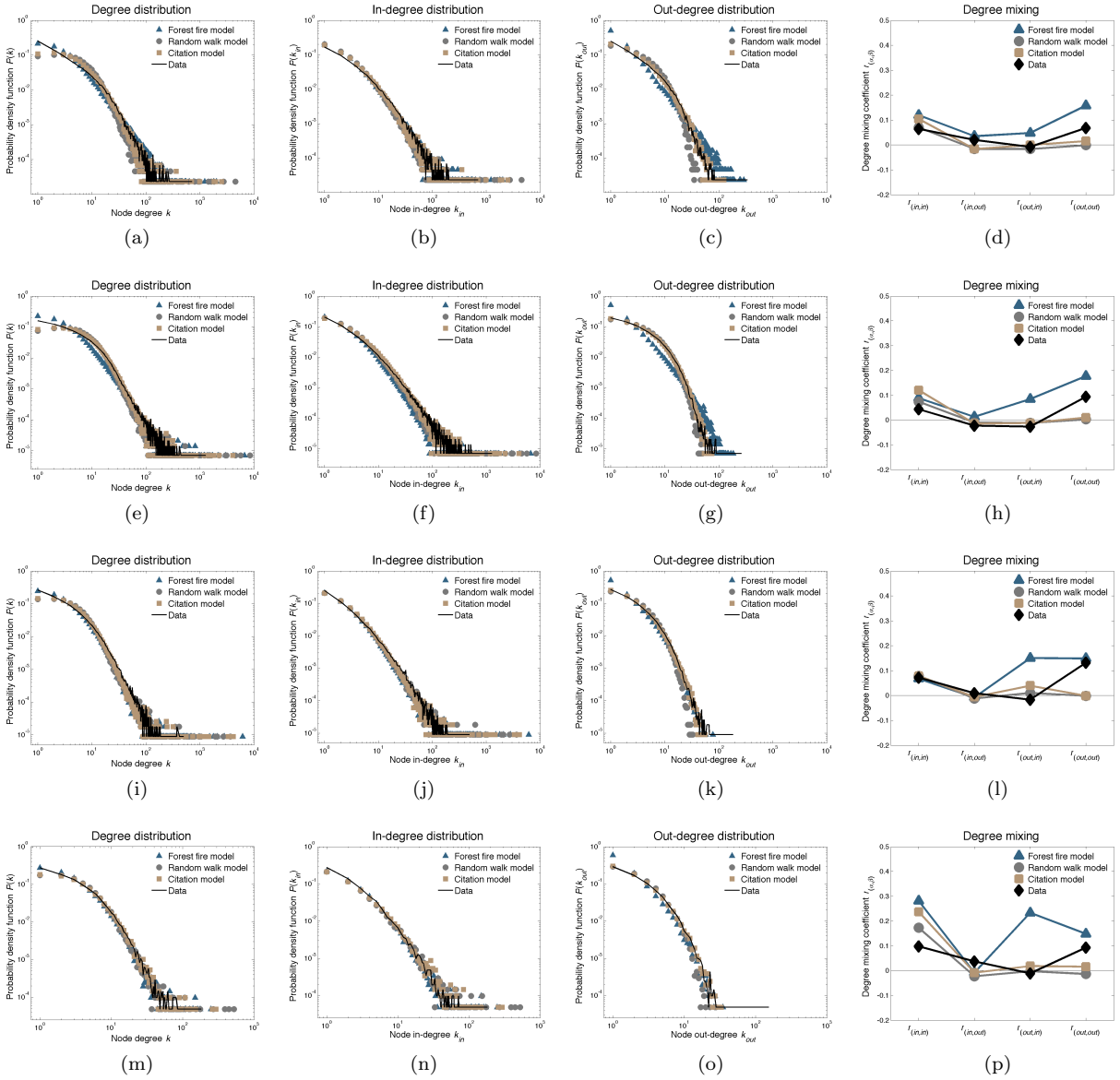

Figure B.9: Degree distributions and mixing in citation networks of WoS categories (a)–(d) *Information Science & Library Science*, (e)–(h) *Computer Science, Artificial Intelligence*, (i)–(l) *Computer Science, Software Engineering* and (m)–(p) *Computer Science, Cybernetics*, and graphs generated with random models (see also Table B.9).

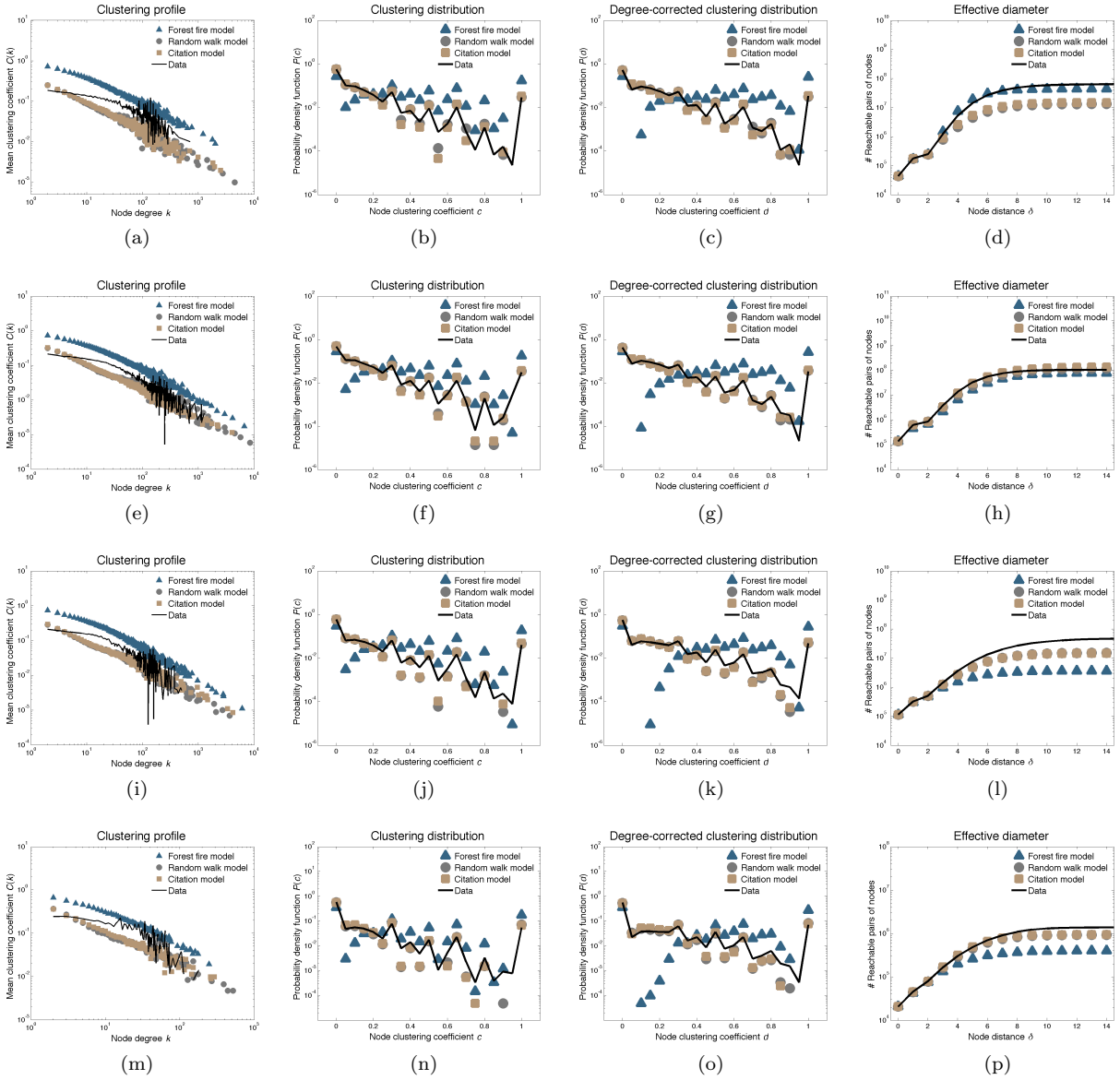

Figure B.10: Clustering profile and distributions in citation networks of WoS categories (a)–(d) *Information Science & Library Science*, (e)–(h) *Computer Science, Artificial Intelligence*, (i)–(l) *Computer Science, Software Engineering* and (m)–(p) *Computer Science, Cybernetics*, and graphs generated with random models (see also Table B.8).

Table B.8: Standard statistics and clustering in citation networks of WoS categories *Information Science & Library Science* (ILS), *Computer Science, Artificial Intelligence* (AI), *Computer Science, Software Engineering* (SE) and *Computer Science, Cybernetics* (CY), and graphs generated with random models. (Results are estimates of the mean over 25 graph realizations.)

Data	Model	n	m	WCC	$\langle k \rangle$	$\langle c \rangle$	$\langle d \rangle$	δ_e	$\mathcal{F}$
ILS	Data	43808	176219	94.7%	8.05	0.11	0.14	9.6	*1.000*
	Citation (our)	43808	174467	92.5%	7.97	0.11	0.13	8.6	*0.863*
	Random walk [9]	43808	173241	94.7%	7.91	0.11	0.13	8.8	*0.695*
	Random-citing [7]	43808	163743	77.3%	7.48	0.21	0.26	5.6	*0.400*
	Forest fire [5]	43808	172510	100.0%	7.88	0.40	0.52	7.7	*0.087*
	Kronecker [4]	32768	165822	98.6%	10.12	0.00	0.00	7.7	*0.209*
	Small-world [10]	43808	175228	100.0%	8.00	0.12	0.14	11.5	*0.000*
	Scale-free [2]	43808	175226	100.0%	8.00	0.00	0.00	7.3	*0.329*
	Random [3]	43808	176219	100.0%	8.05	0.00	0.00	9.8	*0.001*
AI	Data	140362	639110	97.0%	9.11	0.14	0.16	8.8	*1.000*
	Citation (our)	140362	635007	95.8%	9.05	0.14	0.16	9.7	*0.849*
	Random walk [9]	140362	629652	96.4%	8.97	0.14	0.16	9.9	*0.798*
	Random-citing [7]	140362	590798	76.9%	8.42	0.22	0.26	5.7	*0.486*
	Forest fire [5]	140362	464120	100.0%	6.61	0.41	0.53	9.6	*0.129*
	Kronecker [4]	131072	487535	99.0%	7.44	0.00	0.00	10.0	*0.073*
	Small-world [10]	140362	561445	100.0%	8.00	0.14	0.16	12.7	*0.000*
	Scale-free [2]	140362	561442	100.0%	8.00	0.00	0.00	7.8	*0.309*
	Random [3]	140362	639110	100.0%	9.11	0.00	0.00	9.8	*0.003*
SE	Data	113907	323406	90.5%	5.68	0.13	0.16	12.3	*1.000*
	Citation (our)	113907	316471	91.1%	5.56	0.13	0.16	9.3	*0.841*
	Random walk [9]	113907	316495	91.5%	5.56	0.13	0.16	9.1	*0.718*
	Random-citing [7]	113907	314972	70.9%	5.53	0.21	0.26	5.0	*0.475*
	Forest fire [5]	113907	295989	100.0%	5.20	0.41	0.53	7.5	*0.139*
	Kronecker [4]	131072	410583	88.8%	6.26	0.00	0.00	8.7	*0.253*
	Small-world [10]	113907	227813	74.0%	4.00	0.00	0.00	-	*0.000*
	Scale-free [2]	113907	227812	100.0%	4.00	0.00	0.00	16.1	*0.288*
	Random [3]	113907	323406	99.7%	5.68	0.00	0.00	13.7	*0.000*
CY	Data	20706	45743	83.4%	4.42	0.16	0.20	9.4	*1.000*
	Citation (our)	20706	45196	88.2%	4.37	0.16	0.20	7.9	*0.849*
	Random walk [9]	20706	44731	89.5%	4.32	0.16	0.19	7.6	*0.770*
	Random-citing [7]	20706	44892	72.1%	4.34	0.19	0.24	5.4	*0.608*
	Forest fire [5]	20706	42883	100.0%	4.14	0.37	0.50	7.3	*0.246*
	Kronecker [4]	16384	51143	96.2%	6.24	0.00	0.00	9.2	*0.284*
	Small-world [10]	20706	41412	68.6%	4.00	0.00	0.00	-	*0.000*
	Scale-free [2]	20706	41410	100.0%	4.00	0.00	0.00	14.0	*0.311*
	Random [3]	20706	45743	98.7%	4.42	0.00	0.00	15.4	*0.031*

Table B.9: Degree distributions and mixing in citation networks of WoS categories *Information Science & Library Science* (ILS), *Computer Science, Artificial Intelligence* (AI), *Computer Science, Software Engineering* (SE) and *Computer Science, Cybernetics* (CY), and graphs generated with random models. (Results are estimates of the mean over 25 graph realizations.)

Data	Model	α	α_{in}	α_{out}	$r_{(in,in)}$	$r_{(in,out)}$	$r_{(out,in)}$	$r_{(out,out)}$	$\mathcal{F}$
ILS	Data	2.57	2.47	3.35	0.06	0.02	−0.01	0.07	*1.000*
	Citation (our)	2.83	2.58	3.31	0.12	−0.01	0.01	0.01	*0.863*
	Random walk [9]	3.24	2.61	4.68	0.07	−0.01	−0.02	0.00	*0.695*
	Random-citing [7]	2.72	2.36	3.26	0.16	−0.02	0.08	0.15	*0.400*
	Forest fire [5]	2.39	2.51	2.45	0.13	0.04	0.06	0.17	*0.087*
	Kronecker [4]	3.13	4.75	4.12	0.16	0.16	0.16	0.16	*0.209*
	Small-world [10]	15.51	-	-	−0.07	−0.07	−0.07	−0.07	*0.000*
	Scale-free [2]	3.13	3.06	3.06	−0.03	−0.03	−0.03	−0.03	*0.329*
	Random [3]	8.58	22.30	22.34	0.00	0.00	0.00	0.00	*0.001*
AI	Data	2.74	2.39	3.88	0.04	−0.02	−0.03	0.09	*1.000*
	Citation (our)	2.94	2.60	3.84	0.13	−0.01	−0.01	0.01	*0.849*
	Random walk [9]	3.12	2.61	4.36	0.07	−0.01	−0.01	0.00	*0.798*
	Random-citing [7]	2.57	2.30	2.97	0.11	−0.02	0.08	0.15	*0.486*
	Forest fire [5]	2.67	2.63	2.91	0.08	0.01	0.09	0.17	*0.129*
	Kronecker [4]	4.70	7.88	8.15	0.02	0.02	0.03	0.02	*0.073*
	Small-world [10]	16.52	-	-	−0.07	−0.07	−0.07	−0.07	*0.000*
	Scale-free [2]	3.13	3.06	3.06	−0.02	−0.02	−0.02	−0.02	*0.309*
	Random [3]	7.12	19.03	18.93	0.00	0.00	0.00	0.00	*0.003*
SE	Data	3.01	2.61	4.46	0.07	0.01	−0.02	0.13	*1.000*
	Citation (our)	3.32	2.73	4.47	0.08	−0.01	0.03	0.00	*0.841*
	Random walk [9]	3.50	2.69	6.17	0.06	−0.01	0.01	0.00	*0.718*
	Random-citing [7]	2.95	2.50	3.63	0.15	−0.01	0.11	0.19	*0.475*
	Forest fire [5]	3.19	2.68	4.34	0.07	−0.01	0.16	0.15	*0.139*
	Kronecker [4]	2.96	4.35	3.26	0.04	0.04	0.04	0.04	*0.253*
	Small-world [10]	61.87	-	-	−0.14	−0.14	−0.14	−0.14	*0.000*
	Scale-free [2]	3.13	3.05	3.05	−0.02	−0.02	−0.02	−0.02	*0.288*
	Random [3]	14.19	37.46	37.74	0.00	0.00	0.00	0.00	*0.000*
CY	Data	3.03	3.00	4.05	0.10	0.04	−0.01	0.09	*1.000*
	Citation (our)	3.05	2.81	4.09	0.23	−0.01	0.01	0.02	*0.849*
	Random walk [9]	3.25	2.78	4.98	0.15	−0.01	0.00	0.00	*0.770*
	Random-citing [7]	2.74	2.67	3.26	0.26	−0.02	0.10	0.18	*0.608*
	Forest fire [5]	3.14	2.96	4.06	0.23	−0.00	0.21	0.17	*0.246*
	Kronecker [4]	2.91	4.39	4.16	0.18	0.18	0.18	0.19	*0.284*
	Small-world [10]	5.31	73.77	73.77	−0.13	−0.13	−0.13	−0.13	*0.000*
	Scale-free [2]	2.58	3.10	3.10	−0.05	−0.05	−0.05	−0.05	*0.311*
	Random [3]	5.85	12.91	12.94	0.00	0.00	0.00	0.00	*0.031*

Appendix C. WoS citation dynamics

TEXT

Table C.10: Paper citation, study and discovery dynamics of WoS categories *Information Science & Library Science* (ILS), *Computer Science, Artificial Intelligence* (AI), *Computer Science, Software Engineering* (SE) and *Computer Science, Cybernetics* (CY) as predicted by random graph models. (Results are estimates of the mean over 25 graph realizations.)

		Paper citation		Paper study		Paper discovery by		
Data	Model	# Cite	% Copy	# Read	% Cite	% Citation	% Service	% Other
ILS	Citation (our)	3.98	86.1%	2.14	27.9%	29.2%	41.0%	29.8%
	Random walk [9]	3.95	90.4%	1.54	29.2%	83.7%	-	16.3%
	Random-citing [7]	3.73	65.4%	1.29	100.0%	-	-	-
	Forest fire [5]	3.95	-	3.95	100.0%	62.3%	37.7%	-
AI	Citation (our)	4.52	87.3%	1.47	40.9%	25.8%	47.6%	26.6%
	Random walk [9]	4.49	88.9%	1.35	41.7%	88.1%	-	11.9%
	Random-citing [7]	4.21	69.7%	1.28	100.0%	-	-	-
	Forest fire [5]	3.31	-	3.31	100.0%	73.5%	26.5%	-
SE	Citation (our)	2.78	81.5%	1.58	36.4%	68.8%	2.0%	29.2%
	Random walk [9]	2.78	83.7%	1.54	33.3%	80.2%	-	19.8%
	Random-citing [7]	2.75	61.0%	1.07	100.0%	-	-	-
	Forest fire [5]	2.59	-	2.59	100.0%	84.4%	15.6%	-
CY	Citation (our)	2.18	69.6%	1.59	43.2%	24.5%	37.8%	37.6%
	Random walk [9]	2.16	75.7%	1.34	42.7%	79.0%	-	21.0%
	Random-citing [7]	2.17	49.1%	1.10	100.0%	-	-	-
	Forest fire [5]	2.07	-	2.07	100.0%	68.9%	31.1%	-

Appendix D. WoS citation evolution

TEXT

Table D.11: Paper citation, study and discovery evolution of WoS categories *Information Science & Library Science* (ILS), *Computer Science, Artificial Intelligence* (AI), *Computer Science, Software Engineering* (SE) and *Computer Science, Cybernetics* (CY) as predicted by citation model. (Results are estimates of the mean over 25 graph realizations.)

		Paper citation		Paper study		Paper discovery by		
Data	Period	# Cite	% Copy	# Read	% Cite	% Citation	% Service	% Other
ILS	1945–2013	3.98	86.1%	2.14	27.9%	29.2%	41.0%	29.8%
	1970–1980	2.23	52.1%	3.39	33.5%	41.4%	0.0%	58.5%
	1980–1990	2.62	65.1%	2.96	33.0%	48.3%	1.1%	50.6%
	1990–2000	3.42	81.6%	2.38	29.0%	40.3%	23.2%	36.5%
	2000–2010	5.06	83.6%	2.90	32.2%	40.7%	27.5%	31.7%
AI	1964–2013	4.52	87.3%	1.47	40.9%	25.8%	47.6%	26.6%
	1970–1980	1.23	38.3%	2.11	36.8%	28.9%	19.5%	51.6%
	1980–1990	2.84	78.1%	1.56	42.1%	25.2%	39.8%	35.0%
	1990–2000	3.38	79.7%	1.70	43.4%	36.0%	34.9%	29.1%
	2000–2010	4.27	85.5%	1.53	44.7%	58.9%	18.4%	22.7%
SE	1954–2013	2.78	81.5%	1.58	36.4%	68.8%	2.0%	29.2%
	1970–1980	1.89	25.1%	2.69	54.5%	33.9%	23.3%	42.7%
	1980–1990	2.22	65.0%	2.01	40.9%	40.7%	21.0%	38.3%
	1990–2000	2.54	76.2%	1.70	37.9%	50.7%	14.1%	35.2%
	2000–2010	2.90	83.0%	1.49	36.1%	63.1%	5.8%	31.1%
CY	1961–2013	2.18	69.6%	1.59	43.2%	24.5%	37.8%	37.6%
	1970–1980	1.62	27.6%	1.81	66.9%	27.5%	34.1%	38.4%
	1980–1990	2.06	53.5%	1.76	56.6%	35.8%	25.8%	38.4%
	1990–2000	2.07	67.9%	1.49	45.8%	16.1%	43.4%	40.5%
	2000–2010	2.19	72.3%	1.65	40.0%	43.9%	21.4%	34.7%

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