

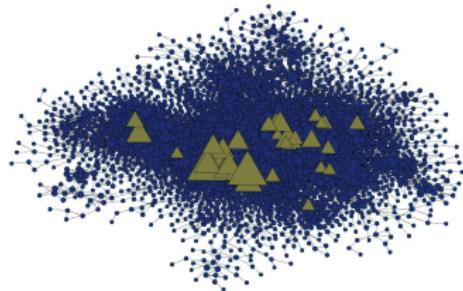
# applications *scientometrics*

introduction to *network analysis* (*ina*)

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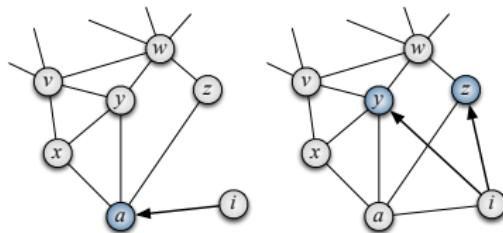
## models *overview*

- *random graph models* for reasoning about *network structure*
  - node mixing [New02], scale-free [BA99] and small-world [WS98] networks
- *generative models* for reasoning also about *network evolution*
  - node aging [HS05], densification laws and shrinking diameters [LKF07]
- next *realistic generative models* of *paper citation networks*



## models *forest fire*

- $G(n, p)$  *forest fire* model [LKF07]
- each new *node  $i$*  *forms links* as follows
  1.  $i$  selects *random ambassador  $a$*  and *links to  $a$*
  2.  $i$  selects  $x_p$  *neighbors  $a_1, \dots, a_{x_p}$*  and *links to  $a_i$*
  3.  $a_1, \dots, a_{x_p}$  are taken as *new ambassadors* of  $i$

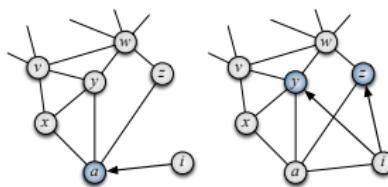


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\*  $p$  is *burning probability* &  $x_p$  from *geometric distribution*  $G(\frac{p}{1-p})$

## models *dynamics*

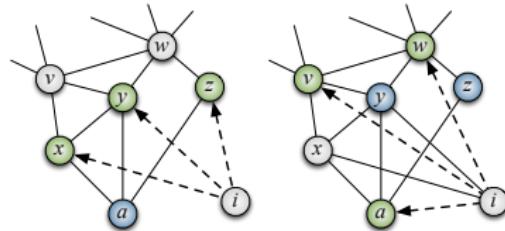
- $G(n, p)$  *author dynamics* model [LKF07]
- each new *paper  $i$*  *forms citations* as follows
  1.  $i$  selects *random read paper  $a$*  and *cites  $a$*
  2.  $i$  selects  $x_p$  *references  $a_1, \dots, a_{x_p}$*  and *cites  $a_i$*
  3.  $a_1, \dots, a_{x_p}$  are taken as *new reading* for  $i$



- then *authors read all cited papers* (vice-versa)
- but only  $\approx 20\%$  of *cited papers are read* [SR03]

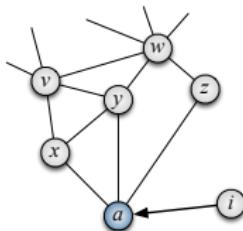
## models *citation*

- $G(n, p, q)$  *citation* model [SB13]
- each new *node  $i$*  *forms links* as follows
  1.  $i$  selects *random ambassador  $a$*  as before
  2.  $i$  selects  $x_p$  *neighbors  $a_1, \dots, a_{x_p}$*  as before
  3.  $i$  selects  $x_q$  *neighbors  $l_1, \dots, l_{x_q}$*  and *links to  $l_i$*
  4.  $a_1, \dots, a_{x_p}$  are taken as *new ambassadors* of  $i$

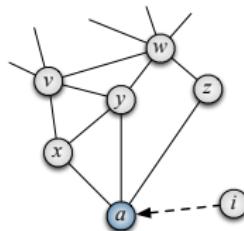
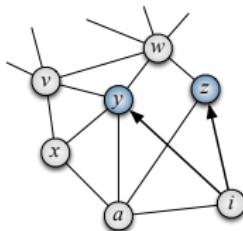


†  $q$  is *linking probability* &  $x_q$  from *geometric distribution*  $G(\frac{q}{1-q})$

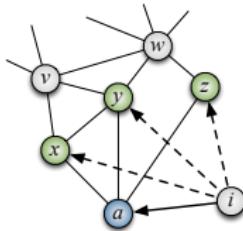
# models *undirected*



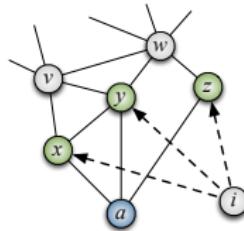
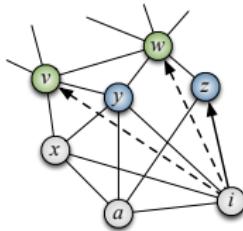
*forest fire* model [LKF07]  $A = L$



*butterfly* model [MAF08]  $A \supseteq L$



*copying* model [KR05]  $A \subseteq L$

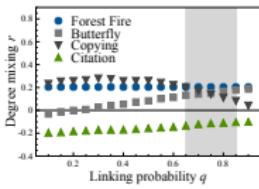
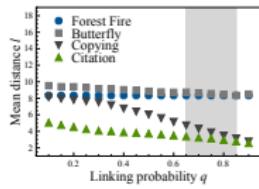
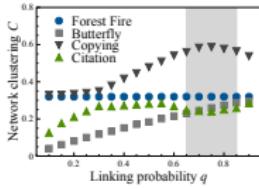
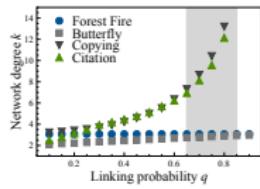
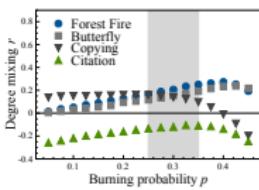
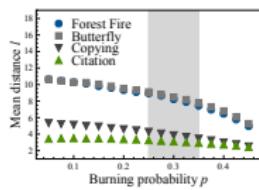
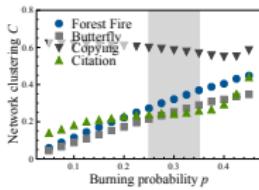
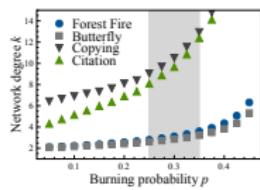


*citation* model [ŠB13]  $A$  &  $L$  arbitrary

‡  $A$  is set of *ambassadors* &  $L$  is set of *linked nodes*

# models *comparison*

- *all models* generate *modular scale-free/small-world* networks
- *only citation* model generates *degree disassortative* networks



§ shaded regions show likely parameter values [LJT<sup>+</sup>11]

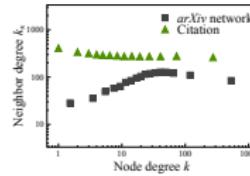
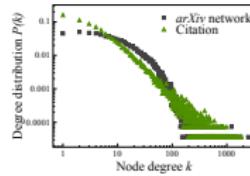
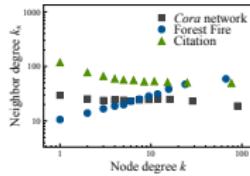
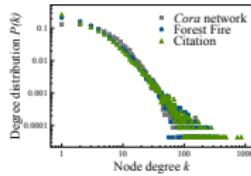
# models examples

- modeling of Cora citation network [MNRS00]

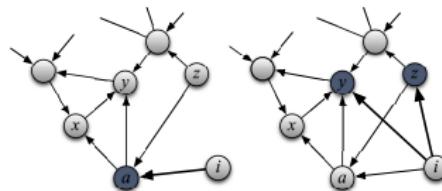
model	nodes $n$	degree $\langle k \rangle$	mixing $r$
forest fire	23,166	7.669	0.211
citation	23,166	7.760	-0.047
	23,166	7.697	-0.055

- modeling of arXiv citation network [LKF05]

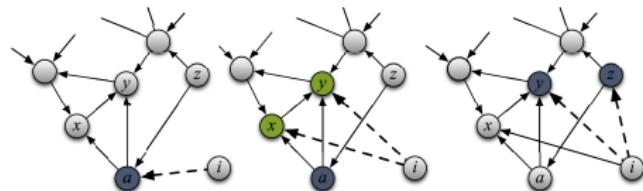
model	nodes $n$	degree $\langle k \rangle$	mixing $r$
citation	27,400	25.598	-0.068
	27,400	25.695	-0.030



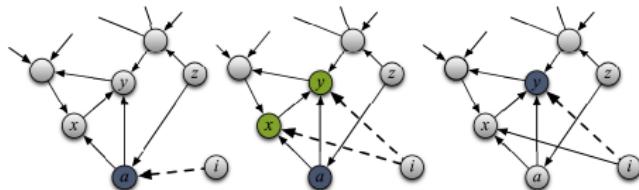
# models *directed*



*forest fire* model [LKF07]



*citation* model [ŠŽB14]



*random walk* model [Váz03]

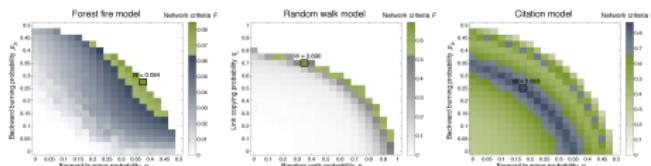
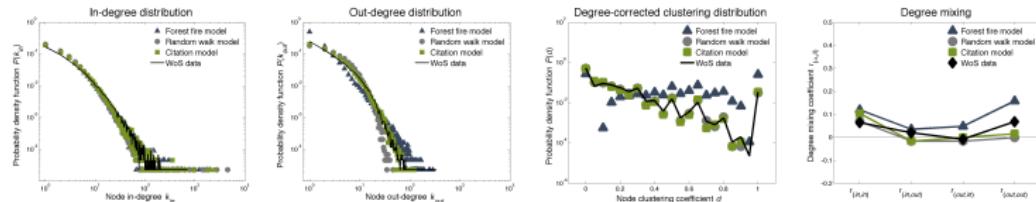


$P_{fwd}/P_{bck}$  are *burning probabilities* &  $q_{amb}/q_{lnk}$  are *linking probabilities*

# models examples

— modeling of WoS citation network [ŠŽB14]

model	degree $\langle k \rangle$	power-law $\gamma$	mixing $r$	clustering $C$			
forest fire	7.88	2.51	2.45	0.13	0.17	0.40	0.52
random walk	7.91	2.61	4.68	0.07	0.00	0.11	0.13
citation	7.97	2.58	3.31	0.12	0.01	0.11	0.13
	8.05	2.47	3.35	0.06	0.07	0.11	0.14

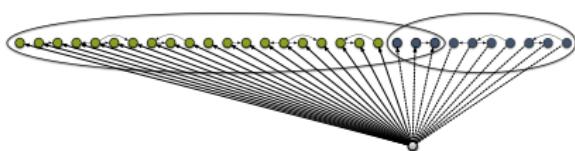


# models *applications*

— evolution/comparison of WoS citation network [ŠŽB14]

period	paper citation		paper study		paper discovery by		
	# cites	% copied	# reads	% cited	% papers	% services	% others
1970–1980	2.23	52.1%	3.39	33.5%	41.4%	0.0%	58.5%
1980–1990	2.62	65.1%	2.96	33.0%	48.3%	1.1%	50.6%
1990–2000	3.42	81.6%	2.38	29.0%	40.3%	23.2%	36.5%
2000–2010	5.06	83.6%	2.90	32.2%	40.7%	27.5%	31.7%

field	paper citation		paper study		paper discovery by		
	# cites	% copied	# reads	% cited	% papers	% services	% others
theory	2.93	79.7%	1.47	45.2%	74.7%	0.5%	24.9%
AI	4.52	87.3%	1.47	40.9%	25.8%	47.6%	26.6%
software	2.78	81.5%	1.58	36.4%	68.8%	2.0%	29.2%
cybernetics	2.18	69.6%	1.59	43.2%	24.5%	37.8%	37.6%



$$\| \langle k_{out} \rangle \text{ papers cited} \& s = (1 - \frac{p_{fwd}}{1-p_{fwd}} - \frac{p_{bck}}{1-p_{bck}})^{-1} \text{ papers read} \& s \frac{q_{lnk}}{1-q_{lnk}} \text{ citations copied etc.}$$

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# models *references*



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