

Measures of centrality, PageRank algorithm

You are given [IMDB actors collaboration network](#) in Pajek format. Your task is to find the most important actors according to different measures of centrality.

1	1 IA	14	1	DC	Degree Centrality	2	2 IIA	13	18	VIIIA	2	EC	Eigenvector Centrality	8																																																																																																																																																																																										
1	3	12	4	BC	Betweenness Centrality	12	4	CC	Closeness Centrality	13	5	13 IIIA	SC	Subgraph Centrality	9	6	4	14 IVA	C _{COEF}	Clustering Coefficient	7	1	15 VA	C _{COEF} ⁻¹	inverse CCOEF	8	1	16 VIA	MNC	max. neighb. comp.	9	3	17 VIIA	EC _{COEF}	edge clustering coefficient	10	2	18	PR	PageRank																																																																																																																																																																
2	11	2	12	8	RL	RangeLimited Betweenness	12	8	IC	Information Centrality	3	3 IIIA	4	IVB	5	VB	6	VIB	7	VIIIB	8	VIIIB	9	VIIIB	10	VIIIB	11	IB	12	IIB	91	1	14	2	15	3	16	3	17	2	18	1																																																																																																																																																														
3	19	4	20	2	21	1	57	1	58	1	59	1	61	1	62	1	63	1	64	1	65	1	89	1	90	1	32	2	33	1	34	1	35	2	36	2	51	1	52	1	53	5	54	1																																																																																																																																																												
4	37	1	38	1	39	1	93	1	95	1	94	1	50	1	97	1	96	1	46	1	47	1	48	1	49	1	50	1	51	1	52	1	53	5	54	1	55	2	56	2	57	1	58	1	59	1	60	1	61	1	62	1	63	1	64	1	65	1	66	1	67	1	68	1	69	1	70	1	71	1	72	1	73	1	74	1	75	1	76	1	77	1	78	1	79	1	80	1	81	1	82	1	83	1	84	1	85	1	86	1	87	1	88	1	89	1	90	1	91	1	92	1	93	1	94	1	95	1	96	1	97	1	98	1	99	1	100	1	101	1	102	1	103	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	0	117	1	118	1																																				
5	19	4	20	2	21	1	57	1	58	1	59	1	61	1	62	1	63	1	64	1	65	1	89	1	90	1	32	2	33	1	34	1	35	2	36	2	51	1	52	1	53	5	54	1																																																																																																																																																												
6	37	1	38	1	39	1	93	1	95	1	94	1	50	1	97	1	96	1	46	1	47	1	48	1	49	1	50	1	51	1	52	1	53	5	54	1	55	2	56	2	57	1	58	1	59	1	60	1	61	1	62	1	63	1	64	1	65	1	66	1	67	1	68	1	69	1	70	1	71	1	72	1	73	1	74	1	75	1	76	1	77	1	78	1	79	1	80	1	81	1	82	1	83	1	84	1	85	1	86	1	87	1	88	1	89	1	90	1	91	1	92	1	93	1	94	1	95	1	96	1	97	1	98	1	99	1	100	1	101	1	102	1	103	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	0	117	1	118	1																																				
7	87	1	88	1	89	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	0	117	1	118	1	119	1	120	1	121	1	122	1	123	1	124	1	125	1	126	1	127	1	128	1	129	1	130	1	131	1	132	1	133	1	134	1	135	1	136	1	137	1	138	1	139	1	140	1	141	1	142	1	143	1	144	1	145	1	146	1	147	1	148	1	149	1	150	1	151	1	152	1	153	1	154	1	155	1	156	1	157	1	158	1	159	1	160	1	161	1	162	1	163	1	164	1	165	1	166	1	167	1	168	1	169	1	170	1	171	1	172	1	173	1	174	1	175	1	176	1	177	1	178	1	179	1	180	1	181	1	182	1	183	1	184	1	185	1	186	1	187	1	188	1	189	1	190	1	191	1	192	1	193	1	194	1	195	1	196	1	197	1	198	1	199	1	200	1
	87	1	88	1	89	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	0	117	1	118	1	119	1	120	1	121	1	122	1	123	1	124	1	125	1	126	1	127	1	128	1	129	1	130	1	131	1	132	1	133	1	134	1	135	1	136	1	137	1	138	1	139	1	140	1	141	1	142	1	143	1	144	1	145	1	146	1	147	1	148	1	149	1	150	1	151	1	152	1	153	1	154	1	155	1	156	1	157	1	158	1	159	1	160	1	161	1	162	1	163	1	164	1	165	1	166	1	167	1	168	1	169	1	170	1	171	1	172	1	173	1	174	1	175	1	176	1	177	1	178	1	179	1	180	1	181	1	182	1	183	1	184	1	185	1	186	1	187	1	188	1	189	1	190	1	191	1	192	1	193	1	194	1	195	1	196	1	197	1	198	1	199	1	200	1
	87	1	88	1	89	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	0	117	1	118	1	119	1	120	1	121	1	122	1	123	1	124	1	125	1	126	1	127	1	128	1	129	1	130	1	131	1	132	1	133	1	134	1	135	1	136	1	137	1	138	1	139	1	140	1	141	1	142	1	143	1	144	1	145	1	146	1	147	1	148	1	149	1	150	1	151	1	152	1	153	1	154	1	155	1	156	1	157	1	158	1	159	1	160	1	161	1	162	1	163	1	164	1	165	1	166	1	167	1	168	1	169	1	170	1	171	1	172	1	173	1	174	1	175	1	176	1	177	1	178	1	179	1	180	1	181	1	182	1	183	1	184	1	185	1	186	1	187	1	188	1	189	1	190	1	191	1	192	1	193	1	194	1	195	1	196	1	197	1	198	1	199	1	200	1
	87	1	88	1	89	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	0	117	1	118	1	119	1	120	1	121	1	122	1	123	1	124	1	125	1	126	1	127	1	128	1	129	1	130	1	131	1	132	1	133	1	134	1	135	1	136	1	137	1	138	1	139	1	140	1	141	1	142	1	143	1	144	1	145	1	146	1	147	1	148	1	149	1	150	1	151	1	152	1	153	1	154	1	155	1	156	1	157	1	158	1	159	1	160	1	161	1	162	1	163	1	164	1	165	1	166	1	167	1	168	1	169	1	170	1	171	1	172	1	173	1	174	1	175	1	176	1	177	1	178	1	179	1	180	1	181	1	182	1	183	1	184	1	185	1	186	1	187	1	188	1	189	1	190	1	191	1	192	1	193	1	194	1	195	1	196	1	197	1	198	1	199	1	200	1
	87	1	88	1	89	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	0	117	1	118	1	119	1	120	1	121	1	122	1	123	1	124	1	125	1	126	1	127	1	128	1	129	1	130	1	131	1	132	1	133	1	134	1	135	1	136	1	137	1	138	1	139	1	140	1	141	1	142	1	143	1	144	1	145	1	146	1	147	1	148	1	149	1	150	1	151	1	152	1	153	1	154	1	155	1	156	1	157	1	158	1	159	1	160	1	161	1	162	1	163	1	164	1	165	1	166	1	167	1	168	1	169	1	170	1	171	1	172	1	173	1	174	1	175	1	176	1	177	1	178	1	179	1	180	1	181	1	182	1	183	1	184	1	185	1	186	1	187	1	188	1	189	1	190	1	191	1	192	1	193	1	194	1	195	1	196	1	197	1	198	1	199	1	200	1
	87	1	88	1	89	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	0	117	1	118	1	119	1	120	1	121	1	122	1	123	1	124	1	125	1	126	1	127	1	128	1	129	1	130	1	131	1	132	1	133	1	134	1	135	1	136	1	137	1	138	1	139	1	140	1	141	1	142	1	143	1	144	1	145	1	146	1	147	1	148	1	149	1	150	1	151	1	152	1	153	1	154	1	155	1	156	1	157	1	158	1	159	1	160	1	161	1	162	1	163	1	164	1	165	1	166	1	167	1	168	1	169	1	170	1	171	1	172	1	173	1	174	1	175	1	176	1	177	1	178	1	179	1	180	1	181	1	182	1	183	1	184	1	185	1	186	1	187	1	188	1	189	1	190	1	191	1	192	1	193	1	194	1	195	1	196	1	197	1	198	1	199	1	200	1
	87	1	88	1	89	1	104	1	105	1	106	1	107	1	108	1	109	1	110	1	111	1	112	1	113	1	114	1	115	1	116	0	117	1																																																																																																																																																																						

kind of actors have the highest C_i^μ (e.g. Hollywood, international, unknown)?

II. Closeness and betweenness centrality

1. **(code)** Find the most important actors according to closeness centrality $\ell_i^{-1} = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$, where n is the number of network nodes and d_{ij} is the distance between nodes i and j . You should use the breadth-first search algorithm from previous labs. What kind of actors have the highest ℓ_i^{-1} (e.g. Hollywood, international, unknown)?
2. **(answer)** Find the most important actors according to betweenness centrality $\sigma_i = \frac{1}{n^2} \sum_{st} \frac{g_{st}^i}{g_{st}}$, where n is the number of network nodes, g_{st} is the number of geodesic paths between nodes s and t , and g_{st}^i is the number of such paths through node i . You should ask the course instructor to do these computations for you. What kind of actors have the highest σ_i (e.g. Hollywood, international, unknown)?

III. Eigenvector centrality and PageRank algorithm

1. **(code)** Find the most important actors according to eigenvector centrality $e_i = \lambda_1^{-1} \sum_j A_{ij} e_j$, where A is the network adjacency matrix and λ_1 is a normalizing constant. You should use the power iteration algorithm shown below. What kind of actors have the highest e_i (e.g. Hollywood, international, unknown)?
2. **(code)** Find the most important actors according to PageRank algorithm $p_i = \alpha \sum_j A_{ij} \frac{p_j}{k_j} + \frac{1-\alpha}{n}$, where A is the network adjacency matrix, n is the number of network nodes, k_i is the degree of node i and α is the damping factor set to 0.85. You should use the PageRank algorithm shown below. What kind of actors have the highest p_i (e.g. Hollywood, international, unknown)?

```
input  graph G, precision  $\epsilon$ 
output eigenvector centrality E
1:  $E \leftarrow$  array of ones
2: do
3:    $U \leftarrow$  array of zeros
4:   for nodes  $i \in N$  do
5:     for neighbors  $j \in \Gamma_i$  do
6:        $U[i] \leftarrow U[i] + E[j]$ 
7:    $u \leftarrow \|U\|$ 
8:   for nodes  $i \in N$  do
9:      $U[i] \leftarrow U[i] \cdot n/u$ 
10:   $\Delta \leftarrow \|E - U\|$ 
11:   $E \leftarrow U$ 
12: while  $\Delta > \epsilon$ 
13: return E
```

```
input  graph G, damping  $\alpha$ , precision  $\epsilon$ 
output PageRank ranks P
1:  $P \leftarrow$  array of  $n^{-1}$ -s
2: do
3:    $U \leftarrow$  array of zeros
4:   for nodes  $i \in N$  do
5:     for predecessors  $j \in \Gamma_i^{in}$  do
6:        $U[i] \leftarrow U[i] + P[j] \cdot \alpha/k_j^{out}$ 
7:    $u \leftarrow \|U\|$ 
8:   for nodes  $i \in N$  do
9:      $U[i] \leftarrow U[i] + (1 - \alpha)/n$ 
10:   $\Delta \leftarrow \|P - U\|$ 
11:   $P \leftarrow U$ 
12: while  $\Delta > \epsilon$ 
13: return P
```