link *bridging*

introduction to *network analysis* (*ina*)

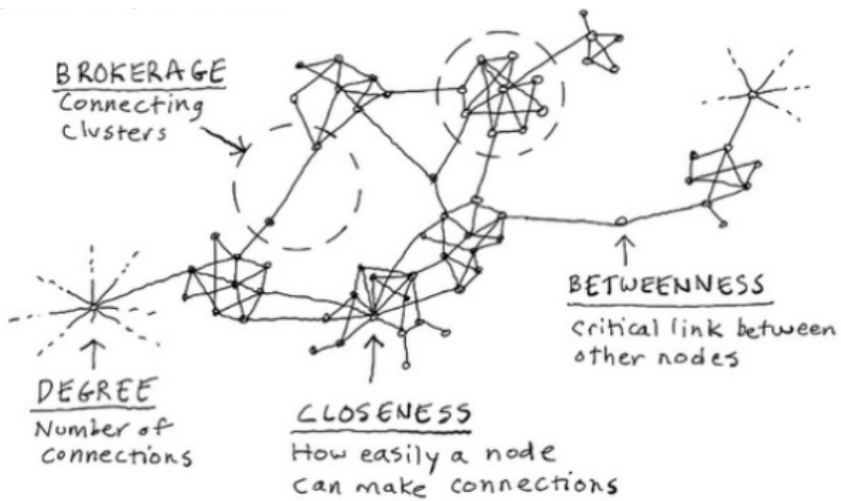
Lovro Šubelj
University of Ljubljana
spring 2021/22

bridging *measures*

which *links* are most *important*?

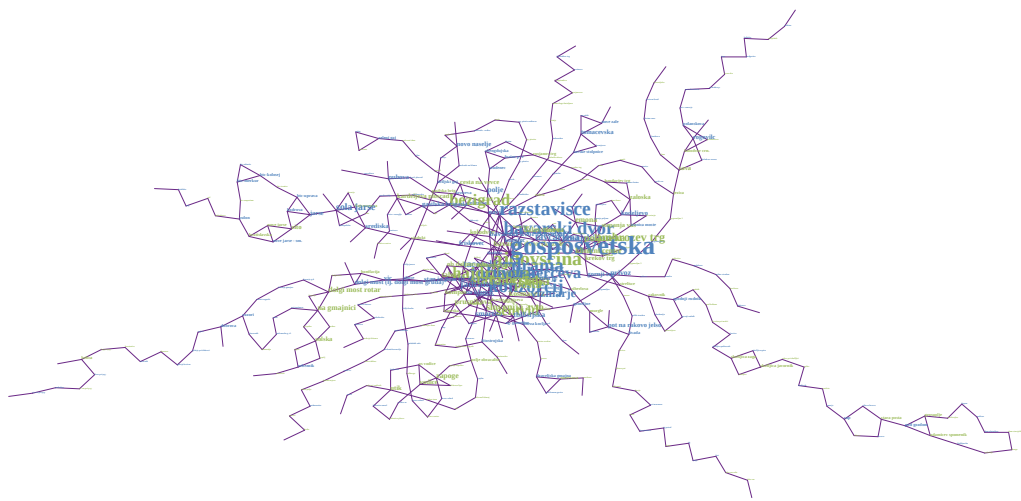
— *link bridging measures* for (*un*)*directed* networks
  – *betweenness-based* centrality [Fre77, FBW91, New05]

— *link embeddedness measures* for (*un*)*directed* networks
  – *topological overlap* measures [RSM+02, OSH+07, dNMB11]



networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 2.72$ connections
- *giant component* 95.4% nodes (6 components)
- *"small-world"* with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- *"scale-free"* with $\gamma = 2.43$ for cutoff $k_{min} = 2$



*reduced to largest connected component of simple undirected graph

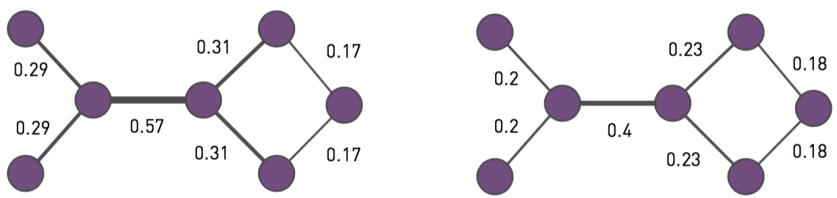
bridging *betweenness*

important *links* are *between other nodes*

— for (*un*)*directed* G *link betweenness* σ [Fre77] of $\{i, j\}$ is
  - g_{st} is number of *geodesic paths between* s and t
  - g_{st}^{ij} is number of *such geodesic paths through* $\{i, j\}$

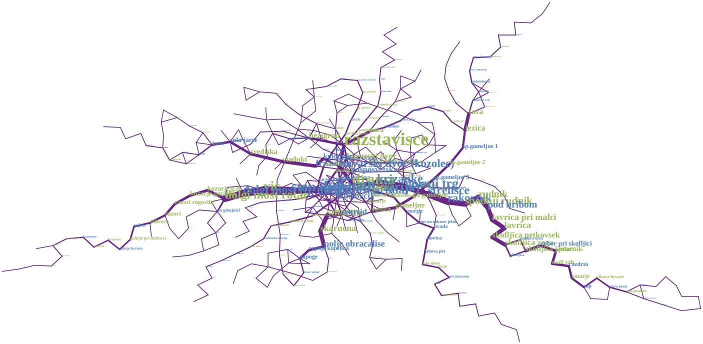
$$\sigma_{ij} = \sum_{st \notin \{i,j\}} \frac{g_{st}^{ij}}{g_{st}}$$

— σ considers *only geodesic paths* [FBW91, New05]



networkology *betweenness*

- *link betweenness* σ in partial LPP network[†]
- *highest* $\sigma_{ij} = 0.176n^2$ link is $\{$*Vič, Stan in dom*$\}$



† reduced to largest connected component of simple undirected graph

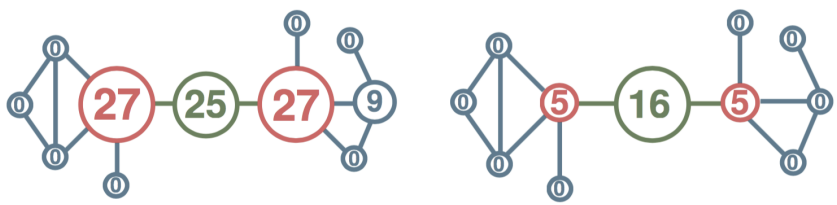
bridging *bridgeness*

important *links* are *bridges between nodes*

— for (*un*)*directed* G *link bridgeness* $\widetilde{\sigma}$ [JMK+16] of $\{i, j\}$ is
  - g_{st} is number of *geodesic paths between* s and t
  - g_{st}^{ij} is number of *such geodesic paths through* $\{i, j\}$

$$\widetilde{\sigma}_{ij} = \sigma_{ij} - \sum_{st \in \Gamma_i \cup \Gamma_j} \frac{g_{st}^{ij}}{g_{st}} = \sum_{st \notin \Gamma_i \cup \Gamma_j} \frac{g_{st}^{ij}}{g_{st}}$$

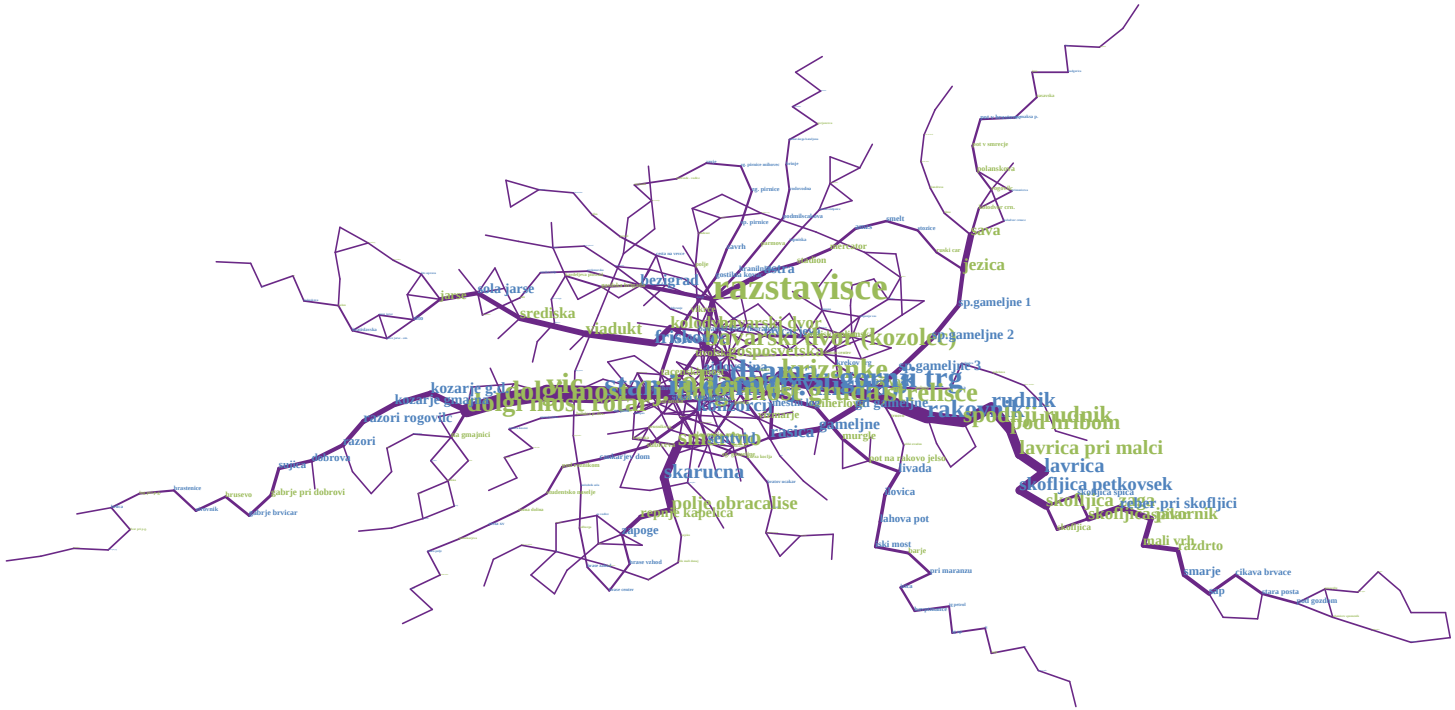
— σ mixes *local centers* with *global bridges* [JMK+16]



networkology *bridgeness*

— *link bridgeness* $\widetilde{\sigma}$ in partial LPP network[‡]

— *highest* $\widetilde{\sigma}_{ij} = 0.169n^2$ link is $\{$*Vič, Stan in dom*$\}$



[‡] reduced to largest connected component of simple undirected graph

bridging *embeddedness*

important *links* are *embedded between nodes*

— for *undirected* G *link embeddedness*[§] θ [OSH+07] of $\{i,j\}$ is

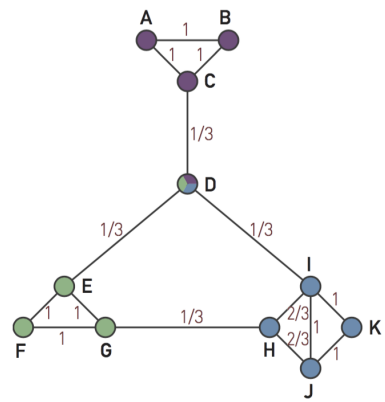
- Γ_i is set of *neighbors* or *neighborhood* of i

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \qquad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$

— μ*-corrected link embeddedness* $\widetilde{\theta}$ [dNMB11] of $\{i,j\}$ is

- μ is *maximum* number of *triangles* over *links*

$$\widetilde{\theta}_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{\mu + \max(k_i, k_j) - 1 - |\Gamma_i \cap \Gamma_j|}$$

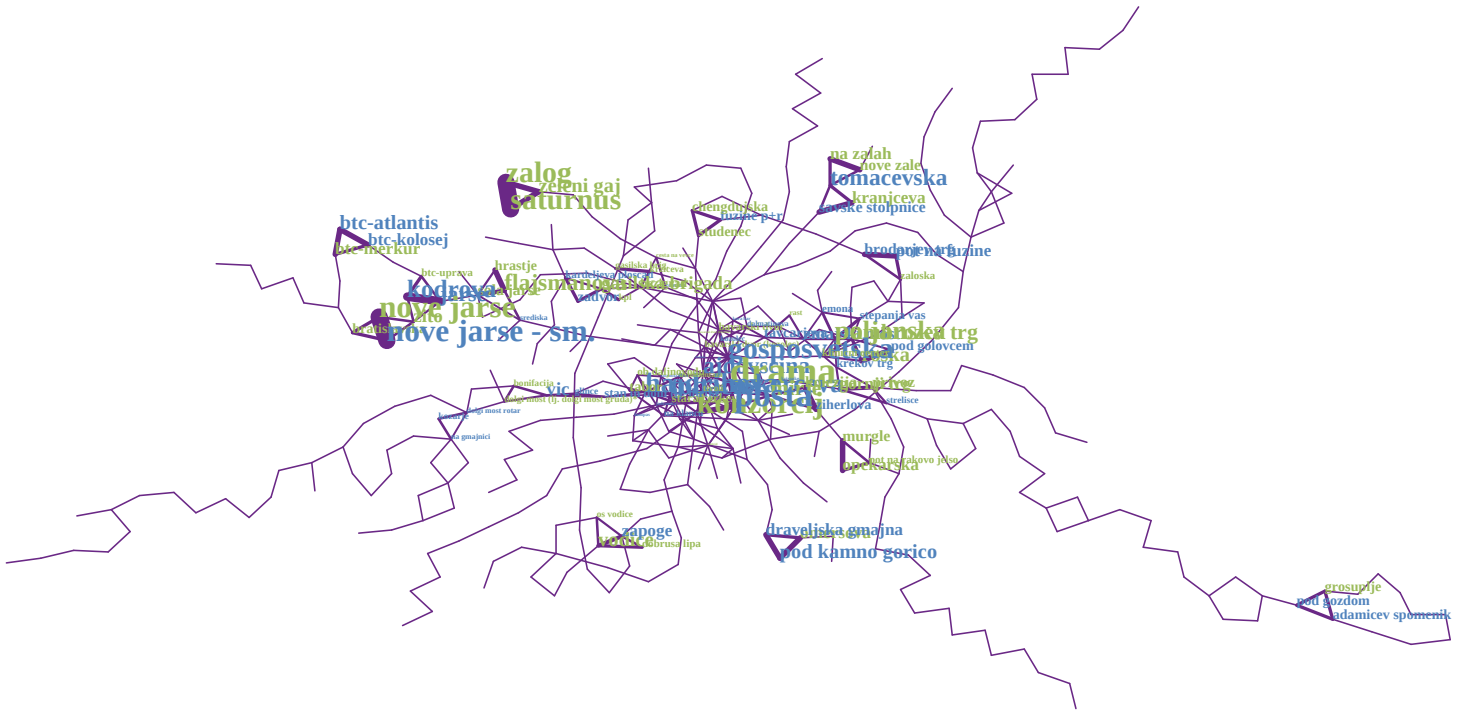


[§] θ & $\widetilde{\theta}$ better known as topological overlap indices/weights

networkology *embeddedness*

— *link embeddedness* θ in partial LPP network[¶]

— *highest* $\theta_{ij} = 1.0$ links are $\{$*Zalog, Saturnus*$\}$ *etc.*

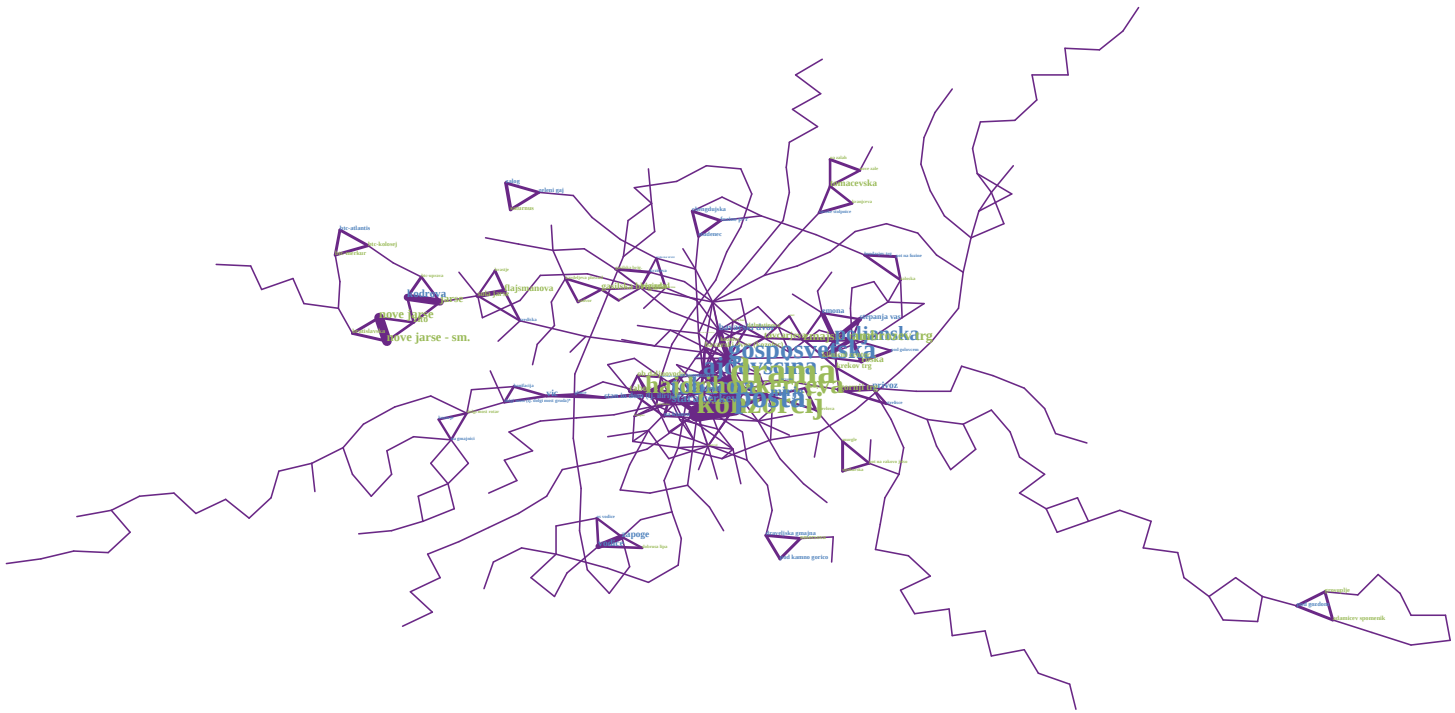


[¶] reduced to largest connected component of simple undirected graph

networkology *μ-embeddedness*

— *μ-corrected embeddedness* $\widetilde{\theta}$ in partial LPP network‖

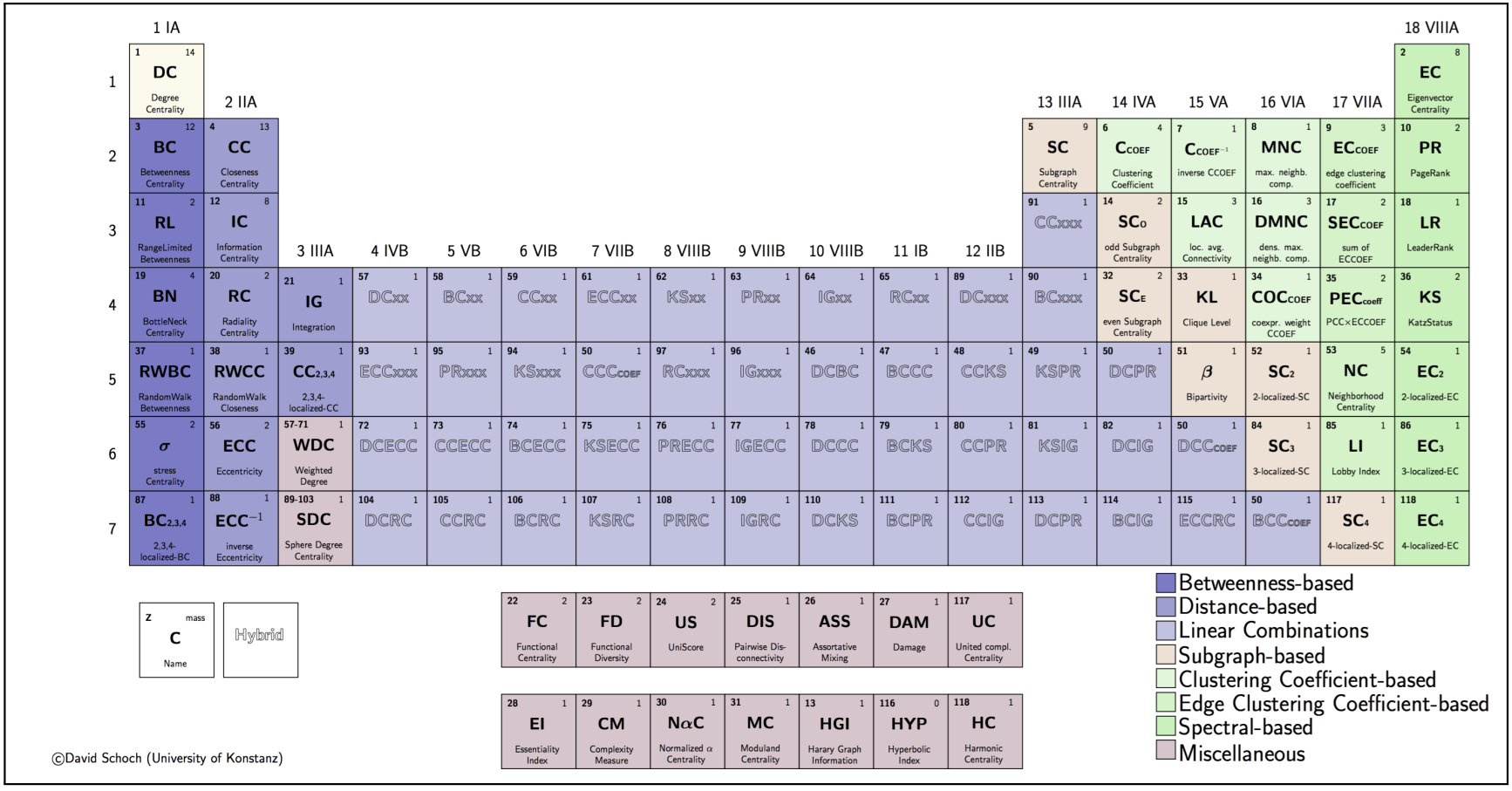
— *highest* $\widetilde{\theta}_{ij} = 0.4$ links are $\{$*Pošta, Konzorcij*$\}$ *etc.*



‖ reduced to largest connected component of simple undirected graph

bridging *overview*

which *links* are most *important*?



bridging *references*

A.-L. Barabási.
Network Science.
Cambridge University Press, Cambridge, 2016.

Wouter de Nooy, Andrej Mrvar, and Vladimir Batagelj.
Exploratory Social Network Analysis with Pajek: Expanded and Revised Second Edition.
Cambridge University Press, Cambridge, 2011.

David Easley and Jon Kleinberg.
Networks, Crowds, and Markets: Reasoning About a Highly Connected World.
Cambridge University Press, Cambridge, 2010.

Ernesto Estrada and Philip A. Knight.
A First Course in Network Theory.
Oxford University Press, 2015.

Linton C. Freeman, Stephen P. Borgatti, and Douglas R. White.
Centrality in valued graphs: A measure of betweenness based on network flow.
Soc. Networks, 13(2):141–154, 1991.

L. Freeman.
A set of measures of centrality based on betweenness.
Sociometry, 40(1):35–41, 1977.

Pablo Jensen, Matteo Morini, Marton Karsai, Tommaso Venturini, Alessandro Vespignani, Mathieu Jacomy, Jean-Philippe Cointet, Pierre Merckle, and Eric Fleury.
Detecting global bridges in networks.
J. Complex Netw., 4(3):319–329, 2016.

bridging *references*

M. E. J. Newman.
A measure of betweenness centrality based on random walks.
Soc. Networks, 27(1):39–54, 2005.

Mark E. J. Newman.
Networks.
Oxford University Press, Oxford, 2nd edition, 2018.

J.-P. Onnela, J. Saramäki, J. Hyvönen, G. Szabó, D. Lazer, K. Kaski, J. Kertész, and A.-L. Barabási.
Structure and tie strengths in mobile communication networks.
P. Natl. Acad. Sci. USA, 104(18):7332–7336, 2007.

E. Ravasz, A. L. Somera, D. A. Mongru, Z. N. Oltvai, and Albert László Barabási.
Hierarchical organization of modularity in metabolic networks.
Science, 297(5586):1551–1555, 2002.