

Homework 1

STT 465, Bayesian Statistical Methods

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Question 1

(a) 10 points

Two factories I and II produce phones for brand ABC. Factory I produces 60% of all ABC phones, and factory II produces 40%. 10% of phones produced by factory I are defective, and 20% of those produced by factory II are defective. You purchase a brand ABC phone, and assume this phone is randomly chosen. Suppose the phone is not defective. What is the probability that it came from factory II?

Answer

First, I establish the events for this problem:

D : A phone is defective.

D^c : A phone is not defective.

F_1 : A phone is from factory I.

F_2 : A phone is from factory II.

I now define the probabilities from the problem statement:

$$P(F_1) = 0.60$$

$$P(F_2) = 0.40$$

$$P(D|F_1) = 0.10$$

$$P(D|F_2) = 0.20$$

I need to find the probability that a phone came from factory II, given that it is not defective. Thus, I need to find $P(F_2|D^c)$.

To go ahead with this problem, I am finding the overall probability of a randomly chosen phone being non-defective. We can do this using the Law of Total Probability.

Before that, we need to find conditional probabilities for each factory when the phone is non-defective.

$$P(D^c|F_1) = 1 - P(D|F_1) = 1 - 0.10 = 0.90$$

$$P(D^c|F_2) = 1 - P(D|F_2) = 1 - 0.20 = 0.80$$

The total probability that a randomly chosen phone is not defective can be written as:

$$\begin{aligned}P(D^c) &= P(D^c|F_1) \cdot P(F_1) + P(D^c|F_2) \cdot P(F_2) \\&= 0.90 \cdot 0.60 + 0.80 \cdot 0.40 \\&= 0.54 + 0.32 \\&= 0.86\end{aligned}$$

Now, we can apply the Bayes' theorem to find the probability that the given phone is from factory II if it is not defective:

$$\begin{aligned}P(F_2|D^c) &= \frac{P(D^c|F_2) \cdot P(F_2)}{P(D^c)} \\&= \frac{0.80 \cdot 0.40}{0.86} \\&= \frac{0.32}{0.86} \\&= 0.372093 \approx 0.37\end{aligned}$$

Answer: There is a 37% chance a randomly selected ABC phone that is not defective was produced in factory II.