CMSE 381, Fundamental Data Science Methods

September 14, 2025

Homework 2

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Question 1: ISLP § 3.7.8

This question involves the use of simple linear regression on the `Auto`data set.

[2]:
```
auto = pd.read_csv('../data/Auto.csv')
auto=auto.replace('?', np.nan)
auto=auto.dropna()
auto['horsepower']=auto['horsepower'].astype('int')
auto=auto.reset_index(drop=True)
```

(a) Use the `sm.OLS()` function to perform a simple linear regression with `mpg` as the response and `horsepower` as the predictor. Use the `summarize()` function to print the results. Comment on the output as guided by the below questions.

1. Is there a relationship between the predictor and the response?
2. How strong is the relationship between the predictor and the response?
3. Is the relationship between the predictor and the response positive or negative?
4. *(modified from the textbook)* What are the predicted values for the inputs? Compute the RSS and MSE using these predicted values.

I commence by creating an ordinary least squares object as the model. All imports have been completed in the file preamble and may not be visible on the final document.

[3]:
```
X = sm.add_constant(auto['horsepower'])
y = auto['mpg']
lin_reg = sm.OLS(y, X).fit()
lin_reg.summary()
```

[3]:

Dep. Variable:	mpg	**R-squared:**	0.606
Model:	OLS	**Adj. R-squared:**	0.605
Method:	Least Squares	**F-statistic:**	599.7
Date:	Sun, 14 Sep 2025	**Prob (F-statistic):**	7.03e-81
Time:	13:23:22	**Log-Likelihood:**	-1178.7
No. Observations:	392	**AIC:**	2361.
Df Residuals:	390	**BIC:**	2369.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	39.9359	0.717	55.660	0.000	38.525	41.347
horsepower	-0.1578	0.006	-24.489	0.000	-0.171	-0.145

Omnibus:	16.432	**Durbin-Watson:**	0.920
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	17.305
Skew:	0.492	**Prob(JB):**	0.000175
Kurtosis:	3.299	**Cond. No.**	322.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

From the model summary, I can conclude that there exists a linear relationship between the predictor, i.e., `horsepower`, and the response, i.e., `mpg`. I can say this since the coefficient of determination, as determined by the R-squared value, is around 0.61. In other words, a little less than 61% of the variance in `mpg` is explained by `horsepower` linearly.

As for the strength of the relationship, we use the R-squared value again. Since the R-squared value is ~0.61, the relationship between the predictor and response values is moderately strong.

The coefficient associated with `horsepower` is negative, leading me to conclude that the relationship between `horsepower` and `mpg` is negative in direction.

I can find the predicted values for the inputs by using the `predict()` method in `sm.OLS()`.

[4]:
```
y_pred = lin_reg.predict(X)
y_pred
```

[4]:
```
0      19.416046
1      13.891480
2      16.259151
3      16.259151
4      17.837598
         ...
387    26.361214
388    31.727935
389    26.676903
390    27.466127
391    26.992593
Length: 392, dtype: float64
```

The Residual Sum of Squares (RSS) and the Mean Squared Error (MSE) can be calculated as follows:

$$\text{RSS} = \sum_{i=0}^{n} (y_i - f(x_i))^2$$

$$\text{MSE} = \frac{\text{RSS}}{n}$$

`scikit-learn` provides a built-in function to calculate MSE, and the length of `X` or `y_pred` can be used to find RSS from there as the value of n. However, I am choosing to calculate RSS via an array's vectorization properties to skip the summation, leading to an extremely easy and intuitive calculation without further imports.

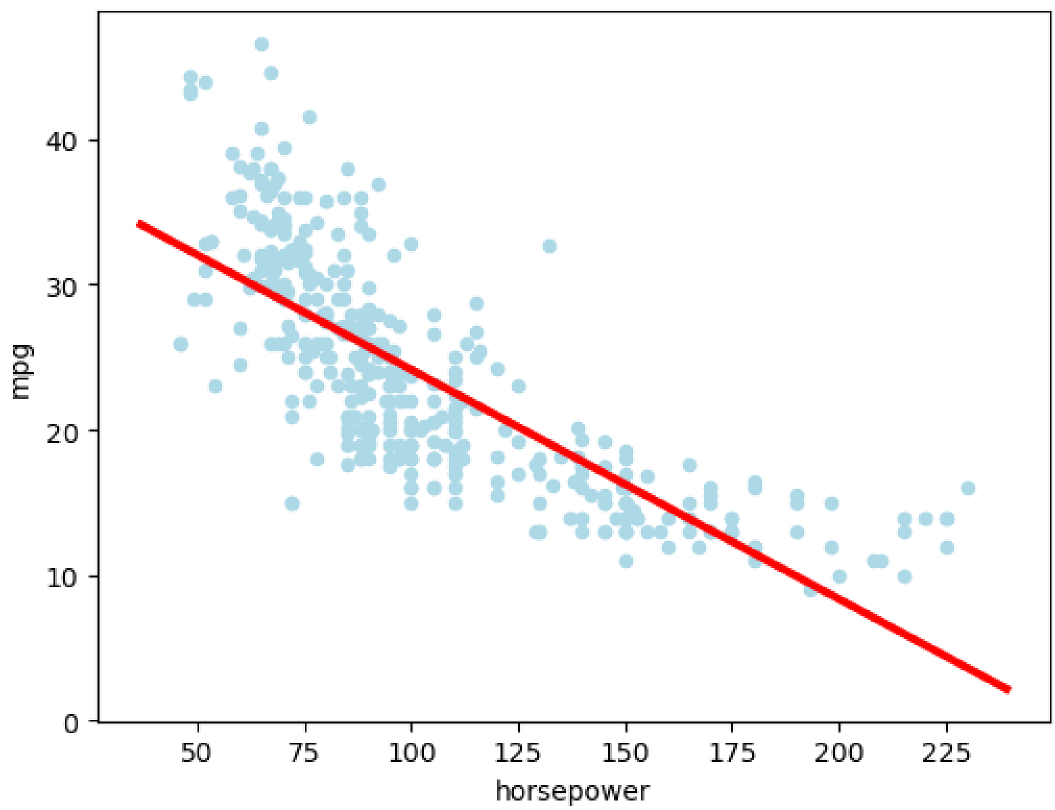
[5]:
```
rss = np.sum((y-y_pred)**2)
print("RSS =",rss)
mse=rss/len(X)
print("MSE =",mse)
```

```
RSS = 9385.915871932419
MSE = 23.943662938603108
```

(b) Plot the response and the predictor in a new set of axes `ax`. Use the `ax.axline()` method or the `abline()` function defined in the lab to display the least squares regression line. Ideally, I would use `sns.regplot()` or use simple `matplotlib.pyplot` commands. However, the instructions are clear here, and I will first define the functions according to the lab in text.

[6]:
```python
def abline(ax, b, m, *args, **kwargs):
    """Add a line with slope m and intercept b to ax
    Adapted from ISLP Section 3.6"""
    xlim = ax.get_xlim()
    ylim = [m * xlim[0] + b, m * xlim[1] + b]
    ax.plot(xlim, ylim, *args, **kwargs)
```

[7]:
```python
ax=auto.plot.scatter('horsepower', 'mpg', color='lightblue')
abline(ax,
       lin_reg.params.iloc[0],
       lin_reg.params.iloc[1],
       'r',
       linewidth=3)
```



Question 2: ISLP § 3.7.13

We will not be doing part (g) in this question.

In this exercise, we will create some simulated data and fit simple linear regression models to it. Make sure to use the default random number generator with a seed set to 1 before starting part (a) to ensure consistent results.

I am defining `rng` as a function rather than a `Generator` object since each use of `rng` advances its state, thus making the `seed` definition moot. I have also decided to combine the cells for (a) and (b) to avoid potential errors since there aren't a lot of textual answers for these sub-questions. This way, the values always stay the same, without a chance for accidentally creating different values for `eps` or `x` when we need them, and I do not have to keep writing the whole code for a new `rng` repeatedly. This has also led me to create a function for the two other repetitions of this process.

[8]:
```
def rng():
    return np.random.default_rng(seed=1)
```

(a) Using the `normal()` method of your random number generator, create a vector, `x`, containing 100 observations drawn from a $\mathcal{N}(0, 1)$ distribution. This represents a feature, X.

(b) Using the `normal()` method, create a vector, `eps`, containing 100 observations drawn from a $\mathcal{N}(0, 0.25)$ distribution–a normal distribution with mean zero and variance 0.25. Here, the `scale` parameter is not variance (σ^2), but standard deviation (σ). However, the notation for a normal distribution is $\mathcal{N}(\mu, \sigma^2)$. Hence, we input the square root of 0.25 to the `normal()` method's `scale` parameter.

[9]:
```
def create_data(var):
    r=rng()
    x=r.normal(0,1,100)
    eps=r.normal(0,np.sqrt(var),100)
    return x, eps
x=create_data(0.25)[0]
eps=create_data(0.25)[1]
```

(c) Using `x` and `eps`, generate a vector `y` according to the given model. What is the length of the vector `y`? What are the values of β_0 and β_1 in this linear model? The model is given by:

$$Y = -1 + 0.5X + \epsilon$$

[10]:
```
y=-1+0.5*x+eps
len(y)
```

[10]: 100

The length of the vector `y` is expected to be 100, since the model formula essentially acted as a transformation on the original vectors `x` and `eps`, which had 100 elements each. The length inquiry using the `len()` function verifies this expectation.

All ideal linear regression models follow the following general formula:

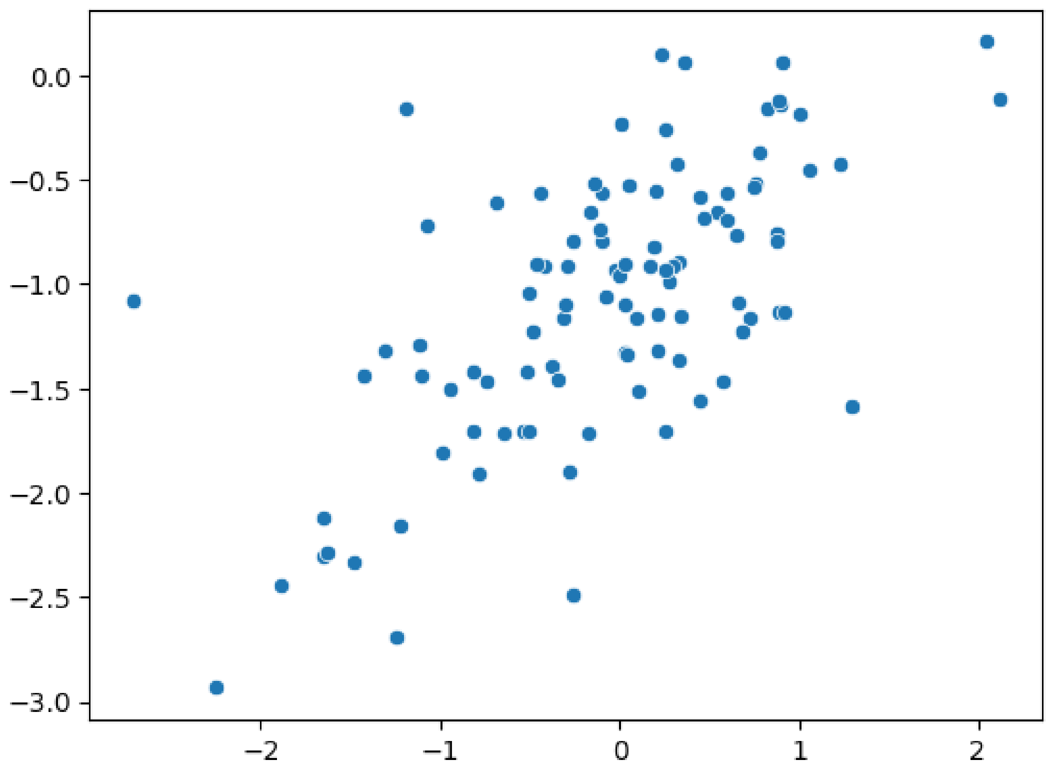
$$y = \beta_0 + \beta_1 x = \epsilon$$

Here, y is the response vector, while β_0 and β_1 are the coefficients. β_0 specifically is the intercept and β_1 is the slope. Upon comparison, $\beta_0 = -1$ and $\beta_1 = 0.5$ in this linear model.

(d) Create a scatterplot displaying the relationship between `x` and `y`. Comment on what you observe.

[11]:
```
sns.scatterplot(x=x, y=y)
```

[11]: <Axes: >



In this scatterplot, I notice a generally increasing linear relationship with a positive relationship between `x` and `y`. Although there are deviations from the general trend, I can observe that an increase in `x` leads to a near equal increase in `y`, with a few deviations from a potential line-of-best-fit (added as a result of `eps`, creating noise in the data). This noise can lead to a relatively smaller R-squared since the residuals appear to be potentially larger than expected.

(e) Fit a least squares linear model to predict `y` using `x`. Comment on the model obtained. How do $\hat{\beta}_0$ and $\hat{\beta}_1$ compare to β_0 and β_1?

[12]:
```
X = sm.add_constant(x)
lm = sm.OLS(y, X).fit()
lm.summary()
```

[12]:

Dep. Variable:	y	**R-squared:**	0.409
Model:	OLS	**Adj. R-squared:**	0.403
Method:	Least Squares	**F-statistic:**	67.79
Date:	Sun, 14 Sep 2025	**Prob (F-statistic):**	8.04e-13
Time:	13:23:22	**Log-Likelihood:**	-71.745
No. Observations:	100	**AIC:**	147.5
Df Residuals:	98	**BIC:**	152.7
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-1.0380	0.050	-20.647	0.000	-1.138	-0.938
x1	0.4843	0.059	8.233	0.000	0.368	0.601

Omnibus:	1.277	**Durbin-Watson:**	2.198
Prob(Omnibus):	0.528	**Jarque-Bera (JB):**	0.759
Skew:	0.114	**Prob(JB):**	0.684
Kurtosis:	3.361	**Cond. No.**	1.20

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The model obtained shows a positive relationship between the predictor and response. Both coefficients are statistically significant with negligible p-values. The R-squared value tells us that only about 41% of the variance in the data can be explained by this model, leading me to conclude that the relationship is fairly strong.

All predicted linear regression models follow the following general formula:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = \epsilon$$

Here, $\hat{y}$ is the predicted response vector, while $\hat{\beta}_0$ and $\hat{\beta}_1$ are the estimated coefficients, and not the true population coefficients/parameters like β_i. Upon comparison, $\hat{\beta}_0 = -1.04$ and $\beta_1 = 0.48$ in this linear model.

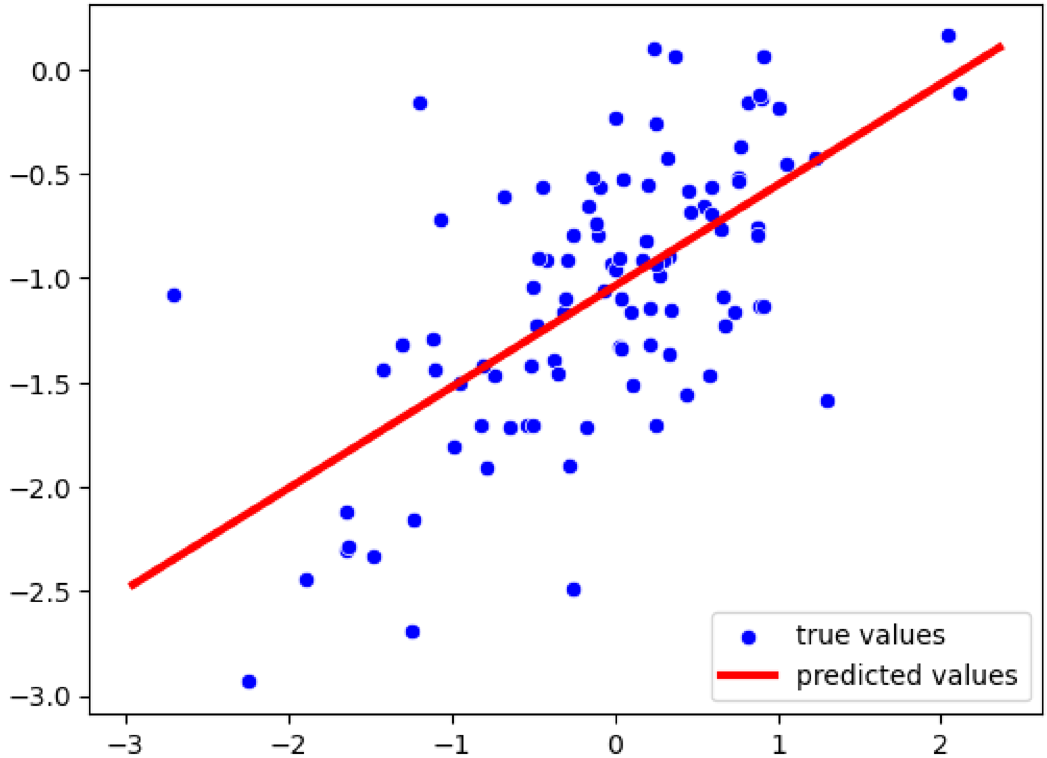
The estimated coefficients $\hat{\beta}_i$ were close to the true values β_i.

(f) Display the least squares line on the scatterplot obtained in (d). Draw the population regression line on the plot, in a different color. Use the `legend()` method of the axes to create an appropriate legend.

[13]:
```
ax = sns.scatterplot(x=x, y=y, color='blue', label='true values')
abline(ax,
       lm.params[0],
       lm.params[1],
```

```
        'r',
        linewidth=3,
        label='predicted values')
ax.legend()
```

[13]: <matplotlib.legend.Legend at 0x7ea247321430>



(h) Repeat (a)-(f) after modifying the data generation process in such a way that there is less noise in the data. The model should remain the same. You can do this by decreasing the variance of the normal distribution used to generate the error term ϵ in (b). Describe your results. I will reuse the `create_data()` function I wrote earlier to easily replicate the process. However, I will decrease the noise in the data by decreasing the variance of the normal distribution used to generate `eps`. Thus, I will update `eps` to $\mathcal{N}(0, 0.1)$, with the `scale` parameter of the `normal()` method of `rng` being $\sqrt{0.1}$.

[14]:
```
x1=create_data(0.1)[0]
eps1=create_data(0.1)[1]
```

The model remains the same:

$$Y = -1 + 0.5X + \epsilon$$

Since `x1` remains the same as `x`, and the number of elements in `eps1` remains the same as earlier, the length of `y1` should remain 100. I apply the model to `y1` again with the new values.

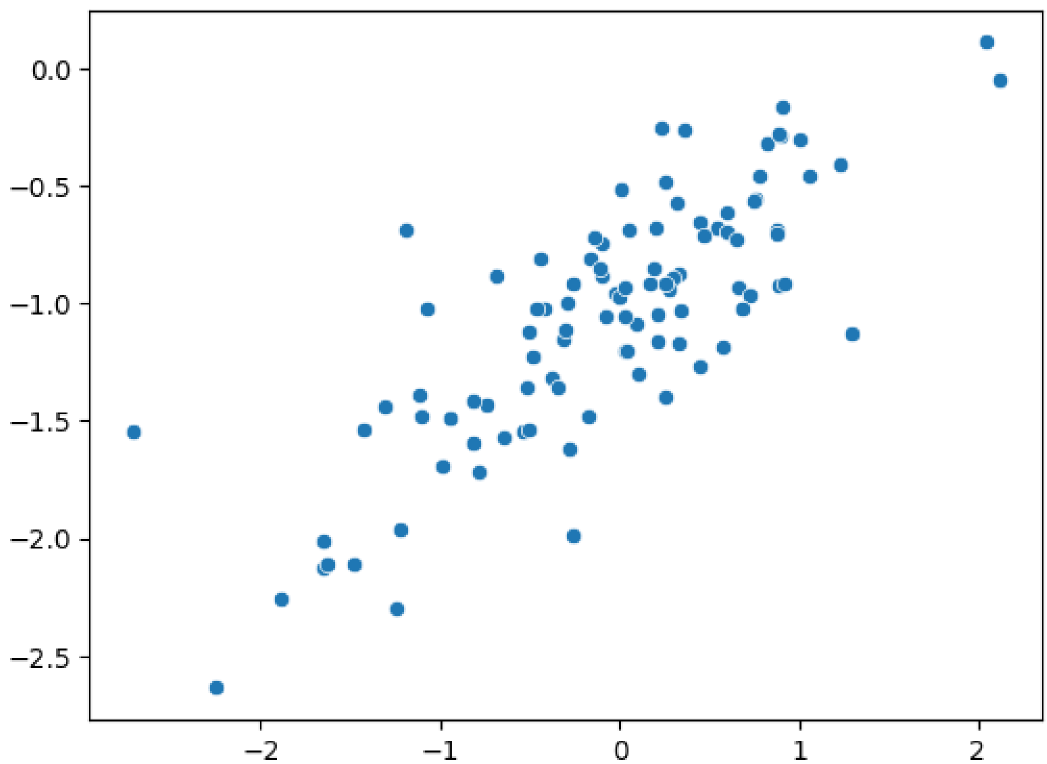
[15]:
```
y1=-1+0.5*x1+eps1
len(y1)
```

[15]:
```
100
```

The values of β_i remain the same as earlier, since the model was not changed. $\beta_0 = -1$ and $\beta_1 = 0.5$ in this linear model.

[16]:
```
sns.scatterplot(x=x1, y=y1)
```

[16]:
```
<Axes: >
```



In this scatterplot, I notice a clearly increasing linear relationship with a positive relationship between `x1` and `y1`. Although there are deviations from the general trend, it is lesser compared to the original model. I can conclude safely that the R-squared value will be fairly higher, as the residuals appear to be substantially smaller compared to the original model.

Finally, we will proceed with creating a linear model for the data.

[17]:
```
X1 = sm.add_constant(x1)
lm1 = sm.OLS(y1, X1).fit()
```

```python
lm1.summary()
```

[17]:

Dep. Variable:	y	**R-squared:**	0.639
Model:	OLS	**Adj. R-squared:**	0.635
Method:	Least Squares	**F-statistic:**	173.5
Date:	Sun, 14 Sep 2025	**Prob (F-statistic):**	2.05e-23
Time:	13:23:23	**Log-Likelihood:**	-25.931
No. Observations:	100	**AIC:**	55.86
Df Residuals:	98	**BIC:**	61.07
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-1.0240	0.032	-32.206	0.000	-1.087	-0.961
x1	0.4901	0.037	13.173	0.000	0.416	0.564

Omnibus:	1.277	**Durbin-Watson:**	2.198
Prob(Omnibus):	0.528	**Jarque-Bera (JB):**	0.759
Skew:	0.114	**Prob(JB):**	0.684
Kurtosis:	3.361	**Cond. No.**	1.20

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The model obtained shows a positive relationship between the predictor and response, with almost exactly a 1-unit increase in response per 2-unit increase in predictor value. Both coefficients are statistically significant with negligible p-values. The R-squared value tells us that nearly 64% of the variance in the data can be explained by this model, leading me to conclude that the relationship is moderately strong.

All predicted linear regression models follow the following general formula:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = \epsilon$$

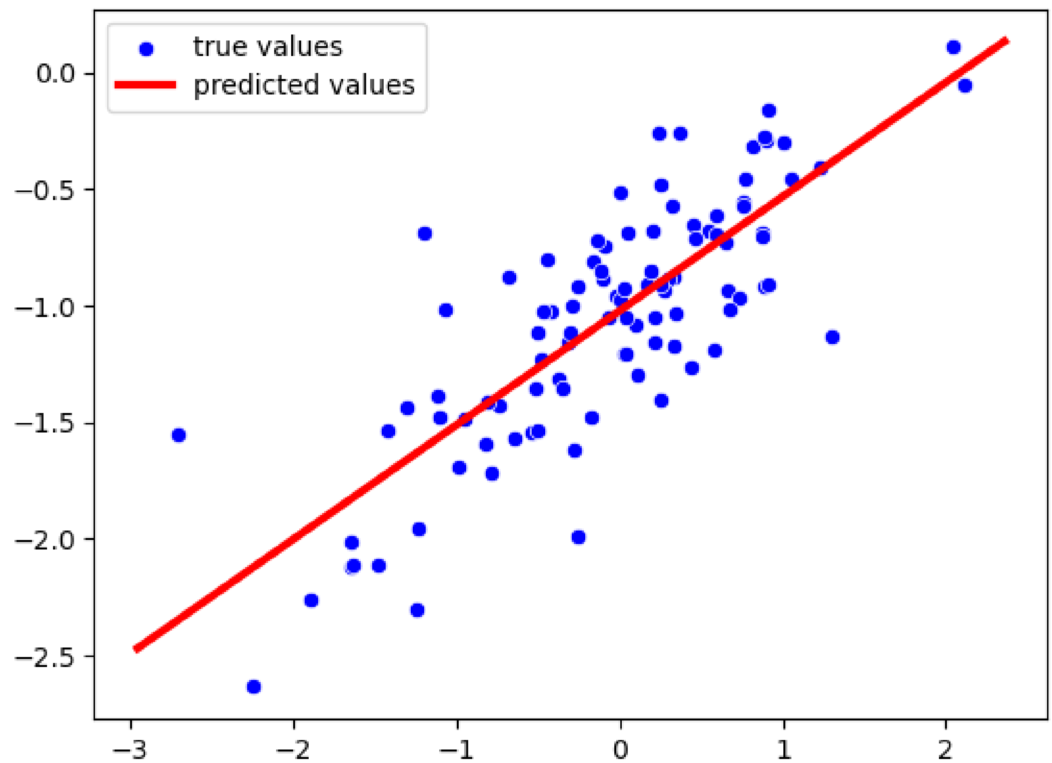
Upon comparison, $\hat{\beta}_0 = -1.02$ and $\beta_1 = 0.49$ in this linear model.

The estimated coefficients $\hat{\beta}_i$ are very close to the true values β_i.

We can now visualize the linear regression model by superimposing the line-of-best-fit on the true data points.

[18]:
```python
ax = sns.scatterplot(x=x1, y=y1, color='blue', label='true values')
abline(ax,
       lm1.params[0],
       lm1.params[1],
       'r',
       linewidth=3,
       label='predicted values')
ax.legend()
```

[18]: <matplotlib.legend.Legend at 0x7ea23e3f5b20>



(i) Repeat (a)-(f) after modifying the data generation process in such a way that there is more noise in the data. The model should remain the same. You can do this by increasing the variance of the normal distribution used to generate the error term ϵ in (b). Describe your results. I will reuse the `create_data()` function I wrote earlier to easily replicate the process. However, I will increase the noise in the data by decreasing the variance of the normal distribution used to generate `eps`. Thus, I will update `eps` to $\mathcal{N}(0, 0.4)$, with the `scale` parameter of the `normal()` method of `rng` being $\sqrt{0.4}$.

[19]:
```
x2=create_data(0.4)[0]
eps2=create_data(0.4)[1]
```

The model remains the same:

$$Y = -1 + 0.5X + \epsilon$$

Since `x2` remains the same as `x`, and the number of elements in `eps2` remains the same as earlier, the length of `y2` should remain 100. I apply the model to `y2` again with the new values.

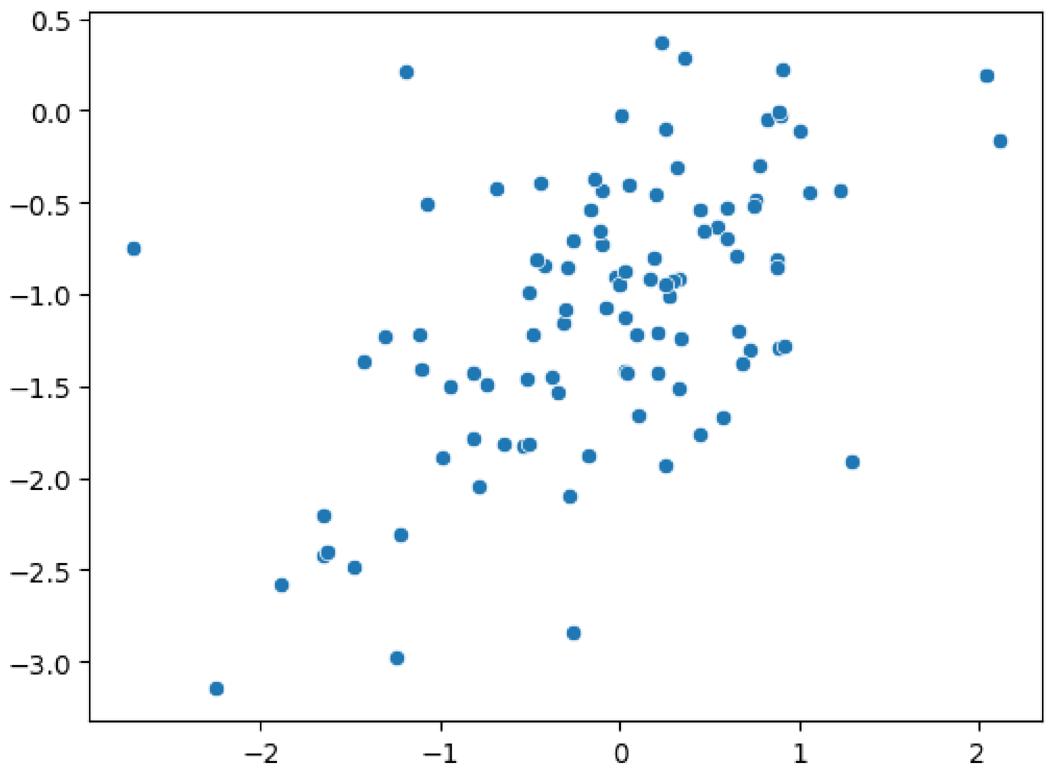
[20]:
```
y2=-1+0.5*x2+eps2
len(y2)
```

[20]: 100

The values of β_i remain the same as earlier, since the model was not changed. $\beta_0 = -1$ and $\beta_1 = 0.5$ in this linear model.

[21]:
```
sns.scatterplot(x=x2, y=y2)
```

[21]: <Axes: >



In this scatterplot, I notice a clearly increasing linear relationship with a positive relationship between x2 and y2. There are noticeable residuals and deviations from a general trend line. I can conclude that this model will have a lower R-squared value.

Finally, we will proceed with creating a linear model for the data.

[22]:
```
X2 = sm.add_constant(x2)
lm2 = sm.OLS(y2, X2).fit()
lm2.summary()
```

[22]:

Dep. Variable:	y	**R-squared:**	0.298
Model:	OLS	**Adj. R-squared:**	0.291
Method:	Least Squares	**F-statistic:**	41.64
Date:	Sun, 14 Sep 2025	**Prob (F-statistic):**	4.19e-09
Time:	13:23:23	**Log-Likelihood:**	-95.246
No. Observations:	100	**AIC:**	194.5
Df Residuals:	98	**BIC:**	199.7
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	-1.0481	0.064	-16.481	0.000	-1.174	-0.922
x1	0.4801	0.074	6.453	0.000	0.332	0.628

Omnibus:	1.277	**Durbin-Watson:**	2.198
Prob(Omnibus):	0.528	**Jarque-Bera (JB):**	0.759
Skew:	0.114	**Prob(JB):**	0.684
Kurtosis:	3.361	**Cond. No.**	1.20

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The model obtained shows a positive relationship between the predictor and response. Both coefficients are statistically significant with negligible p-values. The R-squared value tells us that nearly 30% of the variance in the data can be explained by this model, leading me to conclude that the relationship is fairly weak.

All predicted linear regression models follow the following general formula:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = \epsilon$$

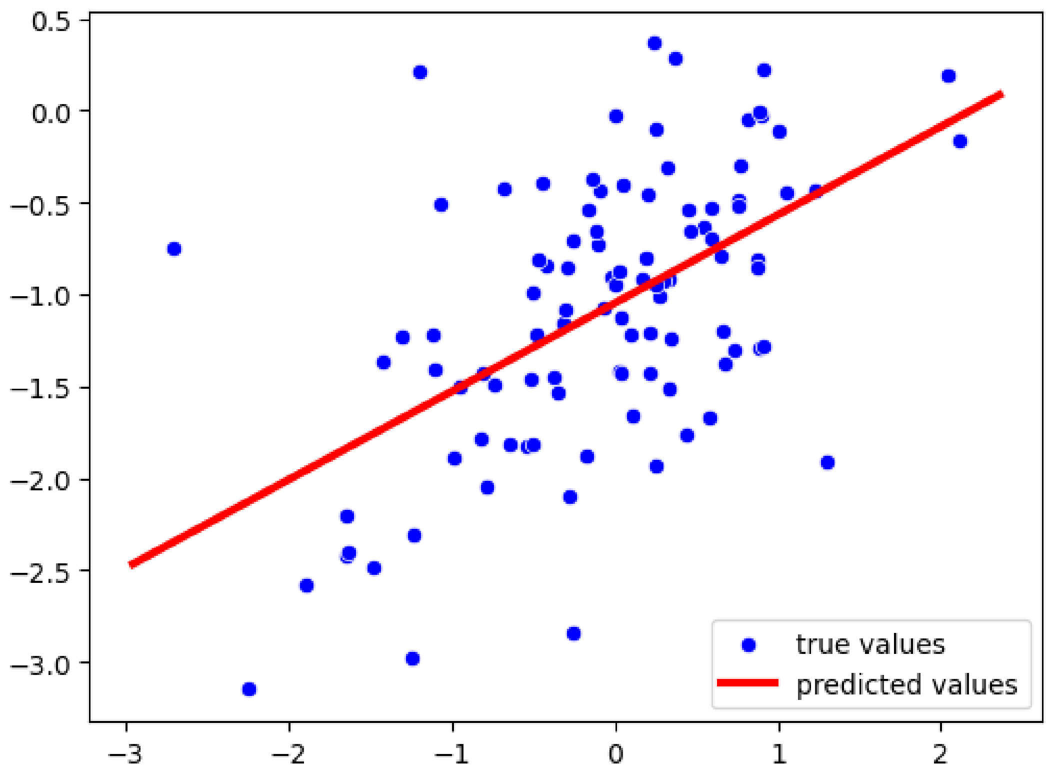
Upon comparison, $\hat{\beta}_0 = -1.05$ and $\beta_1 = 0.48$ in this linear model.

The estimated coefficients $\hat{\beta}_i$ are close to the true values β_i, but the furthest when compared precisely with the other models.

We can now visualize the linear regression model by superimposing the line-of-best-fit on the true data points.

[23]:
```
ax = sns.scatterplot(x=x2, y=y2, color='blue', label='true values')
abline(ax,
       lm2.params[0],
       lm2.params[1],
       'r',
       linewidth=3,
       label='predicted values')
ax.legend()
```

[23]:
```
<matplotlib.legend.Legend at 0x7ea23e1c98b0>
```



(j) What are the confidence intervals for β_0 and β_1 based on the original data set, the noisier data set, and the less noisy data set? Comment on your results. We can compute the confidence intervals of each of the three models using the `conf_int()` method of the OLS objects. We start with writing a function to make this easier to compute:

[24]:
```
def interpretCI(model):
    print('beta_0 =',model.params[0],'is in between',model.conf_int(alpha=0.
↪05)[0][0],'and',model.conf_int(alpha=0.05)[0][1])
    print('beta_1 =',model.params[1],'is in between',model.conf_int(alpha=0.
↪05)[1][0],'and',model.conf_int(alpha=0.05)[1][1])
```

For the original data set:

[25]:
```
interpretCI(lm)
```

```
beta_0 = -1.0380127774981607 is in between -1.1377819826358517 and
-0.9382435723604696
beta_1 = 0.48429101457751783 is in between 0.3675653637721341 and
0.6010166653829015
```

There is a 95% chance that the true values of $\beta_0 = -1.0$ and $\beta_1 = 0.5$ are within their respective confidence intervals as computed above.

$$\beta_0 \in [-1.14, -0.94]$$

$$\beta_1 \in [0.37, 0.60]$$

We now proceed with the less noisy data set:

[26]:
```
interpretCI(lm1)
```

```
beta_0 = -1.0240413914166768 is in between -1.0871409771326122 and
-0.9609418057007415
beta_1 = 0.4900647652669149 is in between 0.4162409816848188 and
0.563888548849011
```

There is a 95% chance that the true values of $\beta_0 = -1.0$ and $\beta_1 = 0.5$ are within their respective confidence intervals as computed above.

$$\beta_0 \in [-1.09, -0.96]$$

$$\beta_1 \in [0.42, 0.56]$$

Finally, we proceed with the noisier data set:

[27]:
```
interpretCI(lm2)
```

```
beta_0 = -1.0480827828333545 is in between -1.1742819542652252 and
-0.9218836114014838
beta_1 = 0.48012953053382973 is in between 0.33248196336963765 and
0.6277770976980218
```

There is a 95% chance that the true values of $\beta_0 = -1.0$ and $\beta_1 = 0.5$ are within their respective confidence intervals as computed above.

$$\beta_0 \in [-1.17, -0.92]$$

$$\beta_1 \in [0.33, 0.63]$$

Upon observation, the confidence intervals become wider with more noise.

Question 3: ISLP § 3.7.1

Describe the null hypotheses to which the p-values given in Table 3.4 correspond. Explain what conclusions you can draw based on these p-values. Your explanation should be phrased in terms of `sales`, `TV`, `radio`, and `newspaper`, rather than in terms of the coefficients of the linear model.

	Coefficient	Std. error	t-statistic	p-value
Intercept	2.939	0.3119	9.42	< 0.0001
TV	0.046	0.0014	32.81	< 0.0001
radio	0.189	0.0086	21.89	< 0.0001
newspaper	-0.001	0.0059	-0.18	0.8599

The following are the corresponding null hypotheses to the p-values provided for each of the predictors: 1. The null hypothesis claims that TV advertising has no effect on sales. 2. The null hypothesis claims that radio advertising has no effect on sales. 3. The null hypothesis claims that newspaper advertising has no effect on sales. 4. The null hypothesis claims that the baseline sales, i.e., when no money is spent on advertising, is zero.

The p-values associated with TV advertising and radio advertising are near zero. This tells me that the chance that there is no effect of TV and radio advertising on sales is also near zero, or that the probability of there being no relation between TV/radio advertising and sales is negligible. Another way to say this is that the chances of the relationship between TV/radio advertisement and sales being random is zero. Thus, there is strong evidence against the null hypothesis, leading us to reject it and accept that there is a statistically significant relationship between TV advertising and sales, as well as radio advertising and sales.

Similarly, he p-value associated with the baseline sales is also near zero. This tells me that the chance that no sales occur when no money is spent on advertisement is negligible. Thus, there is strong evidence against the null hypothesis and we reject it and accept that there is a statistically significant amount of sales happening when advertisement budgets are zero.

Finally, the p-value associated with newspaper advertising is 0.8599. This tells me that the chance that there is no effect of newspaper advertisement on sales is pretty high. Another way to put this is that the probability of the relationship established by the model between the newspaper advert budget and sales being random is about 86%. Thus, there is weak evidence against the null hypothesis, leading us to fail to reject it. Thus, newspaper advertisement does not appear to influence sales in a statistically significant way, and there is a possibility that the relationship is random.

Question 4: ISLP § 3.7.9

The question involves the use of multiple linear regression on the `Auto` data set. However, for this homework, the question has been modified slightly. Using the `Auto` data set, we will predict $Y = \text{mpg}$ using all other variables except name and origin.

The sub-parts have been modified too.

(a) Generate the correlation matrix between all variables. Are there any pairs that are particularly highly correlated? First, we remove all the qualitative variables we do not need from the previous initiation of the data set.

[28]:
```
auto=auto.drop(['name', 'origin'], axis=1)
```

Now, we use the `DataFrame.corr()` method in `pandas` to compute the correlation matrix between all the quantitative variables.

[29]:
```
auto.corr()
```

[29]:
```
                   mpg  cylinders  displacement  horsepower    weight  \
mpg           1.000000  -0.777618     -0.805127   -0.778427 -0.832244
cylinders    -0.777618   1.000000      0.950823    0.842983  0.897527
displacement -0.805127   0.950823      1.000000    0.897257  0.932994
horsepower   -0.778427   0.842983      0.897257    1.000000  0.864538
weight       -0.832244   0.897527      0.932994    0.864538  1.000000
acceleration  0.423329  -0.504683     -0.543800   -0.689196 -0.416839
year          0.580541  -0.345647     -0.369855   -0.416361 -0.309120

              acceleration      year
mpg               0.423329  0.580541
cylinders        -0.504683 -0.345647
displacement     -0.543800 -0.369855
horsepower       -0.689196 -0.416361
weight           -0.416839 -0.309120
acceleration      1.000000  0.290316
year              0.290316  1.000000
```

From the matrix, I can see that the following pairs are highly correlated:

1. `mpg` and `cylinders`; -0.78
2. `mpg` and `displacement`; -0.81
3. `mpg` and `horsepower`; -0.78
4. `mpg` and `weight`; -0.83
5. `cylinders` and `displacement`; 0.95
6. `cylinders` and `horsepower`; 0.84
7. `cylinders` and `weight`; 0.90
8. `displacement` and `horsepower`; 0.90
9. `displacement` and `weight`; 0.93
10. `horsepower` and `weight`; 0.86
11. `horsepower` and `acceleration`; =0.69

(b) Using `statsmodels`, create a linear model predicting `mpg` from all other variables except name and origin.

[30]:
```
y=auto.mpg
X=sm.add_constant(auto.drop('mpg',axis=1))
mlm=sm.OLS(y,X).fit()
mlm.summary()
```

[30]:

Dep. Variable:	mpg	**R-squared:**	0.809
Model:	OLS	**Adj. R-squared:**	0.806
Method:	Least Squares	**F-statistic:**	272.2
Date:	Sun, 14 Sep 2025	**Prob (F-statistic):**	3.79e-135
Time:	13:23:23	**Log-Likelihood:**	-1036.5
No. Observations:	392	**AIC:**	2087.
Df Residuals:	385	**BIC:**	2115.
Df Model:	6		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	-14.5353	4.764	-3.051	0.002	-23.902	-5.169
cylinders	-0.3299	0.332	-0.993	0.321	-0.983	0.323
displacement	0.0077	0.007	1.044	0.297	-0.007	0.022
horsepower	-0.0004	0.014	-0.028	0.977	-0.028	0.027
weight	-0.0068	0.001	-10.141	0.000	-0.008	-0.005
acceleration	0.0853	0.102	0.836	0.404	-0.115	0.286
year	0.7534	0.053	14.318	0.000	0.650	0.857

Omnibus:	37.865	**Durbin-Watson:**	1.232
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	60.248
Skew:	0.630	**Prob(JB):**	8.26e-14
Kurtosis:	4.449	**Cond. No.**	8.53e+04

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 8.53e+04. This might indicate that there are strong multicollinearity or other numerical problems.

(c) Is there a relationship between the predictors and the response? Justify your answer. There is indeed a statistically significant relationship between the predictors and the response, `mpg`.

Consider the F-statistic, 272.2. The probability of the F-statistic is negligible. The null hypothesis associated with the F-statistic is that all coefficients of the predictors are zero, meaning that the predictors are completely unrelated to/have no effect on the response variable. Alternatively, we can say that the null hypothesis is that none of the coefficients of the predictors are significant. The alternative hypothesis is that at least one of the coefficients are non-zero, or that at least one of the coefficients is significant. Considering the near zero p-value, we reject the null hypothesis, telling us there is a possibility that the response depends on at least one of the predictors.

There is also a high R-squared value associated with the model. The value being 0.809 tells us that about 81% of the variability in `mpg` can be explained by the predictors in the model, which is also strong evidence of the existence of a relationship.

(d) Which predictors appear to have a statistically significant relationship to the response? Based on the table of coefficients, only `weight`, `year`, and the constant have a p-value lesser than the significance level 0.05, or 5%. Thus, only `weight` and `year` have a statistically significant relationship to the response.

(e) What does the coefficient for the year variable suggest? The coefficient for the `year` variable suggests that for every one year increase in the vehicle's model year, the `mpg` is estimated to increase by 0.7534 times on an average, holding all other predictors constant. This indicates a positive relationship between a car's model year and its miles-per-gallon fuel efficiency.

Collaborations and Acknowledgments

In Question 2, I used Gemini by Google to try and understand why my values for `x` kept changing every time I ran my random number generator to create a sample from a normal distribution, even though my seed was set in stone. I used the following prompt:

> "This is what I have in my Jupyter notebook:
>
> cell 1: `rng=np.random.default_rng(1)`
>
> cell 2: `x=rng.normal(0,1,100)`
>
> When I run cell 2 after running cell 1, each run of cell 2 gives me different values of x, which is weird because I have a Generator object set to a single seed. What's going wrong?"

The answer I got has been mentioned and incorporated within my answer to the question as justification for why I used a function rather than an object for `rng`. I recognize the fallacies in generative artificial intelligence when it told me to combine both cells, which would technically ruin my flow while trying to answer each question sequentially with added markdown text. It also suggested that I repeat the random number generation for each cell. The logical solution would be to use a function, which the prompt answer failed to provide, even with a more advanced GPT model in the premium version of Gemini.