CCDSTRU Departmental Long Exam 1
Term 2, AY 2022–2023
February 22, 2023 (W)

Some friendly reminders.
1. Switch all communication devices to silent mode. Switch off all other electronic devices. Keep the devices inside your bag. These should not be brought out during the exam.
2. Only writing materials, and distributed papers are allowed on your desks.
3. Communication, in any form, is not allowed during the exam and within the exam venue.
4. If you have any questions, approach your proctor. Not all questions will be given an answer during the exam.
5. All final answers must be written **neatly** and **legibly** using ink, preferably blue or black. Multiple or messy answers will not be checked.
6. Under no circumstances are you permitted to use a pencil or a pen with erasable ink in the answer sheet.
7. Tearing of page/s from the exam papers or other suspicious acts are considered cheating. Cheating in any form is punishable with a grade of 0.0 for the course and a disciplinary offense.
8. When you are done with the exam, submit your paper. **Leave the exam venue quietly and immediately. Remain quiet until all students taking CCDSTRU exam submitted their papers.**

Name: 蔦沢 みさき **Section:** S100

I. (*10 pts*) Write the UPPERCASE letter of your answer. Choose the **BEST** answer. Write **F** if the answer is not among the choices.

C 1. $\neg(p \vee q) \wedge (p \wedge q)$ is a ____.
   A. tautology B. contingency C. contradiction

B 2. The statement $(\neg p \leftrightarrow q) \wedge \neg q$ is true when ____.
   A. p is true and q is true
   B. p is true and q is false
   C. p is false and q is false
   D. p is false and q is true

A 3. $(q \vee \neg q) \vee (q \vee \mathbf{T})$ is a ____.
   A. tautology B. contingency C. contradiction

B 4. $p \wedge \neg(p \vee (p \wedge \mathbf{T}))$ is always ____.
   A. true B. false

D 5. Which of the following statements is correct?
   A. $(p \vee q) \vee r \equiv p \vee (q \vee r)$ ✓
   B. $p \vee q \equiv q \vee p$ ✓
   C. $\neg(p \wedge q) \equiv \neg p \vee \neg q$ ✓
   D. all (A, B, and C) ✓

B 6. $p \leftrightarrow q$ is logically equivalent to ____.
   A. $(p \wedge q) \rightarrow (q \wedge p)$
   B. $(p \rightarrow q) \wedge (q \rightarrow p)$
   C. $(p \rightarrow q) \rightarrow (q \rightarrow p)$
   D. $(p \rightarrow q) \vee (q \rightarrow p)$

A 7. If p is false, q is true, then $p \rightarrow q$ is true.
   A. True B. False

For numbers 8 to 10, use the following choices:

A. $r \wedge p$ C. $p \vee (r \vee q)$ E. $r \rightarrow p$
B. $p \rightarrow r$ D. $(p \vee q) \wedge (r \vee q)$

D 8. $(p \wedge r) \vee q$
E 9. $\neg r \vee p$
B 10. $r \vee \neg p$

$\neg p \wedge \neg q \wedge p \wedge q$
$F \wedge F$

$(\neg p \leftrightarrow q) \wedge \neg q$
$(\neg p \rightarrow q) \wedge (q \rightarrow \neg p) \wedge \neg q$
$(p \vee q) \wedge (\neg q \vee \neg p) \wedge \neg q$
$(p \vee q) \wedge \neg q$
$(p \wedge \neg q) \vee F$

$(q \vee \neg q) \vee (q \vee T)$
T

$p \wedge \neg (p \vee (p \wedge T))$
$p \wedge \neg (p \vee p)$
$p \wedge \neg p \wedge \neg p$
F

II. (*7 pts*) Write **T** if the statement is true or **F** if false.

F 1. By Identity law, $\neg a \vee a \equiv \mathbf{T}$. NEGATION
T 2. By Commutative law, $\neg b \wedge c \equiv c \wedge \neg b$.
F 3. By Negation law, $(a \vee \neg b) \wedge b \equiv a \vee \mathbf{F}$.
F 4. By De Morgan's law, $p \to q \equiv \neg(p \to q)$.
F 5. By Distributivity law, $p \wedge (\neg q \wedge \neg p) \equiv (p \wedge \neg q) \wedge (q \wedge \neg p)$.
T 6. By Material Implication law, $\neg p \vee (r \vee \neg p) \equiv p \to (p \to r)$.
T 7. By Associative law, $a \wedge (b \wedge \neg c) \equiv (a \wedge b) \wedge \neg c$.

III. (*10 pts*) Complete the solution below to simplify the given expression by filling in the appropriate expression/s based on the indicated rule applied, or by indicating the rule applied (Do not use abbreviations).

Expresssion	Rule Applied
$(p \oplus q) \wedge (\neg p \wedge q)$	given
$\equiv \neg(\underline{\ p \leftrightarrow q\ }) \wedge (\neg p \wedge q)$	xor
$\equiv \neg((\underline{\ p \to q\ }) \wedge (\underline{\ q \to p\ })) \wedge (\neg p \wedge q)$	Material Equivalence
$\equiv (\underline{\neg(p \to q)} \vee \underline{\neg(q \to p)}) \wedge \neg(\underline{\ p \vee \neg q\ })$	De Morgan's
$\equiv (\neg(p \to q) \vee \neg(q \to p)) \wedge \neg(q \to p)$	MATERIAL IMPLICATION
$\equiv \underline{\neg(q \to p)}$	ABSORPTION

IV. (*10 pts*) Given $(a \vee b) \to (\neg b \wedge (b \leftrightarrow \neg a))$

1. Simply using **Rules of Logical Equivalences**. Show your complete solution. Do not forget to indicate the rule/s applied at each step. Do not use abbreviations in writing the rules.
2. Based on your answer in 1, determine if the given expression is a *tautology*, *contradiction*, or *contingency*.

V. Rules of Inference

1. Translate the argument (*5 pts*) and use Rules of Inference to derive the conclusion (*5 pts*). Show your complete solution.

   If David studies Discrete Structures, it means that he is in the Computer Science program. If David studies Automata Theory, it also means that he is in the Computer Science program. David is studying Automata Theory or Discrete Structure. Therefore, David is either studying Discrete Structures or he is in the Computer Science program.

2. (*8 pts*) Use Rules of Inference to derive the following. Show your complete solution.

   $p \leftrightarrow q$
   $p \leftrightarrow r$
   $q \vee r$
   $\therefore p$

3. (*8 pts*) Use Rules of Inference to derive the following. Show your complete solution.

   $\forall x\ (Q(x) \vee P(x))$
   $\forall x\ (S(x) \vee \neg Q(x))$
   $\forall x\ (S(x) \to \neg R(x))$
   $\exists x\ \neg P(x)$
   $\therefore \exists x\ \neg R(x)$

$(a \vee \neg b) \wedge b$
$(b \wedge a) \vee (b \vee \neg b)$
$b \wedge a$

IV.

1. $(a \vee b) \to (\neg b \wedge (b \leftrightarrow \neg a))$
   $(a \vee b) \to (\neg b \wedge (b \to \neg a) \wedge (\neg a \to b))$ MAT'L EQUIVALENCE
   $\neg(a \vee b) \vee (\neg b \wedge (\neg b \vee \neg a) \wedge (a \vee b))$ MAT'L IMPLICATION
   $\neg(a \vee b) \vee (\neg b \wedge (a \vee b))$ ABSORPTION
   $\neg(a \vee b) \vee (\neg b \wedge a) \vee (\neg b \wedge b)$ DISTRIBUTIVE
   $\neg(a \vee b) \vee (\neg b \wedge a) \vee \mathbf{T}$ NEGATION
   $\neg(a \vee b) \vee (\neg b \wedge a)$ IDENTITY
   $(\neg a \wedge \neg b) \vee (\neg b \wedge a)$ DE MORGAN'S
   $\neg b \wedge (a \vee \neg a)$ DISTRIBUTIVE
   $\neg b \wedge \mathbf{T}$ NEGATION
   $\neg b$ IDENTITY
2. $\therefore$ CONTINGENCY

V.

1. Let d = David studies Discrete Structures
   c = David is in the Computer Science program
   a = David studies Automata Theory

$d \to c$
$a \to c$
$d \vee a$
$\therefore d \vee c$

1.	$d \to c$	GIVEN
2.	$a \to c$	GIVEN
3.	$d \vee a$	GIVEN
4.	$\neg a \vee c$	MAT'L IMPLICATION (2)
5.	$d \vee c$	RESOLUTION (3, 4)

Q.E.D.

2. $p \leftrightarrow q$
   $p \leftrightarrow r$
   $q \vee r$
   $\therefore p$

1.	$p \leftrightarrow q$	GIVEN
2.	$p \leftrightarrow r$	GIVEN
3.	$q \vee r$	GIVEN
4.	$(p \to q) \wedge (q \to p)$	MAT'L EQUIVALENCE (1)
5.	$q \to p$	SIMPLIFICATION (4)
6.	$\neg q \vee p$	MAT'L IMPLICATION (5)
7.	$p \vee r$	RESOLUTION (3, 6)
8.	$(p \to r) \wedge (r \to p)$	MAT'L EQUIVALENCE (2)
9.	$r \to p$	SIMPLIFICATION (8)
10.	$\neg r \vee p$	MAT'L IMPLICATION (9)
11.	$p \vee p$	RESOLUTION (7, 10)
12.	p	IDEMPOTENCE (11)

Q.E.D.

3. $\forall x\ (Q(x) \vee P(x))$
   $\forall x\ (S(x) \vee \neg Q(x))$
   $\forall x\ (S(x) \to \neg R(x))$
   $\exists x\ \neg P(x)$
   $\therefore \exists x\ \neg R(x)$

1.	$\forall x\ (Q(x) \vee P(x))$	GIVEN
2.	$\forall x\ (S(x) \vee \neg Q(x))$	GIVEN
3.	$\forall x\ (S(x) \to \neg R(x))$	GIVEN
4.	$\exists x\ \neg P(x)$	GIVEN
5.	$\neg P(\text{entity})$	EXISTENTIAL INSTANTIATION (4, entity $\in \mathcal{U}$)
6.	$P(\text{entity}) \vee Q(\text{entity})$	UNIVERSAL INSTANTIATION (1)
7.	$Q(\text{entity})$	DISJUNCTIVE SYLLOGISM (5, 6)
8.	$S(\text{entity}) \vee \neg Q(\text{entity})$	UNIVERSAL INSTANTIATION (2)
9.	$S(\text{entity})$	DISJUNCTIVE SYLLOGISM (7, 8)
10.	$S(\text{entity}) \to \neg R(\text{entity})$	UNIVERSAL INSTANTIATION (3)
11.	$\neg R(\text{entity})$	MODUS PONENS (9, 10)
12.	$\exists x\ \neg R(x)$	EXISTENTIAL GENERALIZATION (11) Q.E.D.

VI. (*15 pts*) Determine the correct logical translation of the following statements for the given domain using Predicates and Quantifiers.

1. Some real numbers are rational. (domain: all numbers)
2. Not every student studied for the exam. (domain: all students)
3. Not all rainy days are cold. (domain: all days)
4. Some excuses are unsatisfactory. (domain: all English texts)
5. The cube of a negative real number is always negative. (domain: all real numbers)

VII. (*10 pts*) Simplify the following expressions by applying Rules of Logical Equivalences. Make sure that no compound expression is negative. State the rule/s applied at each step (Do not use abbreviations). Show your solution and box your final answer.

1. $\neg\exists x\,\neg(P(x) \wedge Q(x) \wedge \neg(Q(x) \vee \neg P(x)))$
2. $\neg\exists x\forall y\,\neg(P(x,y) \vee Q(x,y))$

VIII. (*2 pts each*) Determine the truth values of the following statements.

1. Let $Q(x,y,z)$ be the statement $x + z = y$ where the domain of all the variables is the set of real numbers. Find the truth values of the following statements. Give one counterexample, if necessary
   A. $\forall x\forall y\exists z\ Q(x,y,z)$
   B. $\exists z\forall x\forall y\ Q(x,y,z)$
2. Let x and y be the real numbers and $P(x,y)$ denotes $xy = 27$. Find the truth values of the following statements below. Give one counterexample, if necessary
   A. $\forall x\forall y\ P(x,y)$
   B. $\forall x\exists y\ P(x,y)$

IX. Sets.

1. (*2 pts*) Let $X = \{a,b,c\}$, $Y = \{b,c,d\}$ and $Z = \{a,b,c,d,e,f\}$. Evaluate the following: $Z - (X \cap Y)$.
2. (*2 pts*) Draw a Venn diagram of the following: $A \cup (B - C)$
3. (*3 pts*) Let $S = \{a,b,c,d\}$. Evaluate the following: $\{x \mid x \in \mathcal{P}(S) \wedge |x| = 1\}$.
4. (*3 pts*) Let $A = \{1,2,3\}$ and $B = \{a,b,c\}$. Evaluate $A \times B$ and $|A \times B|$.
5. (*4 pts*) Let $A = \{1,2,3\}$ and $B = \{a,b,c\}$. Evaluate $A \times (B \cap A)$.
6. (*1 pt*) **True or False.** $\{\{one\},\{two\},\{three\}\} \subseteq \{one,two,three\}$.
7. (*3 pts*) Let $A = \{1,2,3\}$, $B = \{a,b,c\}$, $C = \{2,3,4\}$ and $D = \{b,c,d\}$. Evaluate $(A \times B) - (C \times D)$.
8. A travel agency owner wants to know how many of his customers traveled to the following places: Japan, Mainland China, Korea, and Taiwan. In her data, she found out that 30 customers went to Japan, 20 went to Korea and 15 of them traveled to both Japan and Korea. While 50 customers traveled to Mainland China and among those 50 guests, 30 of those also traveled to Taiwan. Interestingly, none of her customers who traveled to Korea or Japan went to Mainland China and vice versa.
   A. (*3 pts*) Construct a Venn Diagram of the owner's customer data.
   B. (*1 pt*) How many customers are being analyzed by the owner?
   C. (*1 pt*) How many customers traveled to Mainland China only?
   D. (*2 pts*) **True or False.** Mainland China Set $\subseteq$ Taiwan Set? If true, is the relation of the two sets a PROPER subset? If false, rewrite the relationship between the two sets.

VI.

1. Let $Q(x): x \in \mathbb{Q}$ $\quad \exists x\,(Q(x) \wedge R(x))$
   $R(x): x \in \mathbb{R}$
2. Let $S(x)$: x studied for the exam.
   $\exists x\,\neg S(x) \equiv \neg\forall x\,S(x)$
3. Let $C(x)$: x is cold. $\quad \exists x\ R(x) \wedge \neg C(x)$
   $R(x)$: x is rainy.
4. Let $S(x)$: x is satisfactory $\quad \exists x\,\neg(R(x) \wedge \neg S(x))$
   $E(x)$: x is an excuse
5. Let $C(x)$: $x<0 \rightarrow x^3<0$ $\quad \forall x\ C(x)$

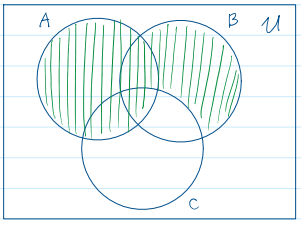
VII.

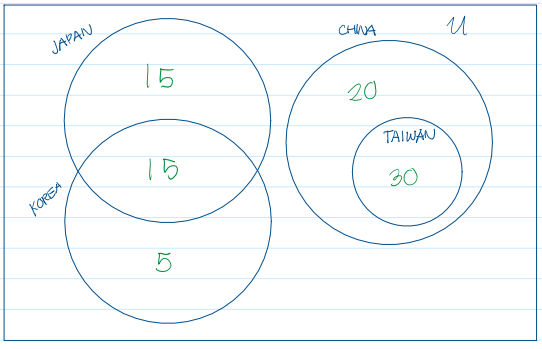
1. $\neg\exists x\ \neg(P(x) \wedge Q(x) \wedge \neg(Q(x) \vee \neg P(x)))$
   $\neg\exists x\ \neg(P(x) \wedge Q(x) \wedge \neg Q(x) \wedge P(x))$ DE MORGAN'S
   $\forall x\ P(x) \wedge Q(x) \wedge \neg Q(x) \wedge P(x)$ DE MORGAN'S
   $\forall x\,(F)$ NEGATION/DOMINATION
2. $\neg\exists x\forall y\,\neg(P(x,y) \vee Q(x,y))$
   $\forall x\exists y\ P(x,y) \vee Q(x,y)$ DE MORGAN'S

VIII.

1. Let $Q(x,y,z): x+z=y$ $\quad U = x \in \mathbb{R}$
   A. $\forall x\forall y\exists z\ Q(x,y,z) = T$
   B. $\exists z\forall x\forall y\ Q(x,y,z) = F$
2. Let $P(x,y): xy=27$ $\quad U = x \in \mathbb{R}$
   A. $\forall x\forall y\ P(x,y) = F$, $x=1$, $y=1$
   B. $\forall x\exists y\ P(x,y) = F$, $x=0$

IX.

1. Let $X = \{a,b,c\}$ $\quad X \cap Y = \{b,c\}$
   $Y = \{b,c,d\}$ $\quad Z - (X \cap Y) = \boxed{\{a,d,e,f\}}$
   $Z = \{a,b,c,d,e,f\}$
2. 

3. Let $S = \{a,b,c,d\}$
   $\{x \mid x \in \mathcal{P}(S) \wedge |x| = 1\} = \boxed{\{\{a\},\{b\},\{c\},\{d\}\}}$
4. Let $A = \{1,2,3\}$ $\quad |A \times B| = |A||B| = (3)(3) = \boxed{9}$
   $B = \{a,b,c\}$
   $A \times B = \boxed{\{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c),(3,a),(3,b),(3,c)\}}$
6. $\{\{one\},\{two\},\{three\}\} \notin \{one,two,three\}$
   $\therefore$ False
5. Let $A = \{1,2,3\}$
   $B = \{a,b,c\}$
   $B \cap A = \emptyset$
   $\therefore A \times (B \cap A) = \boxed{\emptyset}$
7. Let $A = \{1,2,3\}$
   $B = \{a,b,c\}$
   $C = \{2,3,4\}$
   $D = \{b,c,d\}$
   $A \times B = \{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c),(3,a),(3,b),(3,c)\}$
   $C \times D = \{(2,b),(2,c),(2,d),(3,b),(3,c),(3,d),(4,b),(4,c),(4,d)\}$
   $(A \times B) - (C \times D) = \boxed{\{(1,a),(1,b),(1,c),(2,a),(3,a)\}}$
8. A.


   B. 85 customers
   C. 20 customers
   D. False. Taiwan Set $\subset$ Mainland China Set