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CSMATH S16

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x^2}\right)$$

$$=\lim_{x\to 0^+}\left(\frac{x-1}{x^2}\right)$$

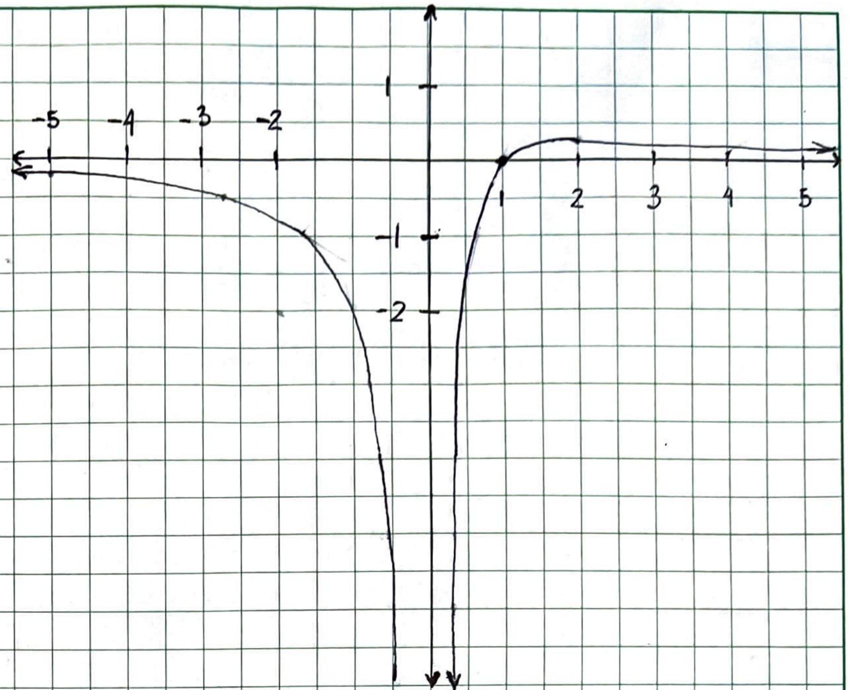
$$=\lim_{x\to 0^+}(x-1)\left(\frac{1}{x^2}\right)$$

$$\lim_{x\to 0^+}(x-1)=0-1=-1$$

$$\lim_{x\to 0^+}\left(\frac{1}{x^2}\right)=+\infty \text{ by Theorem 17 (i)}$$

$$\therefore \lim_{x\to 0^+}(x-1)\left(\frac{1}{x^2}\right)$$

$$=-\infty \text{ by Theorem 20 (ii)}$$



$\therefore$ $x=0$ is a vertical asymptote of the function

$$\lim_{x\to +\infty}\left(\frac{1}{x}-\frac{1}{x^2}\right)=0-0=0=\lim_{x\to -\infty}\left(\frac{1}{x}-\frac{1}{x^2}\right) \text{ by Theorem 22}$$

$\therefore$ $y=0$ is a horizontal asymptote of the function.

x-intercept(s)

$$\frac{1}{x}-\frac{1}{x^2}=0$$

$$\frac{x-1}{x^2}=0$$

$$x-1=0$$

$$x=1$$

$\therefore$ There is an x-intercept at the point $(1,0)$.

Because $x=0$ is a vertical asymptote, there are NO y-intercepts.

$$\lim_{z \to 0^-} \left(\frac{2 - 4z^3}{5z^2 + 3z^3} \right)$$

$$\lim_{z \to 0^-} \left(2 - 4z^3 \right) = 2$$

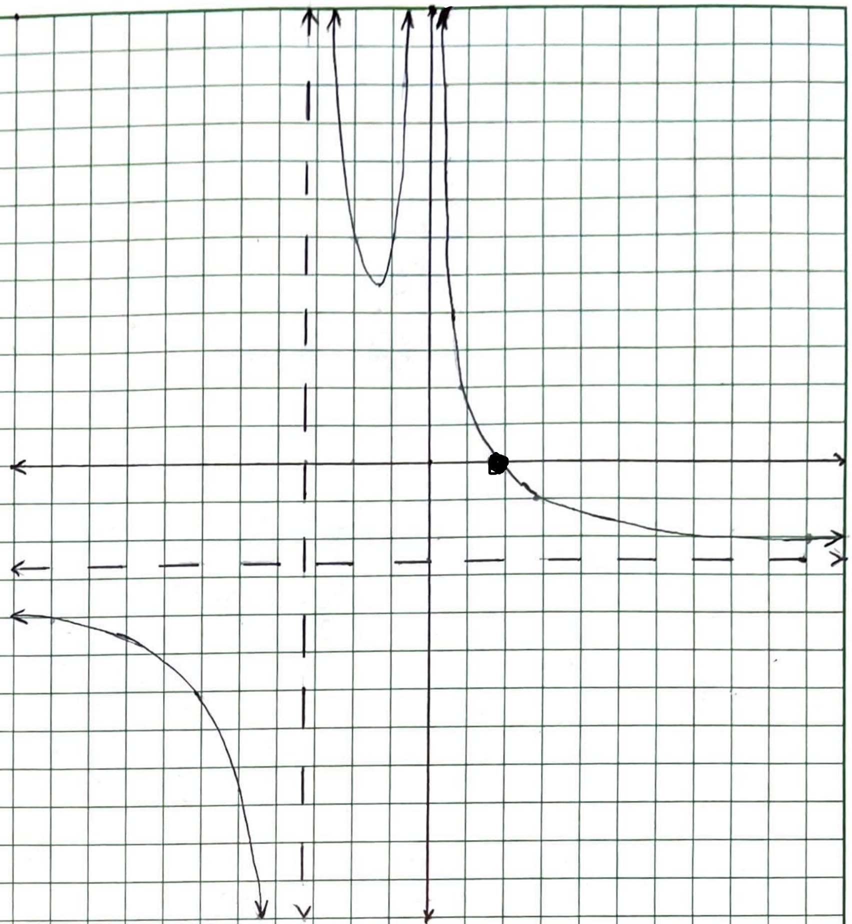
$$\lim_{z \to 0^-} \left(5z^2 + 3z^3 \right) = 0$$

$5(-0.1)^2 + 3(-0.1)^3 > 0$

$\therefore$ The function approaches 0 through positive values

$\therefore \lim_{z \to 0^-} \left(\frac{2 - 4z^3}{5z^2 + 3z^3} \right) = +\infty$ by Theorem 18 (i)

$\therefore$ $z = 0$ is a vertical asymptote of the function



$5z^2 + 3z^3 = z^2(5 + 3z)$

Since $z^2(5 + 3z) = 0$ has the following solutions:

$z = \left\{ 0, -\frac{5}{3} \right\}$

and we know that $z = 0$ is a vertical asymptote:

$$\lim_{z \to -\frac{5}{3}} \left[\frac{2 - 4z^3}{z^2(5 + 3z} \right]$$

$\lim_{z \to -\frac{5}{3}} \left(2 - 4z^3 \right) > 0$ $\quad \lim_{z \to -\frac{5}{3}} \left(5z^2 + 3z^3 \right) = 0$

The function approaches 0 through positive values

$\therefore \lim_{z \to -\frac{5}{3}} \left(\frac{2 - 4z^3}{5z^2 + 3z^3} \right) = +\infty$ by Theorem 18 (i)

$\therefore$ $z = -\frac{5}{3}$ is a vertical asymptote of the function

$$\lim_{x \to +\infty} \left(\frac{\frac{2}{z^3} - 4}{\frac{5}{z} - 3} \right) = \frac{2(0) - 4}{5(0) + 3} = -\frac{4}{3} = \lim_{x \to +\infty} \left(\frac{\frac{2}{z^3} - 4}{\frac{5}{z} - 3} \right)$$ by Theorems 2, 4, 6, 22

$\therefore$ $z = -\frac{4}{3}$ is a horizontal asymptote of the function.

Because $z = 0$ is a vertical asymptote, there are NO y-intercepts

$$\frac{2 - 4z^3}{5z^2 + 3z^3} = 0$$

$$4z^3 = 2$$

$$z^3 = \frac{1}{2} = \frac{\sqrt[3]{4}}{2}$$

$\therefore$ There is an x-intercept at the point $\left(\frac{\sqrt[3]{4}}{2}, 0 \right)$