MP Specs

Friday, February 9, 2024 12:02 AM

Applicable Sets

$\mathbf{U} : \{x \in \mathbf{Z}^+ \mid x < 3\}$ — 1, 2

$\mathbf{T} : \{x \in \mathbf{Z}^+ \mid x < 7\}$ — 1 – 6

$\mathbf{C} : \mathbf{U} \times \mathbf{U}$ — $\{(1,1), (1,2), (2,1), (2,2)\}$

$\mathbf{F} : \mathbf{T} \times \mathbf{T}$ — $\{(1,1), (1,2), \ldots, (6,5), (6,6)\}$

$\mathbf{V} : \{\text{true}, \text{false}\}$

∠ Boolean values

Each element in P is a subset of C

Each element in S is a subset of F

$\mathbf{P} : \{\{(1,1),(2,2)\},\{(1,2),(2,1)\}\}$

$\mathbf{S} : \{\{(1,1),(1,3),(2,2),(3,1),(3,3)\},$
$\{(4,4),(4,6),(5,5),(6,4),(6,6)\},$
$\{(1,5),(2,4),(2,5),(2,6),(3,5)\},$
$\{(4,1),(4,3),(5,1),(5,3),(6,1),(6,3)\}\}$

Boolean variables

System Variables

$good \in \mathbf{V}$ $\qquad over \in \mathbf{V}$ $\qquad next \in \mathbf{V}$

$C_1, C_2 \subseteq \mathbf{C}$ $\qquad F_1, F_2, F_3 \subseteq \mathbf{F}$

P? $\qquad$ S?

F_3 is all elements in F not in F_1 and F_2

System Facts

$F_3 = \mathbf{F} - (F_1 \cup F_2)$

$over \leftrightarrow ((|F_3| = 0) \vee \exists x\,(x \in \mathcal{P}(C_1) \wedge |x| > 0 \wedge x \in \mathbf{P}) \vee \exists x\,(x \in \mathcal{P}(C_2) \wedge |x| > 0 \wedge x \in \mathbf{P}))$

over = "game over"? IFF 1. F_3 is empty, or
2. there exists x in $\mathcal{P}(C_1)$ that is $\neq \emptyset$ and is in P, or
3. there exists x in $\mathcal{P}(C_2)$ that is $\neq \emptyset$ and is in P

there is a subset of C_1 or C_2 that is not null and is in P

We know that each element of P is a subset of C
C_1 and C_2 are also subsets of C

start as null sets

System Initialization

$good = \text{false}$ — true: player 1
$next = \text{false}$ — false: player 2

$C_1 = \emptyset$
$C_2 = \emptyset$

$F_1 = \emptyset$
$F_2 = \emptyset$

leads me to suspect that F_1 and F_2 get "filled up" until all elements of F are exhausted (suspecting it is like "capturing" positions)

System States and Behavior

$\texttt{NextPlayerMove}(pos \in \mathbf{F})$

$(a, b) = pos$

$c = \lfloor \frac{a-1}{3} \rfloor + 1$

$d = \lfloor \frac{b-1}{3} \rfloor + 1$

$(\neg over \wedge next \wedge pos \in F_3) \rightarrow (good = \neg good \wedge\ F_1 = F_1 \cup \{pos\})$

$(\neg over \wedge \neg next \wedge pos \in F_3) \rightarrow (good = \neg good \wedge\ F_2 = F_2 \cup \{pos\})$

$(\neg over \wedge good \wedge next \wedge |\mathcal{P}(F_1) \cap \mathbf{S}| > |C_1|) \rightarrow C_1 = C_1 \cup \{(c, d)\}$

$(\neg over \wedge good \wedge \neg next \wedge |\mathcal{P}(F_2) \cap \mathbf{S}| > |C_2|) \rightarrow C_2 = C_2 \cup \{(c, d)\}$

$(\neg over \wedge good) \rightarrow good = \neg good$

position has not been "taken" in F_1 or F_2

I will assume this is assignment

(a, b) is a position (ordered pair)
a and b are decremented, divided by 3, rounded down, then incremented

If pos is a subset of F, then (c, d) is a subset of C because $1 \leq c \leq 2$ and $1 \leq d \leq 2$

If "move" is valid, invert good and add pos to F_1

These lines occur if good is true, which will be the case after the player makes a "valid" move

$|\mathcal{P}(F_1) \cap S| > |C_1|$

Given that F_1 is all of player 1's "moves":

$\mathbf{S} : \{\{(1,1),(1,3),(2,2),(3,1),(3,3)\},$ A
$\{(4,4),(4,6),(5,5),(6,4),(6,6)\},$ B
$\{(1,5),(2,4),(2,5),(2,6),(3,5)\},$ C D
$\{(4,1),(4,3),(5,1),(5,3),(6,1),(6,3)\}\}$

$\mathcal{P}(F_1) \cap S$ is the set of all "combinations" of "moves" made by the player that are in this list

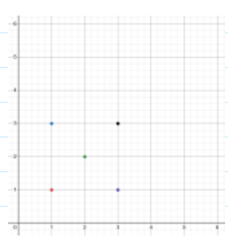
I will label them A, B, K, D:

Game must not be over

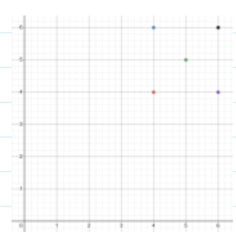
The number of completed patterns will always match the number of "captured quadrants"

If a player's move completes one of these patterns, the "area" or "quadrant" is "captured"

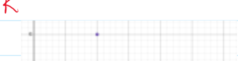
A



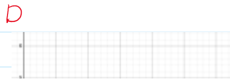
B



K



D



GameOver(over)

$\mathtt{GameOver}(over)$

$result \in \{\text{“B wins”}, \text{“A wins”}\}$

$(over \wedge next \wedge \exists x\, (x \in \mathcal{P}(C_1) \wedge |x| > 0 \wedge x \in \mathbf{P})) \rightarrow result = \text{“A wins”}$

$(over \wedge \neg next \wedge \exists x\, (x \in \mathcal{P}(C_2) \wedge |x| > 0 \wedge x \in \mathbf{P})) \rightarrow result = \text{“B wins”}$

$\neg over \rightarrow (next = \neg next)$

EXAMPLE..?

$C = \{(1,1), (1,2),$
$(2,1), (2,2)\}$

$C_1 = \underbrace{\{(1,2), (2,1)\}}$

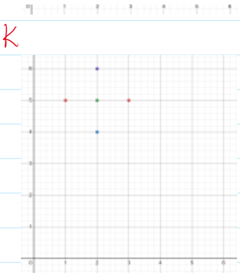
This subset, or "combination" exists in P

Therefore GAME OVER?

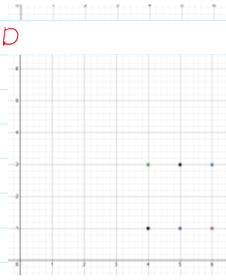
The game ends when both (1,1) and (2,2) OR both (1,2) and (2,1) are in either C_1 or C_2 (OR F_3 is empty)

When diagonally opposite "quadrants" are "captured" by the same player

K



D



If (a, b) is in A, (c, d) will be (1,1)

B ⟶ (2,2)

K ⟶ (1,2)

D ⟶ (2,1)

For reference, here are the points in C: