

$$\text{Given } f(x) = \begin{cases} 2x + 3, & \text{if } x < 1 \\ 4, & \text{if } x = 1 \\ x^2 + 2, & \text{if } x > 1 \end{cases}$$

(a) Find $\lim_{x \rightarrow 1^+} f(x)$.

x	$f(x)$
1.1000	3.2100
1.0100	3.0201
1.0010	3.0020
1.0001	3.0002

As x approaches 1 from the right, $f(x)$ approaches 3.

Thus, $\lim_{x \rightarrow 1^+} f(x) = 3$

(b) Find $\lim_{x \rightarrow 1^-} f(x)$.

x	$f(x)$
0.9000	4.8000
0.9900	4.9800
0.9990	4.9980
0.9999	4.9998

As x approaches 1 from the left, $f(x)$ approaches 5.

Thus, $\lim_{x \rightarrow 1^-} f(x) = 5$

(c) Find $\lim_{x \rightarrow 1} f(x)$.

$$\lim_{x \rightarrow a} f(x) = L \iff \lim_{x \rightarrow a^+} f(x) = L = \lim_{x \rightarrow a^-} f(x) \quad (1)$$

Given that $\lim_{x \rightarrow 1^-} f(x) = 5 \neq \lim_{x \rightarrow 1^+} f(x) = 3$, it follows that the limit of $f(x)$ as x approaches a does not exist.

$$\text{Given } f(x) = \begin{cases} x + 1, & \text{if } x < -1 \\ x^2, & \text{if } -1 \leq x \leq 1 \\ 2 - x, & \text{if } x > 1 \end{cases}$$

(c) Find $\lim_{x \rightarrow -1} f(x)$.

Given that $\lim_{x \rightarrow -1^-} f(x) = 5 \neq \lim_{x \rightarrow -1^+} f(x) = 3$, it follows that the limit of $f(x)$ as x approaches a does not exist (1).