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CSMATH 1 - S16

Find an equation of the tangent line to the curve $y = \sqrt{4x-3} - 1$ that is perpendicular to the line $x + 2y - 11 = 0$

$x + 2y - 11 = 0$

$$y = \frac{-x + 11}{2}$$

The slope of a line given by an equation of the form $y = mx + b$ is m

$\therefore$ The slope of $y = \dfrac{-x+11}{2}$ is $-\dfrac{1}{2}$

Where m_1 and m_2 are the slopes of two perpendicular lines,

$m_1 m_2 = -1$

$\therefore$ Given $m_1 = -\dfrac{1}{2}$, $m_2 = 2$

$\boxed{\therefore \text{ The slope of the line perpendicular to the line } x + 2y - 11 = 0 \text{ is } 2.}$

Let $f(x) = y = \sqrt{4x-3} - 1$

The slope of the tangent line to the graph of f at a point (x_1, y_1) is given by:

$$m(x_1) = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left(\sqrt{4(x_1+\Delta x)-3} - 1\right) - \left(\sqrt{4x_1 - 3} - 1\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{4(x_1+\Delta x)-3} - \sqrt{4x_1 - 3}}{\Delta x} \left(\frac{\sqrt{4(x_1+\Delta x)-3} + \sqrt{4x_1 - 3}}{\sqrt{4(x_1+\Delta x)-3} + \sqrt{4x_1 - 3}}\right)$$

$$= \lim_{\Delta x \to 0} \frac{4x_1 + 4\Delta x - 3 - 4x_1 + 3}{\Delta x\left(\sqrt{4(x_1+\Delta x)-3} + \sqrt{4x_1 - 3}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{4}{\sqrt{4(x_1+\Delta x)-3} + \sqrt{4x_1 - 3}}$$

$$= \frac{4}{\sqrt{4(x_1+0)-3} + \sqrt{4x_1 - 3}} = \frac{4}{2\sqrt{4x_1 - 3}} = \boxed{\frac{2}{\sqrt{4x_1 - 3}}}$$

Given that $m(x_1) = 2$:

$$\frac{2}{\sqrt{4x_1 - 3}} = 2$$

$$1 = \sqrt{4x_1 - 3}$$

$$4x_1 = 4$$

$$\boxed{x_1 = 1}$$

$$y_1 = \sqrt{4x_1 - 3} - 1$$

$$= \sqrt{4(1) - 3} - 1$$

$$\boxed{y_1 = 0}$$

$\therefore$ The tangent line to $y = \sqrt{4x-3} - 1$ with slope 2 (i.e., perpendicular to the line $x + 2y - 11 = 0$) is given by the equation:

$$y - y_1 = m(x - x_1) \equiv y - 0 = 2(x - 1)$$

$$\boxed{y = 2x - 2}$$