2. $f(x) = \begin{cases} 2x+3, & \text{if } x < 1 \\ 4, & \text{if } x = 1 \\ x^2+2, & \text{if } x > 1 \end{cases}$

a.) Find $\lim\limits_{x \to 1^+} f(x)$.

x	f(x)
1.1	3.21
1.01	3.0201
1.001	3.0020
1.0001	3.0002

As x approaches 1 from the right,
f(x) approaches 3.

Thus, $\lim\limits_{x \to 1^+} f(x) = 3$.

b.) Find $\lim\limits_{x \to 1^-} f(x)$.

x	f(x)
0.9	4.8
0.99	4.98
0.999	4.998
0.9999	4.9998

As x approaches 1 from the left,
f(x) approaches 5.

Thus, $\lim\limits_{x \to 1^-} f(x) = 5$.
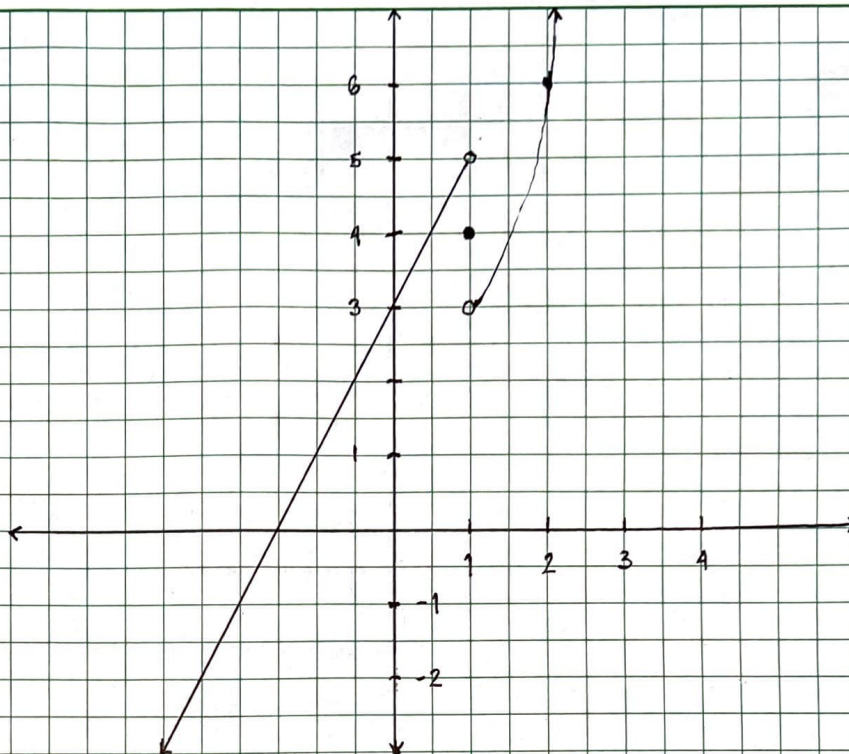
c) Find $\lim\limits_{x \to 1} f(x)$.

$\lim\limits_{x \to a} f(x)$ exists and is equal to $L \Longleftrightarrow \lim\limits_{x \to a^-} f(x) = L = \lim\limits_{x \to a^+} f(x)$.

$\lim\limits_{x \to 1^-} f(x) = 5 \neq 3 = \lim\limits_{x \to 1^+} f(x)$.

Thus, $\lim\limits_{x \to 1} f(x)$ DOES NOT EXIST.

3) $f(x) = \begin{cases} x+1, & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x \leq 1 \\ 2-x, & \text{if } x > 1 \end{cases}$

a) Find $\lim\limits_{x \to -1^+} f(x)$.

x	f(x)
-0.9	0.81
-0.99	0.9801
-0.999	0.9980
-0.9999	0.9998

As x approaches -1 from the right f(x) approaches 1.

Thus, $\lim\limits_{x \to -1^+} f(x) = 1$

b) Find $\lim\limits_{x \to -1^-} f(x)$.

x	f(x)
-1.1	-0.1
-1.01	-0.01
-1.001	-0.001
-1.0001	-0.0001

As x approaches -1 from the left, f(x) approaches 0.

Thus, $\lim\limits_{x \to -1^-} f(x) = 0$

c) Find $\lim\limits_{x \to -1} f(x)$.

$\lim\limits_{x \to -1^-} f(x) = 0 \neq 1 = \lim\limits_{x \to -1^+} f(x)$

Thus, $\lim\limits_{x \to -1} f(x)$ DOES NOT EXIST.
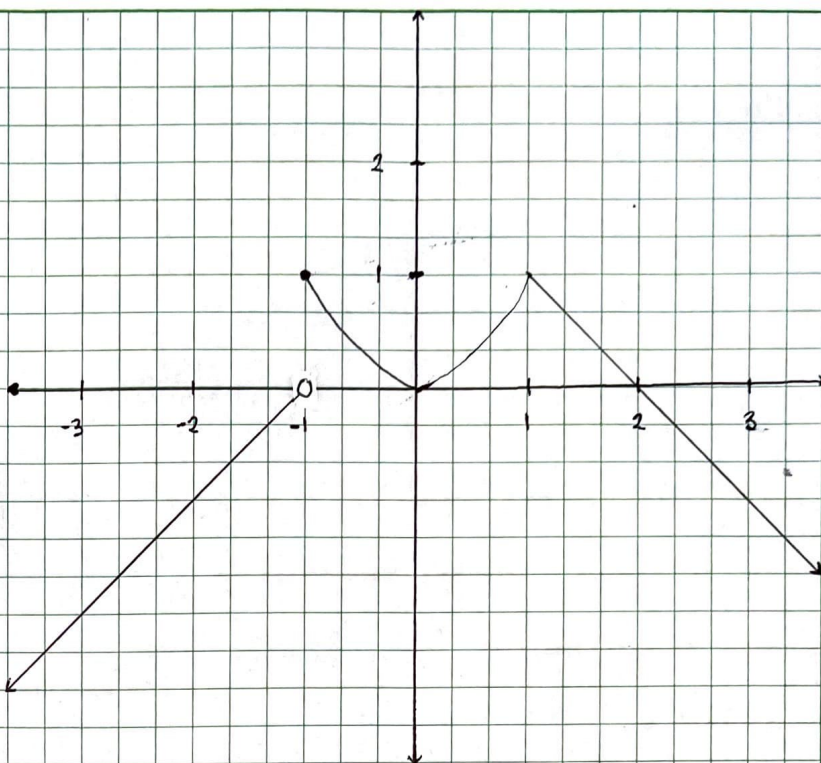
d) Find $\lim\limits_{x \to 1^+} f(x)$

x	f(x)
1.1	0.9
1.01	0.99
1.001	0.999
1.0001	0.9999

As x approaches 1 from the right, f(x) approaches 1

Thus, $\lim\limits_{x \to 1^+} f(x) = 1$.

e) Find $\lim\limits_{x \to 1^-} f(x)$

x	f(x)
0.9	0.81
0.99	0.9801
0.999	0.9980
0.9999	0.9998

As x approaches 1 from the left, f(x) approaches 1.

Thus, $\lim\limits_{x \to 1^-} f(x) = 1$

f) Find $\lim\limits_{x \to 1} f(x)$.

$\lim\limits_{x \to 1^-} f(x) = 1 = \lim\limits_{x \to 1^+} f(x)$.

Thus, $\lim\limits_{x \to 1} f(x) = 1$.