Optimizing Lot Size and Harvest Size

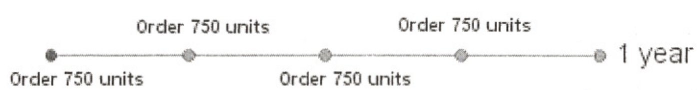
Inventory means the amount of goods held by a company at a particular time. The inventory cost means the cost of holding goods in stock.

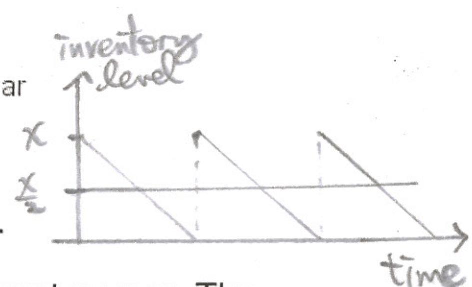
For a company, there are two kinds of costs involved in maintaining inventory. First, it is <u>storage cost</u>, eg. warehouse cost and insurance cost for goods not yet sold. Second, <u>recorder cost</u>, including delivery and bookkeeping costs for each order.

Inventory cost = Storage cost + Reorder cost

For example, the Pitt Shop expects to sell 3000 T-shirts in a year. It could order all 3000 T-shirts at once.

Or it can order 300 T-shirts in many small lots. Say 4 orders of 750 units each. The order size of each lot, is called the <u>lot size</u>.





The best lot size is one that minimizes the inventory cost.

We always assume that T-shirts are sold steadily throughout a year. The store reorders whenever stocks run out. So, throughout a year,

$$\text{average inventories} = \frac{\text{lot size}}{2}.$$

Example. The Pitt Shop expects to sell 3000 T-shirts a year. Each T-shirt costs the store \$10, and there is a fixed charge of \$20 per order. If it costs \$3 to store a T-shirt for a year, how large should each order be, and how often should orders be placed to minimize inventory costs?

Let x be the lot size.

No. of lots $= \frac{3000}{x}$

Storage cost

$$SC(x) = (\text{storage cost per item})(\text{Average no. of items})$$
$$= 3 \cdot \frac{x}{2}$$
$$= \frac{3}{2}x$$

Reorder Cost

$$RC(x) = (\text{Cost per order}) \cdot (\text{No of lots})$$
$$= (20 + 10x)\left(\frac{3000}{x}\right)$$
$$= \frac{60{,}000}{x} + 30{,}000$$

$\therefore$ Inventory cost

$$C(x) = SC(x) + RC(x)$$
$$= \frac{3}{2}x + \frac{60{,}000}{x} + 30{,}000$$

Minimize $C(x)$.

$$C'(x) = \frac{3}{2} - \frac{60{,}000}{x^2}$$
$$C''(x) = \frac{120{,}000}{x^3}$$

$$C'(x) = 0 \Leftrightarrow \frac{60{,}000}{x^2} = \frac{3}{2} \Leftrightarrow x^2 = 40{,}000$$
$$\Leftrightarrow x = 200 \text{ or } \cancel{-200} \text{ (rejected) } x > 0$$

$$C''(200) = \frac{120{,}000}{(200)^3} = 0.015 > 0$$

$\therefore C(x)$ is minimized at $x = 200$.

When $x = 200$, no. of lots $= \frac{3000}{200} = 15$

Therefore, the lot size is 200 T-shirts, and orders are placed 15 times a year.

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Similar to the retail business, in manufacturing business, a company estimates the annual demand of a product, and then chooses to manufacture all at once, or to manufacture the total in several smaller runs. These runs are called production runs.

The total costs in production consists of storage cost, just like what we did before, and the setup cost for every production run, similar to the reorder cost in previous example.

Inventory cost = Storage cost + Setup cost

Example. A book publisher estimates to sell 200 copies of a book per year. Each copy costs the publisher \$6 to print, and the setup cost is \$50 for each printing. It costs \$2 per year to store a book.

How many books should be printed per run in order to minimize costs?

> Let x be the number of books in each run
> No of production runs is $\frac{200}{x}$.

> Storage Cost
> $SC(x) = $ (Storage cost per item)(average no of item)
> $= 2 \cdot \frac{x}{2}$
> $= x$
>
> Setup Cost
> $SU(x) = $ (Cost per run) (No. of runs)
> $= (50 + 6x) \cdot \frac{200}{x}$
> $= \frac{10,000}{x} + 1,200$

> $\therefore$ Inventory Cost $C(x) = SC(x) + SU(x)$
> $C(x) = x + \frac{10,000}{x} + 1200$
>
>

Minimize $C(x)$.

$$C'(x) = 1 - \frac{10,000}{x^2}$$

$$C''(x) = \frac{20,000}{x^3}$$

$$C'(x) = 0 \Leftrightarrow \frac{10,000}{x^2} = 1 \Leftrightarrow x^2 = 10,000$$

$$\Leftrightarrow x = 100$$

$$C''(100) = \frac{20,000}{100^3} = 0.02 > 0$$

$\therefore$ $C(x)$ is minimized at $x = 100$.

When $x = 100$, no. of runs $= \frac{200}{100} = 2$

Therefore, 100 books should be printed per run, [and there are 2 production runs in a year].

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The third application of optimization in this section, is about finding maximum sustainable yield. In fishing (or hunting) industry, we get fish from the mother nature, but it is important to keep the animal population as large as before hunting every year.

The maximum amount of fish that we can harvest this year, while the animal population can still return to the previous level in the next year, is called the maximum sustainable yield.

Reproduction function. A reproduction function $f(p)$ gives the animal population a year from now if the current population is p.

Given reproduction function $f(p)$ and the current population of size p, the amount of growth in the population during this year is

$$(Amount\ of\ growth) = f(p) - p\,.$$

Sustainable yield. For a reproduction function $f(p)$, the sustainable yield is

$$Y(p) = f(p) - p\,.$$

$Y(p)$ is the function to be maximized in our concern.

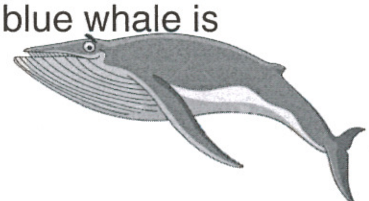
Maximum sustainable yield. For reproduction function $f(p)$, the population p that results in the maximum sustainable yield is the solution to

$$f'(p) = 1\,,$$

provided that $f''(p) < 0$. The maximum sustainable yield is then

$$Y(p) = f(p) - p\,.$$

Example. The reproduction function for the Antarctic blue whale is estimated to be



$$f(p) = -0.0004p^2 + 1.06p$$

where p and $f(p)$ are in thousands. Find the population that gives the maximum sustainable yield, and the size of the yield.

Solution

$f(p) = -0.0004\,p^2 + 1.06\,p$

$f'(p) = -0.0008\,p + 1.06$

$f''(p) = -0.0008$

$f'(p) = 1 \iff -0.0008p + 1.06 = 1$

$0.0008p = 0.06$

$p = \dfrac{0.06}{0.0008} = 75$

$f''(75) = -0.0008 < 0$

$\therefore$ the yield function $Y(p) = f(p) - p$ is maximized at $p = 75$.

When $p = 75$,

$Y(75) = \left(-0.0004\cdot(75^2) + 1.06(75)\right) - 75$

$= 77.25 - 75 = 2.25$

Therefore, the population size for maximum sustainable yield is 75,000, and the yield is 2250 whales. #