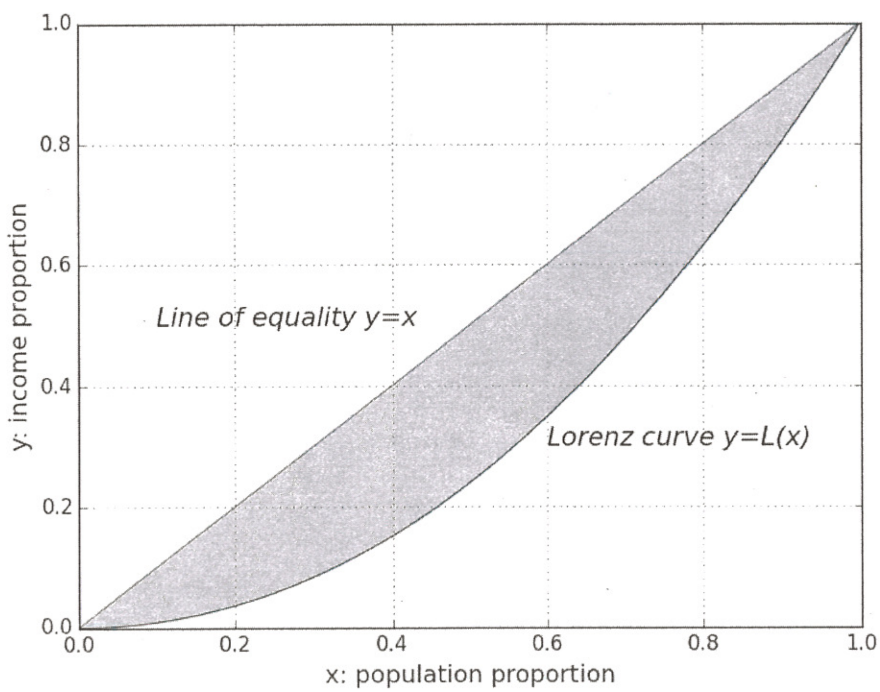
The fourth application is on income distribution. The Lorenz curve and Gini index help us measure income inequality in a city or over a country.

The Lorenz curve $L(x)$ gives us the proportion of total income earned by the lowest $x\%$ of population (or of households) in income.

Suppose in 2016, the city A carries out a census and obtains the following data about household incomes.

Proportion of population (from lowest to highest)	Proportion of income
0.2	0.032
0.4	0.1
0.6	0.226
0.8	0.43
1.0	1

For example, it means the bottom 20% of population earns 3.2% of the total income. We plot the income population against population proportion to obtain the Lorenz curve.



For convenience, we are not using the above data, but use ~~them to~~ an approximate $L(x)$ instead. We get

$$L(x) = x^{2.063}\,.$$

The line $y = x$ above is called the line of equality. Income equality is achieved if the Lorenz curve is exactly the line of equality $y = x$.

For a Lorenz curve $L(x)$, Gini index is two times the area between the line of equality and the Lorenz curve.

$$\text{Gini index} = 2\int_0^1 [x - L(x)]\,dx$$

The Gini index is always between 0 and 1.

Gini = 0 absolute equality
Gini = 1 absolute inequality

The area between the line of equality and the Lorenz curve is shaded in purple in the above graph.

Q. Find the Gini index for the given Lorenz curve.

$$L(x) = x^{2.063}$$

> $\text{Gini index} = 2\int_0^1 x - L(x)\,dx$
>
> $= 2\int_0^1 x - x^{2.063}\,dx$
>
> $= 2\left(\frac{x^2}{2} - \frac{x^{3.063}}{3.063}\right)\Big|_0^1$
>
> $= 2\left[\left(\frac{1}{2} - \frac{1}{3.063}\right) - (0)\right]$
>
> $= 2\left(\frac{1}{2} - \frac{1}{3.063}\right) = 0.347$ #
>
>
>