*Business Calculus Week 10*

Functions of Several Variables

The function is so called *a function of two variables*. These two variables are and . For each ordered pair , the function returns a real number .

Let say . It means and . We then have

The domain of a function is the set of all ordered pairs at which is well defined.

Also, the range of is the set of all values given by ordered pairs inside the domain of .

Example. The domain of is

Q. Let .   
(1) Find the domain of .  
(2) Find .

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Q. Find the domain of the function .

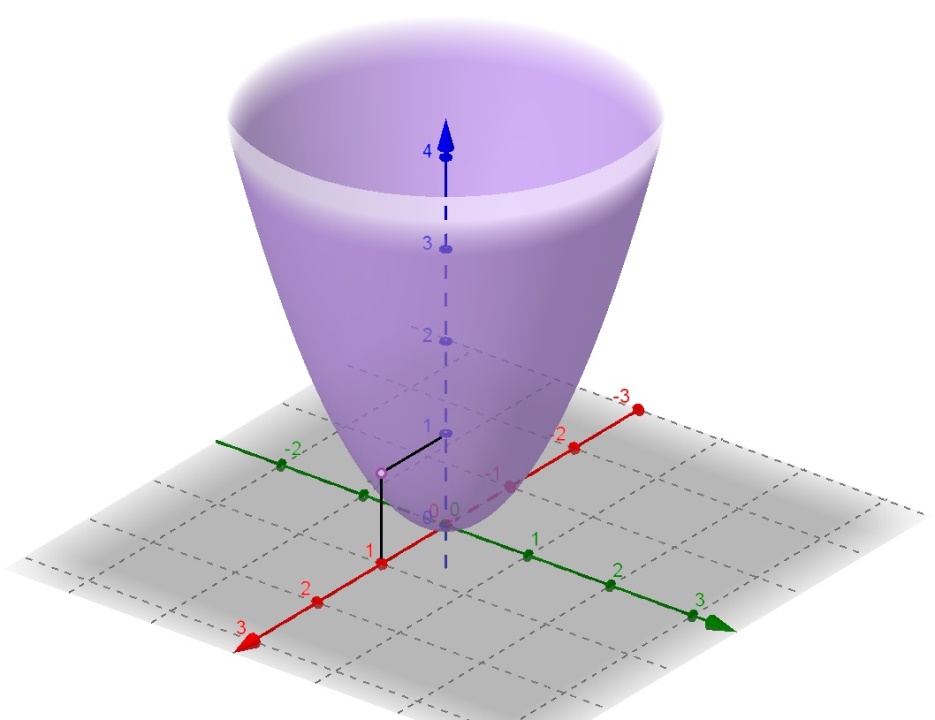
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The graph of a two-variable function is displayed on the three dimensional coordinate system. The graph of is the surface on coordinate space. For example,

In the below figure, the red line (pointing left) is the x-axis, the green line (pointing right) is the y-axis, and the blue line (pointing up) is the z-axis. The plane on the floor is the xy-plane in this (x,y,z) coordinate space.



The purple-shaded surface is the graph of , denoted by . For example, I pick the ordered pair on the xy-plane, and then draw a vertical line above until it hits the surface at a point.

We know that . So the z-coordinate of that point is . That point is .

Differentiation on a two variable function is by taking partial derivatives. Given a function , it has two partial derivatives, one coming from and other coming from , which are

Partial derivatives. At a point in the domain of .

We are finding partial derivatives by differentiation formulas.

1. When finding , we treat as a constant and differentiate everything with respect to . All differentiation formulas apply.
2. When finding , we treat as a constant and differentiate everything with respect to . All differentiation formulas apply.

There are two ways to write down partial derivatives.

Q. Let . Find and .

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Q. Let .

(1) Find and .   
(2) Find and .

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Q. Let . Find and .

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