*Business Calculus – Week 1*

Real Numbers and Inequalities

Real numbers are and so on. They are points on the number line.

0

Q. Mark the numbers and on the number line.

Inequalities express the order of real numbers. For example

|  |  |
| --- | --- |
|  | is smaller than . |
|  | is smaller than , and is smaller than . |
|  | is greater than or equal to . |
|  |  |
|  | is between and . |

Sets and intervals are commonly seen in this course.

|  |  |
| --- | --- |
|  | The set of all such that is greater than . |
|  | The set of all such that is between and . |
|  | The set of all such that is between and , including endpoints. |

Those sets on the left hand side are called intervals.

|  |  |  |
| --- | --- | --- |
|  |  | The closed interval from to . |
|  |  | The open interval from to . |
|  |  | The left-open interval from to . |
|  |  | The right-open interval from to . |

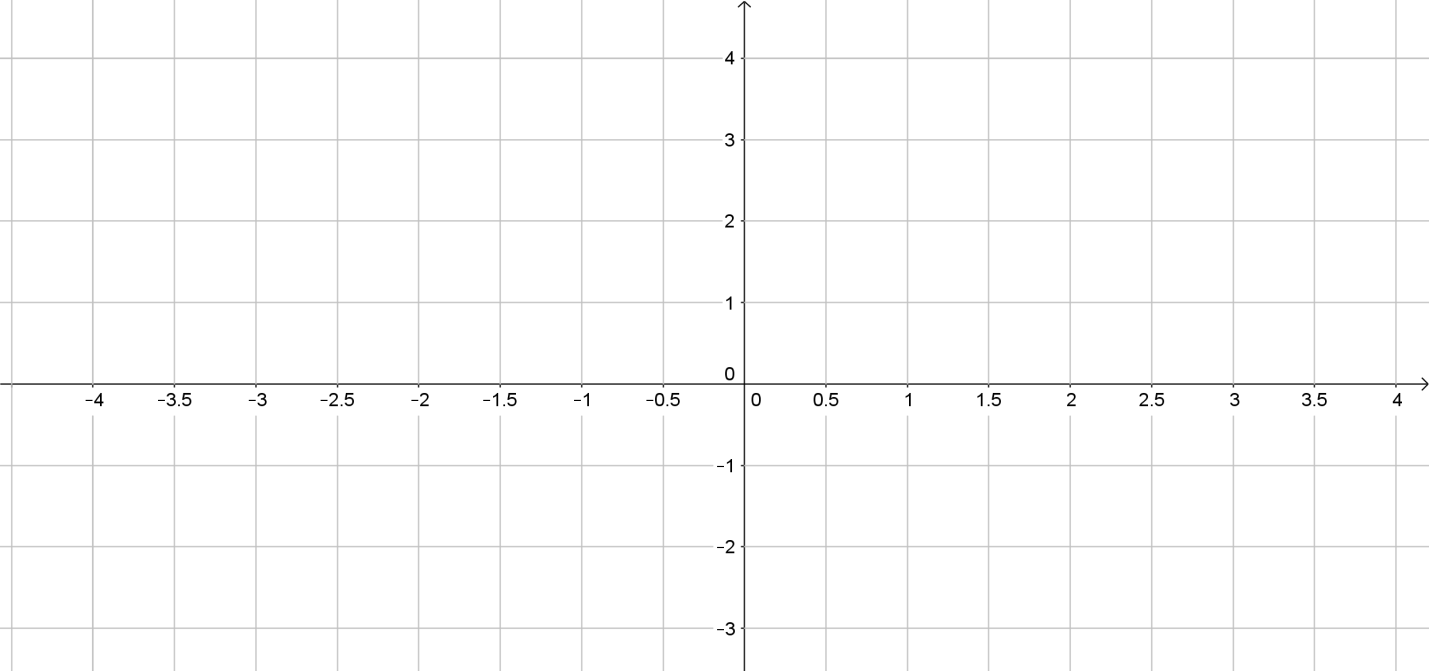
Describe intervals: http://www.mathquickeasy.com/types\_of\_intervals.html

In addition to finite intervals, we have infinite intervals.

|  |  |  |
| --- | --- | --- |
|  |  | The closed interval from to plus infinity. |
|  |  |  |
|  |  |  |
|  |  |  |

\*\*You may hear that is called the closed ray from to plus infinity.

On the Cartesian plane, a point is specified uniquely by an ordered pair . is the x-coordinate, and is the y-coordinate of that point.



Q.Mark and label the point (1,2), (-2,3), (-3,-3) and (3,-1).

Two points and determine a line on the Cartesian plane. The slope of the line through and is

The terms and are called the change in and the change in . If we put to the slope formula, we get . In other words, slope is the amount that the line rises when is increased by.

Q. Find the slope of the line through pairs of points below. Graph the lines.

1. (2,1) and (3,4)
2. (-1,10) and (5,7)

>

>

>

>

>

>

>

>

Very often we have to find the equation of a line. There are two forms of a line equation.

1. Slope-intercept form of a line.

If is the slope of a line and is the y-intercept of the line, then the line equation is given by

1. Point-slope form of a line.

If is the slope of a line and is a point on the line, then the line equation is given by .

A general linear equation of a line is in the form of for some constants . Whenever you see this equation, immediately you know it represents a line on the Cartesian plane.

We also have the following facts about line equations.

1. A horizontal line is given by the equation .

A vertical line is given by the equation .

The slope of a horizontal line is , and that of a vertical line is undefined.

1. Two lines and are   
   (a) parallel to each other if they have the same slope, i.e. .

(b) perpendicular to each other if .

Q. Write down the linear equation of the line passing through (5,3)   
and (7,-1) in the slope-intercept form. >

>

>

>

>

>

>

Q. Given the linear equation , find the slope and the y-intercept . Draw the graph.

>

>

>

>

>

>

Exponents

For any positive integer , for many on the right hand side. In the term , is called the base, and is called the exponent or power.

|  |  |
| --- | --- |
| Formula | Example |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

When , we have the below formulas for zero and negative exponents.

|  |  |
| --- | --- |
| Formula | Example |
|  |  |
|  |  |
|  |  |

Formulas for roots and fractional exponents are stated below.

|  |  |
| --- | --- |
| Formula | Example |
|  |  |
|  |  |
|  |  |
|  |  |

Q. Evaluate each expression.

>

>

>

>

>

>

Q. Write each expression in power form for numbers and .

a.

b.

>

>

>

>

>

>

>

>

Q. Simplify the expression. .

>

>

>

>

>

>

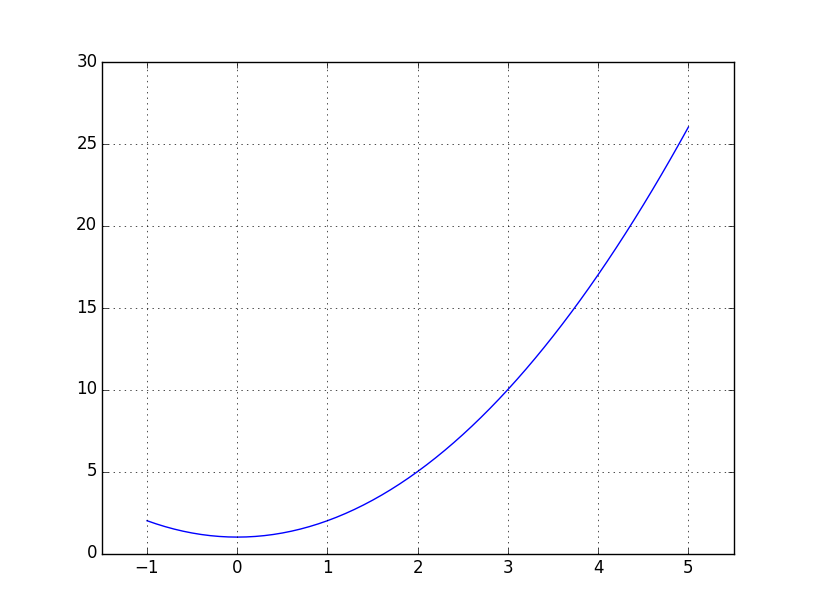
Linear and Quadratic Functions

A function is assigning each number to another number . For example, the function assigns to ,   
 to and so on.

Domain of = the set of all values at which is well defined.

Range of = the set of all values .

Q. Let , . Find the domain and range of .



>

>

Q. Find the domain and range of the following functions.

a) .

b)

>

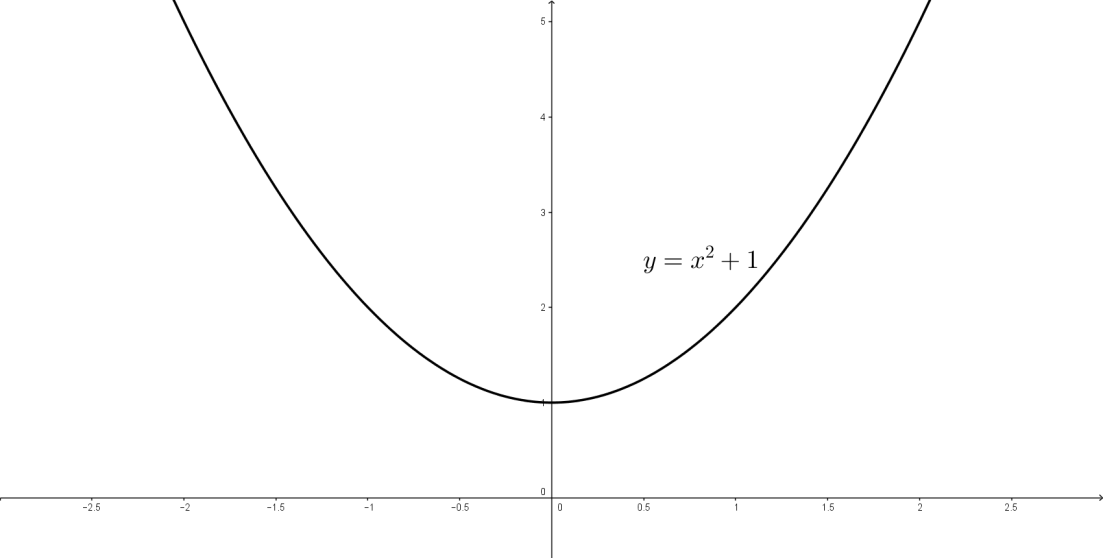
>

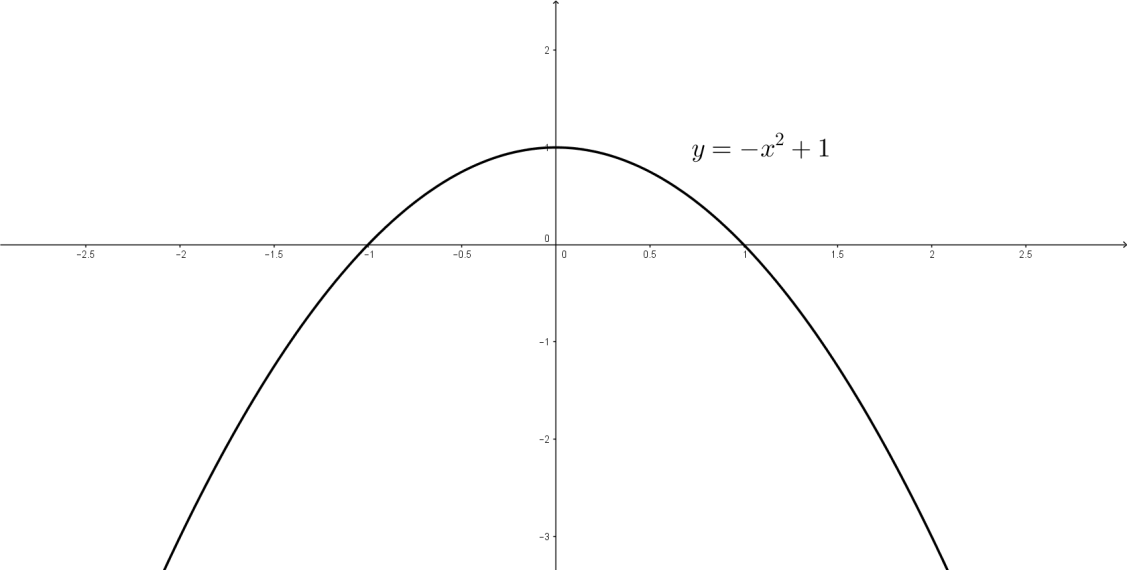
>

A linear function is a function in the form for some constants and . The graph of is a line, of slope and y-intercept .

A quadratic function is a function in the form for some constants , where . Its graph is called a parabola.

Example. The graph of and .





Q. Identify the constants in the function .

>

>

>

Suppose we have a quadratic function , .

|  |  |  |
| --- | --- | --- |
| *If* | *then* | Example |
|  | opens upward. |  |
|  | opens downward. |  |

The vertex of the parabola , has x-coordinate .  
Therefore, its vertex is the point .

Q. Find the x-coordinate of the vertex of .

>

>

>

>

Solving a quadratic equation is exactly finding the x-intercept(s) of the quadratic function .

There are two ways of solving a quadratic equation. By (1) factorization, or by (2) quadratic formula, which is stated as follows.

The solution to is

whenever

Q. Solve the following quadratic equations.

a.

b.

>

>

>

>

>

>

Q. Find the domain of the function .

>

>

>

>

>

>

Example. (P.40) A company that installs automobile CD players finds that, if the company installs CD players per day, then its costs will be  
 and its revenue will be   
(both in dollars). Find the profit function of the company.

>

>

>

Functions: Polynomial, Rational and Exponential

Polynomials are functions in the forms

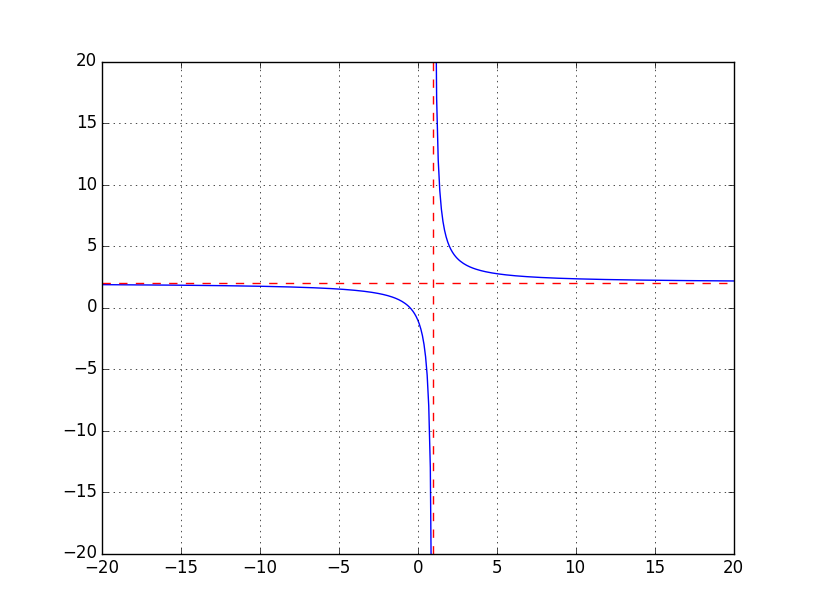
.

For example, is a polynomial of degree 3.

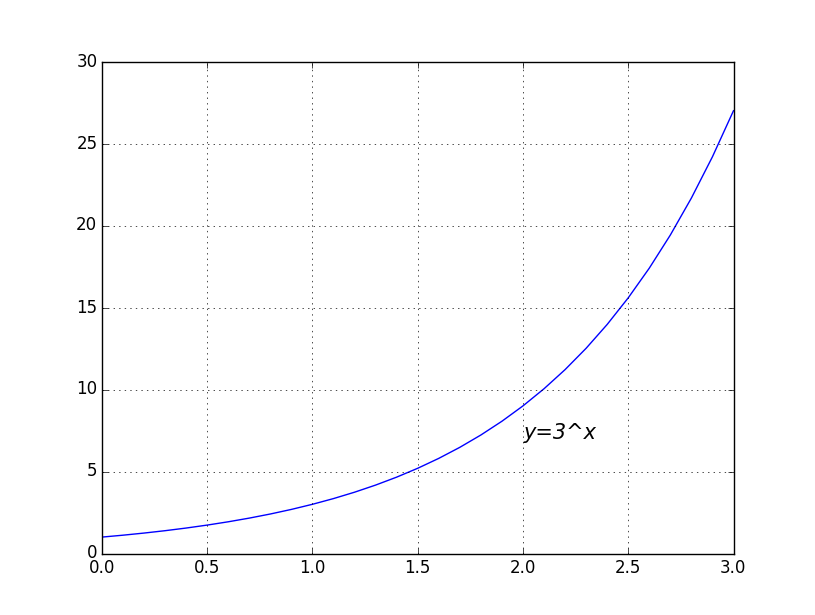
A rational function is a quotient of two polynomials. For example,

The domain of a rational function is the set of numbers at which the bottom polynomial is non-zero. Using the above rational function,

.



An exponential function is a function in the form for some   
non-zero number . For example, or .



The absolute value function is

Q. Draw the graph of the absolute value function.

>

>

>

A piecewise linear function is a function defined by different formulas on different intervals. The function above is a good example of piecewise linear functions.

Q. Express in the form of a piecewise linear function.

>

>

A composite function is a function defined by the composition of two functions. Suppose we have two functions and .

The composition of with , (or the function ) is defined by  
For example, . We are finding (1) g(1) first, and then (2) use x=g(1)=2 as an input and put it to the formula f(x)=x+1. In general,

Note   
 and are basically two different functions.

Q, Let . Find .

>

>

>

>

The difference quotient of a function is the quantity

The top is the change in when increases to . The bottom captures the change in , which is by default.

Q. Let . Find its difference quotient and simplify your answer.

>

>

>

>