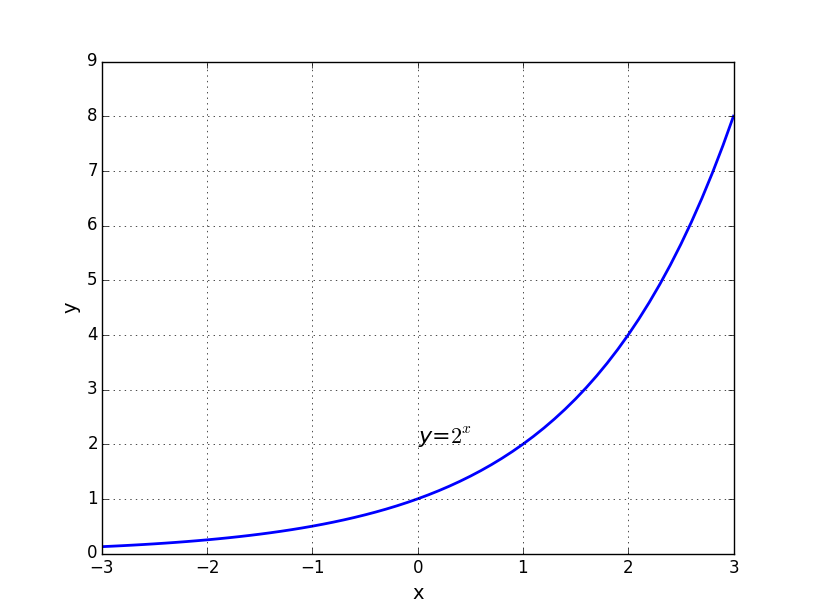
*Business Calculus - Week 7*

Exponential & Logarithmic Functions

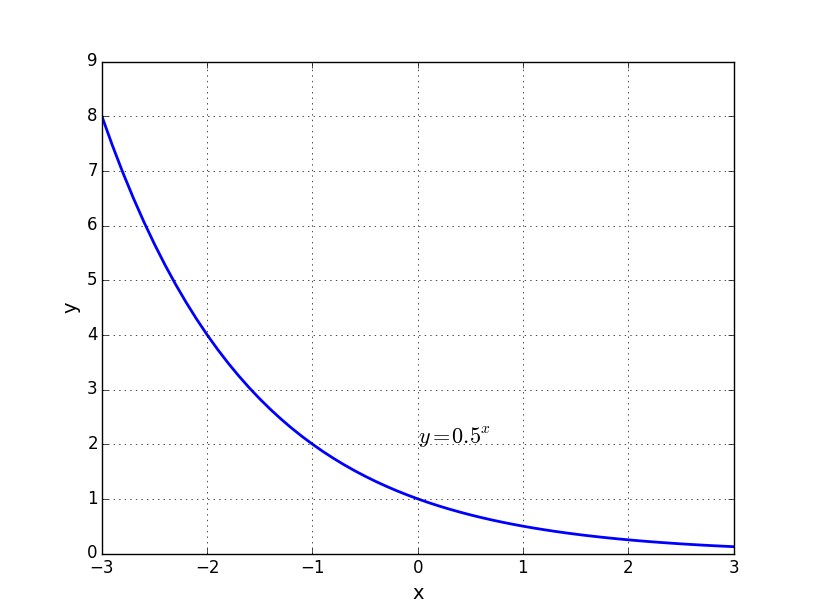
The exponential function with base and exponent is

We require . When this constant , we have for all .

When , for example , the graph of is plotted below.



On the other hand, when , for example , the graph is as below.



*Compound interest* means that, the interest is counted multiple times within a year. Suppose a bank offers an annual interest rate at compounded quarterly, and suppose your starting investment, *the principal*, is .

Starting from January, you put your money in the bank. After a quarter, that means we are on the first day of April, your amount in the bank, principal plus interest, is

Keeping this amount in the bank, on the first day of July, the bank gives you another round of interest. At this moment you have

in your bank account. In this fashion, you also get your interest on the first day of October and on the first day of 2017. In each quarter, the interest rate is .

Compound interest. For dollars invested at annual interest rate compounded times a year for years,

The number is . We have

The bank can offer an annual interest *compounded continuously* at certain rate. It means that at every moment, you gain the interest.

Continuous compounding. For dollars invested at annual interest rate compounded continuously for years

The number gives us a family of exponential functions. For example,

which will appear in the following sections.

Q. Find the value of $1000 deposited in a bank at interest for 8 years compounded

(a) annually, (b) quarterly, (c) continuously.

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In addition to the interest gained after certain years under an investment, we can reverse the order and look at the present value of a future payment at a certain annual interest rate, compounded several times a year.

Present value. For a future payment of dollars at annual interest rate compounded times a year to be paid in years,

For a future payment of dollars at annual interest rate compounded continuously to be paid in years,

Logarithm is the inverse of exponent. For example, we know

so we have . The operator is the logarithm to the base 10. if we know that .

For the logarithm to the base ,

Example. Find .

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The natural logarithm refers to the logarithm to the base . It is denoted by the symbol .

There are many properties of the natural logarithm.

Properties of .

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Q. Simplify the following functions.

1.

2.

3.

4.

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Q. A bank account grows at 8% compounded quarterly. How long will it take to double the amount? [Use natural logarithm to find the number of years . Leave in your answer and simply the result.]

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Differentiation of Logarithmic & Exponential functions

In this section, we are differentiating these two functions and .

9. Derivative of

10. Derivative of

11. Derivative of

12. Derivative of

In particular, for any constant , in item (12).

Q. Differentiate the following functions.

(a)

(b)

(c)

(d)

(e)

(f)

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We always have

Differentiation formula (13)

Differentiation Formula (14)

Q. The United States population (in millions) is predicted to be

where is the number of years after 2013. Find the instantaneous rate of change of population in 2023.

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Relative Rates and Elasticity of Demand

The relative rate of change of a function at a number is the derivative divided by .

Example. We know that GDP stands for gross domestic product.

For example, the GDP of the United States is 17419 billion dollars in 2014. Suppose that it is increasing at a *relative rate* of 3% per year. Then, the instantaneous rate of change of GDP in 2014 is

It is simpler to talk about the relative rate than the actual rate of change in this situation.

In 2014, the GDP of Canada is 1785 billion dollars. Suppose the Canada GDP is increasing at the instantaneous rate of 36 billion dollars per year. Then, the relative rate of change of Canada is

So we may say that in 2014, the US GDP is growing faster than the Canada GDP, by comparing their relative rates of change.

If is in dollars while is in years, is in dollars per year. On the other hand, the relative rate is in “per year”.

The demand function gives the quantity of a product that will be demanded by consumers if the price is .

Elasticity of demand () is the percentage change in demand divided by the percentage change in price.

We have the following.

For a demand function , the elasticity of demand is

Demand is elastic if . Demand is inelastic if .

We say that demand is unit-elastic if .

Elasticity and Revenue.

1. If demand is elastic , lower the price to increase revenue.
2. If demand is inelastic , raise the price to increase revenue.

At maximum revenue, the elasticity of demand must equal 1.

Q. The demand function for a newspaper is ,   
where is the price in cents. The publisher currently charges 40 cents. Should it raise or lower the price to increase revenue?

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