*Business Calculus - Week 8*

Antiderivatives and Indefinite Integrals

Anti-differentiation, or finding anti-derivatives, is to recover a function from its derivative. For example, the derivative of is , so we say:

An antiderivative of is .

Note and are also antiderivatives of . The most general antiderivative of is , where is a constant.

The indefinite integral of a function is the most general antiderivative of , written as

We have the following.

Example. .

We say that the indefinite integral of is .

Is called the integral sign. is the integrand in this example. The constant is called an arbitrary constant, since it can be any value.

Note that we exactly have .

We use integration rules to do integration.

1. Constant rule for integration.

In particular, we have .

2. Power rule for integration.

Example.

Q. Find the indefinite integral

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Q. Find the indefinite integral

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3. Constant-multiple rule for integration. For any constant ,

For example, .

4. Sum-Difference rule for integration. For two functions and ,

Example. Find the indefinite integral,

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Q. Find .

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Q. Find .

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Q. Find .

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Q. Find  
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Q. Find

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There are some application problems in this section.

Example. [Recovering cost function from marginal cost function]

A company’s marginal cost function is

where is the number of units, and fixed costs are $4000. Find the cost function.

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Long time ago we learnt that, if there is a car riding on a straight line,

then we have and .

In this manner, we can recover the distance travelled function by velocity function .

Q. A Porsche 997 Turbo Cabriolet can accelerate from a standing start to a speed of feet per second, after seconds   
.

1. Find a formula for the distance that it will travel from the starting point in the first seconds. [Hints: integrate velocity to find distance, and then use the fact that distance is 0 at time .]

2. Use the formula in (1) to find the distance that the car will travel in the first 10 seconds.

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Integration using Logarithmic and Exponential Functions

5. Integral of .

When is a constant, we have

6. Integral of .

The absolute sign in appears so that the above formula works for positive -values and negative -values.

Q. Find .

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Q. Find .

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Q. Find .

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Q. Find the indefinite integral

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We have three relevant application problems in this section.

Example. [Epidemic, textbook P.321]

An influenza epidemic hits a large city and spreads at the rate of new cases per day, where is the number of days since the epidemic began. The epidemic began with 4 cases.

1. Find a formula for the total number of flu cases in the first days of the epidemic.

2. Use your formula to find the number of cases during the first 30 days.

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Example. [Total sales]

A college bookstore runs a sale on its least popular mathematics books. The sales rate (books sold per day) on day of the sale is predicted to be

where corresponds to the beginning of the sale, at which time none of the inventory of 350 books had been sold.

1. Find a formula for the number of books sold up to day .

2. Will the store have sold its inventory of 350 books by day ?

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Example. [Consumption of natural resources, textbook P.324]

The annual world consumption of silver is predicted to be thousand metric tons per year, where is the number of years since 2014.

Find a formula for the total silver consumption within years of 2014 and estimate when the known world reserves of 540 thousand metric tons will be exhausted.

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