2025 CFA®
Exam Prep

SchweserNotes™
Fixed Income and Derivatives

Level I Book 3

KAPLAN SCHWESER

Book 3: Fixed Income and Derivatives

SchweserNotes™ 2025

Level I CFA®

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SCHWESERNOTES™ 2025 LEVEL I CFA® BOOK 3: FIXED INCOME AND DERIVATIVES

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READING 47

FIXED-INCOME INSTRUMENT FEATURES

MODULE 47.1: FIXED-INCOME INSTRUMENT FEATURES

Video covering this content is available online.

LOS 47.a: Describe the features of a fixed-income security.

Major types of fixed-income instruments include **loans**, which are private (nontradable) agreements between a borrower and lender, and **bonds** (or **fixed-income securities**), which are standardized, tradable securities representing a debt investment.

Investors in bonds are lending capital (referred to as **principal**, **par**, or **face value**) to the issuer of the bond. The issuer of the bond promises to repay this principal amount plus interest, typically in the form of a regular periodic **coupon** that is stated as a percentage of par. The capital raised is usually used to finance the long-term investments of the bond issuer. For a corporate issuer, loans and bonds are classified as long-term liabilities in the balance sheet.

Key features that are specified in a fixed-income security include the following:

- *Issuer*. Major issuers of bonds are sovereign national governments and corporations. Other issuers include local governments, supranational entities (e.g., the International Monetary Fund), quasi-government entities sponsored by the government (e.g., national railways), and special purpose entities, which are corporations set up to purchase financial assets and issue **asset-backed securities**, which are bonds backed by the cash flows from those assets.
- *Maturity*. The maturity date of a bond is the date on which the final cash flow is to be paid. Once a bond has been issued, the time remaining until maturity is referred to as the **tenor** of a bond. Bonds with original maturities (their tenor when they were first issued) of one year or less are referred to as **money market securities**. Bonds with original maturities of more than one year are referred to as **capital market securities**. Bonds that have no stated maturity date are called **perpetual bonds**.
- *Principal* (par or face value). The par value of a bond is the principal amount that will be repaid. Repayment of principal typically occurs at maturity, but debt instruments may specify that principal is paid back gradually over the life of the instrument, such as with a mortgage loan.

- *Coupon rate and frequency*. The coupon rate on a bond is the annual percentage of its par value that will be paid to bondholders. Some bonds make coupon interest payments annually, while others make semiannual, quarterly, or monthly payments. A $1,000 par value semiannual-pay bond with a fixed 5% coupon would pay 2.5% of $1,000, or $25, every six months.
  - Some bonds pay coupons based on a variable market rate of interest at the date of coupon payment. These bonds are called **floating-rate notes (FRNs)** or floaters. The variable market rate of interest is called the **market reference rate (MRR)**, and an FRN promises to pay the variable reference rate plus a fixed margin. This added margin is typically expressed in **basis points**, which are hundredths of 1%.
  - Some bonds pay no interest before maturity and are called **zero-coupon bonds** or **pure discount bonds**. *Pure discount* refers to the fact that these bonds are sold at a discount to their par value, and the interest is all paid at maturity when bondholders receive the par value. A 10-year, $1,000, zero-coupon bond yielding 7% would sell for a bit more than $500 initially and pay $1,000 at maturity. (In our reading on Fixed-Income Bond Valuation we will show how to calculate the exact price.)
- *Seniority*. In the event of bankruptcy or liquidation of an issuer, debt investors' claims on the issuer's assets rank above those of equity investors, making debt *senior* to equity in the capital structure of the issuer. However, not all debt claims rank equally. **Senior debt** ranks higher than **junior debt** (also called **subordinated debt**), making senior debt a less risky investment from a credit risk perspective.
- *Contingency provisions*. A bond may have an **embedded option**, such as a call option, put option, or the right to convert the debt into equity. We will describe these options in later readings.

Yield Measures

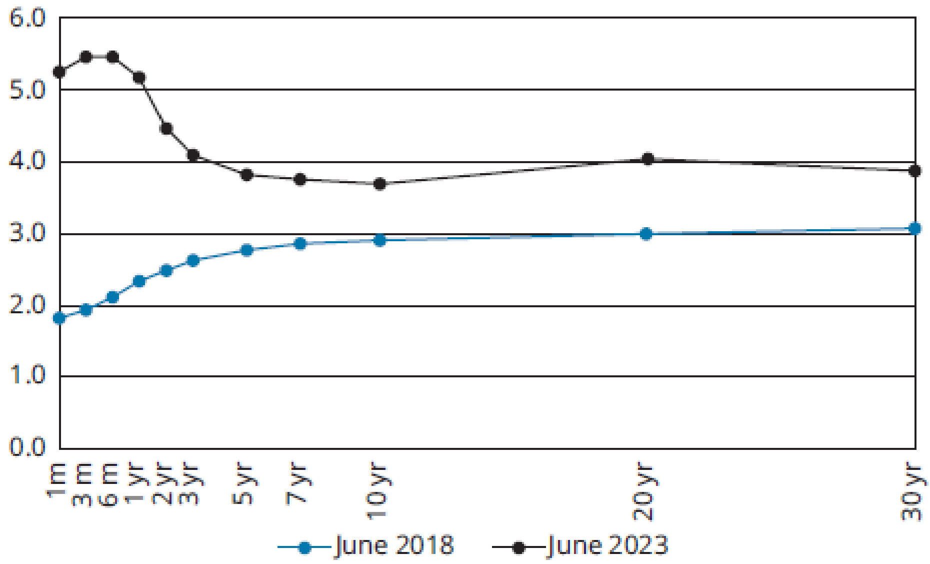
Given a bond's price and its expected cash flows, we can calculate the expected return from investing in the bond, referred to as the bond's **yield**. For a fixed-coupon bond, when prices fall, the bond offers a higher yield, and when prices rise, the bond offers a lower yield. As such, prices and yields are inversely related. We will perform yield calculations in later readings.

PROFESSOR'S NOTE

The inverse price/yield relationship for fixed-coupon bonds is a crucial concept that runs through the whole fixed income topic. If a bond with fixed cash flows is to offer a higher return (yield), the only way this is possible is through investors paying a lower price for the bond today. Hence, increasing bond yields imply decreasing bond prices, and decreasing bond yields imply increasing bond prices.

For a given issuer, we will likely find that bonds of different maturity will offer different yields. A graphical plot of these yields versus maturity is referred to as a **yield curve**. An example of yield curves for U.S. Treasury bonds is displayed in Figure 47.1.

Figure 47.1: U.S. Treasury Yield Curve



An upward-sloping yield curve (i.e., higher expected returns for longer-dated maturities), as U.S Treasuries exhibited in mid-2018, is referred to as a normal yield curve because this is the shape most frequently observed. A normal yield curve reflects investor demand for higher returns for longer-dated maturities due to higher levels of uncertainty (i.e., risk) over longer time frames. A downward-sloping yield curve, as U.S. Treasuries exhibited in mid-2023, is less common and is referred to as an **inverted yield curve**.

Government bonds are often deemed to be of the lowest credit risk (highest credit quality) in a particular market due to the fact the bonds are backed by the tax-raising powers of the government. A government bond yield curve is commonly used as a benchmark to assess the extra returns (called spreads) offered by more risky issuers, such as corporations. For example, if a 5-year corporate bond were yielding 6% and 5-year government bonds were yielding 5%, then the spread offered by the corporate bond is 6% − 5% = 1%. We will discuss credit spreads in more detail in later readings.

LOS 47.b: Describe the contents of a bond indenture and contrast affirmative and negative covenants.

The legal contract between the bond issuer (borrower) and bondholders (lenders) is called the **bond indenture**. The indenture defines obligations of, and restrictions on, the borrower, including the sources of repayment, and it forms the basis for all future interactions between the bondholder and the issuer.

Sources of Repayment

The source of the cash flows required to be paid by the bond issuer depends on the nature of the issuer and type of bond issue.

Sovereign (national government) bonds are repaid from taxes on economic activity and, in some cases, the ability of a government to create new currency. This tends to result in sovereign debt being perceived as the lowest credit risk in a particular region.

Local government bonds are repaid from local government taxes or revenue from operational infrastructure, such as toll roads.

The sources of repayment for a corporate bond depend on the type of bond issue. A **secured bond** is repaid from the operating cash flow of the company, with the added security of a legal claim (called a **lien** or **pledge**) on specific assets of the company (referred to as **collateral**) in the event of issuer default. This contrasts with an **unsecured bond**, which, having no such claim, is repaid only from the operating cash flow of the issuing company.

For an asset-backed security (ABS), financial assets held by the special purpose entity that has issued the ABS provide the cash flows promised to the ABS investors. We will discuss these in more detail in later readings.

Bond Covenants

While debt investments do not provide voting rights in the same way as an equity investment, certain legal rules known as **covenants** can be written into the bond indenture.

Affirmative covenants specify requirements the issuer must fulfill. These may require the issuer to provide timely financial reports to bondholders, specify the use of proceeds from the bond issue, or specify a bondholder's right to redeem at a premium to par if the issuer is acquired in a merger or corporate takeover.

Two examples of affirmative covenants are **cross-default** and **pari passu** provisions. A cross-default clause states that if the issuer defaults on any other debt obligation, the issuer will also be considered in default on this bond. A pari passu clause states that the bond will have the same priority of claims as the issuer's other senior debt issues.

Negative covenants place restrictions on the issuer. These can include restrictions on:

- entering into asset sales and leaseback agreements;
- pledges of collateral (the company cannot use the same assets to back several debt issues simultaneously);
- issuance of debt that ranks more senior than existing debt (referred to as a **negative pledge clause**); and
- additional borrowings, share repurchases, or dividend payments. These actions can be subject to an **incurrence test** relating to the financial ratios of the company—for example, they can only be carried out if debt/EBITDA is below a specified threshold.

Negative covenants protect the interests of bondholders and prevent the issuing firm from taking actions that would increase the risk of default. However, covenants must not be so restrictive that they prevent the firm from taking advantage of opportunities or responding appropriately to changing business circumstances.

MODULE QUIZ 47.1

1. A fixed-coupon bond will pay a coupon equal to its:
   A. yield multiplied by price.
   B. stated coupon rate multiplied by price.

C. stated coupon rate multiplied by face value.

2. When fixed-coupon bond prices fall:
   A. their yields rise.
   B. their yields fall.
   C. their coupon rates fall.
3. A bond's indenture:
   A. contains its covenants.
   B. is only required in the event of a lien on collateral.
   C. relates only to its interest and principal payments.
4. A clause in a bond indenture that requires the borrower to perform a certain action is *most accurately* described as a(n):
   A. trust deed.
   B. negative covenant.
   C. affirmative covenant.

KEY CONCEPTS

LOS 47.a

Basic features of a fixed income security include the issuer, maturity date, par value, coupon rate, coupon frequency, seniority, and contingency provisions.

- Issuers include corporations, governments, quasi-government entities, supranational entities and special purpose entities set up to issue asset-backed securities.
- Bonds with original maturities of one year or less are money market securities. Bonds with original maturities of more than one year are capital market securities. Bonds with no stated maturity are perpetual bonds.
- Par value is the principal amount that will be repaid to bondholders, usually at maturity.
- Coupon rate is the percentage of par value that is paid annually as interest. Coupon frequency may be annual, semiannual, quarterly, or monthly. Zero-coupon bonds pay no coupon interest and are pure discount securities.
- Senior debt ranks above junior (subordinated) debt should an issuer file for bankruptcy or undergo liquidation. Junior bonds with lower credit quality must offer investors higher yields to compensate for the greater probability of default.
- Contingency provisions are rights to take actions in response to some potential future event, such as the right for the issuer to call the bond back earlier than maturity.

The return earned from investing in a bond is referred to as the bond's yield. For a fixed coupon bond, there is an inverse relationship between the price and the yield (return) of the instrument. A plot of yield versus maturity for a certain issuer or class of bond is referred to as a yield curve.

The source of repayment for sovereign bonds is the country's taxing authority. For non-sovereign government bonds, the sources may be taxing authority or revenues from a project. Corporate bonds are repaid with funds from the firm's operations. Securitized bonds are repaid with cash flows from a pool of financial assets.

Bonds are secured if they are backed by specific collateral or unsecured if they represent an overall claim against the issuer's cash flows and assets.

LOS 47.b

A bond indenture is a contract between a bond issuer and the bondholders which defines the bond's features and the issuer's obligations. An indenture specifies the entity issuing the bond, the source of funds for repayment, assets pledged as collateral, credit enhancements, and any covenants with which the issuer must comply. Affirmative covenants specify actions an issuer must take, negative covenants specify restrictions on the issuer.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 47.1

1. **C** A fixed-coupon bond has a stated coupon rate that is applied to the bond's face (principal or par) value. The yield of the bond is the return earned through paying the bond's price today and holding the bond to maturity. (LOS 47.a)
2. **A** For fixed-coupon bonds, prices and yields have an inverse relationship. If the price of the bond is falling, then the return (yield) from buying the bond at the lower price is rising. (LOS 47.a)
3. **A** An indenture is the contract between the company and its bondholders and contains the bond's covenants. (LOS 47.b)
4. **C** Affirmative covenants require the borrower to perform certain actions. Negative covenants restrict the borrower from performing certain actions. Trust deed is another name for a bond indenture. (LOS 47.b)

READING 48

FIXED-INCOME CASH FLOWS AND TYPES

MODULE 48.1: FIXED-INCOME CASH FLOWS AND TYPES

Video covering this content is available online.

LOS 48.a: Describe common cash flow structures of fixed-income instruments and contrast cash flow contingency provisions that benefit issuers and investors.

A typical bond has a **bullet structure**, where principal (par value) is paid back in a single payment at maturity. Periodic payments across the life of the bond (referred to as the bond's **coupons**) are purely interest payments.

Consider a $1,000 par value 5-year bond with an annual coupon rate of 5%, issued at par. With a bullet structure, the bond's promised payments at the end of each year would be as follows.

Year	1	2	3	4	5
PMT	$50	$50	$50	$50	$1,050
Principal remaining	$1,000	$1,000	$1,000	$1,000	$0

A loan structure in which the periodic payments include both interest and some repayment of principal (the amount borrowed) is called an **amortizing loan**. If a bond (loan) is **fully amortizing**, this means the principal is fully paid off when the last periodic payment is made. Typically, automobile loans and home loans are fully amortizing loans. If the 5-year, 5% bond in the previous table had a fully amortizing structure rather than a bullet structure, the payments and remaining principal balance at each year-end would be as follows (final payment reflects rounding of previous payments).

Year	1	2	3	4	5
PMT	$230.97	$230.97	$230.97	$230.97	$230.98
Principal remaining	$819.03	$629.01	$429.49	$219.99	$0

This constant PMT can be calculated using a financial calculator:

N = 5; I/Y = 5; PV = 1,000; FV = 0; CPT → PMT = −230.97

Note that the constant yearly payment of $230.97, here, is partly interest and partly principal loan repayment. For example, in the first year, the interest component is 0.05 × $1,000 = $50; hence, the principal component is $230.97 − $50 = $180.97. The opening principal balance for the second year is, therefore, $1,000 − $180.97 = $819.03. In subsequent years, the interest component of the $230.97 will decrease and the proportion relating to principal repayment will increase.

A bond can also be structured to be **partially amortizing** so that there is a repayment of some principal at maturity (referred to as a **balloon payment**). Unlike a bullet structure, the final payment includes just the remaining unamortized principal amount rather than the full principal amount. In the following table, the final payment includes $200 to repay the remaining principal outstanding.

Year	1	2	3	4	5
PMT	$194.78	$194.78	$194.78	$194.78	$394.78
Principal remaining	$855.22	$703.20	$543.58	$375.98	$0

This constant PMT can be calculated using a financial calculator:

N = 5; I/Y = 5; PV = 1,000; ***FV*** = ***−200***; CPT → PMT = −194.78

Other types of amortization schedules include **sinking fund provisions** for bonds and **waterfall structures** for asset-backed securities (ABSs) and mortgage-backed securities (MBSs).

Sinking fund provisions provide for the repayment of principal through a series of payments over the life of a bond issue. For example, a 20-year issue with a face amount of $300 million may require that the bond trustee redeems $20 million of the principal from investors selected at random every year beginning in the sixth year.

Sinking fund provisions offer both advantages and disadvantages to bondholders. On the plus side, bonds with a sinking fund provision have less credit risk because the periodic redemptions reduce the total amount of principal to be repaid at maturity. The presence of a sinking fund, however, can be a disadvantage to bondholders when interest rates fall due to **reinvestment risk**, which is the possibility of receiving cash flows early and only being able to reinvest them at lower yields.

Waterfall structures are used to establish principal repayments to holders of ABSs and MBSs. These structured products can be split into *tranches* of varying seniority. A common waterfall structure is for junior tranches not to receive any principal payment from the collateral pool until all senior tranches have been fully repaid. Interest payments would still be made to all tranches.

There are several coupon structures besides a fixed-coupon structure. We summarize the most important ones here.

Variable Interest Debt

Some bonds pay periodic interest that depends on the prevailing market rate of interest at the time future coupon payments are made. These bonds are called **floating-rate notes (FRNs)** or **floaters**. The variable market rate of interest is called the **market**

reference rate (MRR), and an FRN promises to pay the MRR plus some fixed margin (called a **credit spread**). This added margin is typically expressed in **basis points**, which are hundredths of 1%. A 120 basis point margin is equivalent to 1.2%.

Most floaters pay quarterly coupons and are based on a quarterly (90-day) reference rate. As an example, consider an FRN that pays a quarterly interest rate of MRR plus 0.75% (75 basis points). If the annualized MRR in the current quarter is 2.3%, the bond will pay (2.3% + 0.75%)/4 = 0.7625% of its par value at the end of the quarter.

Other Coupon Structures

Step-up coupon bonds are structured so that the coupon rate increases over time according to a predetermined schedule, providing protection to investors against interest rates rising over the life of the bond.

Coupon changes could also be linked to future potential events. For example, loans to borrowers of lower credit quality (called **leveraged loans**) often have a coupon that increases if the credit quality of the issuer decreases further. For example, the term sheet of a leveraged loan might specify that the coupon to be paid is MRR + 2.5%; however, should the issuer's debt/EBITDA ratio rise above 3, then the coupon paid will increase to MRR + 3%. A similar provision is often included in a **credit-linked note**, whereby the coupon rate increases if the credit rating of the issuer deteriorates (or decreases if the credit rating improves).

A **payment-in-kind (PIK) bond** allows the issuer to make the coupon payments by increasing the principal amount of the outstanding bonds, essentially paying bond interest with more bonds. Firms that issue PIK bonds typically do so because they anticipate that firm cash flows may be less than required to service the debt, often because of high levels of debt financing (leverage). These bonds typically have higher yields because of the lower perceived credit quality implied by expected cash flow shortfalls, or simply because of the high leverage of the issuing firm.

More recently, **green bonds** have been issued whereby the coupon paid increases if certain environmental goals (for example CO_2 emissions reduction) are not met by the issuer over a specified time frame.

An **index-linked bond** has coupon payments or a principal value that is based on a specified published index. **Inflation-linked bonds** (also called **linkers**) are the most common type of index-linked bonds, which increase their cash flows in line with a specified inflation index, such as the Consumer Price Index (CPI) in the United States, to protect the real value of the cash flows promised to investors.

The different structures of inflation-indexed bonds include the following:

- **Interest-indexed bonds.** The coupon rate is adjusted for inflation, while the principal value remains unchanged. This means the principal value of the debt is not inflation-protected.
- **Capital-indexed bonds.** This is the most common structure. An example is U.S. **Treasury Inflation-Protected Securities (TIPS)**. The coupon rate remains constant, but the principal value is increased by the rate of inflation, or decreased by deflation.

In the case of deflation, TIPS investors receive the maximum of inflation-adjusted principal or the unindexed par amount at maturity.

To better understand the structure of capital-indexed bonds, consider a bond with a par value of $1,000 at issuance, a 3% annual coupon rate paid semiannually, and a provision that the principal value will be adjusted for inflation (or deflation). If, six months after issuance, the reported inflation has been 1% over the period, the principal value of the bonds is increased by 1% from $1,000 to $1,010, and the six-month coupon is calculated as 1.5% of the adjusted principal value of $1,010 (i.e., $1,010 × 1.5% = $15.15).

With this structure, we can view the coupon rate of 3% as a real rate of interest. Unexpected inflation will not decrease the purchasing power of the coupon interest payments, and the principal value paid at maturity will have approximately the same purchasing power as the $1,000 par value did at bond issuance. Thus, investors are fully protected against inflation over the life of the bond.

Zero-coupon bonds are the simplest form of fixed-income instrument, offering only a single payment of par at maturity. These bonds are popular with investors that wish to minimize reinvestment risk. With no periodic coupon, zero-coupon bonds must trade below par to offer investors a positive return.

With a **deferred coupon bond**, regular coupon payments do not begin until a specified time after issuance. These bonds may be appropriate financing for issuers with a low credit rating or with a large project that will not be completed and generating revenue for some period after bond issuance. Zero-coupon bonds can be considered the most extreme type of deferred coupon bond—and, like zero-coupon bonds, deferred coupon bonds often trade below par to provide investors with the yields they demand.

Fixed-Income Contingency Provisions

A **contingency provision** in a contract describes an action that may be taken if an event (the contingency) actually occurs. Contingency provisions in bond indentures are referred to as **embedded options**—embedded in the sense that they are an integral part of the bond contract and are not a separate security. Some embedded options are exercisable at the option of the issuer of the bond, so they are valuable to the issuer; others are exercisable at the option of the purchaser of the bond, so they have value to the bondholder.

We will discuss three types of bonds with embedded options here: callable bonds, putable bonds, and convertible bonds. Bonds that do not have contingency provisions are referred to as *straight bonds* or *option-free* bonds.

Callable Bonds

A **callable bond** gives the *issuer* the right, but not the obligation, to redeem (through buying bonds back from investors before maturity) all or part of a bond issue at a predetermined fixed price (known as a call price).

As an example of a call provision, consider a 6% 20-year bond issued at par on June 1, 2022, for which the indenture includes the following *call schedule*:

- The bonds can be redeemed by the issuer at 102% of par after June 1, 2027.
- The bonds can be redeemed by the issuer at 101% of par after June 1, 2030.
- The bonds can be redeemed by the issuer at 100% of par after June 1, 2032.

For the 5-year period from the issue date until June 2027, the bond is not callable; hence, we say the bond has five years of **call protection**.

June 1, 2027, is referred to as the *first call date* (the start of the **call period** where the bond can be called by the issuer) and the *call price* is 102 (102% of par value) between that date and June 2030. The call price declines to 101 (101% of par) after June 1, 2030. After, June 1, 2032, the bond is callable at par.

A call option has value to the issuer because it gives the issuer the right to redeem the bond early and issue a new bond (borrow) if the market yield on the bond declines. This could occur either because interest rates in general have decreased, or because the credit quality of the bond has increased (default risk has decreased).

Consider a situation where the market yield on the 6% 20-year bond has declined from 6% at issuance to 4% on June 1, 2027 (the first call date). If the bond did not have a call option, it would trade at approximately $1,224. With a call price of 102, the issuer can redeem the bonds at $1,020 each and borrow that amount at the current market yield of 4%, reducing the annual interest payment from $60 per bond to $40.80.

PROFESSOR'S NOTE

This is analogous to refinancing a home mortgage when mortgage rates fall to reduce the monthly payments.

The issuer holding the call option creates **call risk** for the bondholder. Call risk relates to the fact that bondholders face an uncertain redemption date. For a bond that is in its call period, the call price will put an upper limit on the value of the bond in the market. Due to call risk, bondholders will demand a higher yield and will pay a lower price for a callable bond than they would for an otherwise equivalent straight bond. The difference in price between a callable bond and an otherwise identical noncallable bond is equal to the value of the call option to the issuer.

Putable Bonds

A **putable bond** gives the *bondholder* the right to sell the bond back to the issuing company at a prespecified price, typically par. Bondholders are likely to exercise such a put option when the price of the bond is less than the put price because interest rates have risen or the credit quality of the issuer has fallen.

Unlike a callable bond, the embedded option for a putable bond has value to the bondholder because the choice of whether to exercise the option is the bondholder's. For this reason, a putable bond will sell at a higher price (offer a lower yield) than an otherwise equivalent straight bond. The difference in price between an otherwise identical straight bond and a putable bond is equal to the value of the put option to the bondholder.

Convertible Bonds

Convertible bonds give bondholders the option to exchange the bond for a specific number of shares of the issuing corporation's common stock. This gives bondholders the opportunity to profit from increases in the value of the common shares. Because the conversion option is valuable to bondholders, convertible bonds can be issued with higher prices (and, therefore, lower yields, which is an advantage to the issuer) compared to otherwise identical straight bonds.

Some terms related to convertible bonds are as follows:

- **Conversion price.** This is the par amount per share at which the bond may be converted to common stock.
- **Conversion ratio.** This is equal to the par value of the bond divided by the conversion price. If a bond with a $1,000 par value has a conversion price of $40, its conversion ratio is 1,000 / 40 = 25 shares per bond.
- **Conversion value.** This is the market value of the shares that would be received upon conversion. A bond with a conversion ratio of 25 shares when the current market price of a common share is $50 would have a conversion value of 25 × $50 = $1,250.

PROFESSOR'S NOTE
Valuation of convertible bonds is addressed in the Level II CFA curriculum.

Warrants

An alternative way to give bondholders an opportunity for additional returns when the firm's common shares increase in value is to attach **warrants** to straight bonds when they are issued. Warrants give their holders the right to buy the firm's common shares at a fixed price over a given period. As an example, warrants that give their holders the right to buy shares for $40 will have value if the common shares increase in value above $40 before the warrants expire. Warrants can be detached from the bond issue and traded on securities exchanges.

Contingent Convertible Bonds

Contingent convertible bonds (referred to as *CoCos*) are bonds that convert from debt to common equity automatically if a specific event occurs. This type of bond has been issued by some European banks. Banks must maintain specific levels of equity financing. If a bank's equity falls below the required level, they must somehow raise more equity financing to comply with regulations. CoCos are often structured so that if the bank's equity capital falls below a given level, they are automatically converted to common stock. This has the effect of decreasing the bank's debt liabilities and increasing its equity capital at the same time, which helps the bank to meet its minimum equity requirement.

LOS 48.b: Describe how legal, regulatory, and tax considerations affect the issuance and trading of fixed-income securities.

Bonds are subject to different legal and regulatory requirements that depend on where they are issued and traded. Bonds of issuers domiciled in the same country as the market in which the bonds are issued and traded are referred to as **domestic bonds**. Bonds of issuers from countries other than the market in which the bond trades are referred to as **foreign bonds**. For example, a U.K. company raising U.S. dollars to expand their U.S. operations by issuing bonds that trade in the United States is a foreign bond issuance.

Eurobonds are issued outside the jurisdiction of any one country and can be issued in any currency. They are subject to less regulation than domestic bonds in most jurisdictions and were initially introduced to avoid U.S. regulations. Eurobonds should not be confused with bonds denominated in euros or thought to originate in Europe, although they can be both. Eurobonds got the "euro" name because they were first introduced in Europe, and most are still traded by firms in European capitals. A bond issued by a Chinese firm that is denominated in yen and traded in markets outside Japan would fit the definition of a Eurobond (it would be referred to as a Euroyen bond). Eurobonds that trade in at least one domestic bond market and in the Eurobond market are referred to as **global bonds**.

Eurobonds are referred to by the currency they are denominated in. Eurodollar bonds are denominated in U.S. dollars, and Euroyen bonds are denominated in yen. At one time, most Eurobonds were issued in **bearer bond** form. Ownership of bearer bonds was not officially recorded by the issuer; hence, ownership was evidenced simply by possessing the bonds. Today Eurobonds, like most other bonds, are issued as **registered bonds** with a record of ownership.

Foreign bonds, global bonds, and Eurobonds that involve more than one country are often collectively referred to as **international bonds** to distinguish them from domestic bonds, which involve only a single country.

While domestic and international bonds are subject to varying laws and regulations in different jurisdictions and have different conventions relating to coupon frequency and calculation methods, the factor that is likely to best describe differences in yields across different markets is the currency of the bond. This is because the interest rates that determine the bond's yield will be driven by the market interest rates of the country in whose currency the bond is denominated.

Sukuk bonds are Sharia-compliant bonds with specific restrictions on the payment of interest and use of the proceeds of the bond issue to comply with Islamic law. The periodic payments on these bonds are considered to be cash flows from rent on underlying assets.

Taxation of Bond Income

Most often, the interest income paid to bondholders is taxed as ordinary income at the same rate as wage and salary income. The interest income from bonds issued by municipal governments in the United States, however, is most often exempt from national income tax and often from state income tax in the state of issue.

When a bondholder sells a coupon bond before maturity, it may be at a gain or a loss relative to its purchase price. Such gains and losses are considered capital gains income (rather than ordinary taxable income). Capital gains are often taxed at a lower rate than ordinary income. Capital gains on the sale of an asset that has been owned for more than some minimum amount of time may be classified as long-term capital gains and taxed at an even lower rate.

Zero-coupon bonds and other bonds sold at significant discounts to par when issued are termed **original issue discount (OID) bonds**. Because the gains over an OID bond's tenor as its price moves toward par value are really interest income, these bonds can generate a tax liability even when no cash interest payment has been made. In many tax jurisdictions, a portion of the discount from par at issuance is treated as taxable interest income each year.

Some tax jurisdictions provide a symmetric treatment for bonds issued at a premium to par, allowing part of the premium to be used to reduce the taxable portion of coupon interest payments.

MODULE QUIZ 48.1

1. Compared to a fully amortizing loan, an equivalent loan with a balloon payment will *most likely* have:
   A. lower regular periodic payments and a higher final payment amount.
   B. higher regular periodic payments and a lower final payment amount.
   C. lower regular periodic payments and a lower final payment amount.
2. With which of the following features of a corporate bond issue does an investor *most likely* face the risk of redemption before maturity?
   A. Floating-rate notes.
   B. Sinking fund.
   C. Term maturity structure.
3. A 10-year bond pays no interest for three years, then pays $229.25, followed by payments of $35 semiannually for seven years, and an additional $1,000 at maturity. This bond is *most likely* a:
   A. step-up bond.
   B. zero-coupon bond.
   C. deferred coupon bond.
4. Which of the following *most accurately* describes the maximum price for a currently callable bond?
   A. Its par value.
   B. The call price.
   C. The present value of its par value.
5. An investor buys a pure-discount bond, holds it to maturity, and receives its par value. For tax purposes, the increase in the bond's value is *most likely* to be treated as:
   A. a capital gain.
   B. interest income.

C. tax-exempt income.

KEY CONCEPTS

LOS 48.a

A bond with a bullet structure pays coupon interest periodically and repays the entire principal value at maturity, along with the final coupon interest payment.

A bond with an amortizing structure repays part of its principal at each payment date. A fully amortizing structure makes equal payments throughout the bond's life. A partially amortizing structure has a balloon payment at maturity, which repays the remaining principal as a lump sum.

A sinking fund provision requires the issuer to retire a portion of a bond issue at specified times during the bond's life.

Floating-rate notes have coupon rates that adjust based on a variable market reference rate. Other coupon structures include step-up coupon notes, credit-linked coupon bonds, payment-in-kind bonds, deferred coupon bonds, and index-linked bonds.

Callable bonds allow the issuer to redeem bonds at a specified call price.

Putable bonds allow the bondholder to sell bonds back to the issuer at a specified put price.

Convertible bonds allow the bondholder to exchange bonds for a specified number of shares of the issuer's common stock.

Embedded options benefit the party who has the right to exercise them. Embedded call options benefit the issuer, while embedded put and conversion options benefit the bondholder.

LOS 48.b

Legal and regulatory matters that affect fixed-income securities vary by the places where they are issued and traded, and the location of the issuing entities.

Domestic bonds trade in the issuer's home country and currency. Foreign bonds are from foreign issuers but denominated in the currency of the country where they trade. Eurobonds are issued outside the jurisdiction of any single country and can be issued in any currency. Global bonds are traded in the Eurobond market and at least one domestic market.

Interest income is typically taxed at the same rate as ordinary income, while gains or losses from selling a bond are taxed at the capital gains tax rate. However, the increase in value toward par of original issue discount bonds is considered interest income. In the United States, interest income from municipal bonds is usually tax exempt at the national level and in the issuer's state.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 48.1

1. **A** A balloon payment in a loan schedule is a partial payment of principal that is made at the end of the loan's life. Compared to an otherwise equivalent fully amortizing loan, the existence of the balloon payment will lead to lower periodic payments over the life of the loan because the borrower has to repay less principal before maturity. There will be a higher final payment, however, at maturity. (LOS 48.a)
2. **B** With a sinking fund, the issuer must redeem part of the issue before maturity, but the specific bonds to be redeemed are not known. Floating-rate notes have an unknown future coupon because it relates to a variable market reference rate; however, they have a known maturity date. In an issue with a term maturity structure, all the bonds are scheduled to mature on the same date. (LOS 48.a)
3. **C** This pattern describes a deferred-coupon bond. The first payment of $229.25 is the value of the accrued coupon payments for the first three years. (LOS 48.a)
4. **B** If the price of the bond increases above the call price stipulated in the bond indenture, it will benefit the issuer to call the bond. Theoretically, the price of a currently callable bond should never rise above its call price. (LOS 48.a)
5. **B** Tax authorities typically treat the increase in value of a pure-discount bond toward par as interest income to the bondholder. In many jurisdictions, this interest income is taxed periodically during the life of the bond, even though the bondholder does not receive any cash until maturity. (LOS 48.b)

READING 49

FIXED-INCOME ISSUANCE AND TRADING

MODULE 49.1: FIXED-INCOME ISSUANCE AND TRADING

Video covering this content is available online.

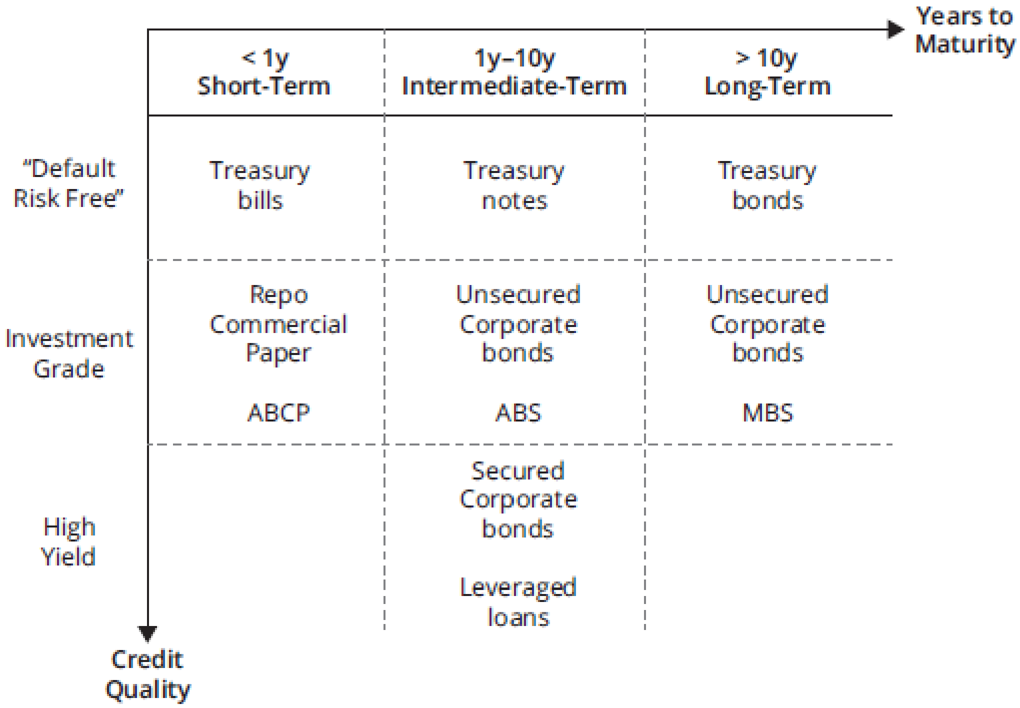
LOS 49.a: Describe fixed-income market segments and their issuer and investor participants.

Global bond markets are primarily segmented by the type of issuer (or *sector*), credit quality, and time to maturity. Other classifications used include currency, the issuer's geography, and environmental, social, and governance (ESG) features.

- *Type of issuer.* Major classifications of bond issuers are governments (sovereign and non-sovereign), corporates, and special purpose entities issuing asset-backed securities (ABSs).
- *Credit quality.* Standard & Poor's (S&P) and Moody's are major examples of **credit rating agencies** that provide credit ratings on bonds. Ratings from AAA down to BBB− (for S&P) and Aaa through Baa3 (Moody's) are considered **investment-grade bonds**. Bonds BB+ or lower (Ba1 or lower for Moody's) are termed **high-yield bonds** (speculative, or "junk" bonds).
- *Original maturities.* Fixed-income markets are usually segmented into short-term investments with original maturities of less than 1 year (known as money market securities), intermediate-term securities with original maturities of 1 to 10 years, and long-term securities with original maturities over 10 years.

The credit/maturity spectrum for issuers is summarized in Figure 49.1.

Figure 49.1: Issuer Credit/Maturity Spectrum



Note: ABCP is asset-backed commercial paper.

Source: Reproduced from Level I CFA Curriculum learning module, "Fixed-Income Issuance and Trading," with permission from CFA Institute.

PROFESSOR'S NOTE

Repos here refers to sale and repurchase agreements, which we will describe in a later reading. For now, it is enough to know that repos are short-term secured borrowing agreements that can be used for short-term financing. Asset-backed commercial paper is a form of short-term asset-backed securities.

Secured corporate bonds appear in the "high-yield" category because we are referring to *new* issues. Companies with less reliable operating cash flows will have to offer security to investors when issuing debt. The high-yield sector also includes the bonds of previously investment-grade issuers that have been downgraded by credit rating agencies due to deteriorating credit quality (such bonds are known as **fallen angels**).

The type of bond that a corporate issuer chooses to issue is generally driven by the access the issuer has to capital markets and by the intended use of the proceeds of the issue.

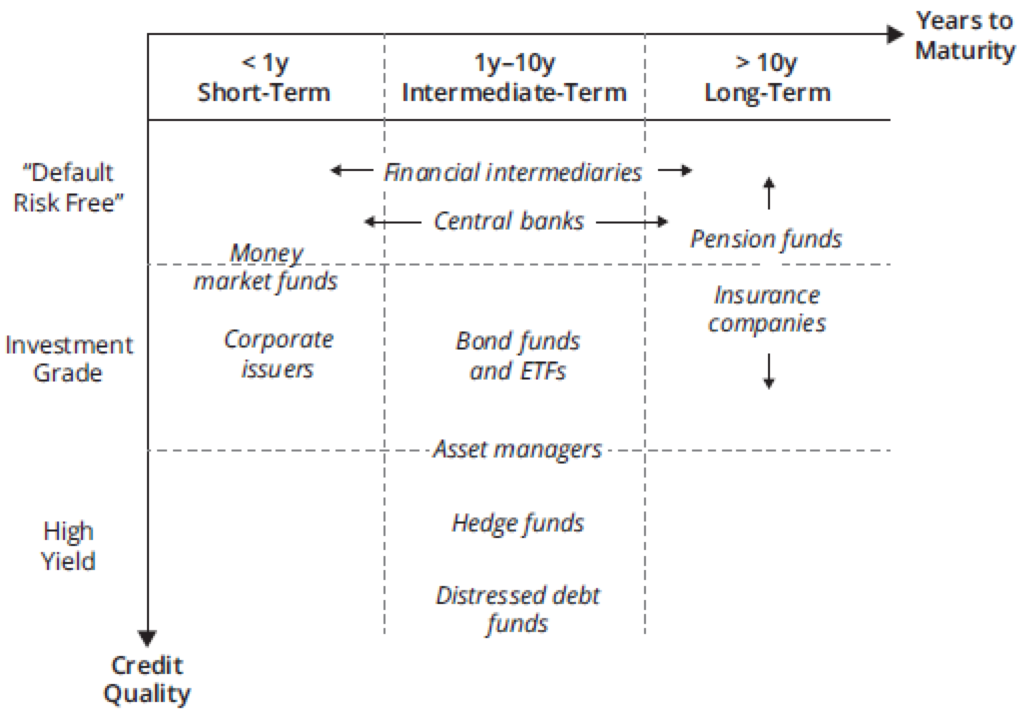
A well-established, investment-grade company could choose to issue commercial paper to fund short-term working capital requirements, intermediate-term debt to fund medium-term investments and permanent working capital, and long-term debt to fund capital investment in fixed assets. The short- and medium-term issues could be arranged for a fee with a **syndicate** of banks offering **credit facilities**, allowing the issuer to issue securities when required for their business operations. However, a riskier company with less stable operating cash flows is likely to have limited access to secured short-term financing and leveraged loans.

Investor Positioning in the Credit/Maturity Spectrum

Where in the credit/maturity spectrum investors choose to invest will depend on their desired interest rate and credit risk exposure, and the maturity of any obligations they need to meet with the cash flows from the bonds.

Common positioning for different types of fixed-income investors is summarized in Figure 49.2.

Figure 49.2: Investor Positioning in Credit/Maturity Spectrum



Source: Reproduced from Level I CFA Curriculum learning module, "Fixed-Income Issuance and Trading," with permission from CFA Institute.

Key points here are the following:

- Pension funds and insurance companies invest in long-term, investment-grade securities to match their long-term liabilities (paying pensions and claims on insurance policies). These institutions are often prohibited by regulations from owning high-yield securities.
- Corporations seek to earn returns on excess liquidity by investing in commercial paper, repos, and ABCP.
- Central banks use intermediate-term Treasury notes as a monetary policy tool to increase or decrease the monetary reserves of commercial banks.
- Bond funds and ETFs will position according to their stated mandate, usually in investment-grade intermediate securities excluding Treasuries. Asset managers seeking higher returns would invest in riskier high-yield intermediate securities, alongside hedge funds and, at the lowest end of the credit spectrum, distressed debt funds (discussed later in this reading).

- Financial intermediaries (banks) use Treasuries across the whole maturity spectrum to manage interest rate and liquidity risks.

LOS 49.b: Describe types of fixed-income indexes.

Fixed-income indexes differ from equity indexes in the following three ways:

1. Corporate bond issuers can, and often do, have many different bonds outstanding, while they will likely have no more than two or three different classes of shares. This leads to fixed-income indexes having many more constituents than equity indexes. Consequently, bond tracker funds employ sampling techniques rather than purchasing all the constituents of a fixed-income index to keep transaction complexity reasonable.
2. Bonds maturing and being issued more frequently cause a higher frequency of removal and replacement of constituents (called turnover) in fixed-income indexes versus in equity indexes.
3. Bonds are issued across multiple sectors. Usually, governments are the largest issuer of bonds; hence, broad bond indexes have large weights in sovereign bonds. Changes in debt issuance trends in terms of maturity and credit quality affect the weights of fixed-income indexes over time.

Bond indexes that contain a broad selection of bonds are called **aggregate indexes**. An example of an aggregate index is the Bloomberg Barclays Aggregate Index, which includes bonds from all sectors across 28 currencies. The index excludes high-yield and unrated issues and bonds that do not meet minimum size for inclusion.

Indexes can have a narrower focus on geography, credit quality, sector, or maturity. For example, the JP Morgan Emerging Markets Bond Index Plus contains U.S. dollar-denominated emerging market sovereign debt with a rating of Baa1/BBB+ or below, with minimum size and maturity limits.

Indexes can also incorporate ESG factors in their construction. For example, the Bloomberg Barclays MSCI Euro Corporate Sustainable SRI Index has a minimum credit rating of Baa3/BBB− and a minimum ESG rating of BBB. The index also screens out certain business sectors deemed incompatible with sustainable investing (e.g., alcohol or generation of thermal coal).

An index chosen to act as a benchmark for a bond fund should match the exposure of the fund in terms of sector focus, credit quality, and maturity.

LOS 49.c: Compare primary and secondary fixed-income markets to equity markets.

Primary Markets

Sales of newly issued bonds are referred to as **primary market** transactions, whereby the issuer sells new securities to investors and receives new capital in return. Newly issued bonds can be registered with securities regulators for sale to the public (a *public* *offering*) or sold only to selected investors (a *private placement*). Both are usually carried out through financial intermediaries (i.e., investment banks). An issuer that is offering its first-ever bond is referred to as a **debut issuer** and is typically a growing and maturing firm that is replacing bank loans in its capital structure with the proceeds from the bond issue.

Bonds can be sold through an **underwritten offering** or a **best-efforts offering**. In an underwritten offering, the bond issue price is guaranteed by the financial intermediaries conducting the bond sale to investors. Issues that are not underwritten are said to be conducted on a best-efforts basis. An issue price is not guaranteed by the intermediaries, but they will charge a commission for placing the bonds with investors at the best price possible.

In a **shelf registration**, a bond issue is registered with securities regulators in its aggregate value with a master prospectus. The bonds can then be issued over time when the issuer needs to raise funds.

Debut issues usually require weeks of “roadshows” by underwriters prior to the issue date to introduce investors to the debut issuer. Subsequent **repeat issues** of fixed-income securities usually take much less time. For an investment-grade frequent issuer with a fully underwritten shelf registration, the time between the transaction being agreed and being completed and allocated to investors may be a matter of hours, because bond funds and other investors are likely already familiar with the issuer and regulatory registrations have already been made. Issuance is likely to take significantly longer for a smaller high-yield issue with more detailed covenants and collateral provisions, made on a best-efforts basis.

Some bonds, especially government bonds, are sold through a public auction, a process we will describe in our reading on Fixed-Income Markets for Government Issuers.

Secondary Markets

Secondary markets refer to the trading of previously issued bonds among investors. While some electronic trading platforms and exchange-based trading exist, the majority of trading in the secondary market remains in the dealer, or over-the-counter (OTC), market.

Dealers post quotes comprising *bid* (purchase) prices and *ask* or *offer* (selling) prices for various bond issues. The difference between the bid and ask prices is the dealer’s spread. The spread varies across individual bonds according to their liquidity, ranging from a fraction of a basis point for liquid, recently issued (“on-the-run”), developed market sovereign bonds and high-credit-quality corporate frequent issuers, to 10–20 basis points or more for less liquid, smaller, or older (“seasoned”) corporate issues.

Distressed debt is a name given to the bonds of issuers that are in, or expected to file for, bankruptcy. A distressed debt investor might buy the debt from other institutions that are prohibited from owning securities with low credit ratings, and aims to profit from the issuer’s fortunes reversing, higher-than-expected recovery rates in liquidation, or value-enhancing restructuring of the issuer. For an otherwise infrequently traded issue, entering a distressed situation may temporarily increase its trading activity.

MODULE QUIZ 49.1

1. Funds required by a corporation to finance investment in seasonal working capital are *most likely* raised through issuing:
   A. secured bonds.
   B. Treasury notes.
   C. commercial paper.
2. Compared to equity indexes, aggregate fixed-income indexes are *most likely* to have a lower:
   A. turnover.
   B. weight in the corporate sector.
   C. number of constituents.
3. In which type of primary market transaction does an investment bank sell bonds on a commission basis?
   A. Single-price auction.
   B. Best-efforts offering.
   C. Underwritten offering.
4. Secondary market bond transactions *most likely* take place:
   A. in dealer markets.
   B. in brokered markets.
   C. on organized exchanges.
5. Sovereign bonds are described as "on the run" when they:
   A. are the most recent issue in a specific maturity.
   B. have increased substantially in price since they were issued.
   C. receive greater-than-expected demand from auction bidders.

KEY CONCEPTS

LOS 49.a

Global bond markets can be classified by the following:

- *Type of issuer.* Sovereign, corporate, and special purpose entities issuing ABSs.
- *Credit quality.* Investment-grade (Baa3/BBB− and above), non-investment grade (Ba1/BB+ and below).
- *Original maturity.* Short term/money market (one year or less), intermediate term (one year to 10 years), long term (more than 10 years).

Well-established investment-grade corporations may issue commercial paper to fund short-term working capital requirements; intermediate-term debt to fund medium-term investments and permanent working capital; and long-term debt to fund capital investment in fixed assets.

Where in the credit/maturity spectrum investors choose to invest will depend on their desired interest rate and credit risk exposure and the maturity of any obligations they need to meet.

- Pension funds and insurance companies: long-term investment grade
- Corporations: short-term investment grade (excluding Treasury bills)

- Central banks: intermediate-term Treasury notes used to conduct monetary policy
- Bond funds and ETFs: intermediate investment-grade (excluding Treasury notes)
- Asset managers seeking higher returns: high-yield intermediate securities, distressed debt
- Financial intermediaries (banks): Treasuries across the maturity spectrum

LOS 49.b

Relative to equity indexes, fixed-income indexes have more constituents, higher turnover, and weights more affected by issuer sector and changes in borrowing trends over time.

Broad bond indexes that include all relevant bonds are referred to as aggregate indexes. Narrower indexes focus on sector, credit quality, geography, or ESG considerations.

LOS 49.c

Bonds may be issued in the primary market through a public offering or a private placement. Bond issues that are underwritten have a price guaranteed by financial intermediaries; those on a best-efforts basis have no such guarantee. Public offerings of government debt commonly take place through auctions. Debut issues are likely to be more time consuming and costly than subsequent repeat issues.

In secondary markets, while some bonds trade on electronic platforms or exchanges, most are traded in OTC dealer markets. Spreads between bid and ask prices are narrower for liquid issues and wider for less liquid issues.

Distressed debt refers to bonds of issuers that are in, or expected to file for, bankruptcy.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 49.1

1. **C** Corporations are most likely to fund short-term seasonal investment in working capital by issuing short-term commercial paper because its maturity (less than a year) will match the required investment period in working capital. Intermediate-term corporate bonds are usually issued to invest in longer-term working capital requirements and medium-term fixed capital investments. A corporation cannot issue Treasury notes, which are intermediate-term securities issued by governments. (LOS 49.a)

2. **B** Aggregate fixed-income indexes include constituents from all sectors. Hence, fixed-income indexes will have higher weights allocated to sovereign issuers and lower weights to corporate issuers than equity indexes, which are based purely on corporate issuers of equity. A fixed-income index is expected to have higher turnover (due to maturing bonds and frequent re-issues) and a higher number of constituents (due to a single issuer typically having more bond issues in existence than equity issues). (LOS 49.b)

3. **B** In a best-efforts offering, the investment bank or banks do not underwrite (i.e., purchase all of) a bond issue, but rather sell the bonds on a commission basis. Bonds sold by auction are offered directly to buyers by the issuer, typically a government. (LOS 49.c)

4. **A** The secondary market for bonds is primarily a dealer market in which dealers post bid and ask prices. (LOS 49.c)

5. **A** Sovereign bonds are described as "on the run" when they represent the most recent issue in a specific maturity. (LOS 49.c)

READING 50

FIXED-INCOME MARKETS FOR CORPORATE ISSUERS

MODULE 50.1: FIXED-INCOME MARKETS FOR CORPORATE ISSUERS

Video covering this content is available online.

LOS 50.a: Compare short-term funding alternatives available to corporations and financial institutions.

Short-Term Funding for Nonfinancial Corporations

A nonfinancial company usually raises external funds for investment in short-term assets (cash, short-term investments, accounts receivable, and inventory) through financial intermediaries that provide either loan financing or security-based financing.

External Loan Financing

External loan financing, or **bank lines of credit**, refers to agreements between borrowers and banks to draw down funds as required. These primarily consist of three types, which are uncommitted, committed, or revolving lines of credit:

1. *Uncommitted line of credit*. A bank extends an offer of credit for a principal amount (credit line), usually charging a floating market reference rate (MRR) plus a fixed credit spread on funds drawn down. The credit is "uncommitted" in the sense that the bank may refuse to lend if circumstances change. As a result, this is a less reliable source of funds than the other two types. However, it is likely to be a flexible agreement with no fees outside of interest charges. These agreements can be offered unsecured if the borrower maintains stable cash balances with the bank.
2. *Committed (regular) line of credit*. A bank commits to an offer of credit for a specific time period, providing a more reliable source of funding for borrowers than an uncommitted line of credit. Banks charge a commitment fee, typically about 50 basis points, on either the full or unused amount over the commitment period. Regulators require banks to hold a higher level of reserves to cover potential defaults on committed lines versus uncommitted lines. Banks can mitigate these default risks by offering commitments for less than one year or by acting in syndicate with other banks to provide such agreements. Although banks commit to extend the line of credit for the stated maturity of the agreement, they may withdraw the agreement at maturity should credit conditions worsen, leading to **renewal risk** for borrowers.

3. *Revolving (operating) line of credit*. An even more reliable source of short-term financing than a committed line of credit, "revolvers" are typically for a longer term, sometimes years (with potential medium-term loan facilities). Banks typically place restrictive covenants on borrowers under such agreements. Fees and rates are similar to a committed line of credit.

Companies with weaker credit ratings typically have to pledge assets as collateral for bank borrowings. **Secured (asset-backed) loans** are backed by collateral (e.g., fixed assets, receivables, or inventory). Companies can assign receivables to lenders to act as collateral for loans. **Factoring** refers to the actual transfer of credit granting and collection of receivables to a lender ("factor") at a discount from their face value. The size of the discount, which represents the interest rate on the loan from the factor, depends on the creditworthiness of the firm's customers, and on collection costs.

External Security-Based Financing

Large corporations with high credit ratings can reduce their funding costs by issuing short-term unsecured debt securities, referred to as **commercial paper (CP)**. For these firms, the interest cost of CP is less than the interest on a bank loan. With maturity of typically less than three months, CP is issued by firms to fund working capital and as a temporary source of funds before issuing longer-term debt. Debt that is temporary until permanent financing can be secured is referred to as **bridge financing**.

CP is often reissued, or "rolled over," when it matures. The risk that a company will not be able to sell new CP to replace maturing paper is termed **rollover risk**. To manage this risk, borrowers maintain **backup lines of credit** with banks. These are sometimes referred to as liquidity enhancement or backup liquidity lines, whereby lenders agree to provide funds to make repayments on CP when it matures, if needed.

Similar to U.S. T-bills, CP in the United States is typically issued as a pure discount security, making a single payment equal to the face value at maturity. A smaller, less liquid international market in CP also exists, referred to as **Eurocommercial paper** (ECP).

Short-Term Funding for Financial Institutions

Commercial and retail deposits are a major short-term funding source for banks.

- *Checking accounts* ("demand deposits") provide transactions services and immediate availability of funds but typically pay no interest.
- *Operational deposits* are made by larger customers who require cash management, custody, and clearing services.
- *Savings deposits* have a stated term and interest rate. These may take the form of an interest-bearing **certificate of deposit (CD)**, which pays interest at a specified maturity of less than a year. *Nonnegotiable CDs* cannot be sold before maturity, and early withdrawal of funds incurs a penalty. *Negotiable CDs* can be sold in the open market before maturity as a means of early withdrawal of funds. At the wholesale (institutional investor) level, negotiable CDs are an important funding source for banks. They trade in domestic bond markets as well as in the Eurobond market.

Funds that are loaned by one bank to another are referred to as **interbank funds**. Banks lend to each other for periods of one day to a year, on either a secured or unsecured basis, at an interest rate based on a market reference rate (MRR) that varies across markets. The most common type of secured interbank borrowing and lending is carried out through repurchase agreements (repos), which we discuss later in this reading.

PROFESSOR'S NOTE

We saw this interbank MRR when we defined the coupon paid by FRNs as being equal to a variable MRR plus a fixed margin. In a stable banking system, the short-term interbank rate is likely to be of very low credit risk. This MRR can be used as a starting point for other interest payments that need to be made on riskier loans (e.g., the coupon on FRNs issued by corporations). An example of an MRR is the Secured Overnight Funding Rate (SOFR) in the United States.

Another source of short-term funding for banks is to borrow excess reserves from other banks in the **central bank funds market**. Banks in most countries must maintain a portion of their funds as reserves on deposit with the central bank. Banks with excess reserves lend them to banks that need funds at the **central bank funds rate**, which is strongly influenced by the central bank's open market operations and by the availability of short-term funds. The central bank may act as the lender of last resort ("discount window lending") to banks struggling to access liquidity. This will likely be made at a higher rate than the central bank funds rate and may bring extra scrutiny and restrictions on the activities of the borrower.

Financial institutions issue more commercial paper than nonfinancial companies. Financial institutions often sponsor **asset-backed commercial paper (ABCP)**, a type of short-term asset-backed security. The ABCP creation process is as follows:

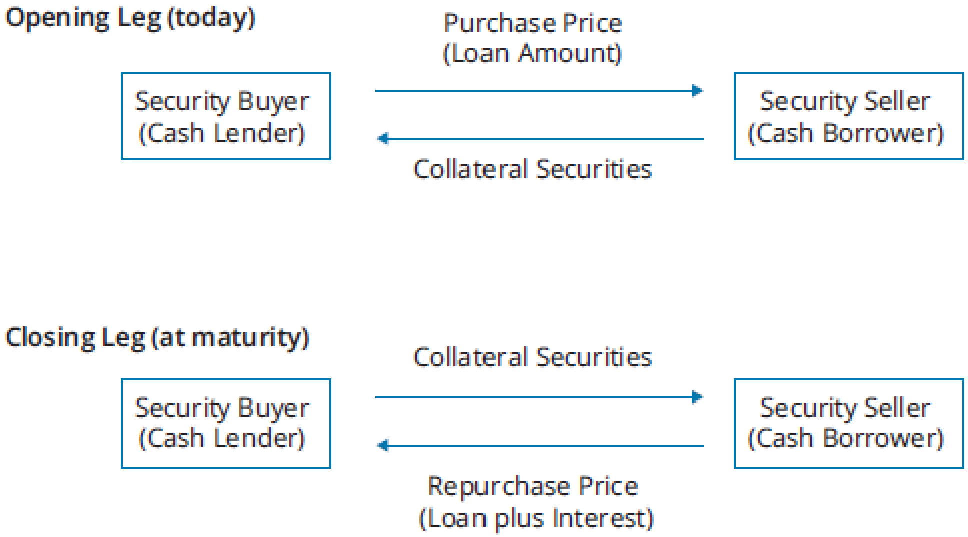
1. A financial institution transfers collateral (usually existing short-term loans made by the bank) to a separate legal entity called a special purpose entity (SPE), in return for cash. The SPE is set up as an off-balance-sheet vehicle with respect to the financial institution.
2. The SPE sells ABCP to investors, who accept the risk and return of the collateral backing the ABCP. The sponsoring financial institution provides a backup credit liquidity line.

LOS 50.b: Describe repurchase agreements (repos), their uses, and their benefits and risks.

A **repurchase agreement (repo)** is an arrangement by which one party sells a security to a counterparty with a commitment to buy it back at a later date at a prespecified higher price. The original purchase price is effectively a loan by the security buyer to the security seller, with the security as collateral. The difference between the repurchase price and the original purchase price accounts for the interest paid to the security buyer. The annualized interest rate implied by the difference between the two prices is called the **repo rate**.

The basic structure of a repo is displayed in Figure 50.1.

Figure 50.1: Repurchase Agreement



To protect the lender against a potential decrease in the value of the securities posted as collateral, the borrower typically must post extra collateral above the loan amount (the purchase price) by an amount known as the **initial margin**. In practice this means the loan amount will be a discount to the value of the securities.

As an example, consider a firm that wishes to borrow by entering into a repo agreement to sell a bond today with a market value of \$1 million and repurchase it 90 days later (the **repurchase date**). The lender requires a repo rate of 2% and initial margin of 103%.

The initial purchase price (the initial loan amount) is calculated as the market value of the securities posted as collateral divided by the initial margin:

$$\text{purchase price (loan amount)} = \frac{\text{market value of securities}}{\text{initial margin}} = \frac{\$1{,}000{,}000}{1.03}$$
$$= \$970{,}874$$

The repurchase price after 90 days is calculated as the purchase price (loan amount) multiplied by one plus the de-annualized repo rate. Assuming a 360-day count convention, this is calculated as follows for our example:

repurchase price = \$970,874 × [1 + (0.02 × (90 / 360)] = \$975,728

The discount applied to the market value of collateral to get the purchase price is referred to as a **haircut**. In this example, the haircut is calculated as follows:

$$\text{haircut} = \frac{(\$1{,}000{,}000 - \$970{,}874)}{\$1{,}000{,}000} = 2.91\%$$

The haircut can also be calculated quickly as $1 - \frac{1}{\text{initial margin}}$.

The loan value increases during its life at the repo rate. Should the market value of the collateral fall below this value times the initial margin, the lender will ask the borrower for more collateral, known as **variation margin**.

For example, given the repo details just listed, assume that after 30 days the market value of the bond has fallen to $990,000:

$$\text{adjusted loan amount after 30 days} = \$970{,}874 \times [1 + (0.02 \times (30 / 360))]$$
$$= \$972{,}492$$

$972,492 × 1.03 = $1,001,667. Because the collateral value is less than this, the borrower must provide variation margin.

$$\text{variation margin} = (\text{initial margin} \times \text{adjusted purchase price}) - \text{market value of collateral}$$
$$= \$1{,}001{,}667 - \$990{,}000 = \$11{,}667$$

In this case, the security buyer/lender will ask the security seller/borrower to post an extra $11,667 of securities as collateral. If the variation margin is negative, then the loan is overcollateralized, and the borrower can request a release of collateral equal in value to the variation margin amount.

A repurchase agreement for one day is called an **overnight repo**, and an agreement covering a longer period is called a **term repo**. Due to the short-term collateralized nature of a repo and the fact that collateral is usually high-quality, liquid, sovereign bonds, interest rates are customarily less than the rate on bank loans or other short-term borrowing.

Although the transactions underlying the repo are described as a "sale" and "repurchase" of securities, the seller/borrower retains rights to the benefits of holding the bond over the repo term. The collateral used in the repo may involve a specific security or a general type of security (e.g., Treasury bonds of a certain range of maturities), in which case it is known as a **general collateral repo**. The details of the contractual terms of the repo are contained in the **master repurchase agreement** between the counterparties.

Repo Applications

The main uses of repurchase agreements are as follows:

- Financial institutions enter repos as security sellers/borrowers to finance positions in securities held in their trading activities.
- Banks and institutional investors, such as mutual funds and pension funds, enter repos as security buyers/lenders to earn the repo rate on excess short-term funds.
- Central banks may use repos to enact monetary policy, buying securities/lending to increase the money supply and selling securities/borrowing to decrease the money supply.
- Short sellers, such as hedge funds, can use repo agreements to borrow securities that they intend to short sell, speculating that the security value will decrease. This is executed through the following trades:
  1. Buy securities/lend in a repo.
  2. Short sell the securities in the open market.
  3. Buy back the securities in the open market later (before the repo term ending).
  4. Deliver the securities back to the repo counterparty at the maturity of the repo.

The hedge fund earns the repo rate from being the lender in the repo, and gains if the security value decreases over the repo term. When the motivation to enter a repo is to borrow a security in this way, the participant is said to be entering a **reverse repo**.

A short seller engaging in the type of transaction just described will need to specify the security they wish to borrow (known as a special trade). If the specified security is difficult to borrow, the hedge fund will be willing to accept a lower repo rate than that earned on general collateral, or may even be willing to accept a negative repo rate for extremely-hard-to-source collateral.

Factors Affecting the Repo Rate

The repo rate is usually:

- Higher, when interest rates for alternative sources of short-term (money market) funds are higher
- Lower, the higher the credit quality of the collateral security
- Higher, the longer the repo term (when longer-term rates are generally higher than short-term rates)
- Lower, when the collateral security is in high demand or low supply
- Higher, if the repo is undercollateralized, or if collateral is specified but not actually delivered to the lender

Repo Risks

While a repo agreement is a safer form of lending than most sources of short-term funds, it remains a source of debt financing to the borrower, and overuse can lead to financial distress or insolvency. Like other forms of collateralized borrowing, the major risks include the following:

- Default risk, that the lender of the security (borrower of cash) fails to make the repurchase payment at the end of the repo
- Collateral risk, relating to the value that can be generated for collateral in event of default
- Margining risk, relating to the timely and accurate calculation and payment of margin
- Legal risk, that the contracts cannot be legally enforced
- Netting and settlement risk, relating to the ability to net off payments across different contracts with the same nondefaulting counterparty, and the ability to settle the cash and collateral transactions underlying the repo

Many of the risks just listed can be mitigated through using **tri-party repos**, which employ a third-party intermediary (usually a custodian bank or clearinghouse) as an agent to arrange and administer repo transactions. While this does not reduce credit risk, it does likely improve cost efficiencies with respect to access to collateral and counterparties, and the valuation and safekeeping of assets. A repo agreement that is struck directly between two parties without a third party is referred to as a **bilateral repo**.

LOS 50.c: Contrast the long-term funding of investment-grade versus high-yield corporate issuers.

Recall that in a normal yield curve environment, bond yields are higher for longer-dated maturities, reflecting higher risk-free rates and credit spreads over longer time frames. In this environment, both investment-grade and high-yield corporate issuers must offer higher yields on longer-maturity bond issues. This difference in yields across maturity is likely to be greater, however, for high yield issuers due to the higher spreads offered on high yield debt. Companies that choose to issue shorter-dated bonds to reduce the yield they pay on their debt assume rollover risk.

Other major differences between issuances of investment grade debt versus high yield debt include the following:

- Default risk, and loss given default, are primary concerns for high yield investors due to the lower credit quality of the issuers. For investment grade investors, the primary credit-related concern is the chance of a ratings downgrade and the probability of future default increasing, rather than an imminent default.
- Credit spreads are likely to be a smaller proportion of yield for investment grade issues, where yields are largely tied to benchmark rates (e.g., sovereign debt).
- Investment grade issuers usually face only a few restrictive covenants on debt issues, typically limiting liens and sale and leaseback arrangements on the core operating assets that are expected to generate the operating cash flows required to repay the debt. High yield issuers are likely to face a larger number of restrictive covenants relating to debt-based ratios, issuance of additional debt, and distributions of capital to equity investors (e.g., limits on dividend payments). High yield issuers are also likely to need to provide collateral as security for bond issues.
- Investment grade issues are somewhat standardized (similar across different issues) and typically issued across multiple maturities, which reduces rollover risk for the issuer. This choice of maturity allows investment grade issuers to take advantage of changes in market conditions. High yield issues are likely to have more specific covenants and liens, making issues less standardized. High yield issuers also typically have less flexibility with regard to maturity, with bonds usually being issued with maturity of 10 years or less. With less flexibility and less standardization, high yield issuers are less able to take advantage of opportunities to refinance debt when borrowing costs fall.
- High yield issuers are more likely to structure their debt so that it can be repaid earlier if their credit quality improves. This is done either by taking leveraged loans with prepayment options or by issuing callable debt.
- Due to the higher uncertainty in the cash flows used to repay high yield debt, returns are likely to be more equity-like than the returns of investment grade debt.

MODULE QUIZ 50.1

1. Restrictive covenants are *most likely* to be placed on borrowers under a:
   A. revolving line of credit.
   B. factoring arrangement.

C. committed line of credit.

2. A borrower pledges $100 million of securities as collateral for an overnight repo with a repo rate of 4% and an initial margin of 101%. The purchase price of the repo is *closest* to:
   A. $99,000,000.
   B. $99,009,901.
   C. $101,000,000.
3. A borrower pledges $100 million of securities as collateral for an overnight repo with a repo rate of 4% and an initial margin of 101%. Assuming 360 days in a year, the interest paid under the repo is *closest* to:
   A. $10,891.
   B. $11,001.
   C. $11,111.
4. A borrower pledges $100 million of securities as collateral for an overnight repo with a repo rate of 4% and an initial margin of 101%. The haircut on collateral is *closest* to:
   A. 0.01%.
   B. 0.99%.
   C. 1.01%.
5. Relative to a long-term high-yield bond issue, an investment-grade bond issue is *most likely* to have a:
   A. longer maturity.
   B. greater number of covenants.
   C. higher proportion of its yield related to credit spreads.

KEY CONCEPTS

LOS 50.a

Forms of short-term funding for nonfinancial corporations include lines of credit (uncommitted, committed, and revolving in order of increasing reliability and cost), secured loans, factoring arrangements, and commercial paper issuance.

A major form of short-term funding for financial institutions is deposits from customers. Savings deposits can take the form of certificates of deposit, which can be nonnegotiable (no early withdrawal without penalty), or negotiable (can be sold to a third party). Other sources of short-term financing for financial institutions include interbank funds (including repos), central bank funds, and commercial paper (including asset-backed commercial paper).

LOS 50.b

A repurchase agreement is a collateralized loan whereby a borrower sells a security to a counterparty with a commitment to buy it back later at a higher price. The annualized percentage difference between the two prices represents the repo rate.

Borrowers under a repo must post collateral, which must maintain a value greater than the initial margin or the borrower must provide variation margin.

The main uses of repos are for borrowers to finance positions in securities, for lenders to earn the repo rate on excess liquidity, for central banks to engage in monetary policy, and for short sellers to temporarily borrow securities (as the lending party in the repo).

The repo rate will be higher for longer-term repos, lower quality collateral, or when the repo is undercollateralized. The repo rate will be lower for a specific security that is difficult to source.

Repo risks include default risk, collateral risk, margining risk, legal risk, and netting and settlement risk. The use of a tri-party repo can help mitigate these risks.

LOS 50.c

Relative to investment-grade debt, high-yield debt is likely to have a higher proportion of its yield related to credit spread, a greater number of covenants, shorter maturity, and more equity-like returns, and is likelier to have call or prepayment provisions.

Probability of default risk, and loss given default, are a greater concern for high-yield investors than investment-grade investors, whose primary concern is the prospect of credit downgrades.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 50.1

1. **A** Restrictive covenants are most likely to be placed on borrowers using revolving lines of credit because these agreements require banks to commit funds over the longest terms. A factoring arrangement involves selling accounts receivable to a third party. (LOS 50.a)

2. **B** Repo purchase price (loan amount) = market value of securities / initial margin

   = $100,000,000 / 1.01 = $99,009,901

   (LOS 50.b)

3. **B** Repo purchase price (loan amount) = market value of securities / initial margin

   = $100,000,000 / 1.01 = $99,009,901

   Repo interest = $99,009,901 × 0.04 × (1 / 360) = $11,001.10

   (LOS 50.b)

4. **B** 1 – 1 / 1.01 = 0.0099, or 0.99%.

   (LOS 50.b)

5. **A** An investment grade bond is likely to have a longer maturity than a high yield bond because high yield issues tend to be restricted to 10-year maturity or less, while investment grade issuers can issue bonds of any maturity. Investment-grade bond issues typically have fewer restrictive covenants than high yield issues. The proportion of yield that is credit spread is likely to be lower for investment grade issues, where yield is primarily composed of benchmark rates. (LOS 50.c)

READING 51

FIXED-INCOME MARKETS FOR GOVERNMENT ISSUERS

MODULE 51.1: FIXED-INCOME MARKETS FOR GOVERNMENT ISSUERS

Video covering this content is available online.

LOS 51.a: Describe funding choices by sovereign and non-sovereign governments, quasi-government entities, and supranational agencies.

Sovereign Debt

National governments issue bonds to raise funds for spending on public goods and services and investment in public infrastructure. These sovereign issues are backed by the power to collect taxes and therefore typically carry the highest credit rating in their domestic market. Sovereign issuers are usually the largest debt issuers in their domestic market.

Public sector issuers such as governments prepare financial reports, but the accounting standards that apply differ from those in the private sector. Generally, public sector accounting standards are based more on cash transactions and less on accruals (e.g., depreciation, unfunded liabilities). When assessing the financial statements of a government, an analyst should consider the issuer to have an "economic balance sheet" that includes implied assets (e.g., expected future tax revenues) and implied liabilities (e.g., promised future expenditures) in addition to the financial assets and liabilities reported in the public accounts.

A key distinction in sovereign debt markets is the difference between **developed market** versus **emerging market** issuers.

Developed market sovereign issuers have stable, diversified economies with consistent and transparent fiscal policy. Debt is denominated in a **reserve currency** (i.e., a major currency held as reserves by central banks across the world and widely used in international trade, such as the U.S. dollar or the Chinese renminbi).

Emerging market sovereign issuers typically have faster growing, less stable, and more concentrated economies and, consequently, less stable tax revenues, which are sometimes tied to a dominant industry or commodity. Emerging market debt is often raised to fund investment in economic growth and can be **domestic debt** or **external debt**.

- Domestic debt is issued in the nation's home currency and is held by domestic investors. That currency might not be freely convertible into other currencies due to illiquidity or restrictions on capital flows.
- External debt is debt owed to foreign creditors and may be denominated in the government's home currency or a foreign reserve currency. When external debt is denominated in a reserve currency, foreign investors avoid the direct currency risk of the issuer's currency weakening, but the investor still faces indirect currency risk related to the emerging market government generating enough flows of the reserve currency to make repayments on its external debt.

A government's **debt management policy** sets out the amount and type of securities the government intends to issue. As we discussed in the Economics topic area, fiscal policy is often used as a tool to manage the business cycle. A government that believes the economy is below full employment may tax less and spend more to stimulate economic activity. This may require the government to issue new debt. Analysts who wish to forecast a government's debt issuance needs should focus on its fiscal policy, as well as how cyclical and inflation-sensitive its revenues and spending are. Analysts should also be aware of debt features such as floating rates or inflation indexing, as well as whether the government guarantees any non-sovereign debt.

Recall from the Economics topic area the condition of Ricardian equivalence, which suggests taxpayers expect that future taxes will have to repay any debt the government issues. This would imply that a government should be indifferent among the possible maturities of its debt, but only under all of the following assumptions about taxpayer behavior:

- They will increase savings when they expect higher future taxes.
- They have rational expectations, expecting tax decreases in the present to be offset by tax increases in the future.
- They can borrow and lend in capital markets that have no transactions costs.
- They can and will pass tax savings on to future generations.

Because these assumptions do not hold in practice, governments must manage how much of their debt is short-term or longer-term. Governments typically issue securities across maturities to maintain a stable split between long-term and short-term debt over time.

Different maturity sectors for sovereign debt have some benefits for market participants. For example, many investors view short-term sovereign debt as safe and highly liquid, and it may function as an alternative to bank deposits. This liquidity benefit to investors likely causes short-term government debt yields to be lower than they would be otherwise. While governments have a great deal of flexibility as to how much short-term debt they issue, relying too heavily on it creates rollover risk.

Having liquid markets in longer maturities of government debt also has benefits. Market participants use government debt yields as benchmarks against which to measure the credit risk of non-government debt. Asset managers and financial institutions often rely on government debt in their interest rate risk management, and use it as collateral in transactions such as repurchase agreements. Central banks buy and sell government debt of various maturities to conduct monetary policy.

Nonsovereign Government Debt

Nonsovereign government bonds are issued by states, provinces, counties, and entities created to fund and provide services (e.g., for the construction of hospitals, airports, and other municipal services).

Agency bonds or **quasi-government bonds** are issued by entities that national governments create for specific purposes, such as financing infrastructure investment or providing mortgage financing. An example of such an agency is the Government National Mortgage Association (Ginnie Mae) in the United States, which securitizes and guarantees mortgage loans to facilitate home ownership. Ginnie Mae issues debt securities to finance its operations, the repayment sources for which are guarantor fees from their business alongside U.S. government backing. When they are backed by the sovereign entity, agency bonds typically have yields and credit ratings closely aligned with those of the government.

Local and regional government authorities may issue debt raised for general public spending backed by local tax raising powers (referred to as **general obligation bonds** or GO bonds), or debt issued to fund a specific project (called **revenue bonds**) where the source of repayment is fees from the use of the infrastructure funded by the bond issue (e.g., a toll road or bridge).

Supranational bonds are issued by international institutions such as the World Bank, the IMF, and the Asian Development Bank, which have been set up by multiple sovereign governments to promote economic cooperation, trade, or economic growth. Bonds issued by supranational agencies typically have high credit quality and some issues are highly liquid.

LOS 51.b: Contrast the issuance and trading of government and corporate fixed-income instruments.

While corporate issuers of debt raise debt finance as required, sovereign issuers use regular public auctions to issue government debt securities.

Buyers can make **competitive bids** or **noncompetitive bids** at government debt auctions. Competitive bids are used to set the price of the debt issue, while noncompetitive bids are guaranteed to have their allocation met at the price determined by the competitive bids. The auction is conducted by first allocating bonds to noncompetitive bids. Then, competitive bids are ranked in order of highest price (lowest yield). Bonds are allocated to competitive bids starting with the highest price and moving through the auction order book until the offering amount is met. The yield of the successful competitive bid with the lowest price is referred to as the **cut-off yield**. Under a **single-price auction**, all investors pay the price associated with this cut-off yield, regardless of the yield they actually bid. Under a **multiple-price auction**, successful competitive bidders actually pay the price that they bid.

A government issuer that wishes to minimize yield volatility would likely choose to conduct single-price auctions because all the bonds will be issued at a single yield. This decrease in yield volatility may increase the chance of a successful auction, distribute bonds more broadly among investors, and result in a lower cost of funds for the issuer. Because successful competitive bidders in a multiple-price auction pay what they actually bid, it is likely that bids will be close together and large in size.

A sovereign issuer typically designates certain financial institutions as **primary dealers** that are required to make competitive bids in auctions, submit bids in auctions on behalf of third parties, and act as counterparty to the central bank when it buys and sells securities to carry out monetary policy.

Once issued, sovereign debt typically trades in quote-driven OTC dealer markets in a similar fashion to corporate bonds. Trading is most active, and prices most informative, for the most recently issued government securities of a particular maturity. These issues are referred to as **on-the-run** bonds and their yields are typically used to represent default-risk-free "benchmark" yields when constructing yield curves.

Investors in government securities may have "noneconomic" objectives. For example, government bonds are often used by central banks to conduct monetary policy; foreign governments purchase sovereign bonds of other nations as reserves; and some financial institutions are required to hold government bonds to comply with regulations. The presence of such investors decreases the yields of sovereign bonds relative to those of non-sovereign issuers.

MODULE QUIZ 51.1

1. Bonds issued by the World Bank are *best* described as:
   A. quasi-government bonds.
   B. global bonds.
   C. supranational bonds.
2. A foreign investor who invests in the USD-denominated external debt of an emerging market government has currency risk that is *best* described as:
   A. direct.
   B. indirect.
   C. hedged.
3. Investors are guaranteed a bond allocation in a Treasury bond auction when:
   A. they submit a noncompetitive bid.
   B. the auction is a single-price auction.
   C. they submit a competitive bid in a multiple-price auction.

KEY CONCEPTS

LOS 51.a

Sovereign issuers are usually the highest credit quality and largest issuers of debt in any given bond market.

Developed market sovereign issuers have stable, diversified economies with consistent and transparent fiscal policy, with debt denominated in a major reserve currency. Emerging market sovereign issuers typically have faster growing, less stable, and more concentrated economies and, consequently, less stable tax revenues, sometimes tied to a dominant industry or commodity. Emerging market debt can be domestic (issued in domestic currency and held by domestic investors) or external (owed to foreign creditors).

Ricardian equivalence theory states that governments should be indifferent about raising taxes today or issuing debt of any maturity. The high rollover risk of short-term debt and the need for predictable fiscal policy means that, in practice, governments tend to issue debt evenly across the maturity spectrum.

Benefits of having a broad range of maturities of government securities in existence include the identification of benchmark government yield curves; the ability of investors to hedge interest rate risk; the ability to pledge the securities as collateral for repos; and the ability of the central bank to use government bonds to execute monetary policy.

Agency or quasi-government bonds are issued by entities created by national governments for specific purposes such as financing infrastructure investment or providing mortgage financing.

Local and regional government authorities may issue debt for general public spending backed by local tax raising powers (general obligation bonds) or to fund a specific project (revenue bonds).

Supranational bonds are issued by international institutions that promote economic cooperation, trade, or economic growth.

LOS 51.b

Sovereign issuers use regular public auctions to issue government debt securities.

Auctions bids can be either competitive or noncompetitive. Competitive bids are used to set the price of the debt issue, while noncompetitive bids are guaranteed to have their allocation met at the auction price.

In a single-price auction, all investors pay the price associated with the cut-off yield. In a multiple-price auction, successful competitive bidders actually pay the price that they bid.

Primary dealers are financial institutions required to make competitive bids in auctions, submit bids in auctions on behalf of third parties, and act as counterparties to the central bank.

Once issued, sovereign debt typically trades in quote-driven OTC dealer markets. Trading is most active and prices most informative for on-the-run bonds.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 51.1

1. **C** Bonds issued by the World Bank, which is a multilateral agency operating globally, are termed supranational bonds. (LOS 51.a)

2. **B** While the U.S. investor will not have the direct currency exposure of holding foreign-denominated debt, they still face indirect currency exposure from the emerging market government having to raise USD through international transactions to repay their external debt. (LOS 51.a)

3. **A** Investors who make noncompetitive bids in government bond auctions are guaranteed to have their allocations met at a price determined by the competitive bids. Competitive bidders are not guaranteed allocations in either single-price or multiple-price auctions. (LOS 51.b)

READING 52

FIXED-INCOME BOND VALUATION: PRICES AND YIELDS

MODULE 52.1: FIXED-INCOME BOND VALUATION: PRICES AND YIELDS

Video covering this content is available online.

LOS 52.a: Calculate a bond's price given a yield-to-maturity on or between coupon dates.

Calculating the Value of an Annual Coupon Bond

The value of a coupon bond can be calculated by summing the present values of all of the bond's promised cash flows. The market discount rate appropriate for discounting a bond's cash flows is called the bond's **yield to maturity (YTM)**. If we know a bond's YTM, we can calculate its value, and if we know its value (market price), we can calculate its YTM.

Consider a newly issued 5-year, 10% coupon, annual-pay bond. For \$100 of par value, the coupon payments will be \$10 at the end of each year, and the \$100 par value will be paid at the end of Year 5. First, let's value this bond assuming that the appropriate discount rate (also called "yield") is 10%. The present value of the bond's cash flows discounted at 10% is as follows:

$$\frac{10}{(1.10)^1}+\frac{10}{(1.10)^2}+\frac{10}{(1.10)^3}+\frac{10}{(1.10)^4}+\frac{110}{(1.10)^5}$$

The calculator solution is as follows:

$N = 5$; $PMT = 10$; $FV = 100$; $I/Y = 10$; $CPT \rightarrow PV = -100$

where:
- N = number of years
- PMT = the *annual* coupon payment
- I/Y = the *annual* discount rate (YTM)
- FV = the par value or face value of the bond received at maturity

This calculation shows that when the coupon of a bond is equal to its yield, the bond's price is equal to par.

PROFESSOR'S NOTE

Take note of a couple points here. The discount rate is entered as a whole number in a percentage, 10, not 0.10. The five coupon payments of $10 each are taken care of in the N = 5 and PMT = 10 entries. The principal repayment is in the FV = 100 entry. Lastly, note that the PV is negative; it will be the opposite sign to the sign of PMT and FV. The calculator is just "thinking" that to receive the payments and future value (to own the bond), you must pay the present value of the bond today (you must buy the bond). That's why the PV amount is negative; it is a cash outflow to a bond buyer.

Now, let's value that same bond with a discount rate of 8%:

$$\frac{10}{(1.08)^1}+\frac{10}{(1.08)^2}+\frac{10}{(1.08)^3}+\frac{10}{(1.08)^4}+\frac{110}{(1.08)^5}$$

The calculator solution is as follows:

N = 5; PMT = 10; FV = 100; I/Y = 8; CPT → PV = −107.99

If the market discount rate for this bond were 8%, it would sell at a **premium** of $7.99 above its par value. When bond yields decrease, the present value of a bond's payments, its market value, increases. Here, we also see that a bond with a coupon greater than its yield will be trading above par.

If we discount the bond's cash flows at 12%, this is the present value of the bond:

$$\frac{10}{(1.12)^1}+\frac{10}{(1.12)^2}+\frac{10}{(1.12)^3}+\frac{10}{(1.12)^4}+\frac{110}{(1.12)^5}$$

The calculator solution is as follows:

N = 5; PMT = 10; FV = 100; I/Y = 12; CPT → PV = −92.79

If the market discount rate for this bond were 12%, it would sell at a **discount** of $100 − $92.79 = $7.21 to its par value. When bond yields increase, the present value of a bond's payments, its market value, decreases. Here, we also see that a bond with a coupon less than its yield will be trading below par.

PROFESSOR'S NOTE

It's worth noting here that a 2% decrease in YTM increases the bond's value by more ($7.99) than a 2% increase in yield decreases the bond's value ($7.21). This illustrates that the bond's price-yield relationship is convex, as we will explain in more detail in a later reading.

Calculating the Value of a Semiannual Coupon Bond

Let's calculate the value of the same bond with semiannual payments. Rather than $10 per year, the security will pay $5 every six months. Assuming an annual YTM of 8%, we need to discount the coupon payments at 4% per period, which results in this present value:

$$\frac{5}{(1.04)^1}+\frac{5}{(1.04)^2}+\frac{5}{(1.04)^3}+\ldots+\frac{5}{(1.04)^9}+\frac{105}{(1.04)^{10}}$$

The calculator solution is as follows:

N = 10; PMT = 5; FV = 100; I/Y = 4; CPT → PV = −108.11

The stated annualized YTM is equal to the periodic return of the bond multiplied by the number of periods in the year. In this case, the bond actually earns 4% every six months, hence the stated annualized YTM = 4% × 2 = 8%.

Calculating Yield to Maturity

Now let's calculate the stated yield of the same bond after changes in market conditions have caused the bond price to move to 105. (This price is stated as a percentage of par quote—in other words, investors will pay 105% of the par value they purchase. The easiest way to interpret this price is as the price of $100 of par.)

We solve for the semiannual return of the bond using the following calculator inputs:

N = 10; PMT = 5; FV = 100; PV = −105; CPT → I/Y = 4.37%

This is the true semiannual return of the bond. To quote a YTM, we need to multiply by two to give a stated YTM of 4.37% × 2 = 8.74%. Also note the negative sign for the PV input: we must respect the direction of cash flows (negative for the cash outflow today, positive for the cash inflow later), or else the calculator will not be able to calculate a rate of return.

To actually earn the YTM over the life of a bond, the investor must hold the bond to maturity, the issuer must make all the promised payments, and the investor must be able to reinvest the periodic cash flows and earn the same YTM.

Accrued Interest, Flat Price, and Full Price

So far we have been calculating bond values on the date a coupon is paid, as the present value of the remaining coupons. For most actual bond trades, the settlement date, which is when cash is exchanged for the bond, will fall between coupon payment dates.

Bond pricing has to account for the fact that the next coupon will be paid to the buyer, but a portion of it (the **accrued interest**) is owed to the seller. The accrued interest since the last payment date can be calculated as the coupon payment times the portion of the coupon period that has passed between the last coupon payment date and the settlement date of the transaction. That is:

$$\text{accrued interest} = \text{coupon payment} \times \frac{\text{days from last coupon to settlement}}{\text{days in coupon period}}$$

Financial markets use a variety of **day count methods**. Two of the most common are the **actual/actual convention**, which uses the actual number of days between coupon payments and the actual number of days between the last coupon date and the settlement date; and the **30/360 convention**, which assumes each month has 30 days and a year has 360 days. Government bonds typically use actual/actual, while corporate bonds typically use 30/360.

EXAMPLE: Accrued interest

An investor buys a 4% annual-pay bond that pays its coupons on May 15. The investor's order settles on August 10. Calculate the accrued interest that is owed to the bond seller, using the 30/360 method and the actual/actual method.

Answer:

The annual coupon payment is 4% × $100 = $4.

Using the 30/360 method, interest is accrued for 30 − 15 = 15 days in May; 30 days each in June and July; and 10 days in August, or 15 + 30 + 30 + 10 = 85 days:

$$\text{accrued interest (30/360 method)} = \frac{85}{360} \times \$4 = \$0.944$$

Using the actual/actual method, interest is accrued for 31 − 15 = 16 days in May; 30 days in June; 31 days in July; and 10 days in August, or 16 + 30 + 31 + 10 = 87 days.

$$\text{accrued interest (actual/actual method)} = \frac{87}{365} \times \$4 = \$0.953$$

Bond prices are quoted without accrued interest. This is because, holding yield constant, including accrued interest would make a bond's price appear to increase on each day of a coupon period and drop suddenly on the coupon payment date. A bond's quoted price is known as its **flat price** (or clean price).

A bond's **full price** (also known as its invoice price or dirty price) is the sum of its flat price and the accrued interest. However, we cannot simply calculate a flat price and add accrued interest to it. Instead we must calculate the full price and derive the flat price from it:

flat price = full price − accrued interest

The method for calculating the full price is as follows:

Step 1: Calculate the value of the bond on the last coupon date.

Step 2: Compound this value at the YTM per period, over the number of days since the last coupon payment:

$$\text{full price} = \text{PV on last coupon date} \times \left(1 + \frac{\text{YTM}}{\text{periods per year}}\right)^{\frac{\text{days since last coupon}}{\text{days in coupon period}}}$$

Let's work an example for a specific bond.

EXAMPLE: Calculating the full and flat prices of a bond

A 5% bond makes coupon payments on June 15 and December 15, and is trading with a YTM of 4%. The bond is purchased, and will settle on August 21 when there will be four coupons remaining until maturity. Calculate the full price, accrued interest, and the flat price of the bond using actual days.

Answer:

Step 1: Calculate the value of the bond on the last coupon date (coupons are semiannual, so we use 4 / 2 = 2% for the periodic discount rate):

N = 4; PMT = 2.5; FV = 100; I/Y = 2; CPT → PV = −101.904

Step 2: Adjust for the number of days since the last coupon payment:

days between June 15 and December 15 = 183 days

days between June 15 and settlement on August 21 = 67 days

full price = $101.904 \times (1.02)^{67/183} = 102.645$

Accrued interest on the settlement date of August 21 is as follows:

$\$2.5 \times (67 / 183) = \$0.915$

Flat price = 102.645 − 0.915 = 101.73.

Note that the flat price is *not* the present value of the bond on its last coupon payment date, 101.73 < 101.904.

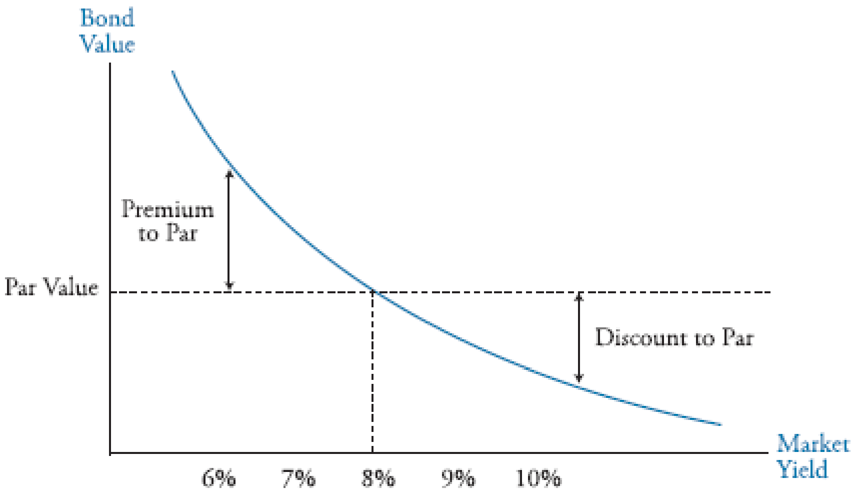
LOS 52.b: Identify the relationships among a bond's price, coupon rate, maturity, and yield-to-maturity.

We can summarize the relationships between price and specific bond features as follows:

1. At a point in time, a decrease (increase) in a bond's YTM will increase (decrease) its price. That is, there is an inverse relationship between yield and price.
2. Other things equal, the price of a bond with a lower coupon rate is more sensitive to a change in yield than is the price of a bond with a higher coupon rate.
3. Other things equal, the price of a bond with a longer maturity is more sensitive to a change in yield than is the price of a bond with a shorter maturity.
4. The percentage decrease in value when the YTM increases by a given amount is smaller than the increase in value when the YTM decreases by the same amount (the price-yield relationship is convex).

Figure 52.1 illustrates the convex relationship between a bond's price and its YTM.

Figure 52.1: Market Yield vs. Bond Value for an 8% Coupon Bond



Relationship Between Price and Maturity

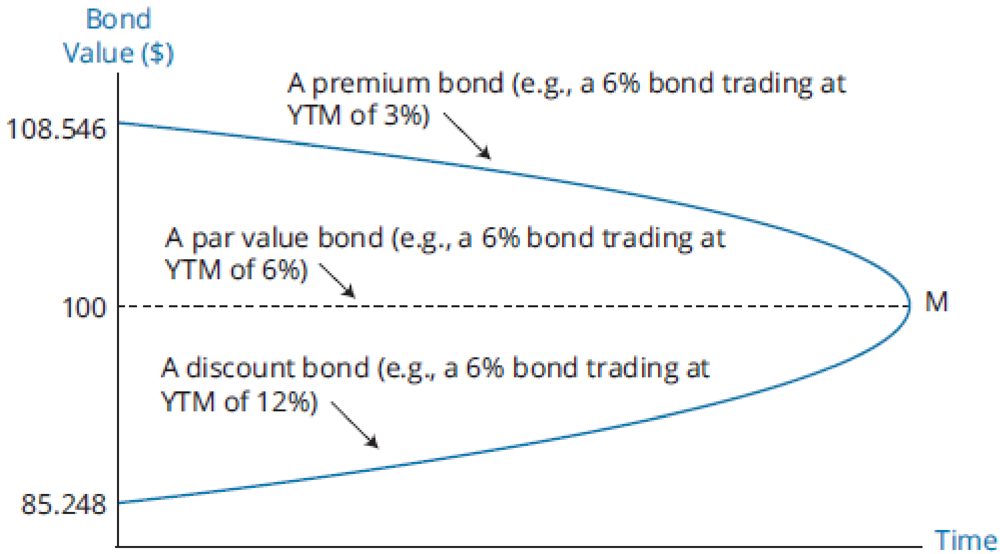
Before maturity, a bond can be selling at a significant discount or premium to par value. However, regardless of its required yield, the price will converge to par value as maturity approaches. Consider a bond with a 3-year life paying 6% semiannual coupons. The bond values corresponding to required yields of 3%, 6%, and 12% as the bond approaches maturity are presented in Figure 52.2.

Figure 52.2: Bond Values and the Passage of Time

Time to Maturity (In Years)	YTM = 3%	YTM = 6%	YTM = 12%
3.0	108.546	100	85.248
2.5	107.174	100	87.363
2.0	105.782	100	89.605
1.5	104.368	100	91.981
1.0	102.934	100	94.500
0.5	101.478	100	97.169
0.0	100.000	100	100.000

The change in value associated with the passage of time for the three bonds represented in Figure 52.2 is presented graphically in Figure 52.3. This convergence to par value ("pull to par") at maturity is known as the **constant-yield price trajectory** because it shows how the bond's price would change as time passes if its YTM remained constant.

Figure 52.3: Premium, Par, and Discount Bonds



LOS 52.c: Describe matrix pricing.

Matrix pricing is a method of estimating the required YTM (or price) of bonds that are currently not traded, or infrequently traded. The procedure is to use the YTMs of traded bonds that have credit quality very close to that of a nontraded or infrequently traded bond and are similar in maturity and coupon, to estimate the required YTM.

EXAMPLE: Pricing an illiquid bond

Rob Phelps, CFA, is estimating the value of a nontraded 4% annual-pay, A+ rated bond that has three years remaining until maturity. He has obtained the following yields to maturity on similar corporate bonds:

- A+ rated, 2-year annual-pay, YTM = 4.3%
- A+ rated, 5-year annual-pay, YTM = 5.1%
- A+ rated, 5-year annual-pay, YTM = 5.3%

Estimate the value of the nontraded bond.

Answer:

Step 1: Take the average YTM of the 5-year bonds:

(5.1 + 5.3) / 2 = 5.2%

Step 2: Interpolate the 3-year YTM based on the 2-year and average 5-year YTMs:

4.3% + (5.2% − 4.3%) × [(3 years − 2 years) / (5 years − 2 years)] = 4.6%

Step 3: Price the nontraded bond with a YTM of 4.6%:

N = 3; PMT = 4; FV = 100; I/Y = 4.6; CPT → PV = −98.354

The estimated value is $98.354 per $100 par value.

In Step 2 in the preceding example, we have used simple linear interpolation. Because the maturity of the nontraded bond is three years, we estimate the YTM on the 3-year bond as the yield on the 2-year bond, plus one-third of the difference between the YTM of the 2-year bond and the average YTM of the 5-year bonds. (The difference between the maturities of the 2-year bond and the 3-year bond is one year, and the difference between the maturities of the 2-year and 5-year bonds is three years.)

A variation of matrix pricing used for new bond issues focuses on spreads. The required yield spread to a benchmark for a new issue bond can be estimated by observing spreads on existing similar securities, as demonstrated in the following example.

EXAMPLE: Estimating the spread for a new 6-year, A rated bond issue

Consider the following market yields:

- 4-year, U.S. Treasury bond, YTM 1.48%
- 5-year, A rated corporate bond, YTM 2.64%
- 6-year, U.S. Treasury bond, YTM 2.15%

Estimate the required yield spread on a newly issued 6-year, A rated corporate bond.

Answer:

We will use the existing 5-year, A rated corporate bond to estimate the required yield spread of the issuer by comparing the YTM of the 5-year corporate bond to the interpolated 5-year Treasury bond YTM.

Interpolated 5-year Treasury bond YTM:

= 1.48% + (2.15% − 1.48%) × [(5 years − 4 years) / (6 years − 4 years)] = 1.815%

Note: Because the target maturity of the existing bond (5 years) is midway between the two Treasury bond maturities (4 and 6 years), we could have simply averaged the two Treasury bond yields here (1.48% + 2.15%) / 2 = 1.815%.

The yield spread on existing 5-year corporate debt is 2.64% − 1.815% = 0.825%.

We will apply this yield spread to the new 6-year corporate debt issue:

YTM for the new 6-year corporate bond = 2.15% + 0.825% = 2.975%

MODULE QUIZ 52.1

1. A 20-year bond has a 10% annual-pay coupon. What is the price of the bond if it has a yield to maturity of 15%?
   A. 68.514.
   B. 68.703.
   C. 82.839.
2. An analyst observes a 5-year, 10% semiannual-pay bond. The face amount is £1,000. The analyst believes that the yield to maturity on a semiannual bond basis should be 15%. Based on this yield estimate, the value of this bond is:
   A. £828.40.
   B. £1,189.53.

C. £1,193.04.

3. An analyst observes a 20-year, 8% option-free bond with semiannual coupons. The required yield to maturity on a semiannual bond basis was 8%, but suddenly it decreased to 7.25%. As a result, the price of this bond:

   A. increased.
   B. decreased.
   C. stayed the same.

4. A $1,000 par, 5% coupon, 20-year annual-pay bond has a YTM of 6.5%. If the YTM remains unchanged, how much will the bond value increase over the next three years?

   A. $13.62.
   B. $13.78.
   C. $13.96.

5. An investor paid a full price of 105.904 for $1 million face value of a bond issue. The purchase was between coupon dates, and accrued interest was 2.354. What is each bond's flat price?

   A. 100.000.
   B. 103.550.
   C. 108.258.

6. Cathy Moran, CFA, is estimating a value for an infrequently traded bond with six years to maturity, an annual coupon of 7%, and a single-B credit rating. Moran obtains yields to maturity for more liquid bonds with the same credit rating:

   - 5% coupon, eight years to maturity, yielding 7.20%.
   - 6.5% coupon, five years to maturity, yielding 6.40%.

   The infrequently traded bond is *most likely* trading at:

   A. par value.
   B. a discount to par value.
   C. a premium to par value.

KEY CONCEPTS

LOS 52.a

The price of a bond is the present value of its future cash flows, discounted at the bond's yield to maturity.

For an annual coupon bond with *N* years to maturity:

$$\text{price} = \frac{\text{coupon}}{(1+\text{YTM})} + \frac{\text{coupon}}{(1+\text{YTM})^2} + \ldots + \frac{\text{coupon} + \text{principal}}{(1+\text{YTM})^N}$$

For a semiannual coupon bond with *N* years to maturity:

$$\text{price} = \frac{\text{coupon}}{\left(1+\frac{\text{YTM}}{2}\right)} + \frac{\text{coupon}}{\left(1+\frac{\text{YTM}}{2}\right)^2} + \ldots + \frac{\text{coupon} + \text{principal}}{\left(1+\frac{\text{YTM}}{2}\right)^{N\times 2}}$$

The full price of a bond includes interest accrued between coupon dates. The flat price of a bond is the full price minus accrued interest.

Accrued interest for a bond transaction is calculated as the coupon payment times the portion of the coupon period from the previous payment date to the settlement date.

Methods for determining the period of accrued interest include actual days or 30-day months and 360-day years.

LOS 52.b

A bond's price and YTM are inversely related. An increase in YTM decreases the price, and a decrease in YTM increases the price.

Bond prices are convex with respect to yield movements, which means price increases when yields fall are greater in magnitude than the fall in prices caused by an equivalent yield rise.

A bond will be priced at a discount to par value if its coupon rate is less than its YTM (deficient coupon), and at a premium to par value if its coupon rate is greater than its YTM (excessive coupon).

Prices are more sensitive to changes in YTM for bonds with lower coupon rates and longer maturities, and less sensitive to changes in YTM for bonds with higher coupon rates and shorter maturities.

A bond's price moves toward par value as time passes and maturity approaches.

LOS 52.c

Matrix pricing is a method used to estimate the yield to maturity for bonds that are not traded or infrequently traded. The yield is estimated based on the yields of traded bonds with the same credit quality. If these traded bonds have different maturities than the bond being valued, linear interpolation is used to estimate the subject bond's yield.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 52.1

1. **B** N = 20; I/Y = 15; FV = 100; PMT = 10; and CPT → PV = −68.703 (LOS 52.a)
2. **A** N = 10; I/Y = 7.5; FV = 1,000; PMT = 50; and CPT → PV = −$828.40 (LOS 52.a)
3. **A** The price-yield relationship is inverse. If the required yield decreases, the bond's price will increase, and vice versa. (LOS 52.b)
4. **A** With 20 years to maturity, the value of the bond with an annual-pay yield of 6.5% is N = 20; PMT = 50; FV = 1,000; I/Y = 6.5; and CPT → PV = −834.72. With N = 17, CPT → PV = −848.34, so the value will increase $13.62. (LOS 52.a, 52.b)
5. **B** The full price includes accrued interest, while the flat price does not. Therefore, the flat (or clean) price is 105.904 − 2.354 = 103.550. (LOS 52.b)
6. **C** Using linear interpolation, the yield on a bond with six years to maturity should be 6.40% + (1 / 3)(7.20% − 6.40%) = 6.67%. A bond with a 7% coupon and a yield of 6.67% is at a premium to par value. (LOS 52.c)

READING 53

YIELD AND YIELD SPREAD MEASURES FOR FIXED-RATE BONDS

MODULE 53.1: YIELD AND YIELD SPREAD MEASURES FOR FIXED-RATE BONDS

Video covering this content is available online.

LOS 53.a: Calculate annual yield on a bond for varying compounding periods in a year.

Given a bond's price in the market, we can say that the yield to maturity (YTM) is the discount rate that makes the present value of a bond's cash flows equal to its price.

For a 5-year, annual-pay 7% bond that is priced in the market at 102.078, the YTM will satisfy the following equation:

$$\frac{7}{(1+\text{YTM})^1}+\frac{7}{(1+\text{YTM})^2}+\frac{7}{(1+\text{YTM})^3}+\frac{7}{(1+\text{YTM})^4}+\frac{107}{(1+\text{YTM})^5}$$
$$=102.078$$

We can calculate the YTM (discount rate) that satisfies this equality as follows:

N = 5; PMT = 7; FV = 100; PV = −102.078; CPT → I/Y = 6.5%

By convention, the YTM on a semiannual coupon bond is stated as two times the semiannual discount rate. For a 5-year, semiannual-pay 7% coupon bond, we can calculate the semiannual discount rate as YTM / 2 and then double it to get the YTM expressed as an annualized yield:

$$\frac{3.5}{\left(1+\frac{\text{YTM}}{2}\right)^1}+\frac{3.5}{\left(1+\frac{\text{YTM}}{2}\right)^2}+\frac{3.5}{\left(1+\frac{\text{YTM}}{2}\right)^3}+\ldots+\frac{3.5}{\left(1+\frac{\text{YTM}}{2}\right)^9}+$$
$$\frac{103.5}{\left(1+\frac{\text{YTM}}{2}\right)^{10}}=102.078$$

$$\text{N}=10;\ \text{PMT}=3.5;\ \text{FV}=100;\ \text{PV}=-102.078;\ \text{CPT}\rightarrow \text{I/Y}=3.253\%$$

The quoted annualized YTM is 3.253 × 2 = 6.506%.

Yield Measures for Fixed-Rate Bonds

The number of bond coupon payments per year is referred to as the **periodicity** of a bond. A bond with a periodicity of 2 will have its YTM quoted on a **semiannual bond basis**. For a given coupon rate, the greater the periodicity, the more compounding periods, and the greater the effective annual yield (which reflects compounding).

In general, the effective annual yield for a bond with its YTM stated for a periodicity of *n* (*n* compounding periods per year) is as follows:

$$\text{annual yield} = \left(1 + \frac{\text{YTM}}{n}\right)^n - 1$$

EXAMPLE: Effective annual yields

What is the effective annual yield for a bond with a stated YTM of 10%:

1. When the periodicity of the bond is 2 (pays semiannually)?
2. When the periodicity of the bond is 4 (pays quarterly)?

Answer:

1. $\text{annual yield} = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 1.05^2 - 1 = 0.1025 = 10.25\%$
2. $\text{annual yield} = \left(1 + \frac{0.10}{4}\right)^4 - 1 = 1.025^4 - 1 = 0.1038 = 10.38\%$

It may be necessary to adjust the quoted yield on a bond to make it comparable with the yield on a bond with a different periodicity. This is illustrated in the following example.

EXAMPLE: Adjusting yields for periodicity

An Atlas Corporation bond is quoted with a YTM of 4% on a semiannual bond basis. What yields should be used to compare it with a quarterly-pay bond and an annual-pay bond?

Answer:

The first thing to note is that a YTM of 4% on a semiannual bond basis means a periodic return of 2% per 6-month period.

To compare this with the yield on an annual-pay bond, which is an effective annual yield, we need to calculate the effective annual yield on the semiannual coupon bond, which is $1.02^2 - 1 = 4.04\%$.

For the quoted annual YTM on the quarterly-pay bond, we need to calculate the effective quarterly yield and multiply by four. The quarterly yield (yield per quarter) that is equivalent to a yield of 2% per six months is $1.02^{1/2} - 1 = 0.995\%$. The quoted annual rate for the equivalent yield on a quarterly bond basis is $4 \times 0.995\% = 3.98\%$.

Note that we have shown that the effective annual yields are the same for the following:

- An annual coupon bond with a yield of 4.04% on an annual basis (periodicity of one)
- A semiannual coupon bond with a yield of 4.0% on a semiannual basis (periodicity of two)
- A quarterly coupon bond with a yield of 3.98% on quarterly basis (periodicity of four)

Bond yields calculated using the stated coupon payment dates are referred to as following the **street convention**. When coupon dates fall on weekends and holidays, coupon payments will actually be made the next business day. The yield calculated using these actual coupon payment dates is referred to as the **true yield**. Because coupon payments will be made later when holidays and weekends are taken into account, true yields are usually slightly lower than street convention yields, if only by a few basis points.

Current yield (also called **income yield** or **running yield**) looks at just one source of return, which is *a bond's annual interest income*—it does not consider capital gains or losses or reinvestment income. The formula for the current yield is as follows:

$$\text{current yield} = \frac{\text{annual cash coupon payment}}{\text{bond price}}$$

EXAMPLE: Computing current yield

Consider a 20-year, $1,000 par value, 6% semiannual-pay bond that is currently trading at a flat price of $802.07. Calculate the current yield.

Answer:

The annual cash coupon payments total:

annual cash coupon payment = par value × stated coupon rate = $1,000 × 0.06 = $60

Because the bond is trading at $802.07, the current yield is:

$$\text{current yield} = \frac{60}{802.07} = 0.0748\text{, or } 7.48\%$$

Note that current yield is based on *annual* coupon interest so that it is the same for a semiannual-pay and annual-pay bond with the same coupon rate and price.

A bond's **simple yield** takes a discount or premium into account by assuming that any discount or premium declines evenly over the remaining years to maturity. The sum of the annual coupon payment plus (minus) the straight-line amortization of a discount (premium) is divided by the flat price to get the simple yield.

EXAMPLE: Computing simple yield

A 3-year, 8% coupon, semiannual-pay bond is priced at 90.165. Calculate the simple yield.

Answer:

The discount from par value is 100 − 90.165 = 9.835. Annual straight-line amortization of the discount is 9.835 / 3 = 3.278:

$$\text{simple yield} = \frac{8 + 3.278}{90.165} = 12.51\%$$

For a callable bond, an investor's yield will depend on whether and when the bond is called. The **yield to call** can be calculated for each possible call date and price. The lowest of YTM and the various yields to call is termed the **yield to worst**. The following example illustrates these calculations.

EXAMPLE: Yield to call and yield to worst

Consider a 5-year, semiannual-pay 6% bond trading at 102 on January 1, 20X4. The bond is callable according to the following schedule:

- Callable at 102 on or after January 1, 20X7
- Callable at 101 on or after January 1, 20X8

Calculate the bond's YTM, yield to first call, yield to second call, and yield to worst.

Answer:

The yield to maturity on the bond is calculated as:

N = 10; PMT = 3; FV = 100; PV = −102; CPT → I/Y = 2.768%

2 × 2.768 = 5.54% = YTM

To calculate the *yield to first call*, we calculate the YTM using the number of semiannual periods until the first call date in 20X7 (6) for *N* and the call price (102) for *FV*:

N = 6; PMT = 3; FV = 102; PV = −102; CPT → I/Y = 2.941%

2 × 2.941 = 5.88% = yield to first call

To calculate the *yield to second call*, we calculate the YTM using the number of semiannual periods until the second call date in 20X8 (8) for *N* and the call price (101) for *FV*:

N = 8; PMT = 3; FV = 101; PV = −102; CPT → I/Y = 2.830%

2 × 2.830 = 5.66% = yield to second call

The lowest yield, 5.54%, is realized if the bond is held to maturity and not called, so the *yield to worst* is 5.54%.

A callable bond can be viewed as an equivalent straight (option-free) bond combined with a *short* call option position (because the right to call the bond lies with the issuer, not the investor):

callable bond value = straight bond value − call option value

Stated differently, we can view the value of an equivalent straight bond (known as the **option-adjusted price**) as the value of the callable bond *plus* the value of the call option embedded in the callable bond (which could be derived from an option pricing model).

This option-adjusted price can then be used to calculate an **option-adjusted yield**, which represents the yield that the bond would be offering if it were not callable. Because the existence of the call option decreases the bond price and increases an investor's required yield, "removing" the option in this manner will cause the option-adjusted yield to be *lower* than the yield of the callable bond. The option-adjusted yield is useful because it can be used to compare the yields of bonds with various embedded options to each other and to similar option-free bonds on a consistent basis.

PROFESSOR'S NOTE

Take care to understand that option-adjusted prices and yields remove the impact of the option from a bond with an embedded option. It is a common mistake when first encountering these measures to incorrectly think they are adjusting to incorporate the impact of the option, when in fact they are doing the opposite. When you see "option-adjusted," think "option removed" or "option taken away."

LOS 53.b: Compare, calculate, and interpret yield and yield spread measures for fixed-rate bonds.

A **yield spread**, or **benchmark spread**, is the difference between the yields of a bond and a benchmark security. For example, if a 5-year corporate bond has a yield of 6.25% and its benchmark, the 5-year Treasury note, has a yield of 3.50%, the corporate bond has a benchmark spread of 625 − 350 = 275 basis points.

For fixed-coupon bonds, on-the-run government bond yields for the same or nearest maturity are frequently used as benchmarks because they are the most actively traded bonds, and therefore give the most useful price and yield information. A yield spread in basis points over a government bond is also known as a **G-spread**. If a benchmark government bond with exactly the same maturity as the riskier bond does not exist, interpolation should be used to estimate the appropriate maturity benchmark yield.

EXAMPLE: G-spread

A 3-year, 8% coupon, semiannual-pay bond is priced at 103.165, and 1-year and 4-year U.S. Treasury yields are 3% and 5%, respectively. Calculate the G-spread of the bond.

Answer:

The YTM of the bond is calculated as:

$N = 6$; $PMT = 4$; $FV = 100$; $PV = -103.165$; CPT → $I/Y = 3.408\%$

Quoted YTM $= 2 \times 3.408\% = 6.82\%$

The interpolated 3-year government bond yield is:

$$3\% + [(3 - 1) / (4 - 1)] \times (5\% - 3\%) = 4.33\%$$

> Then, the G-spread = 6.82% − 4.33% = 2.49%, or 249 basis points.

An alternative to using government bond yields as benchmarks is to use rates for interest rate swaps in the same currency and with the same tenor as a bond. Yield spreads relative to swap rates are known as **interpolated spreads** or **I-spreads** and represent the extra return of a bond in excess of the interbank market reference rates (MRRs) used in swap contracts. I-spreads are frequently stated for bonds denominated in euros.

Yield spreads are useful for analyzing the factors that affect a bond's yield. If a corporate bond's yield increases from 6.25% to 6.50%, this may have been caused by factors that affect all bond yields (macroeconomic factors) or by firm-specific or industry-specific (microeconomic) factors. If a bond's yield increases but its spread remains the same, the yield on its benchmark must have also increased, which suggests macroeconomic factors caused bond yields in general to increase. However, if the yield spread increases, this suggests the increase in the bond's yield was caused by microeconomic factors, such as credit risk of the issuer increasing or the issue's liquidity deteriorating.

PROFESSOR'S NOTE

Recall from Quantitative Methods that an interest rate is composed of the real risk-free rate, the expected inflation rate, and a risk premium. We can think of macroeconomic factors as those that affect the real risk-free rate and expected inflation (which make up the benchmark yield), and microeconomic factors as those that affect credit and liquidity risk premiums (which make up the yield spread). Differences in taxation of the returns from bonds of different issuers can also affect yield spreads.

Zero-Volatility and Option-Adjusted Spreads

The G-spread and I-spread are based on the difference between the yields of a specific bond and a benchmark. Recall that the YTM of a bond is the *single* discount rate that sets the present value of the cash flows of the bond equal to its market price.

We can observe from the prices of zero-coupon bonds that different individual cash flows occurring at different maturities can have different yields. Yields earned by individual cash flows at different maturities are referred to as **spot rates**. The single YTM of a coupon-paying bond represents a weighted average of the different spot rates offered by the individual cash flows of the bond.

PROFESSOR'S NOTE

Spot rates are formally introduced and discussed later in our reading on The Term Structure of Interest Rates. For now, it is enough to know that calculating spreads over benchmark spot rates is a more precise way of calculating spreads than basing the measure on YTMs because it better captures how rates vary over different maturities (referred to as the term structure of rates).

A method for deriving a bond's yield spread to a benchmark spot yield curve that accounts for the shape of the yield curve is to add an equal amount to each benchmark spot rate and value the bond with those rates. When we find an amount which, when added to the benchmark spot rates, produces a value equal to the market price of the bond, we have the appropriate yield curve spread. A yield spread calculated this way is known as a **zero-volatility spread** or **Z-spread**.

EXAMPLE: Zero-volatility spread

The 1-, 2-, and 3-year spot rates on Treasuries are 4%, 8.167%, and 12.377%, respectively. Consider a 3-year, 9% annual coupon corporate bond trading at 89.464. The YTM is 13.50%, and the YTM of a 3-year Treasury is 12%. Calculate the G-spread and the Z-spread of the corporate bond.

Answer:

The G-spread is $YTM_{bond} - YTM_{Treasury} = 13.50 - 12.00 = 1.50\%$.

To compute the Z-spread, set the present value of the bond's cash flows equal to today's market price. Discount each cash flow at the appropriate zero-coupon bond spot rate *plus* a fixed-spread ZS. Solve for ZS in the following equation, and you have the Z-spread:

$$89.464 = \frac{9}{(1.04 + ZS)^1} + \frac{9}{(1.08167 + ZS)^2} + \frac{109}{(1.12377 + ZS)^3}$$

$$\Rightarrow ZS = 1.67\%, \text{ or } 167 \text{ basis points}$$

Note that this spread is found by trial and error. In other words, pick a number "ZS," plug it into the right-hand side of the equation, and see if the result equals 89.464. If the right-hand side equals the left, then you have found the Z-spread. If not, adjust ZS in the appropriate direction and recalculate.

An **option-adjusted spread (OAS)** is used for bonds with embedded options. Loosely speaking, the OAS takes the option yield component out of the Z-spread measure; the OAS is the spread to the government spot rate curve that the bond would have if it were option free. This is similar to the option-adjusted yields discussed earlier, the only difference being OAS respects the term structure of rates instead of being based on YTM.

If we calculate an OAS for a callable bond, it will be less than the bond's Z-spread. The difference is the extra yield required to compensate bondholders for the call option. That extra yield is referred to as the option value. Thus, we can write the following:

option value = Z-spread − OAS
OAS = Z-spread − option value

For example, if a callable bond has a Z-spread of 180 bp and the value of the call option is 60 bp, the bond's OAS is 180 − 60 = 120 bp.

The interpretation here is that investors are demanding the Z-spread (180 bp) for the credit, liquidity, taxation, and optionality risks of the callable bond. When the component of the spread relating to optionality is *removed*, we are left with the OAS (120 bp) that rewards investors for facing credit, liquidity, and taxation risks.

MODULE QUIZ 53.1

1. Based on semiannual compounding, what is the YTM of a 15-year, zero-coupon, $1,000 par value bond that is currently trading at $331.40?
   A. 3.750%.
   B. 5.151%.
   C. 7.500%.
2. An analyst observes a Widget & Co. 7.125%, 4-year, semiannual-pay bond trading at 102.347. The bond is callable at 101 in two years. The bond's yield to call is *closest* to:
   A. 3.2%.
   B. 6.3%.
   C. 9.4%.
3. Holding the effective annual yield constant, if the periodicity of a bond is increased, its stated YTM will:
   A. decrease.
   B. stay the same.
   C. increase.
4. A corporate bond is quoted at a spread of +235 basis points over an interpolated 12-year U.S. Treasury bond yield. This spread is a(n):
   A. G-spread.
   B. I-spread.
   C. Z-spread.
5. For a callable bond, relative to its option-adjusted spread, its Z-spread is *most likely* to be:
   A. lower.
   B. the same.
   C. higher.

KEY CONCEPTS

LOS 53.a

The effective yield of a bond depends on its periodicity, or frequency of coupon payments. For an annual-pay bond, the effective yield is equal to the yield to maturity (YTM). For bonds with greater periodicity, the effective yield is greater than the YTM.

A YTM quoted on a semiannual bond basis is two times the semiannual discount rate.

Bond yields that follow street convention use the stated coupon payment dates. A true yield accounts for coupon payments that are delayed by weekends or holidays and may be slightly lower than a street convention yield.

Current yield is the ratio of a bond's annual coupon payments to its price. Simple yield adjusts current yield by using straight-line amortization of any discount or premium.

For a callable bond, a yield to call may be calculated using each of its call dates and prices. The lowest of these yields or its YTM is a callable bond's yield to worst.

LOS 53.b

A yield spread or benchmark spread is the difference between a bond's yield and a benchmark yield or yield curve. If the benchmark is a government bond yield, the spread is known as a government spread or G-spread. If the benchmark is a swap rate, the spread is known as an interpolated spread or I-spread.

A zero-volatility spread or Z-spread is the percentage spread that must be added to each spot rate on the benchmark yield curve to make the present value of a bond's cash flows equal to its price.

An option-adjusted spread (OAS) is used for bonds with embedded options and represents the spread the bond would offer if it had no embedded options. For a callable bond, the OAS is equal to the Z-spread minus the call option value in basis points.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 53.1

1. **C** N = 30; FV = 1,000; PMT = 0; PV = −331.40; CPT → I/Y = 3.750 × 2 = 7.500%

   Alternatively, $\left[\left(\frac{1{,}000}{331.4}\right)^{\frac{1}{30}} - 1\right] \times 2 = 7.5\%.$

   (LOS 53.a)

2. **B** N = 4; FV = 101; PMT = 3.5625; PV = −102.347; CPT → I/Y = 3.167 × 2 = 6.334%

   (LOS 53.a)

3. **A** Due to their increased compounding frequency, bonds with higher periodicity will have a lower stated YTM for a certain level of effective annual yield (EAY). For example, for an EAY of 5% and periodicity of 2, the stated YTM must solve $[1 + (\text{YTM} / 2)]^2 = 1.05$. Hence, YTM = $2\,(1.05^{1/2} - 1)$ = 4.94%. With periodicity of 4, the stated YTM must solve $[1 + (\text{YTM} / 4)]^4 = 1.05$. Hence, YTM = $4\,(1.05^{1/4} - 1)$ = 4.91%. (LOS 53.a)

4. **A** G-spreads are quoted relative to an actual or interpolated government bond yield. I-spreads are quoted relative to swap rates. Z-spreads are calculated based on the shape of the benchmark yield curve. (LOS 53.b)

5. **C** A callable bond will offer a higher yield than an equivalent straight bond because the investor faces the call risk of the option. Hence, the Z-spread, which includes the impact of the option, will be higher than the OAS, which has removed the impact of the option. (LOS 53.b)

READING 54

YIELD AND YIELD SPREAD MEASURES FOR FLOATING-RATE INSTRUMENTS

MODULE 54.1: YIELD AND YIELD SPREAD MEASURES FOR FLOATING-RATE INSTRUMENTS

Video covering this content is available online.

LOS 54.a: Calculate and interpret yield spread measures for floating-rate instruments.

Floating-Rate Note Yields

The values of floating-rate notes (FRNs) are more stable than those of fixed-rate debt of similar maturity because the coupon rate is reset periodically based on a variable market reference rate (MRR). Recall that the coupon rate on an FRN consists of a relatively risk-free (usually interbank) MRR plus a fixed margin based on the credit risk of the issuer (at the time of issuance) relative to the credit risk of the MRR. The coupon rate for the next period is set using the current MRR for the reset period, and the payment at the end of the period is based on this rate. For this reason, we say that interest is paid *in arrears*.

If an FRN is issued by a company that has more (less) credit risk than the financial institutions from which the MRR is derived, a margin is added to (subtracted from) the MRR. The liquidity of an FRN and its tax treatment can also affect the margin.

The fixed margin above the MRR actually paid in the coupon is referred to as the **quoted margin (QM)**. The margin required to price the FRN at par is called the **required margin** or the **discount margin (DM)**.

FRNs are usually issued at par with the QM equal to the DM at issuance. If the credit quality of an FRN remains unchanged after issuance, the QM will remain equal to the DM and the FRN will trade at par on its coupon reset dates.

If the credit quality of the issuer decreases after issuance of the FRN, investors will demand a higher DM in compensation for increased credit risk. This will cause the DM to be greater than the fixed QM, and the FRN will trade at a discount to its par value.

Similarly, if the issuer's credit quality improves during an FRN's life, the DM will be less than the fixed QM, and the FRN will sell at a premium to its par value.

PROFESSOR'S NOTE

This is analogous to the relationship between the coupon and yield for fixed-coupon instruments. If investors demand more yield from a fixed-coupon bond than the regular coupon, then the coupon is said to be deficient, and the bond trades at a discount to par. With an FRN, the bond pays a coupon of MRR + QM and investors demand a yield of MRR + DM. When the MRR + DM is greater than MRR + QM, the QM is deficient and the FRN will trade below par.

A simplified way of calculating the value of an FRN on a reset date is to use the current MRR plus the QM to estimate the future cash flows for the FRN, and discount these future cash flows at the MRR + DM. More complex models produce better estimates of value.

EXAMPLE: Valuation of an FRN

A $100,000 FRN with a semiannual coupon pays a 180-day MRR plus a quoted margin of 120 basis points. On a reset date with five years remaining to maturity, the 180-day MRR is quoted as 3.0% (annualized), and the discount margin (based on the issuer's current credit rating) is 1.5% (annualized). Estimate the value of the FRN.

Answer:

The current annualized coupon rate on the note is 3.0% + 1.2% = 4.2%, so the next semiannual coupon payment will be 4.2% / 2 = 2.1% of face value.

The required return in the market (MRR + discount margin) as an effective 180-day discount rate is 4.5% / 2 = 2.25%.

Using a face value of 100%, 10 coupon payments of 2.1%, and a discount rate per period of 2.25%, we can calculate the present value of the FRN as:

N = 10; I/Y = 2.25; FV = 100; PMT = 2.1; CPT ® PV = −98.67

By this method, we can estimate current value of the note as 98.67% of its face value, or $98,670.

LOS 54.b: Calculate and interpret yield measures for money market instruments.

For money market securities (debt securities maturing in a year or less), yields are quoted using various conventions. Some yield quotes are based on a 360-day year, while others are based on a 365-day year. Some yield quotes are add-on yields, and others are discount yields. Add-on yields are simply the interest to be earned on the amount paid or deposited today. Discount yields are annualized current discounts from the face values of money market securities received at maturity.

Bank CDs, repos, and market reference rates are typically quoted as annualized add-on rates. U.S. Treasury bills (T-bills) and commercial paper are quoted as their annualized discount from face value, based on a 360-day year.

The relation between a security's yield quoted as an annualized add-on yield based on a 365-day year and its holding period yield (HPY) is as follows:

quoted add-on yield = HPY × 365 / days to maturity

Consider a 100-day bank CD with an add-on yield (annualized) of 1.5%, based on a 365-day year. We can calculate the HPY of the CD as the quoted yield of 1.5% × 100 / 365 = 0.41%. The purchase of a $1,000 CD would provide a payment of $1,004.10 in 100 days.

The relation between a quoted discount and the actual unannualized discount based on a 360-day year is as follows:

quoted discount yield = actual discount on the security × 360 / days to maturity

Consider a 180-day U.S. T-bill quoted at a 2.2% (annualized) discount yield based on a 360-day year. The actual discount from face value on the T-bill is 180 / 360 × 2.2% = 1.1%. A $1,000 T-bill would be priced at (1 − 0.011) × 1,000 = $989. The HPY of the T-bill is 1,000 / 989 − 1 = 1.11%, slightly higher than its discount from face value of 1.1%.

An analyst should be able to convert the yield of a security calculated on one basis to its yield on another basis. Such adjustments allow us to compare the yields of two money market securities for which quoted yields are calculated differently. The following provides some examples of converting a yield to one based on a different convention.

EXAMPLE: Money market yields

1. A $1,000 90-day T-bill is priced with an annualized discount of 1.2%. Calculate its market price and its annualized add-on yield based on a 365-day year.
2. A $1 million negotiable CD with 120 days to maturity is quoted with an add-on yield of 1.4% based on a 365-day year. Calculate the payment at maturity for this CD and its bond equivalent yield.
3. A bank deposit for 100 days is quoted with an add-on yield of 1.5% based on a 360-day year. Calculate the bond equivalent yield and the yield on a semiannual bond basis.

Answer:

1. The discount from face value is 1.2% × 90 / 360 × 1,000 = $3, so the current price is 1,000 − 3 = $997.

   The equivalent add-on yield for 90 days is 3 / 997 = 0.3009%. The annualized add-on yield based on a 365-day year is 365 / 90 × 0.3009% = 1.2203%. This add-on yield based on a 365-day year is referred to as the *bond equivalent yield* for a money market security.
2. The add-on interest for the 120-day period is 120 / 365 × 1.4% = 0.4603%.

   At maturity, the CD will pay $1 million × (1 + 0.004603) = $1,004,603.

   The quoted yield on the CD of 1.4% is already the bond equivalent yield by definition because it is an add-on yield annualized based on a 365-day year.
3. Because the yield of 1.5% is an annualized yield calculated based on a 360-day year, the bond equivalent yield, which is based on a 365-day year, is:

(365 / 360) × 1.5 % = 1.5208%

We may want to compare the yield on a money market security to the YTM of a semiannual-pay bond. The method is to convert the money market security's holding period return to an effective semiannual yield, and then double it.

Because the quoted yield of 1.5% is calculated as the add-on yield for 100 days times 360 / 100, the 100-day holding period return is 1.5% × 100 / 360 = 0.4167%. The effective annual yield is $1.004167^{365/100} - 1 = 1.5294\%$, the equivalent semiannual yield is $1.015294^{1/2} - 1 = 0.7618\%$, and the annual yield on a semiannual bond basis is 2 × 0.7618% = 1.5236%.

Because the periodicity of the money market security, 365 / 100, is greater than the periodicity of 2 for a semiannual-pay bond, the simple annual rate for the money market security, 1.5%, is less than the yield on a semiannual bond basis.

MODULE QUIZ 54.1

1. A floating-rate note has a quoted margin of +50 basis points and a required margin of +75 basis points. On its next reset date, the price of the note will be:
   A. equal to par value.
   B. less than par value.
   C. greater than par value.
2. Which of the following money market yields is a bond equivalent yield?
   A. Add-on yield based on a 365-day year.
   B. Discount yield based on a 360-day year.
   C. Discount yield based on a 365-day year.
3. Which of the following money market instruments has the highest bond equivalent yield?
   A. A 90-day Treasury bill quoted with a discount of 1% on a 360-day basis.
   B. A 183-day commercial paper quoted with a discount of 1% on a 365-day basis.
   C. A 91-day certificate of deposit offering an add-on rate of 1% on a 365-day basis.

KEY CONCEPTS

LOS 54.a

Floating-rate notes (FRNs) pay a coupon equal to a fixed quoted margin relative over a market reference rate (MRR). The required margin (or discount margin) on an FRN is the extra return over MRR demanded by investors due to credit and liquidity risk of the issuer. At issuance, FRNs usually have a quoted margin equal to the discount margin; hence, the FRN is issued at par value.

When credit conditions deteriorate and the discount margin rises above the quoted margin, the FRN will trade below par.

When credit conditions improve and the discount margin falls below the quoted margin, the FRN will trade above par.

LOS 54.b

For money market instruments, yields may be quoted on a discount basis or an add-on basis, and they may use 360-day or 365-day years. A bond equivalent yield is an add-on yield based on a 365-day year.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 54.1

1. **B** If the required margin is greater than the quoted margin, the credit quality of the issue has decreased, and the price on the reset date will be less than par value. (LOS 54.a)

2. **A** An add-on yield based on a 365-day year is a bond equivalent yield. (LOS 54.b)

3. **A** The bond equivalent yield is defined as the add-on yield quoted on a 365-day basis.

   The 90-day Treasury is trading at a 1% × 90 / 360 = 0.25% discount to par. Hence, it is trading at a price of 99.75. The HPR is, therefore, 100 / 99.75 − 1 = 0.2506%, and the bond equivalent yield is 0.2506% × 365 / 90 = 1.016%.

   The commercial paper is trading at a 1% × 183 / 365 = 0.5014% discount to par. Hence, it is trading at a price of 99.4986. The HPR is, therefore, 100 / 99.4986 − 1 = 0.5039%, and the bond equivalent yield is 0.5039% × 365 / 183 = 1.005%.

   The quoted return of the certificate of deposit is already in bond equivalent form; hence, no translation is needed here. (LOS 54.b)

READING 55

THE TERM STRUCTURE OF INTEREST RATES: SPOT, PAR, AND FORWARD CURVES

MODULE 55.1: THE TERM STRUCTURE OF INTEREST RATES: SPOT, PAR, AND FORWARD CURVES

Video covering this content is available online.

LOS 55.a: Define spot rates and the spot curve, and calculate the price of a bond using spot rates.

The yield to maturity of a bond is calculated as if the discount rate for every bond cash flow is the same. In reality, discount rates vary according to the time period in which a future cash flow is made. These discount rates for a single payment to be received in the future are called **spot rates** and can be observed by calculating the discount rates for zero-coupon bonds (hence, spot rates are sometimes referred to as *zero-coupon rates* or simply *zero rates*).

To price a bond with spot rates, we sum the present values of the bond's payments, each discounted at the spot rate for the number of periods before it will be paid. The general equation for calculating a bond's value using spot rates (S_i) is as follows:

$$\frac{CPN_1}{1+S_1}+\frac{CPN_2}{(1+S_2)^2}+\ldots+\frac{CPN_N+FV_N}{(1+S_N)^N}=PV$$

EXAMPLE: **Valuing a bond using spot rates**

Given the following spot rates, calculate the value of a 3-year, 5% annual coupon bond.

Spot rates:

1-year: 3%

2-year: 4%

3-year: 5%

Answer:

$$\frac{5}{(1.03)^1} + \frac{5}{(1.04)^2} + \frac{105}{(1.05)^3} = 100.180$$

This price, calculated using spot rates, is sometimes called the *no-arbitrage price* of a bond because if a bond is priced differently, there will be a profit opportunity from arbitrage among bonds.

Because the bond value is slightly greater than its par value, we know its YTM is slightly less than its coupon rate of 5%. Using the price of 100.180, we can calculate the YTM for this bond as follows:

N = 3; PMT = 5; FV = 100; PV = −100.180; CPT → I/Y = 4.93%

PROFESSOR'S NOTE

It's useful to think of a spot rate as a return offered by a single cash flow occurring at a certain time in the future. A weighted average of these spot rates gives us an idea of what the yield is likely to be. For the bond in the previous example, we have the first coupon returning 3%, the second coupon returning 4%, and the third coupon and par payment returning 5%. Given that most of the cash flows occur at Time 3 when the par payment is made, the average is going to be heavily weighted to 5%. This means we can easily see that the yield must be just below 5%.

The **spot curve** displays spot rates versus maturity for a particular type of bond or issuer (e.g., U.S. Treasury government spot rates). We will construct and use spot curves later in this reading.

LOS 55.b: Define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.

Par Yields

Par yields reflect the coupon rate that a hypothetical bond at each maturity would need to have to be priced at par, given a specific spot curve. Alternatively, they can be viewed as the YTM of a hypothetical par bond at each maturity.

Consider a 3-year annual-pay bond and spot rates for one, two, and three years of S_1, S_2, and S_3. The following equation can be used to calculate the coupon rate, PMT, necessary for the bond to be trading at par:

$$\frac{\text{PMT}}{1 + S_1} + \frac{\text{PMT}}{(1 + S_2)^2} + \frac{\text{PMT} + 100}{(1 + S_3)^3} = 100$$

With spot rates of 1%, 2%, and 3%, a 3-year annual par bond will have a payment that will satisfy the following:

$$\frac{\text{PMT}}{1.01} + \frac{\text{PMT}}{(1.02)^2} + \frac{\text{PMT} + 100}{(1.03)^3} = 100$$

In this case, the payment is 2.96 and the par bond coupon rate is 2.96%.

PROFESSOR'S NOTE

If this type of calculation appears on the exam, the best tactic is to plug in the middle answer choice and see if it gives a value of 100. If it results in a value less than 100, the larger choice must be correct. If it produces a value greater than 100, the smaller choice must be correct. Algebraically solving for PMT can be done, but it takes much more time than simply trying the coupon rates in the answers.

Forward Rates

A **forward rate** is a borrowing/lending rate for a loan to be made at some future date. The notation used must identify both the start and length of the lending/borrowing period. The most common convention (and the one used on the Level I CFA exam) is to denote a forward period using two numbers, each followed by a letter indicating the compounding period (*y* for years, *m* for months). The first number represents the future period in which the loan begins, and the second denotes the length of the loan. Hence, 2y1y is the rate for a 1-year loan to be made two years from now; 3y2y is the 2-year forward rate three years from now; 1y1y is the rate for a 1-year loan one year from now; and so on.

When linking forward rates to spot rates—say, for example, over the next three years—a key no-arbitrage idea is that *borrowing for three years at the 3-year spot rate, or borrowing for one-year periods in three successive years, should have the same cost*.

This relation is illustrated as $(1 + S_3)^3 = (1 + S_1)(1 + 1y1y)(1 + 2y1y)$.

Thus, $S_3 = [(1 + S_1)(1 + 1y1y)(1 + 2y1y)]^{1/3} - 1$, which is a basic geometric mean periodic return.

More generally, the spot rate earned between now and a future maturity must be equal to the compounded forward rates that apply to each period out to that maturity.

EXAMPLE: Computing spot rates from forward rates

If the current 1-year spot rate is 2%, the 1-year forward rate one year from today (1y1y) is 3%, and the 1-year forward rate two years from today (2y1y) is 4%, what is the 3-year spot rate?

Answer:

$$S_3 = [(1.02)(1.03)(1.04)]^{1/3} - 1 = 2.997\%$$

This can be interpreted to mean that a dollar compounded at 2.997% for three years would produce the same ending value as a dollar that earns compound interest of 2% the first year, 3% the next year, and 4% for the third year.

PROFESSOR'S NOTE

You can get a very good approximation of the 3-year spot rate with the simple average of the forward rates. In the previous example, we calculated 2.997%, and the simple average of the three annual rates is as follows:

$$\frac{2+3+4}{3} = 3\%$$

We can use the same relationships we use to calculate spot rates from forward rates to calculate forward rates from spot rates.

This is our basic relation between forward rates and spot rates (for two periods):

$$(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$$

This again tells us that an investment has the same expected yield (borrowing has the same expected cost) whether we invest (borrow) for two periods at the 2-period spot rate, S_2, or for one period at the current 1-year rate, S_1, and for the next period at the forward rate, 1y1y. Given two of these rates, we can solve for the other.

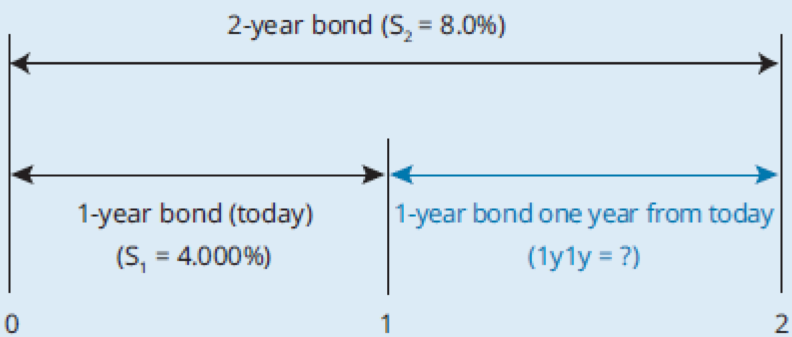
EXAMPLE: Computing a forward rate from spot rates

The 2-period spot rate, S_2, is 8%, and the 1-period spot rate, S_1, is 4%. Calculate the forward rate for one period, one period from now, 1y1y.

Answer:

The following figure illustrates the problem.

Finding a Forward Rate



From our original equality, $(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$, we can get:

$$\frac{(1+S_2)^2}{(1+S_1)} = (1 + 1y1y)$$

Or, because we know that both choices have the same payoff in two years:

$$(1.08)^2 = (1.04)(1 + 1y1y)$$

$$(1 + 1y1y) = \frac{(1.08)^2}{(1.04)}$$

$$1y1y = \frac{(1.08)^2}{(1.04)} - 1 = \frac{1.1664}{1.04} - 1 = 12.154\%$$

In other words, investors are willing to accept 4.0% on the 1-year bond today (when they could get 8.0% on the 2-year bond today) only because they can get 12.154% on a 1-year bond one year from today. This future rate that can be locked in today is a forward rate.

Similarly, we can back other forward rates out of the spot rates. We know that:

$$(1 + S_3)^3 = (1 + S_1)(1 + 1y1y)(1 + 2y1y)$$

And that:

$(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$, so we can write
$(1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y)$.

This last equation says that investing for three years at the 3-year spot rate should produce the same ending value as investing for two years at the 2-year spot rate—and then for a third year at 2y1y, the 1-year forward rate, two years from now.

Solving for the forward rate, 2y1y, we get:

$$\frac{(1 + S_3)^3}{(1 + S_2)^2} - 1 = 2y1y$$

EXAMPLE: Forward rates from spot rates

Let's extend the previous example to three periods. The current 1-year spot rate is 4.0%, the current 2-year spot rate is 8.0%, and the current 3-year spot rate is 12.0%. Calculate the 1-year forward rate two years from now.

Answer:

We know the following relation must hold:

$$(1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y)$$

Substituting values for S_3 and S_2, we have:

$$(1.12)^3 = (1.08)^2 \times (1 + 2y1y)$$

This is so that the 1-year forward rate two years from now is:

$$2y1y = \frac{(1.12)^3}{(1.08)^2} - 1 = 20.45\%$$

We can check our results by calculating:

$$S_3 = [(1.04)(1.12154)(1.2045)]^{1/3} - 1 = 12.00\%$$

This may all seem a bit complicated, but the basic relation, that borrowing for successive periods at 1-period rates should have the same cost as borrowing at multiperiod spot rates, can be summed up as follows:

$(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$ for two periods, and
$(1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y)$ for three periods

PROFESSOR'S NOTE

Simple averages also give decent approximations for calculating forward rates from spot rates (particularly when rates are small). In the preceding example, we had spot rates of 4% for one year and 8% for two years. Two years at 8% is 16%, so if the first-year rate is 4%, the second-year forward rate is close to 16 − 4 = 12% (actual is 12.154). Given a 2-year spot rate of 8% and a 3-year spot rate of 12%, we could approximate the 1-year forward rate from Time 2 to Time 3 as (3 × 12) − (2 × 8) = 20. That may be close enough (actual is 20.45) to answer a multiple-choice question—and, in any case, it serves as a good check to make sure the exact rate you calculate is reasonable.

We can also calculate implied forward rates for loans for more than one period. Given spot rates (1-year = 5%, 2-year = 6%, 3-year = 7%, and 4-year = 8%), we can calculate 2y2y.

The implied forward rate on a 2-year loan two years from now, 2y2y, is as follows:

$$\left[\frac{(1+S_4)^4}{(1+S_2)^2}\right]^{1/2} - 1 = \left(\frac{1.08^4}{1.06^2}\right)^{1/2} - 1 = 10.04\%$$

PROFESSOR'S NOTE

The approximation works for multiperiod forward rates as well.

The difference between four years at 8% (= 32%) and two years at 6% (= 12%) is 20%. Because that difference is for two years, we divide by two to get an annual rate of 10%, $\frac{(4 \times 8 - 6 \times 2)}{2} = 10$, which is very close to the exact solution of 10.04%.

EXAMPLE: Computing a bond value using forward rates

The current 1-year rate, S_1, is 4%, the 1-year forward rate for lending from time = 1 to time = 2 is 1y1y = 5%, and the 1-year forward rate for lending from time = 2 to time = 3 is 2y1y = 6%. Value a 3-year annual-pay bond with a 5% coupon and a par value of $1,000.

Answer:

$$\text{bond value} = \frac{50}{1+S_1} + \frac{50}{(1+S_1)(1+1y1y)} + \frac{1{,}050}{(1+S_1)(1+1y1y)(1+2y1y)}$$

$$= \frac{50}{1.04} + \frac{50}{(1.04)(1.05)} + \frac{1{,}050}{(1.04)(1.05)(1.06)} = \$1{,}000.98$$

PROFESSOR'S NOTE

If you think this looks a little like valuing a bond using spot rates, as we did for arbitrage-free valuation, you are correct. The discount factors are equivalent to spot rate discount factors.

If we have a semiannual coupon bond, the calculation methods are the same, but we would use the semiannual discount rate rather than the annualized rate—and the number of periods would be the number of semiannual periods.

LOS 55.c: Compare the spot curve, par curve, and forward curve.

The **spot rate yield curve** (spot curve) for U.S. Treasury bonds, also referred to as the zero curve or the strip curve (because zero-coupon U.S. Treasury bonds are also called stripped Treasuries), is a plot of spot rates versus maturity.

Usually, a spot curve is upward sloping, with higher spot rates for longer maturities (referred to as a normal yield curve), reflecting investor demand for higher returns over longer time frames. When spot rates are lower for longer-dated maturities, the spot curve is downward-sloping and is said to be inverted. Spot rates are usually quoted on a semiannual bond basis, so they are directly comparable to YTMs quoted for coupon government bonds.

A **yield curve for coupon bonds** shows the YTMs for a similar type of actively traded coupon bonds at various maturities (e.g., U.S. Treasury bonds). Yields are calculated for several available maturities, and yields for bonds with maturities between these are estimated by linear interpolation. Yields are usually expressed on a semiannual bond basis.

A practical issue with constructing yield curves directly from traded coupon bond prices is that distortions in yields can occur due to illiquidity and taxation differences between interest income and gains and losses from buying bonds at a discount or premium. To avoid the illiquidity issue, “on-the-run” (most recently issued) bond yields are used; however, there may not be enough on-the-run securities to construct a meaningful curve, and the ones that do exist may trade at a premium or discount to par, causing taxation distortions.

To avoid these practical issues when constructing a coupon bond yield curve, a **par bond yield curve**, or *par curve*, can be constructed from spot curves. As we discussed earlier, par yields are hypothetical yields of bonds that would trade at par for a specific maturity. A par yield curve displays the yields of par bonds versus maturity.

A **forward yield curve** shows forward rates for bonds or money market securities for annual periods in the future. Typically, the forward curve would show the yields of 1-year securities for each future year, quoted on a semiannual bond basis.

As we have seen, the spot rate for a given maturity is a geometric average of the forward rates that apply to each period between now and that maturity. Therefore, when the forward curve is upward sloping, the spot curve is also upward sloping, but less so. We also saw earlier how a par yield at a certain maturity is a weighted average of the spot rates that apply to the individual cash flows of the bond (most heavily weighted toward the longest-dated spot rate when par payment takes place). Hence, par yields will also be upward sloping, and very close to spot rates (but slightly below them) in a normal (upward-sloping) forward curve environment.

PROFESSOR’S NOTE

It might be helpful, once again, to approximate using simple averages to get a clear picture. Assume the 1-year spot rate is 1% and the 1y1y forward rate is 3%. Note that we have rising periodic rates here (the forward curve is rising with maturity). The annualized 2-year spot is approximated as (1% + 3%) / 2 = 2%, so we can see the spot curve is also rising with maturity, but at a slower pace than forward rates. The yield of a 1-year annual coupon bond would be 1% because there would only be one cash flow at maturity for this bond, and it would therefore earn the 1-year spot rate. The 2-year yield would be a weighted average of the 1-year spot of 1% and the 2-year spot of 2%, but mostly 2% because that is where most of the 2-year bond's cash flows occur. So, we can see that par yields, too, are rising with maturity—and are close to, but slightly below, spot rates. The key takeaway here is that forward rates drive spot rates, which in turn drive par yields.

Likewise, when the forward curve is downward sloping, the spot curve will also be downward sloping, but less so; and par yields will also be downward sloping, and close to spot rates (but slightly above them). One potential explanation for an inverted yield curve is that interest rates are expected to decrease.

When forward rates are constant, it means that all future periodic rates are the same. This means that spot rates to all maturities will be the same—and therefore, bond yields will be the same for all maturities. We describe this as a flat yield curve environment.

MODULE QUIZ 55.1

1. If spot rates are 3.2% for one year, 3.4% for two years, and 3.5% for three years, the price of a $100,000 face value, 3-year, annual-pay bond with a coupon rate of 4% is *closest* to:
   A. $101,420.
   B. $101,790.
   C. $108,230.
2. A market rate of discount for a single payment to be made in the future is a:
   A. spot rate.
   B. simple yield.
   C. forward rate.
3. Which of the following yield curves is *least likely* to consist of observed yields in the market?
   A. Forward yield curve.
   B. Par bond yield curve.
   C. Coupon bond yield curve.
4. The 4-year spot rate is 9.45%, and the 3-year spot rate is 9.85%. What is the 1-year forward rate three years from today?
   A. 8.258%.
   B. 9.850%.
   C. 11.059%.
5. Given the following spot and forward rates:
   - Current 1-year spot rate is 5.5%.
   - 1-year forward rate one year from today is 7.63%.
   - 1-year forward rate two years from today is 12.18%.
   - 1-year forward rate three years from today is 15.5%.

The value of a 4-year, 10% annual-pay, $1,000 par value bond is *closest* to:

A. $870.
B. $996.
C. $1,009.

6. The 1-year spot rate is 1% and the 1y1y rate is 3%. Which of the following statements is *most accurate*?

   A. The 2-year spot is just below 2%, and the 2-year par yield is just below the 2-year spot rate.
   B. The 2-year spot is just below 2%, and the 2-year par yield is just above the 2-year spot rate.
   C. The 2-year spot is just below 4%, and the 2-year par yield is just below the 2-year spot rate.

KEY CONCEPTS

LOS 55.a

Spot rates are market discount rates for single payments to be made in the future.

The no-arbitrage price of a bond is calculated using no-arbitrage spot rates as follows:

$$\text{no-arbitrage price} = \frac{\text{coupon}}{(1 + S_1)} + \frac{\text{coupon}}{(1 + S_2)^2} + \cdots + \frac{\text{coupon} + \text{principal}}{(1 + S_N)^N}$$

LOS 55.b

Spot curves can be used to derive the par yields of hypothetical bonds trading at par across different maturities. It is useful to interpret the par yield of a bond as a weighted average of the spot rates applying to each individual cash flow of the bond.

Forward rates are current lending/borrowing rates for short-term loans to be made in future periods.

A spot rate for a maturity of *N* periods is the geometric mean of forward rates over the *N* periods. The same relation can be used to solve for a forward rate given spot rates for two different periods.

To value a bond using forward rates, discount the cash flows at Periods 1 to *N* by the product of one plus each forward rate for Periods 1 to *N*, and sum them.

This is for a 3-year annual-pay bond:

$$\text{price} = \frac{\text{coupon}}{(1 + S_1)} + \frac{\text{coupon}}{(1 + S_1)(1 + 1\text{y}1\text{y})} + \frac{\text{coupon} + \text{principal}}{(1 + S_1)(1 + 1\text{y}1\text{y})(1 + 2\text{y}1\text{y})}$$

LOS 55.c

The spot curve plots spot rates versus maturity. It can be derived from the prices of instruments offering single payments in the future, such as zero-coupon bonds or stripped Treasury bonds.

The par curve shows the coupon rates for bonds of various maturities that would result in bond prices equal to their par values.

A forward curve is a yield curve composed of forward rates, such as 1-year rates, available at each year over a future period.

In an upward-sloping (normal) yield curve environment, forward rates will be higher than spot rates, which will be higher than par yields. In a downward-sloping (inverted) yield curve environment, forward rates will be lower than spot rates, which will be lower than par yields. In a flat yield curve environment, forward rates will be equal to spot rates, which will be equal to par yields across all maturities.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 55.1

1. **A** bond value $= \frac{4{,}000}{1.032} + \frac{4{,}000}{(1.034)^2} + \frac{104{,}000}{(1.035)^3} = \$101{,}419.28$

   (LOS 55.a)

2. **A** A spot rate is a discount rate for a single future payment. Simple yield is a measure of a bond's yield that accounts for coupon interest and assumes straight-line amortization of a discount or premium. A forward rate is an interest rate for a future period, such as a 3-month rate six months from today. (LOS 55.a)

3. **B** Par bond yield curves are based on the theoretical yields that would cause bonds at each maturity to be priced at par. Coupon bond yields and forward interest rates can be observed directly from market transactions. (LOS 55.b)

4. **A** $(1.0945)^4 = (1.0985)^3 \times (1 + 3y1y)$

   $3y1y = \frac{(1.0945)^4}{(1.0985)^3} - 1 = 8.258\%$

   approximate forward rate = 4(9.45%) − 3(9.85%) = 8.25%

   (LOS 55.b)

5. **C** bond value $= \frac{100}{1.055} + \frac{100}{(1.055)(1.0763)} + \frac{100}{(1.055)(1.0763)(1.1218)} + \frac{1{,}100}{(1.055)(1.0763)(1.1218)(1.155)} = 1{,}009.03$

   (LOS 55.b)

6. **A** The 2-year spot rate must reflect the periodic rates of 1% in the first period and 3% in the second period—hence, the 2-year spot will be approximately 2%. Given that forward rates are rising with maturity, spot rates will be rising with maturity at a slower pace, and par yields will also be rising, but slightly below spot rates. So, the correct answer is that the 2-year spot is just below 2%, and the 2-year par yield is just below the 2-year spot rate. (LOS 55.c)

READING 56

INTEREST RATE RISK AND RETURN

MODULE 56.1: INTEREST RATE RISK AND RETURN

Video covering this content is available online.

LOS 56.a: Calculate and interpret the sources of return from investing in a fixed-rate bond.

There are **three sources of returns** from investing in a fixed-rate bond:

1. Coupon and principal payments
2. Interest earned on coupon payments that are reinvested over the investor's holding period for the bond
3. Any capital gain or loss if the bond is sold before maturity

We will assume that a bond makes all of its promised coupon and principal payments on time (i.e., we are not addressing credit risk). Additionally, we assume that the *interest rate earned on reinvested coupon payments is the same as the prevailing yield to maturity (YTM) on the bond*.

Given the assumptions just listed, we may draw five key results:

1. An investor who holds a fixed-rate bond to maturity will earn an annualized rate of return equal to the YTM of the bond when purchased, if the YTM of the bond (and hence the reinvestment rate) does not change over the life of the bond.
2. An investor who sells a bond before maturity will earn a rate of return equal to the YTM at purchase if the YTM has not changed since purchase.
3. If the market YTM for the bond, our assumed reinvestment rate, increases (decreases) after the bond is purchased but before the first coupon date, an investor who holds the bond to maturity will earn a realized return that is higher (lower) than the original YTM of the bond when purchased.
4. If the market YTM for the bond, our assumed reinvestment rate, *increases* after the bond is purchased but before the first coupon date, a bond investor will earn a rate of return that is lower than the YTM at bond purchase if the bond is held for a *short* period.
5. If the market YTM for the bond, our assumed reinvestment rate, *decreases* after the bond is purchased but before the first coupon date, a bond investor will earn a rate of

return that is lower than the YTM at bond purchase if the bond is held for a *long* period.

We will present mathematical examples to demonstrate each of these results as well as some intuition as to why these results must hold.

We call the time that the bond will be held the investor's **investment horizon**, which may be shorter than the bond's maturity. A bond investor's **horizon yield** is the compound annual return earned from the bond over the investment horizon. It is calculated by comparing the purchase price of the bond to the end value derived from holding the bond, which includes coupons, interest earned on reinvested coupons, and the sale price (or principal payment amount, if the bond is held to maturity).

Unchanged YTM, Bond Held to Maturity

We will illustrate this calculation (and the first result listed earlier) with a 6% annual-pay 3-year bond purchased at a YTM of 7% and held to maturity.

With an annual YTM of 7%, the bond's purchase price is 97.376:

N = 3; I/Y = 7; PMT = 6; FV = 100; CPT → PV = –97.376

At maturity, the investor will have received coupon income and reinvestment income equal to the future value of an annuity of three $6 coupon payments calculated with an interest rate equal to the bond's YTM. This is the amount:

$6(1.07)^2 + 6(1.07) + 6 = \$19.289$

N = 3; I/Y = 7; PV = 0; PMT = 6; CPT → FV = –19.289

We can easily calculate the amount earned from reinvestment of the coupons as follows:

19.289 – 3(6) = $1.289

Adding the principal payment at maturity of $100 to $19.289, we can calculate the investor's rate of return over the 3-year holding period as $\left(\frac{\$119.289}{\$97.376}\right)^{1/3} - 1 = 7\%$ and demonstrate that $97.376 invested at a compound annual rate of 7% would return $119.289 after three years.

With this example, we have demonstrated our first result: for a fixed-rate bond that does not default and has a reinvestment rate equal to the YTM, an investor who holds the bond until maturity will earn a rate of return equal to the YTM at purchase.

Unchanged YTM, Bond Sold Before Maturity

Now let's examine the second result—that an investor who sells a bond before maturity will earn a rate of return equal to the YTM *as long as the YTM has not changed since purchase*. If the YTM of the bond remains unchanged, the value of a bond will move toward the par value of the bond by the maturity date. At dates between the purchase and the maturity, the value of a bond at the same YTM as when it was purchased is its **carrying value**, and it reflects the amortization of the discount or premium since the bond was purchased.

PROFESSOR'S NOTE

Carrying value is a price along a bond's constant-yield price trajectory (in other words, it is the value of the bond at a certain time after purchase, assuming the original yield of the bond has not changed). It is referred to as carrying value because it is the value that is often shown on the balance sheet in financial reporting when a bond is held to maturity. This is not the same as the market value of the bond if its yield *has* changed.

Capital gains or losses at the time a bond is sold are measured relative to this carrying value, as illustrated in the following example.

EXAMPLE: Capital gain or loss on a bond

An investor purchases a 20-year bond with a 5% semiannual coupon and a yield to maturity of 6%. Five years later, the investor sells the bond for a price of 91.40. Determine whether the investor realizes a capital gain or loss, and calculate its amount.

Answer:

Any capital gain or loss is based on the bond's carrying value at the time of sale, when it has 15 years (30 semiannual periods) to maturity. The carrying value is calculated using the bond's YTM at the time the investor purchased it:

N = 30; I/Y = 3; PMT = 2.5; FV = 100; CPT → PV = –90.20

Because the selling price of 91.40 is greater than the carrying value of 90.20, the investor realizes a capital gain of 91.40 – 90.20 = 1.20 per 100 of face value.

Bonds held to maturity have no capital gain or loss. Bonds sold before maturity at the same YTM as at purchase will also have no capital gain or loss. Using the 6% 3-year bond from our earlier examples, we can demonstrate this for an investor with a two-year holding period (investment horizon).

After the bond is purchased at a YTM of 7% (for 97.376), we have the following.

Price at sale (at end of Year 2, YTM = 7%):

$106 / 1.07 = 99.065$

N = 1; I/Y = 7; FV = 100; PMT = 6; CPT → PV = –99.065

which is the carrying value of the bond.

Coupon interest and reinvestment income for two years:

$6(1.07) + 6 = \$12.420$

N = 2; I/Y = 7; PV = 0; PMT = 6; CPT → FV = –12.420

Investor's annual compound rate of return over the two-year holding period:

$$\left(\frac{12.420 + 99.065}{97.376}\right)^{1/2} - 1 = 7\%$$

This demonstrates the second key result we listed: that for a bond investor with a horizon less than the bond's term to maturity, the annual return will be equal to the YTM at purchase if the bond is sold at that YTM and all coupons are reinvested at the original YTM.

Changed YTM, Bond Held to Maturity

Next let's examine our third result—that if rates rise (fall) before the first coupon date, an investor who holds a bond to maturity will earn a rate of return greater (less) than the YTM at purchase.

Based on our previous result that an investor who holds a bond to maturity will earn a rate of return equal to the YTM at purchase if the reinvestment rate is also equal to the YTM at purchase, the intuition of the third result is straightforward. If the YTM, which is also the reinvestment rate for the bond, increases (decreases) after purchase, the return from coupon payments and reinvestment income will increase (decrease) as a result, which will increase (decrease) the investor's rate of return on the bond above (below) its YTM at purchase. The following calculations demonstrate these results for the 3-year 6% bond in our previous examples.

For a 3-year 6% bond purchased at 97.376 (YTM of 7%), first assume that the YTM and reinvestment rate increases to 8% after purchase but before the first coupon payment date. The bond's annualized holding period return is calculated as follows.

Coupons and reinvestment interest:

$6(1.08)^2 + 6(1.08) + 6 = \$19.478$

N = 3; I/Y = 8; PV = 0; PMT = 6; CPT → FV = –19.478

Investor's annual compound holding period return:

$$\left(\frac{119.478}{97.376}\right)^{1/3} - 1 = 7.06\%$$

which is greater than the 7% YTM at purchase.

If the YTM decreases to 6% after purchase but before the first coupon date, we have the following.

Coupons and reinvestment interest:

$6(1.06)^2 + 6(1.06) + 6 = \$19.102$

N = 3; I/Y = 6; PV = 0; PMT = 6; CPT → FV = –19.102

Investor's annual compound holding period return:

$$\left(\frac{119.102}{97.376}\right)^{1/3} - 1 = 6.94\%$$

which is less than the 7% YTM at purchase.

Note that in both cases, the investor's rate of return is between the YTM at purchase and the assumed reinvestment rate (the new YTM).

Changed YTM, Bond Sold Before Maturity

We now turn our attention to the fourth and fifth results concerning the effects of the length of an investor's holding period on the rate of return for a bond that experiences an increase or decrease in its YTM before the first coupon date.

Consider a 3-year 6% bond purchased at 97.376 by an investor with a one-year investment horizon. If the YTM increases from 7% to 8% after purchase and the bond is sold after one year, the rate of return can be calculated as follows.

Bond price just after first coupon has been paid with YTM = 8%:

N = 2; I/Y = 8; FV = 100; PMT = 6; CPT → PV = –96.433

There is no reinvestment income and only one coupon of $6 received, so the holding period rate of return is simply:

$$\left(\frac{6 + 96.433}{97.376}\right) - 1 = 5.19\%$$

which is less than the YTM at purchase.

If the YTM decreases to 6% after purchase and the bond is sold at the end of one year, the investor's rate of return can be calculated as follows.

Bond price just after first coupon has been paid with YTM = 6%:

N = 2; I/Y = 6; FV = 100; PMT = 6; CPT → PV = –100.00

And the holding period rate of return is simply:

$$\left(\frac{6 + 100.00}{97.376}\right) - 1 = 8.86\%$$

which is greater than the YTM at purchase.

The intuition of this result is based on the idea of a tradeoff between **price risk** (the uncertainty about a bond's price due to uncertainty about the prevailing market YTM at a time of sale) and **reinvestment risk** (uncertainty about the total of coupon payments and reinvestment income on those payments due to the uncertainty about future reinvestment rates).

Previously, we showed that for a bond held to maturity, the investor's rate of return increased with an increase in the bond's YTM and decreased with a decrease in the bond's YTM. For an investor who intends to hold a bond to maturity, there is no price risk as we have defined it (because they do not intend to sell the bond). Assuming no default, the bond's value at maturity is its par value regardless of interest rate changes, so that the investor has only reinvestment risk. Her realized return will increase when interest earned on reinvested cash flows increases, and decrease when the reinvestment rate decreases.

For an investor with a short investment horizon, because they intend to sell the bond before maturity, price risk increases and reinvestment risk decreases. For the investor with a one-year investment horizon, there was no reinvestment risk because the bond was sold before any interest on coupon payments was earned. The investor had only price risk, so an increase in yield decreased the rate of return over the one-year holding period because the sale price was lower. Conversely, a decrease in yield increased the one-year holding period return to more than the YTM at purchase because the sale price was higher.

To summarize:

- Short investment horizon: price risk > reinvestment risk
- Long investment horizon: reinvestment risk > price risk

LOS 56.b: Describe the relationships among a bond's holding period return, its Macaulay duration, and the investment horizon.

LOS 56.c: Define, calculate, and interpret Macaulay duration.

Is there an investment horizon where price risk and reinvestment risk are in balance? There is, as demonstrated by the following example.

EXAMPLE: Investment horizon yields

Consider a 5-year, 11% annual coupon bond priced at 86.59 to yield 15% to maturity. Calculate the horizon yield for an investment horizon of 4 years, assuming that the YTM does the following:

a) Falls to 14% before the first coupon date

b) Rises to 16% before the first coupon date

Answer:

a) **Falls to 14% before the first coupon date**

Sale after 4 years

Bond price:

N = 1; PMT = 11; FV = 100; I/Y = 14; CPT → PV = 97.368

Coupons and interest on reinvested coupons:

N = 4; PMT = 11; PV = 0; I/Y = 14; CPT → FV = 54.133

Horizon return:

$[(97.368 + 54.133) / 86.59]^{1/4} - 1 = 15.0\%$

b) **Rises to 16% before the first coupon date**

Sale after 4 years

Bond price:

N = 1; PMT = 11; FV = 100; I/Y = 16; CPT → PV = 95.690

Coupons and interest on reinvested coupons:

N = 4; PMT = 11; PV = 0; I/Y = 16; CPT → FV = 55.731

Horizon return:

$[(95.690 + 55.731) / 86.59]^{1/4} - 1 = 15.0\%$

> For an investment horizon of 4 years, the horizon return is equal to the original YTM of 15%, regardless of changes in the YTM of the bond.

This example shows that for a particular fixed-coupon bond, we can find an investment horizon that is neither short enough to face price risk nor long enough to face reinvestment risk. For this bond it was an investment horizon of four years. At this horizon, if the YTM decreases, losses on reinvestment income are just balanced by a gain in price. If the YTM increases, gains in reinvestment income are just offset by a loss in price.

How can we easily find this "sweet spot" investment horizon? We can determine the average time until the receipt of the cash flows of the bond, referred to as its **Macaulay duration**.

A bond's (annual) Macaulay duration is calculated as the weighted average of the number of years until each of the bond's promised cash flows is to be paid, where the weights are the present values of each cash flow as a percentage of the bond's full value.

Consider the bond in the example. The present values of each of the bond's promised payments, discounted at the bond's YTM of 15%, and their weights in the calculation of Macaulay duration, are shown in the following table.

$C_1 = 11$	$PV_1 = 11 / 1.15 = 9.565$	$W_1 = 9.565 / 86.59 = 0.1105$
$C_2 = 11$	$PV_2 = 11 / 1.15^2 = 8.318$	$W_2 = 8.318 / 86.59 = 0.0961$
$C_3 = 11$	$PV_3 = 11 / 1.15^3 = 7.233$	$W_3 = 7.233 / 86.59 = 0.0835$
$C_4 = 11$	$PV_4 = 11 / 1.15^4 = 6.289$	$W_4 = 6.289 / 86.59 = 0.0726$
$C_5 = 111$	$PV_5 = 111 / 1.15^5 = \underline{55.187}$	$W_5 = 55.187 / 86.59 = \underline{0.6373}$
	86.59	1.0000

The present values of all the promised cash flows sum to 86.59 (the full price of the bond) and the weights sum to 1.

Now that we have the weights, and because we know the time until each promised payment is to be made, we can calculate the Macaulay duration for this bond:

$$0.1105(1) + 0.0961(2) + 0.0835(3) + 0.0726(4) + 0.6373(5) = 4.03 \text{ years}$$

Our interpretation of this Macaulay duration is that, as demonstrated in the example, if we have an investment horizon of four years, then we still earn the original YTM of the bond out to this horizon—even if the YTM of the bond immediately changes after purchasing the bond.

The Macaulay duration of a semiannual-pay bond can be calculated in the same way: as a weighted average of the number of *semiannual periods* until the cash flows are to be received. In this case, the result is the number of semiannual periods rather than years (to express this in annualized terms, divide the number of semiannual periods by two).

The difference between a bond's Macaulay duration and the bondholder's investment horizon is referred to as a **duration gap**. A positive duration gap (Macaulay duration greater than the investment horizon) exposes the investor to price risk from increasing interest rates. A negative duration gap (Macaulay duration less than the investment horizon) exposes the investor to reinvestment risk from decreasing interest rates.

MODULE QUIZ 56.1

1. The largest component of returns for a 7-year zero-coupon bond yielding 8% and held to maturity is:
   A. capital gains.
   B. interest income.
   C. reinvestment income.
2. An investor buys a 10-year bond with a 6.5% annual coupon and a YTM of 6%. Before the first coupon payment is made, the YTM for the bond decreases to 5.5%. Assuming coupon payments are reinvested at the YTM, the investor's return when the bond is held to maturity is:
   A. less than 6.0%.
   B. equal to 6.0%.
   C. greater than 6.0%.
3. Assuming coupon interest is reinvested at a bond's YTM, what is the interest portion of an 18-year, $1,000 par, 5% annual coupon bond's return if it is purchased at par and held to maturity?
   A. $576.95.
   B. $1,406.62.
   C. $1,476.95.
4. An investor buys a 15-year, £800,000, zero-coupon bond with an annual YTM of 7.3%. If she sells the bond after three years for £346,333, she will have:
   A. a capital gain.
   B. a capital loss.
   C. neither a capital gain nor a capital loss.
5. An investor with an investment horizon of six years buys a bond with a Macaulay duration of seven years. This investment has:
   A. no duration gap.
   B. a positive duration gap.
   C. a negative duration gap.
6. The Macaulay duration (in years) of a 2-year semiannual-pay 7% coupon bond yielding 5% is *closest* to:
   A. 0.38.
   B. 1.90.
   C. 3.81.

KEY CONCEPTS

LOS 56.a

Sources of return from a bond investment include the following:

- Coupon and principal payments
- Reinvestment of coupon payments
- Capital gain or loss if bond is sold before maturity

Changes in yield to maturity (YTM) produce price risk (uncertainty about a bond's price) and reinvestment risk (uncertainty about income from reinvesting coupon payments). An increase (a decrease) in YTM decreases (increases) a bond's price but increases (decreases) its reinvestment income.

LOS 56.b

Over a short investment horizon, a change in YTM affects price more than it affects reinvestment income.

Over a long investment horizon, a change in YTM affects reinvestment income more than it affects price.

The Macaulay duration may be interpreted as the investment horizon for which a bond's price risk and reinvestment risk offset each other:

duration gap = Macaulay duration − investment horizon

LOS 56.c

Macaulay duration is calculated as the weighted average of the number of years until each of the bond's promised cash flows is to be paid, where the weights are the present values of each cash flow as a percentage of the bond's full value.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 56.1

1. **B** The increase in value of a zero-coupon bond over its life is interest income. A zero-coupon bond has no reinvestment risk over its life. A bond held to maturity has no capital gain or loss. (LOS 56.a)

2. **A** The investment horizon is maturity, which means that the investor faces reinvestment risk (on average, the cash flows of the bond are received before maturity) and zero price risk. The decrease in the YTM to 5.5% will decrease the reinvestment income over the life of the bond so that the investor will earn less than 6%, the YTM at purchase. (LOS 56.a)

3. **B** The interest portion of a bond's return is the sum of the coupon payments and interest earned from reinvesting coupon payments over the holding period:

   N = 18; PMT = 50; PV = 0; I/Y = 5%; CPT → FV = −1,406.62

   (LOS 56.a)

4. **A** The price of the bond after three years that will generate neither a capital gain nor a capital loss is the price if the YTM remains at 7.3%. After three years, the present value of the bond is $800{,}000 / 1.073^{12} = 343{,}473.57$, so she will have a capital gain relative to the bond's carrying value. (LOS 56.a)

5. **B** Duration gap is the Macaulay duration minus the investment horizon. This bond has a Macaulay duration greater than six years, and the investment has a positive duration gap. (LOS 56.b)

6. **B**

$C_1 = 3.5$	$PV_1 = 3.5 / 1.025$	$= 3.415$	$W_1 = 3.415 / 103.762$	$= 0.0329$
$C_2 = 3.5$	$PV_2 = 3.5 / 1.025^2$	$= 3.331$	$W_2 = 3.331 / 103.762$	$= 0.0321$
$C_3 = 3.5$	$PV_3 = 3.5 / 1.025^3$	$= 3.250$	$W_3 = 3.250 / 103.762$	$= 0.0313$
$C_4 = 103.5$	$PV_4 = 103.5 / 1.025^4$	$= \underline{93.766}$	$W_4 = 93.766 / 103.762$	$= \underline{0.9037}$
		103.762		1.0000

The Macaulay duration is, therefore, calculated as weighted average time as follows:

0.0329(1) + 0.0321(2) + 0.0313(3) + 0.9037(4) = 3.806

Then, the annualized Macaulay duration is 3.806 / 2 = 1.90 years. (LOS 56.c)

READING 57

YIELD-BASED BOND DURATION MEASURES AND PROPERTIES

MODULE 57.1: YIELD-BASED BOND DURATION MEASURES AND PROPERTIES

Video covering this content is available online.

LOS 57.a: Define, calculate, and interpret modified duration, money duration, and the price value of a basis point (PVBP).

Modified duration (ModDur) is calculated as Macaulay duration (MacDur) divided by one plus the bond's periodic rate of return (YTM divided by periodicity).

For an annual-pay bond, this is the general form of ModDur:

$$\text{ModDur} = \text{MacDur} / (1 + \text{YTM})$$

For a semiannual-pay bond with a YTM quoted on a semiannual bond basis, this is the form:

$$\text{ModDur}_{\text{SEMI}} = \text{MacDur}_{\text{SEMI}} / (1 + \text{YTM} / 2)$$

EXAMPLE: Modified duration

A 5-year, 11% annual coupon bond priced at 86.59 to yield 15% to maturity has a Macaulay duration of 4.03. Calculate the modified duration of this bond.

Answer:

Because it is an annual coupon bond (periodicity = 1), its modified duration can be calculated as follows:

$$\text{ModDur} = 4.03 / 1.15 = 3.50$$

Modified duration provides an estimate for the percentage change in a bond's price given a 1% change in YTM:

$$\text{approximate percentage change in bond price} = -\text{ModDur} \times \Delta\text{YTM}$$

Based on a ModDur of 3.50, in response to an 0.5% increase in YTM the price of the bond should fall by approximately 3.50 × 0.5% = 1.75%. The resulting price estimate of

$86.59 \times (1 - 0.0175) = 85.075$ is close to the value of the bond calculated directly using a YTM of 15.5%, which is 85.092:

N = 5; I/Y = 15.5; FV = 100; PMT = 11; CPT → PV = −85.092

For a semiannual-pay bond, $\text{ModDur}_{\text{SEMI}}$ can be annualized (from semiannual periods to annual periods) by dividing by two, and then used as the approximate change in price for a 1% change in a bond's YTM.

Approximate Modified Duration

We can approximate modified duration directly using bond values for an increase in YTM and for a decrease in YTM of the same size.

The calculation of approximate ModDur is based on a given change in YTM. V_- is the price of the bond if YTM is *decreased* by ΔYTM, and V_+ is the price of the bond if the YTM is *increased* by ΔYTM. Approximate ModDur is given by the following:

$$\text{approximate ModDur} = \frac{V_- - V_+}{2V_0 \Delta \text{YTM}}$$

The formula uses the average of the magnitudes of the price increase and the price decrease, which is why $V_- - V_+$ in the numerator is divided by two in the denominator.

V_0, the current price of the bond, is in the denominator to convert this average price change to a percentage, and the ΔYTM term is in the denominator to scale the duration measure to a 1% change in yield by convention. Note that the ΔYTM term in the denominator must be entered as a decimal (rather than in a whole percentage) to properly scale the duration estimate.

EXAMPLE: Calculating approximate modified duration

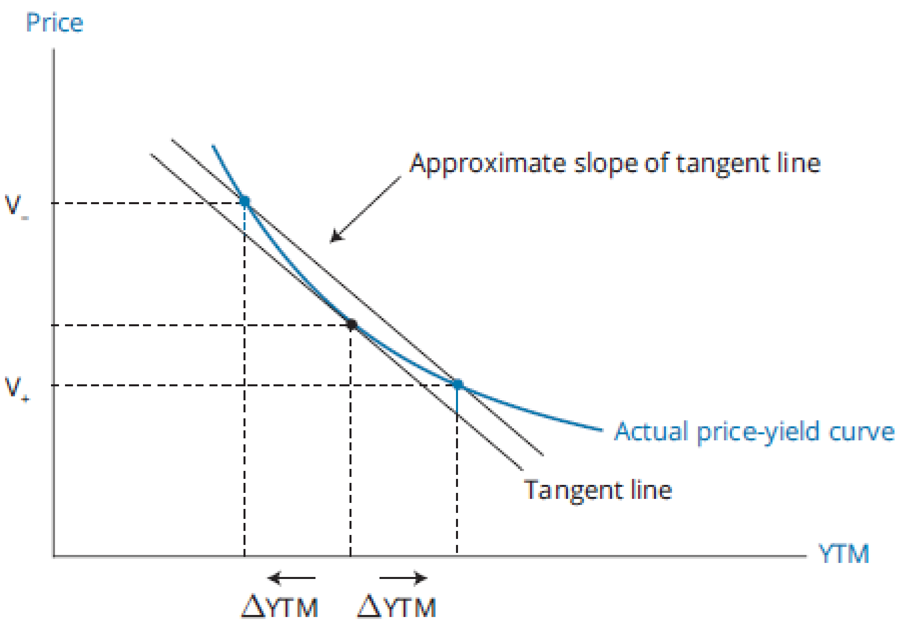
Consider a 5-year, 11% annual coupon bond priced at 86.59 to yield 15% to maturity. If its YTM increases by 50 basis points, its price will decrease to 85.092. If its YTM decreases by 50 basis points, its price will increase to 88.127. Calculate the approximate ModDur.

Answer:

The approximate ModDur is $\frac{88.127 - 85.092}{2 \times 86.59 \times 0.005} = 3.505$ and the

Modified duration is a *linear estimate* of the relation between a bond's price and YTM, whereas the actual relation is convex—not a straight line. This means that the modified duration measure provides good estimates of bond prices for small changes in yield, but increasingly poor estimates for larger changes in yield as the effect of the curvature of the price-yield curve is more pronounced. We illustrate this in Figure 57.1.

Figure 57.1: Approximate ModDur



Money Duration

The **money duration** of a bond position (also called dollar duration) is expressed in currency units:

money duration = annual ModDur × full price of bond position

Multiplying the money duration of a bond by a given change in YTM (as a decimal) will provide an estimate for the change in bond value for that change in YTM.

EXAMPLE: Money duration

1. Calculate the money duration on a coupon date of a $2 million par value bond that has a ModDur of 7.42 and a full price of 101.32, expressed for the whole bond and per $100 of face value.
2. What will be the impact on the value of the bond of a 25 basis point increase in its YTM?

Answer:

1. The money duration for the bond is ModDur times the full value of the bond:

   7.42 × $2,000,000 × 101.32% = $15,035,888

   The money duration per $100 of par value is:

   7.42 × 101.32 = $751.79

   Or, $15,035,888 / ($2,000,000 / $100) = $751.79.
2. $15,035,888 × 0.0025 = $37,589.72

   The bond value decreases by $37,589.72.

The **price value of a basis point (PVBP)** is the money change in the full price of a bond when its YTM changes by one basis point, or 0.01%. We can calculate the PVBP directly for a bond by calculating the average of the decrease in the full value of a bond when its YTM increases by one basis point, and the increase in the full value of the bond when its YTM decreases by one basis point.

EXAMPLE: Calculating the PVBP

A newly issued, 20-year, 6% annual-pay straight bond is priced at 101.39. Calculate the PVBP for this bond, assuming it has a par value of $1 million.

Answer:

First, we need to find the YTM of the bond:

N = 20; PV = –101.39; PMT = 6; FV = 100; CPT→ I/Y = 5.88

Now, we need the values for the bond with YTMs of 5.89 and 5.87:

I/Y = 5.89; CPT → PV = –101.273 (V_+)

I/Y = 5.87; CPT → PV = –101.507 (V_-)

PVBP (per $100 of par value) = (101.507 – 101.273) / 2 = 0.117

For the $1 million par value bond, each 1 basis point change in the YTM will change the bond's price by 0.117 × $1 million × 0.01 = $1,170.

LOS 57.b: Explain how a bond's maturity, coupon, and yield level affect its interest rate risk.

Other things equal, *bonds with longer maturity* will (usually) have higher interest rate risk. The present values of payments made further in the future are more sensitive to changes in the discount rate used to calculate present value than are the present values of payments made sooner.

We must say *usually* because there are instances where an increase in a discount coupon bond's maturity will decrease its Macaulay duration. For a discount bond, duration first increases with longer maturity and then decreases over a range of relatively long maturities until it approaches the duration of a perpetuity, which is (1 + YTM) / YTM.

Between coupon dates, if the YTM of a coupon bond remains constant, its Macaulay duration decreases smoothly with the passage of time, and then goes back up slightly at each coupon payment date as the time to the next coupon resets to a full coupon period.

Other things equal, a *higher coupon rate* on a bond will decrease its interest rate risk. For a given maturity and YTM, the duration of a zero-coupon bond will be greater than that of a coupon bond. Increasing the coupon rate means more of a bond's value will be from payments received sooner, so that the value of the bond will be less sensitive to changes in yield.

For floating-rate notes (FRNs) where coupons are periodically reset to a market reference rate (MRR), when interest rates rise, the coupon will also rise, limiting the price risk of the bond. Macaulay duration for an FRN can be calculated as the time to the next coupon reset date at the end of the current coupon period.

Other things equal, an *increase (decrease) in a bond's YTM* will decrease (increase) its interest rate risk. To understand this, we can look to the convexity of the price-yield curve and use its slope as our proxy for interest rate risk. At lower yields, the price-yield curve has a steeper slope, indicating that price is more sensitive to a given change in yield.

MODULE QUIZ 57.1

1. A 14% annual-pay coupon bond has six years to maturity. The bond is currently trading at par. Using a 25 basis point change in yield, the approximate modified duration of the bond is *closest* to:
   A. 0.392.
   B. 3.888.
   C. 3.970.
2. The current price of a $1,000, 7-year, 5.5% semiannual coupon bond is $1,029.23. The bond's price value of a basis point is *closest* to:
   A. $0.05.
   B. $0.60.
   C. $5.74.
3. The modified duration of a bond is 7.87. The approximate percentage change in price using duration only for a yield decrease of 110 basis points is *closest* to:
   A. −8.657%.
   B. +7.155%.
   C. +8.657%.
4. All else equal, which of the following bonds is likely to have the highest price risk?
   A. A 10-year maturity semiannual-pay floating-rate note.
   B. A 2-year zero-coupon bond.
   C. A 2-year 10% semiannual-pay bond.

KEY CONCEPTS

LOS 57.a

Modified duration is a linear estimate of the percentage change in a bond's price that would result from a 1% change in its YTM:

ModDur = MacDur / (1 + periodic return of bond)

For an expected change in yield, ΔYTM, the expected change in the bond's price is given by the following:

approximate percentage change in bond price = −ModDur × ΔYTM

Modified duration can be approximated by repricing the bond at different yields:

$$\text{approximate modified duration} = \frac{V_{-} - V_{+}}{2V_0 \Delta \text{YTM}}$$