## 微分方程

参考《Quaternion kinematics for the error-state Kalman filter》公式234

$$
\begin{aligned}
\dot{p} &= v \\
\dot{v} &= R(a_m - b_a - n_a) + g \\
\dot{q} &= \frac{1}{2} q \otimes (w_m - b_g - n_g) \\
\dot{b_a} &= n_{b_a} \\
\dot{b_g} &= n_{b_g}
\end{aligned}
$$

## 状态向量

$$
x = \begin{bmatrix} \dot{p} \\ \dot{v} \\ \dot{q} \\ \dot{b_a} \\ \dot{b_g} \end{bmatrix} \in \mathbb{R}^{16}
$$

$$
w = \begin{bmatrix} n_a \\ n_b \\ n_{b_a} \\ n_{b_g} \end{bmatrix} \in \mathbb{R}^{12}
$$

## 状态转移矩阵

$$
\left. \frac{\partial f(x_{k-1}, 0)}{x_{k-1}} \right|_{\hat{x}_{k-1},0} = F_{k-1}
$$

$$
\left. \frac{\partial f(x_{k-1}, w_k)}{x_{k-1}} \right|_{\hat{x}_{k-1},0} = B_{k-1}
$$

$$
F_{k-1} = \begin{bmatrix} 0 & I_3 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial \dot{v}}{\partial q} & -R & 0 \\ 0 & 0 & \frac{\partial \dot{q}}{\partial q} & 0 & \frac{\partial \dot{q}}{\partial b_g} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{16 \times 16}
$$

其中参考《Quaternion kinematics for the error-state Kalman filter》公式174

$$
\frac{\partial \dot{v}}{\partial q} = \frac{\partial R(a_m - b_a)}{\partial q} = \frac{\partial R \bar{a}}{\partial q} = 2 \left[ w\bar{a} + v \times \bar{a} | v^\top \bar{a} I_3 + v \bar{a}^\top - \bar{a} v^\top - w[\bar{a}]_\times \right] \in \mathbb{R}^{3 \times 4}
$$

$$
\frac{\partial \dot{q}}{\partial q} = \frac{1}{2} \frac{\partial q \otimes [w_m - b_g]}{\partial q} = \frac{1}{2} \frac{\partial q \otimes \bar{w}}{\partial q} = \frac{1}{2} \frac{\partial [\bar{w}]_R q}{\partial q} = \frac{1}{2} [\bar{w}]_R \in \mathbb{R}^{4 \times 4}
$$

$$\frac{\partial \dot{q}}{\partial b_g} = \frac{1}{2}\frac{\partial q \otimes [w_m - b_g]}{\partial q} = -\frac{1}{2}\frac{\partial [q]_L b_g}{\partial b_g} \in \mathbb{R}^{4\times 3}$$

$$\frac{\partial [q]_L b_g}{\partial b_g} = \begin{bmatrix} -q_x & -q_y & -q_z \\ q_w & -q_z & q_y \\ q_z & q_w & -q_x \\ -q_y & q_x & q_w \end{bmatrix}$$

## 误差转移矩阵

$$B_{k-1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -R & 0 & 0 & 0 \\ 0 & \frac{\partial \dot{q}}{\partial n_g} & 0 & 0 \\ 0 & 0 & I_3 & 0 \\ 0 & 0 & 0 & I_3 \end{bmatrix} \in \mathbb{R}^{16\times 12}$$

其中$\frac{\partial \dot{q}}{\partial n_g} = \frac{\partial \dot{q}}{\partial b_g}$

## 观测方程

$$y = Gx + Cn$$

其中，C为7x7的单位阵

$$y = \begin{bmatrix} p_{wb} \\ q_{wb} \end{bmatrix} \in \mathbb{R}^{7\times 1}$$

$$G = \begin{bmatrix} I_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_4 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{7\times 17}$$

## 流程总结

$$\check{x}_k = f(\hat{x}_{k-1}, 0)$$
$$\check{P}_k = F_{k-1}\hat{P}_{k-1}F_{k-1}^\top + B_{k-1}Q_kB_{k-1}^\top$$
$$K_k = \check{P}_kG_k^\top(G_k\check{P}_kG_k^\top + C_kR_kC_k^\top)^{-1}$$
$$\hat{P}_k = (I - K_kG_k)\check{P}_k$$
$$\hat{x}_k = \check{x}_k \oplus K_k(y_k \ominus g(\check{x}_k, 0))$$